

Theoretical Maximum Value of Lorentz Factor:

The frontiers between relativistic physics and superluminal physics

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Abstract: In previous work [Comm. in Phys. **24**, 313 (2014)], we have established the foundations of superluminal relativistic mechanics which is actually a basic step toward the superluminalization of special relativity theory (SRT). In the present paper that is partly based on the aforementioned work, the theoretical maximum value of Lorentz factor γ_{\max} is proposed in order to determine the validity limits of SRT in its proper domain of applicability, and situate the frontiers between relativistic physics and superluminal physics for the conceptual and practical purpose at microscopic and macroscopic levels. Among the consequences of the developed formalism, a helpful formula $v/c = E/\gamma_{\max}E_0$ for luminal and superluminal velocities, and the concept of luxonic energy for massive particles, which may be used as a criterion to investigate the high and ultra-high energy cosmic rays, also another formula $m_\gamma \propto v$ is derived to estimate the non-zero photon rest mass.

Keywords: superluminal relativistic mechanics; SRT; Lorentz factor; light speed in local vacuum; ultra-high energy; non-zero photon rest mass

“Theories are only hypotheses, verified by more or less numerous facts. Those verified by the most facts are the best, but even then they are never final, never to be absolutely believed.”

Claude Bernard (1813–1878)

1. Introduction

1.2. Concept of infinity/singularity is absolutely irrelevant to the Nature

One of most fundamental and profound distinction between a physical theory and a mathematical theory is relative to the concept of *infinity/singularity*. While in Mathematics we can associate and attribute, in perfectly logical and coherent way, the infinite value to a parameter, a dimension, or to a limit or boundary conditions, such associations are meaningless when related as results to a physical theory. And this is because in Nature nothing is infinite; all physical parameters of phenomena and material objects (time, space, dimension, mass, energy, temperature, pressure, volume, density, force, velocity...etc) are defined and characterized by finite values and only finite values like: minimum, average, maximum, critical and limit values. Nature cannot be described through infinite concepts and values as they are devoid of any meaning in the physical world. Nevertheless, the concept of infinity/singularity is suited only during mathematical treatment into the realm of the theories of natural sciences in order to obtain equations with finite parameters.

Indeed, any physical theory predicting, at some special upper limit conditions, infinite values for any of its physical parameters is a theory based on fundamental flawed principles and concepts. But what Mathematics is to be used in particular study of Nature is in reality the critical question, which needs to be elucidated before embarking into any credible physical theory. Therefore, to use willy-nilly mathematical models for attempting to describe a particular phenomenon of Nature

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without physical justification for such an undertaking is an illogical act. So, we need constantly to be remained that all ways provided by Mathematics are abstract ways with no counterpart in the real physical world. The clever way therefore is to be able to find a foundation of Mathematics through which we can communicate with the real physical world and show a convincing justification for its employment. Hence, according to the foundations of superluminal relativistic mechanics [1] and the present work, any claim such as: «*The total kinetic energy of the moving material body becomes infinite, when $v = c$.*» becomes completely meaningless because in Nature; none can prevent any free moving material body from reaching or exceeding light speed in (classical) vacuum.

1.3. Motivation

In our previous paper [1], we have conceptually shown that the theoretical maximal possible velocity of an ordinary massive particle or of a physical signal is not necessary equal to that of light speed, c , in local vacuum but can be higher than c . This consideration does not violate special relativity theory (SRT) since it is physically and exclusively valid at subluminal kinematical level for relativistic velocities ($v < c$) and also because we are very convinced of the real existence of a physical world beyond the light speed as a conventional maximum limit in SRT-context. Thus, our principal motivation behind the present work is largely drawn from the principle of kinematical levels [1], which stated that conceptually, there are *three* kinematical levels (KLs) namely subluminal, luminal and superluminal level, such that:

« i) Each KL is characterized by a set of inertial reference frames (IRFs) moving with respect to each other at a constant subluminal velocity ($-c < v < c$) in the first KL; at a constant luminal velocity ($v = c$) in the second KL and at a constant superluminal velocity ($v > c$) in the third KL.

ii) Each IRF has, in addition to its relative velocity of magnitude v , its proper specific kinematical parameter (SKP), which having the physical dimensions of a constant speed defined as

$$\left. \begin{aligned} \mathfrak{G}(v) &= c, & -c < v < c \\ \mathfrak{G}(v) &> v, & c \leq v < \infty \\ \mathfrak{G}^2(-v) &= \mathfrak{G}^2(v), & \forall v \end{aligned} \right\}, \quad (1)$$

iii) All the subluminal IRFs are linked with each other *via* Galilean transformations and/or Lorentz transformations.

iv) All the luminal IRFs are linked with each other *via* luminal (spatio-temporal) transformations.

v) All the superluminal IRFs are linked with each other *via* superluminal (spatio-temporal) transformations.

vi) All the IRFs belonging to the same KL are equivalent.»

According to the principle of kinematical levels and the relations (1), the luminal velocity ($v = c$) and/or superluminal velocity ($v > c$) of any moving particle can be equal to zero only in subluminal kinematical level, that's, the first KL. Furthermore, in our earlier work [1], we have derived the general spatio-temporal transformations (STTs) that ensuring the link between any two IRFs, which are of the form:

“The metre is the length of the path traveled by light in vacuum during a time interval of $1/299\,792\,458$ of a second.”

Therefore, the current numerical value of light speed in vacuum is selected by *recommendation* and fixed by *definition* for purpose of metrology because the real empirical numerical value, from direct frequency and wavelength measurements of the methane-stabilized laser [2], is $299\,792\,456.2(1.1)\text{ms}^{-1}$. In the present work we call the consensually recommended numerical value and the experimentally determined numerical value of light speed in local (classical) vacuum: *theoretical* numerical value and *experimental* numerical value, respectively:

$$c_{\text{theoretical}} = 2.997\,924\,58 \times 10^8 \text{ ms}^{-1}, \quad (5)$$

$$c_{\text{experimental}} = 2.997\,924\,56.2 \times 10^8 \text{ ms}^{-1}. \quad (6)$$

2.2. Conceptual motivation behind the preference for $c_{\text{experimental}} = 2.997\,924\,56.2 \times 10^8 \text{ ms}^{-1}$

It is worthwhile to note that the main conceptual motivation behind the preference for (6) is the strong need to be at any rate close to the physical reality *via* experimental result(s) and also to avoid the infinity/singularity ($\gamma \rightarrow \infty$ as $v \rightarrow c$). Therefore, as we shall see soon, it is judged very convenient for us to combine (5) with (6) to get the desired expression for the theoretical maximum (numerical) value of Lorentz factor. This strategy is absolutely justifiable since, as we know, (5) itself is selected by recommendation and its numerical value fixed by definition, and also its approximate numerical value ($3 \times 10^8 \text{ ms}^{-1}$) used in many textbooks and peer-reviewed articles. Thus, in this sense, we adopt and adapt the experimental numerical value $2.997\,924\,56.2 \times 10^8 \text{ ms}^{-1}$ at the same time as empirically and mathematically a good approximation of the recommended numerical value $2.997\,924\,58 \times 10^8 \text{ ms}^{-1}$.

2.3. Upper limit for Lorentz factor

With the help of the recommended numerical value (5) and its approximation (6), we can determine the theoretical maximum (numerical) value for Lorentz factor *via* its upper limit. To this end, let us rewrite Lorentz factor (4) in terms of v and $c_{\text{theoretical}} = 2.997\,924\,58 \times 10^8 \text{ ms}^{-1}$ as follows:

$$\gamma = \frac{1}{\sqrt{1 - (v/c_{\text{theoretical}})^2}}, \quad (7)$$

consequently, the upper limit for Lorentz factor (7) should be

$$\lim_{v \rightarrow c_{\text{experimental}}} \gamma = \frac{1}{\sqrt{1 - (c_{\text{experimental}}/c_{\text{theoretical}})^2}} = 9125.5511. \quad (8)$$

Therefore, from (8) we can affirm that, in the framework of the present work, the theoretical maximum (numerical) value of Lorentz factor is

$$\gamma_{\text{max}} = 9125.5511, \quad (9)$$

From the viewpoint of practicality, the theoretical maximum value of Lorentz factor (9) should play the role of *criterion* to situate the frontiers between relativistic physics and superluminal physics. Hence, the answer to the central question should be as follows:

- a) if $E/E_0 < \gamma_{\max}$, the material point is moving in subluminal KL,
- b) if $E/E_0 = \gamma_{\max}$, the material point is moving in luminal KL,
- c) if $E/E_0 > \gamma_{\max}$, the material point is moving in superluminal KL.

Logically, the above answer leads to another question, *viz.* –what's the average magnitude of velocity of the material point in each KL? If we take into account the fact that in Nature nothing is infinite; all physical parameters of phenomena and material objects are defined and characterized by finite values and only finite values, and also none can prevent any freely moving material body from reaching or exceeding light speed in vacuum; we get the answer, namely in terms of the average magnitude, the material point's velocity in unit of c is given by the following relations:

$$\frac{v}{c} = \begin{cases} \sqrt{1 - (E_0/E)^2} & \text{if } E/E_0 < \gamma_{\max} \\ E/\gamma_{\max} E_0 & \text{if } E/E_0 \geq \gamma_{\max} \end{cases}, \quad E_0 = mc^2. \quad (10)$$

The first relation in (10) for the case $E/E_0 < \gamma_{\max}$ is well-known in SRT-context whereas the second one for the case $E/E_0 \geq \gamma_{\max}$ is theoretically suggested as an approach *via* a supposed realistic approximation to the luminal and superluminal velocities.

It is clear from the relations (10), that the material point's velocity may be treated as a function of the total kinetic energy. Furthermore, as we can remark it, the present formalism is exclusively based on the recommended numerical value of light speed in local (classical) vacuum (5) and its experimental approximation (6); such an approach is not new since the numerical approximation and symbolic approximation are essential part of experimental and theoretical physics. In this sense, Dirac, one of the founders of quantum mechanics, quantum field theory and particle physics, said: «*I owe a lot to my engineering training because it [taught] me to tolerate approximations. Previously to that I thought ... one should just concentrate on exact equations all time. Then I got the idea that in the actual world all our equations are only approximate. We must just tend to greater and greater accuracy. In spite of the equations being approximate, they can be beautiful.*» [M. Berry, *Physics World* February 1998 p36].

3. Consequences

3.1. Explicit expressions of η and $\mathfrak{S} \equiv \mathfrak{S}(v)$

Now, let us determine an explicit expression for the Eta-factor η . For this purpose, the generalization of the STTs implies $\eta_{v=c} = \gamma_{\max}$ and $\mathfrak{S}_{v=c} \equiv c$, which leads to the most general case

$$\eta_{v \geq c} = \frac{v}{c} \gamma_{\max}. \quad (11)$$

The relation (11) is the expected explicit expression. The second one concerning $\mathfrak{S} \equiv \mathfrak{S}(v)$ may be deduced from (3) and (11), and we find after some algebraic manipulation

$$\mathfrak{G} \equiv \mathfrak{G}(v) = \frac{v}{\sqrt{1 - \frac{c^2}{v^2 \gamma_{\max}^2}}}. \quad (12)$$

3.2. Luminal Lorentz transformations

Once again, the reader is certainly aware that the Lorentz (gamma) factor (4) diverges as $v \rightarrow c$ and the inequality $v > c$ leads to a purely imaginary γ and unphysical-LTs, therefore, in the SRT-context, the relative velocity of two IRFs must be strictly smaller than c . Consequence: Since an IRF can be associated with any non-accelerated particle or material object moving with subluminal velocity, this statement translates into the requirement that the magnitude of particles' velocity and of all physical signals should be limited by c . This consideration justifies the prohibition of the existence of luminal IRFs (*i.e.*, when the IRFs S and S' are in relative motion at luminal velocity of magnitude c with respect to each other) in the SRT-context.

However, the determination of the upper limit for Lorentz factor or equivalently the theoretical existence of the maximum (numerical) value for Lorentz factor (9) render the above mentioned prohibition completely unnecessary since the general STTs (2) reduce to the luminal-LTs for the case $\eta_{v=c} = \gamma_{\max}$ and $\mathfrak{G}_{v=c} \cong c$:

$$S \rightarrow S' : \begin{cases} x' = \gamma_{\max} (x - \beta_{\max} ct) \\ y' = y \\ z' = z \\ t' = \gamma_{\max} \left(t - \beta_{\max} \frac{x}{c} \right) \end{cases}, \quad S' \rightarrow S : \begin{cases} x = \gamma_{\max} (x' + \beta_{\max} ct') \\ y = y' \\ z = z' \\ t = \gamma_{\max} \left(t' + \beta_{\max} \frac{x'}{c} \right) \end{cases} \quad (13)$$

Here, $c = c_{\text{theoretical}}$ or $c = c_{\text{experimental}}$ since $\beta_{\max} \approx 1$ and $\gamma_{\max}^2 (1 - \beta_{\max}^2) = 1$. Hence, contrary to the old belief, the existence of luminal-LTs and luminal IRFs implies, among other things, that the luxons in general and the photons in particular should behave as ordinary particles (bradyons) because according to the principle of KTs any photon may be characterized by its proper luminal IRF, that's, an IRF in which the photon is at (relative) rest or equivalently an IRF in which the momentum of a photon is zero.

3.3. Validity limits of SRT in its own domain of applicability

We have previously shown in [1], and again in the present work, that the existence of the luminal IRFs which are connected to one another by luminal-LTs constitutes the upper limit of validity of LTs and SRT. Now, from all that it will follow that the theoretical existence of the maximum (numerical) value for Lorentz factor (9) determines, among other things, the validity limits of SRT in its proper domain of applicability, that is to say, SRT is theoretically valid only if

$$\gamma \lesssim \gamma_{\max}, \quad (14)$$

where γ is defined by (4). Therefore, the supposed existence of γ_{\max} and the inequality (14) together should indicate the frontiers between relativistic physics and superluminal physics. Since

SRT is exclusively destined to study the relativistic physical phenomena, *i.e.*, a set of natural and/or artificial physical events that may be occurred at relativistic velocities. For this reason, any attempt to apply SRT to superluminality of physical phenomena would be a complete waste of time since this theory has the light speed in vacuum as an upper limiting speed in its proper validity domain of applicability. That's why Einstein himself was clear on this matter because, in order to separate SRT from superluminality, he had repeatedly claimed in his papers the following statement «*For velocities greater than that of light our deliberations become meaningless; we shall, however, find in what follows, that the velocity of light in our theory plays the part, physically, of an infinitely great velocity.*» [11]. Note, however, the occurrence of the expression '*in our theory*' this means that the light speed in vacuum is, in fact, seen as an upper limiting speed only in the framework of SRT.

In the framework of the present work, the theoretical existence of the maximum Lorentz factor (9) implies, among other things, the hypothetical existence of the massive *luxons*, *i.e.*, particles having real non-zero rest mass and capable of moving at exactly the light speed. As illustration, we have selected some important particles and evaluated the value of their luxonic energy $E = \gamma_{\max} E_0$. These values are listed in Table1.

Table 1: Set of six particles is selected and the value of luxonic energy $E = \gamma_{\max} E_0$ of each particle is computed and listed.

Particle	rest energy E_0 (MeV)	luxonic energy E (MeV)
electron	0.511	4.663156×10^3
proton	938.28	8.562322×10^6
neutron	939.57	8.574094×10^6
muon	105.70	9.645707×10^5
pion π^\pm	139.60	1.273926×10^6
pion π^0	135.00	1.231949×10^6

The data contained in Table 1 may be used to test experimentally the hypothesis of the massive luxons. Furthermore, the concept of *luxonic energy* and the second relation in (10), that's, $(v/c) = E/\gamma_{\max} E_0$, may possibly play a useful role particularly for high and ultra-high energy cosmic rays.

In the framework of [1] and the present work, we explain the detected ultra-high energy cosmic rays as a result of the following hypothetical physical mechanism: When a free moving material particle – which may be an electron, neutrino, proton, neutron etc.– is in translational motion in the subluminal KL and just during its instantaneous presence between the end of this subluminal KL and the immediate beginning of luminal KL, the initial (total kinetic) energy of the material particle suddenly undergoes a huge increase afterward becomes progressively stable during its presence in the luminal KL; the second huge increase occurs instantly during the instantaneous presence of the material point between the end of luminal KL and the immediate beginning of superluminal KL.

4. Causality principle

The causality principle in sense of common conventional belief is in fact an assumption according to which the information traveling faster than light speed in vacuum represents a violation of *causality*. According to the superluminal relativistic mechanics [1], such a postulation remains valid only in the context of SRT as a direct consequence of LTs, which are exclusively applicable to the IRFs in relative uniform motion with subluminal velocities.

Therefore, if the causality is really a universal principle, in this case, it would be valid for subluminal, luminal and superluminal velocities because, after all, causality simply means that the cause of an event precedes the effect of the event. For instance, a massive particle is emitted before it is absorbed in a detector. If the particle's velocity was one trillion times faster than c , the cause (emission) would still precede the effect (absorption), and causality would not be violated since, here, LTs should be replaced with STTs (2) for the reason that the particle in question was moving in superluminal space-time not in Minkowski space-time. Consequently, in superluminal space-time, *“the superluminal signals do not violate the causality principle but they can shorten the luminal vacuum time span between cause and effect.”*

From all that, we arrive, again, at the following result regarding causality. If causality is really a universal principle, it would be valid in all the KLs. Consequently, in such a case, we can say that there is in fact three kinds of causality, *viz.*, subluminal causality, luminal causality and superluminal causality, and each kind is characterized by its proper circumstances.

5. Applications: Estimation of the (non-zero) photon rest mass

For a long time, the standard model of particle physics assumed that neutrinos are massless particles, propagating at the light speed. However, with the relatively recent empirical evidence from Super-Kamiokande [12] that the neutrinos are able to oscillate among the three available flavors (electron neutrino, muon neutrino, tau neutrino) while they propagate through space, such a discovery implies neutrinos to have non-zero masses. Moreover, the neutrino oscillations support the above mentioned principle of kinematical levels [1], particularly the concepts of luminal IRFs and luminal spatio-temporal transformations; and also may be regarded as a reinforcement to our reasonable believe already cited, namely, in Nature; none can prevent any free moving material body from reaching or exceeding light speed in vacuum. As repeatedly said in [1] and also in the present work, the existence of luminal and superluminal physical phenomena does not mean that SRT is incorrect or should be modified, on the contrary, this indicates that SRT is only valid in its proper domain of applicability, *i.e.*, in subluminal KL for relativistic velocities.

In view of the fact that the neutrino has a mass, thus the question of the mass of the photon should be re-examined because the formalism of superluminal relativistic mechanics [1] implies that the photons and tachyons should be naturally treated as ordinary particles with non-zero rest mass. But, some authors unscientifically justified, in their textbooks and research article, that the photon is a massless particle because *«A free photon cannot be slowed down to a subluminal speed or just stopped in vacuum.»* this naive argument is similar to very old claim: *«Nothing heavier than air can fly.»* Nevertheless, in 1999, Hau and her team have already produced the remarkable observation of light pulses traveling at velocities of only 17 ms^{-1} [13].

There is a huge number of research articles in which has been proved that the photon has non-zero rest mass, although such infinitesimal mass is extremely difficult to be experimentally detected [14], the deviations of Coulomb's law [15] and Ampère's law [16], the existence of longitudinal electromagnetic waves [17], and the additional Yukawa potential of magnetic dipole fields [18,19], were seriously studied. These consequences are the useful approaches for the cosmological observations [18,20] or the laboratory experiments to determine the upper limit on the photon mass. The fully consistent theory of massive electromagnetic fields is described by the Proca equations [21], which are in fact the generalization of Maxwell's equations. Vigier shown *via* relativistic interpretation (with non-zero photon mass) of the small ether drift velocity detected by Michelson, Morley and Miller [22]. Historically, the introduction of a non-zero photon mass was extensively discussed by the following authors [23-32]. Moreover, any open-minded theoretical physicist may arrive at the following conclusion after having attentively analyzed the famous Compton's scattering experiment [33]: when a photon of wavelength λ collides with a target at rest, and a new photon of wavelength λ' emerges at an angle θ . Just during this collision, the incident photon was *instantaneously* at relative rest.

Now, we arrive at the main subject matter of this subsection, *viz.*, the estimation of the (non-zero) photon rest mass. For this purpose, we shall deduce from the relations (10), an approximate general formula for the rest mass m_γ of a photon. So, for the case of a photon propagating in a local vacuum at light speed, $v=c$, we have from the second relation in (10):

$$E = \gamma_{\max} E_0, \quad E_0 = m_\gamma c^2 . \quad (15)$$

Furthermore, according to Planck's law, we have for the photon's energy

$$E = h\nu , \quad (16)$$

where $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$ is Planck's constant and ν is the supposed observed frequency in laboratory reference frame. Thus, from (15) and (16), we get the required expression

$$m_\gamma = \frac{h\nu}{\gamma_{\max} c^2} . \quad (17)$$

It is worthwhile to notice that according to the formula (17), the rest mass of the photon depends only on the observed frequency ν in the laboratory reference frame. Therefore, m_γ is explicitly a function of frequency $m_\gamma \equiv m_\gamma(\nu)$. *Theoretical minimum (non-zero) rest mass of the photon:* The knowledge, even approximate, of the photon rest mass is important because it may play a role in particle physics and cosmology. To this end, we must select an *ideal* minimum numerical value for frequency, which for convenience should be 1Hz, *i.e.*, one oscillation *per* second. Now, if in the formula (17) we substitute the accepted values of h , γ_{\max} , c and $\nu \equiv \nu_{\min} = 1\text{Hz}$, we obtain

$$m_\gamma^{\min} = 8.08 \times 10^{-55} \text{ kg} = 8.08 \times 10^{-52} \text{ g} . \quad (18)$$

And from (18), we can deduce the ratio of the rest mass of the electron m_e to the rest mass of the photon as follows:

$$m_e/m_\gamma^{\min} = 1.1274 \times 10^{24}, \quad (19)$$

where $m_e = 9.109382 \times 10^{-31}$ kg. Statistically, the ratio (19) is important for the cosmology.

It seems our theoretical result (18) is in good accordance with the experimental results of Refs.[34,35], which led to the upper limit on photon rest mass of 2×10^{-50} g and 1.2×10^{-51} g, respectively. As we can remark it, according to our conceptual approach, this extremely small rest mass of the photon can serve as a fundamental solution to some problems, particularly the observed anisotropy of the cosmic microwave background (CMB). This possibility has been already proposed in 1983, by Georgi, Ginsparg and Glashow [36]. In their seminal paper, the authors suggested as a solution to the apparent discrepancy between theoretical and observed CMB-spectra, a rest mass of 8.913×10^{-51} g.

6. Conclusion

Basing on previous work [1], we have determined the theoretical maximum (numerical) value for Lorentz factor $\gamma_{\max} = 9125.5511$ and marked out the validity limits of SRT in its proper domain of applicability, these validity limits allowed us to situate the frontiers between relativistic physics and superluminal physics for the conceptual and practical purpose at microscopic and macroscopic levels. The established formalism combined with superluminal relativistic mechanics [1] should serve as the foundations of new physics: *superluminal particle physics*.

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