Multiplication Table

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X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Factors and Multiples

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Factors of a number are values that will go in to that number exactly.

Examples The factors of 10 are 1, 2, 5 and 10.
The factors of 15 are 1, 3, 5 and 15.
7 is not a factor of 12.

Multiples of a number are values in that number's times table.

Examples Multiples of 5 are 5, 10, 15, 20, 25, ...
Multiples of 7 are 7, 14, 21, 28, 35, ...
500 is a multiple of 10 (because $50 \times 10 = 500$)
13 is not a multiple of 2.

Fractions, Decimals and Percentages

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Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.10	10%
$\frac{1}{3}$	0.3333333333...	33.3%
$\frac{3}{10}$	0.3	30%
$\frac{1}{5}$	0.2	20%

Multiplying and Dividing with Negative Numbers

Positive	× ÷	Positive	=	Positive
Positive	× ÷	Negative	=	Negative
Negative	× ÷	Positive	=	Negative
Negative	× ÷	Negative	=	Positive

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Order of Operations (BIDMAS)

This is the order in which calculations should be done.

Brackets
Indices
Division
Multiplication
Addition
Subtraction

Examples:

$$(5\times4)\div2$$
$$20\div2$$
$$10$$

$$20-2\times3^2$$
$$20-2\times9$$
$$20-18$$
$$2$$

Finding percentages of amounts

The 'building blocks' of percentages:
To find 50% of a number, just half it.
To find 10% of a number, divide it by 10.
To find 1% of a number, divide it by 100.

You can then use these to find other percentages, for example...
To find 5%, just half your 10%.
To find 60%, just add your 50% and 10% together.

Or if it's on the calculator paper...

To find 35% of 600 $\boxed{3}\ \boxed{5}$ (%) $\boxed{\times}\ \boxed{6}\ \boxed{0}\ \boxed{0}\ \boxed{=}$

Finding fraction of amounts

To find $\frac{1}{2}$ of a number divide it by 2.

To find $\frac{1}{5}$ of a number divide it by 5.

To find $\frac{3}{4}$ of a number divide it by 4 and then multiply it by 3.

Example

Find $\frac{2}{5}$ of 20 $\frac{1}{5} = 20 \div 5 = 4$, so $\frac{2}{5} = 4 \times 2 = 8$

Prime Numbers

A prime number is a number that has only 2 factors – 1 and itself.

Here are the first 12 prime numbers...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Note that 1 is not a prime number – this is because 1 only has 1 factor!

Squares, Cubes and Indices

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When you square a number, you times it by itself.
e.g. 3^2 means '3 squared' which means 3×3 so $3^2 = 9$.

When you cube a number, you times a number by itself 3 times.
e.g. $4^3 = 4 \times 4 \times 4 = 64$

Whatever the power (small number) is, this is how many times you multiply the number by itself.

Example Exam Question
Which is bigger 5^2 or 2^5? You must show your working.

$5^2 = 5 \times 5 = 25$
$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ so 2^5 is bigger

Square & Cube Roots

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Square roots

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

Cube roots

$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$
$\sqrt[3]{27} = 3$
$\sqrt[3]{64} = 4$
$\sqrt[3]{125} = 5$

Square numbers & Cube Numbers

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Square numbers are the result of squaring a number.

e.g.
$1^2 = 1 \times 1 = 1$, so 1 is a square number
$2^2 = 2 \times 2 = 4$, so 4 is a square number
$3^2 = 3 \times 3 = 9$, so 9 is a square number
$10^2 = 10 \times 10 = 100$, so 100 is a square number

Similarly, cube numbers are the result of cubing a number.

e.g.
$1^3 = 1 \times 1 \times 1 = 1$, so 1 is a cube number
$2^3 = 2 \times 2 \times 2 = 8$, so 8 is a cube number
$3^3 = 3 \times 3 \times 3 = 27$, so 27 is a cube number

Adding and Subtracting Fractions

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When adding or subtracting fractions, the denominators must be the same. One way to make them the same is to multiply the first fraction by the denominator of the second fraction and vice versa.

e.g. $\frac{1}{4} + \frac{2}{5} = \frac{5}{20} + \frac{8}{20} = \frac{13}{20}$

We multiplied $\frac{1}{4}$ by 5 (top and bottom) to get $\frac{5}{20}$

We multiplied $\frac{2}{5}$ by 4 (top and bottom) to get $\frac{8}{20}$

We then added the numerators. The denominator stays the same.

If subtracting fractions, the only difference is that we must subtract the numerators instead of adding them.

Multiplying Fractions

Multiplying fractions is easy. Just times the numerators together and times the denominators together.

Examples

$$\frac{1}{3}\times\frac{4}{5}=\frac{1\times 4}{3\times 5}=\frac{4}{15}$$

$$1\frac{1}{2}\times\frac{2}{3}=\frac{5}{2}\times\frac{2}{3}=\frac{10}{6}=\frac{5}{3}=1\frac{2}{3}$$

Dividing Fractions

To divide fractions, flip the second fraction then multiply the fractions together.

Example

$$\frac{2}{9}\div\frac{2}{3}=\frac{2}{9}\times\frac{3}{2}=\frac{2\times 3}{9\times 2}=\frac{6}{18}=\frac{1}{3}$$

Rounding and Estimating

Significant figures are the first digits you reach in a number that aren't zero.

For example...

To 1 s.f. 597 = 600 To 2 s.f. 0.00918 = 0.0092

Estimating

When asked to estimate the answer to a problem, that is what you must do. You will not get credit for working out the actual answer.
Generally, when estimating, round all numbers to 1 significant figure and find the answer.

e.g. Estimate $\frac{(19.84^2+4.12)}{1.87}=\frac{(20^2+4)}{2}=\frac{(400+4)}{2}=\frac{404}{2}=202$

e.g. Estimate $\frac{3.12\times 9.76}{0.49}=\frac{3\times 10}{0.5}=\frac{30}{0.5}=60$

Exchange Rates

Questions to do with exchange rates always involve either dividing or multiplying the values given.

Example

Given that £1 = €1.24

a) Convert £25.99 to euros

b) If a pair of jeans costs €69, how many pounds is this?

a) 25.99 × 1.24 = 32.2276 = €32.23
(Always round money to 2d.p. unless told otherwise).

b) 69 ÷ 1.24 = 55.64516129 = £55.65

When going from 'left to right' – pounds to euros, we multiplied.
When going the other way, we divided.

Percentage Change

To work out a percentage change, use the following formula:

$$\text{percentage change} = \frac{\text{actual change}}{\text{original amount}}\times 100$$

Example

A television is reduced from £800 to £520 in a sale.
What percentage saving is this?

Answer

Actual change = £800-£520 = £280

$$\text{Percentage change} = \frac{280}{800}\times 100 = 0.35\times 100 = 35\%$$

Ratio

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If given the total amount, find the total number of shares first.

Example
Divide £80 in the ratio 7:2:1

Total shares = 7 + 2 + 1 = 10
1 share = £80 ÷ 10 = £8

7	:	2	:	1
× £8		× £8		× £8
£56	:	£16	:	£8

When not given the total amount, you need a different method.

Example 2
Steve divides money between his son and daughter in the ratio 2:5. His daughter is given £35, how much does the son receive?

Son		Daughter
2	:	5
× £7		× £7
£14	:	£35

The daughter has received 7 'lots' of 5 so 1 share must be worth £7. Therefore the son must receive 2 shares of £7, total £14.

Metric Units

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You are expected to know 'by heart' (and be able to use) all of the conversions below:

	Length	
1 kilometre	=	1,000 metres
1 metre	=	100 centimetres (cm)
1 centimetre	=	10 millimetres (mm)
	Weight	
1 kilogram (kg)	=	1,000 grams (g)
1 tonne	=	1,000 kilograms (kg)
	Capacity	
1 litre (l)	=	1,000 millilitres (ml)
1 litre (l)	=	100 centilitres (cl)
1 ml	=	1cm^3

Metric and Imperial Equivalents

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You should know and be able to use the following imperial and metric equivalents:

	Length	
5 miles	=	8km
1 foot	=	30cm
1 inch	=	2.5cm
	Mass/Weight	
2.2lb (pounds)	=	1kg
1 tonne	=	1,000 kilograms (kg)
	Capacity	
1 pint	=	1.75 litres
1 gallon	=	4.5 litres

Polygons and Prisms

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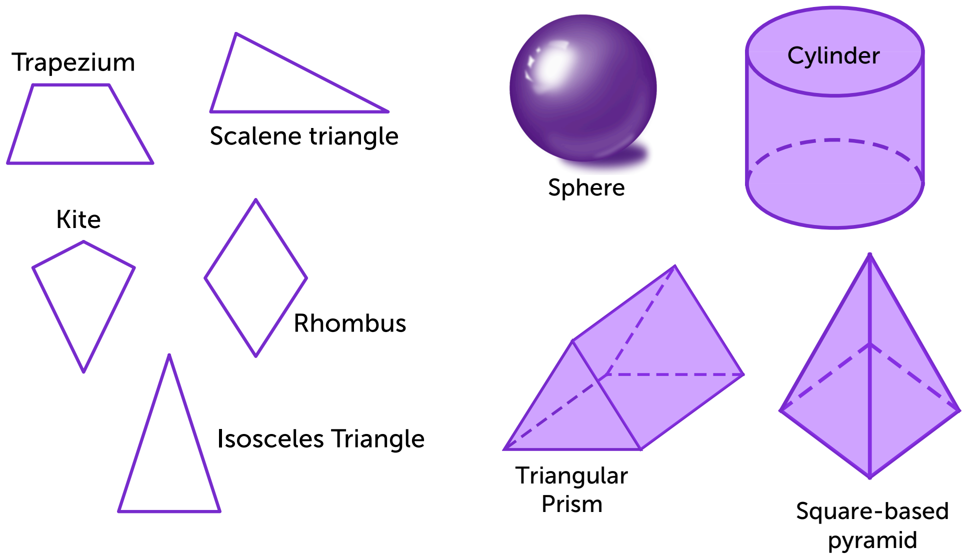

Area Formulas

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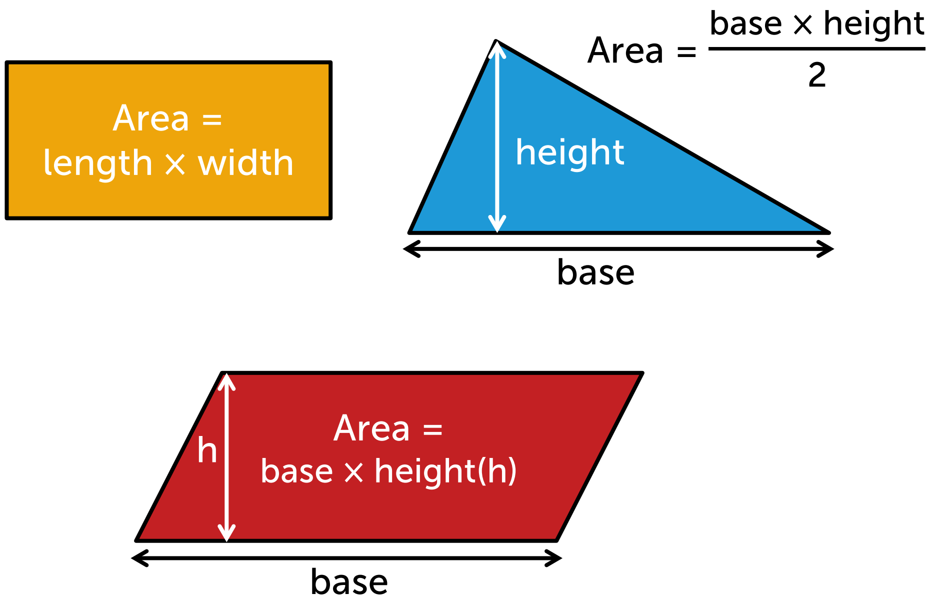

Area of a circle

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The area of a circle is found using the formula:

$$\text{Area} = \pi r^2$$

Where r is the radius* of the circle.

So if a circle has a radius of 5.2cm...

$$\begin{aligned}\text{Area} &= \pi \times 5.2 \times 5.2 \\ &= 84.9\text{cm}^2\end{aligned}$$

Notice the units of the answer are square units as it is an area.

TIP: Check you are given the radius. If you're given the diameter, just half it.

**Don't know what the radius is? See the card 'Parts of a circle 1'.*

Circumference of a circle

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The circumference of a circle is found using the formula:

$$\text{Area} = \pi d$$

Where d is the diameter* of the circle.

So if a circle has a diameter of 7.8m...

$$\begin{aligned}\text{Circumference} &= \pi \times 7.8 \\ &= 24.5\text{m}\end{aligned}$$

Notice the units of the answer are not square units as circumference is a length.

TIP: Check you are given the diameter. If you're given the radius, double it first.

**Don't know what the diameter is? See the card 'Parts of a circle 2'.*

Parts of a circle 1

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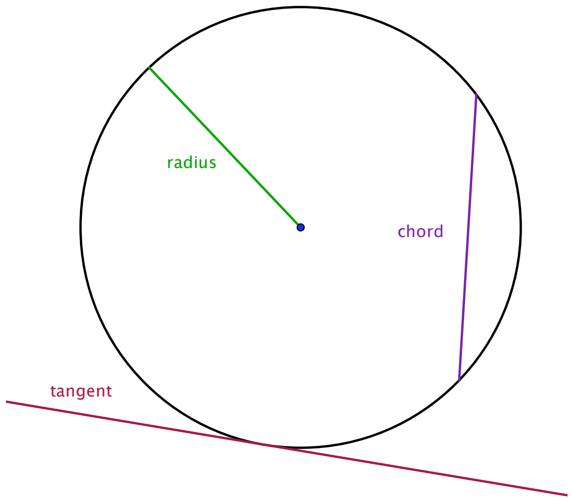
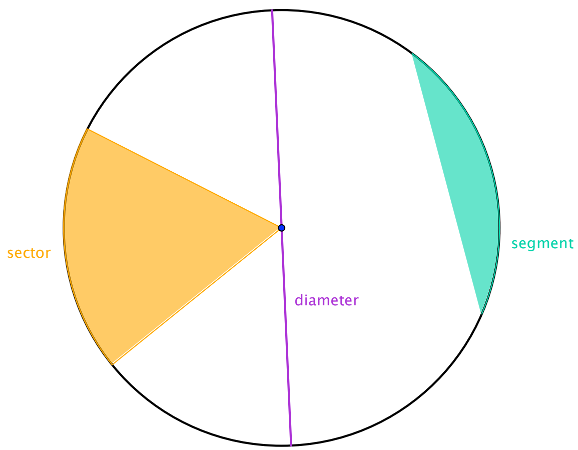


The radius of a circle is the distance from the centre of the circle to the edge.

A chord of a circle is a line joining 2 points on the circumference of a circle.

A tangent is a line that touches a circle on its circumference.

Parts of a circle 2

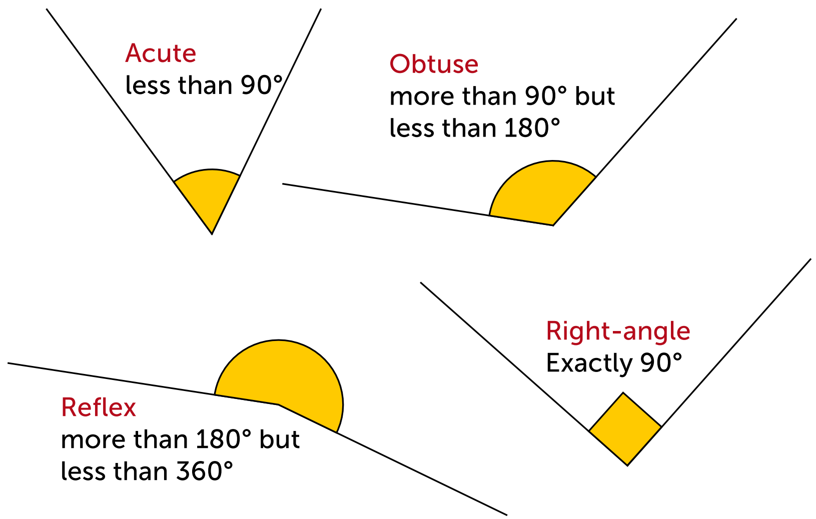


A sector is a 'slice' of a circle taken from the centre to the circumference.

The diameter of a circle is the distance from edge to edge through the centre.

A segment is a slice of a circle that is taken from the circumference. Or it can be described as the area between a chord and the circumference.

Types of angle

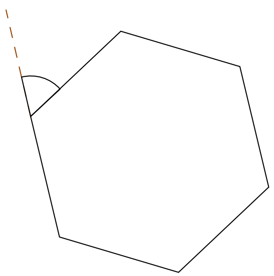


Angle Facts 1

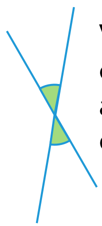
- Angles on a straight line add up to 180°
- Angles at a point add up to 360°
- Angles in a triangle add up to 180°
- Angles in a quadrilateral add up to 360°
- The total of the (interior) angles of an n-sided polygon can be found using this formula:

$$(n - 2) \times 180$$

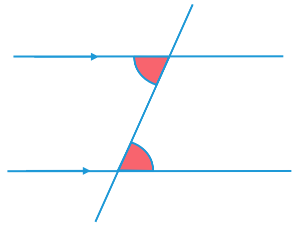
- The exterior (outside) angles of any shape always add up to 360°
- In a regular hexagon, each exterior angle (shown on the right) would be 60° (360° ÷ 6)



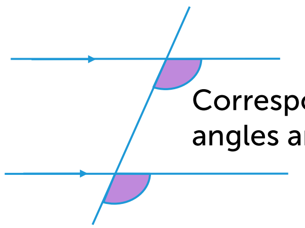
Angle Facts 2



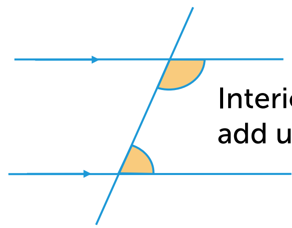
Vertically opposite angles are equal.



Alternate angles are equal.



Corresponding angles are equal



Interior angles add up to 180°

Pythagoras' Rule

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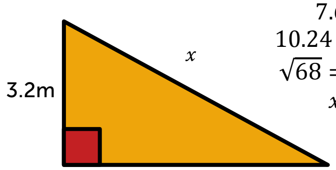
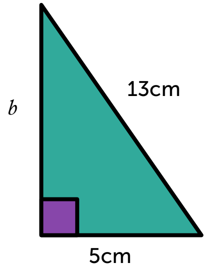
The hypotenuse of a right-angled triangle is the longest side. This is always opposite the right-angle.

- Square both sides
- Looking for the longest side? Add them together. Looking for a short side? Subtract one from the other.
- Square root your answer.



$3.2^2 = 10.24$
$7.6^2 = 57.76$
$10.24 + 57.76 = 68$
$\sqrt{68} = 8.2\text{m(1d.p)}$
$x = 8.2\text{m}$



$13^2 = 169$
$5^2 = 25$
$169 - 25 = 144$
$\sqrt{144} = 12\text{cm}$
$b = 12\text{cm}$

Some Common Constructions

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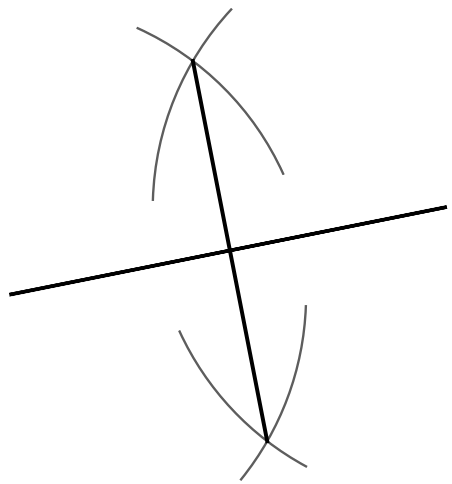
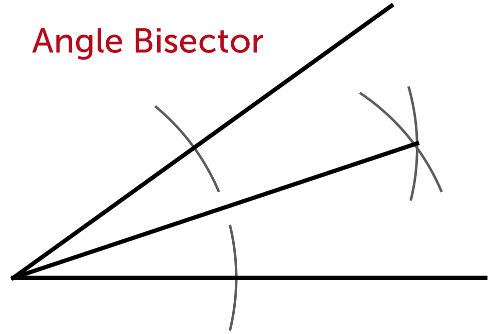
Perpendicular Bisector



Angle Bisector



Translations

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Translations

A translation is when a shape just moves by a given amount.

This is sometimes stated in words. e.g. 'Translate the shape 4 squares right and 3 squares up'.

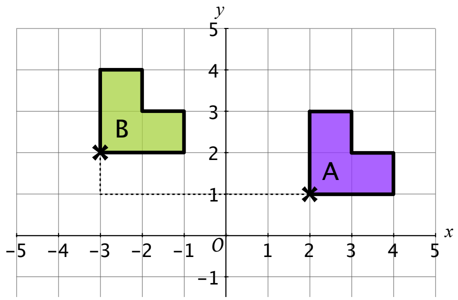
At other times this is given as a vector.

e.g. $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ This means move the shape 2 squares to the left and 5 squares up.

Example

Translate shape A by vector $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$.

Label the new shape B.



Notice how we pick a point on the shape and move that point 5 squares to the left and 1 square up. This point was originally the bottom-left corner on A and still is on B.

Rotations

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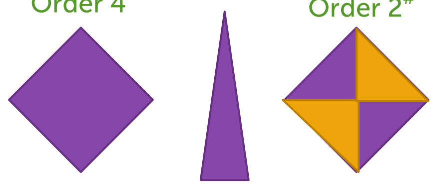
A rotation refers to a shape being turned or spun.

The order of rotation of a shape is how many times that shape fits in to itself when spun 360°.

Order 4 Order 1* Order 2#



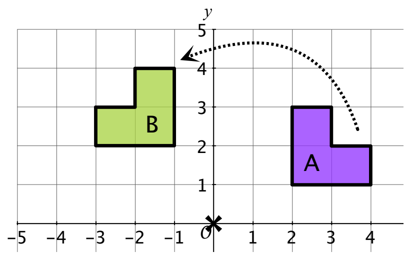
**Every shape has at least order of rotation 1. If a shape only has order 1, we say it has no rotational symmetry.*
#Notice the colours affect the order of rotation.

When told to describe a rotation, you must usually give:

- The degree of the rotation
- The direction of the rotation
- The centre of rotation.

Example

Describe the transformation that maps Shape A to Shape B.



A rotation 90° anti-clockwise, centre (0,0).

Similar and Congruent Shapes

Congruent shapes are exactly the same as each other (though they may have been 'turned around' or flipped).

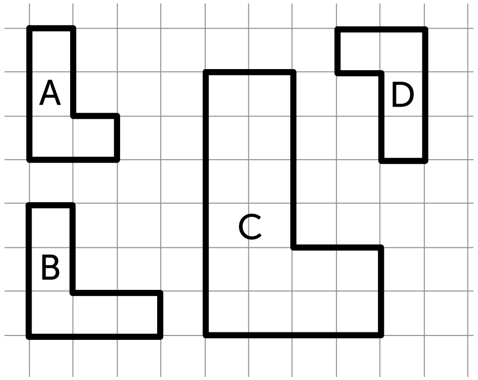
Similar shapes are 'similar' to each other! Sides are still in the same proportion though they may be bigger or smaller. Equivalent angles in similar shapes will be the same.

Example

Shapes A and D are congruent.

Shapes A and C are similar.
(Shape C is an enlargement of scale factor 2).

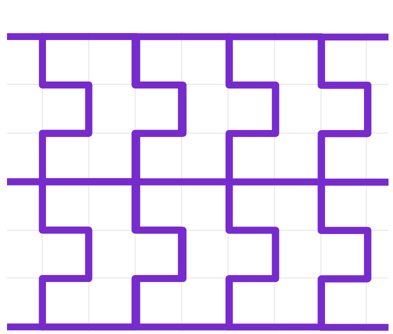
Shapes D and C are also similar.

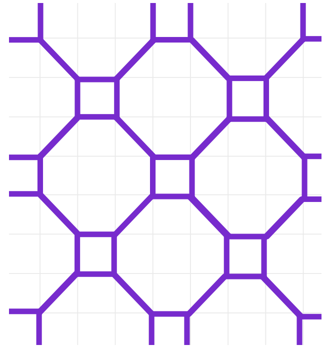


Tessellations

A tessellation is when a pattern is made from one or more shapes that fit together in such a way that no gaps are left.

Some examples of tessellations...





Simplifying expressions

Important Examples

$b + b \ + b = 3b$

$b \times b \times b = \ b^3$

$a + b = a + b$ (You cannot simplify expressions like this)

$5s + 7t + 4s - 9t = 9s - 2t$ (Because 5s + 4s = 9s and 7t – 9t = -2t)

$a \times b = ab$

$4p \times 5q = 20pq$

$4c \times 5c = 20c^2$ (Because 4 × 5 = 20 and c × c = c^2)

Expanding Brackets

Remember that the word 'expand' in this case means to remove the brackets by multiplying terms.

Expand $3(x+5)$

$3x + 15$

Expand $a(b-c)$

$ab - ac$

Expand & Simplify $(x+4)(x+5)$

$x^2 + 5x + 4x + 20$

$x^2 + 9x + 20$

Expand & Simplify $(x+7)(x-9)$

$x^2 - 9x + 7x - 63$

$x^2 - 2x - 63$

Solving Equations

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$$b + 3 = 7$$
$$(-3) \quad (-3)$$
$$b = 4$$

$$5x = 10$$
$$(\div 5) \quad (\div 5)$$
$$x = 2$$

$$\frac{x}{4} = 5$$
$$(\times 4) \quad (\times 4)$$
$$x = 20$$

$$6x + 5 = 35$$
$$(-5) \quad (-5)$$
$$6x = 30$$
$$(\div 6) \quad (\div 6)$$
$$x = 5$$

$$4(x-2) = 12$$
$$4x - 8 = 12$$
$$(+8) \quad (+8)$$
$$4x = 20$$
$$(\div 4) \quad (\div 4)$$
$$x = 5$$

$$7x - 2 = 3x + 10$$
$$(-3x) \quad (-3x)$$
$$4x - 2 = 10$$
$$(+2) \quad (+2)$$
$$4x = 12$$
$$(\div 4) \quad (\div 4)$$
$$x = 3$$

$$x^2 = 9$$
$$(\sqrt{}) \quad (\sqrt{})$$
$$x = 3$$

Sequences (n^{th} term)

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Finding the n^{th} term

+5 +5 +5 +5

4, 9, 14, 19, 24

As the sequence goes up by 5 each time, part of the formula is 5n.

If there were another number 'in front' of the sequence, it would be -1.

Therefore, the nth term for this sequence is 5n-1

Using the n^{th} term

If the n^{th} term of a sequence is $n^2 + 3$

The 1^{st} term is $(1 \times 1) + 3 = 1 + 3 = 4$
The 2^{nd} term is $(2 \times 2) + 3 = 4 + 3 = 7$
The 3^{rd} term is $(3 \times 3) + 3 = 9 + 3 = 12$
and so on...

Drawing Graphs of Equations

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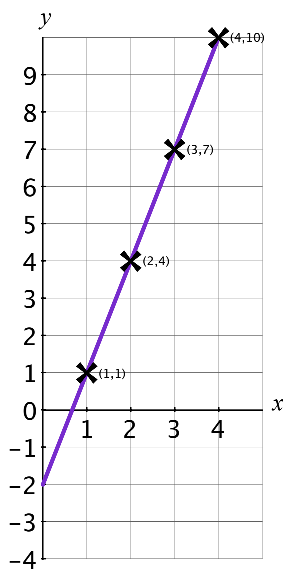
Draw the graph of $y = 3x - 2$

You will normally be given a table of values to fill in.
If not, make your own! Pick easy values for x if you do.

x	1	2	3	4
y	1	4	7	10

y = 10 because...
when x = 4,
y = (3 × 4) – 2
= 12 - 2
= 10

Plot the points as coordinates, then join them together and extend the line as far as possible.



Trial and Improvement

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You will be told when to use trial and improvement in the question.

Example
Solve to 1 decimal place, using trial and improvement:
$$x^3 + \sqrt{x} = 33$$

x	$x^3 + \sqrt{x}$	Too big/small
3	28.73...	Too small
4	66	Too big
3.2	34.55...	Too big
3.1	31.55...	Too small
3.15	33.03	Too big

So, to 1 decimal place x = 3.1

Notice that the final trial was to 1 extra decimal place. As 3.15 was too big, we know the answer was definitely closer to 3.1 than to 3.2

Finding the Mean, Median, Mode and Range

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Mean
Add all the values together.
Divide this answer by the number of values.

e.g. Mean of 2, 4, 5, 5 is $\frac{2+4+5+5}{4} = \frac{16}{4} = 4$

Median
Put the values in order of size.
Find the middle value. If there are 2 middle values, add them together and half it.

Examples

- Median of 1, 2, 3, 4, 5 is 3 as this is the middle value.
- Median of 2, 4, 6, 8 is 5.
  4 and 6 are the 2 middle numbers.
  $(4+6) \div 2 = 10 \div 2 = 5$

Mode
The mode is the most common value.

- The mode of 2, 3, 3, 5, 6 is 3
- The mode of *dog, cat, dog, bird* is dog.

You can have more than one mode or no mode.

- The mode of 1, 1, 3, 4, 4, 5, 6 is 1&4
- 1, 2, 3, 4, 5 has no mode

Range
The range is the difference between the largest and smallest values.

e.g. 6, 7, 3, 5, 4, 9, 4
The range of these numbers is 9-3 = 6

Mean from tables

teemaths.com

Number of pets (x)	Frequency (f)	fx
0	2	$0 \times 2 = 0$
1	3	$1 \times 3 = 3$
2	8	$2 \times 8 = 16$
3	4	$3 \times 4 = 12$
4	1	$4 \times 1 = 4$
Total	18	35

Mean = $35 \div 18 = 1.94$ (2d.p.)

Multiply the first 2 columns.
Find the totals shown.

You always divide by total frequency, though the column isn't always called this!

Height, cm (h)	Midpoint (x)	Freq. (f)	fx
$140<h\le150$	145	2	290
$150<h\le160$	155	8	1240
$160<h\le170$	165	5	825
$170<h\le190$	180	1	180
	Total	16	2535

Mean = $2535 \div 16 = 158.4$cm (1d.p.)

This time we have to add a midpoint column to give us values to use. Then we do the same as for a 'normal' table like that on the right.

Drawing a pie chart

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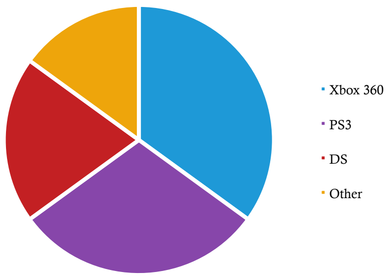
You are normally asked to draw a pie chart from a table of data.
You need to work out what angle each section needs to be.
Add up all your frequencies and then see what you need to times/divide these by to make the total 360.

Favourite Console	Frequency	Angle
Xbox 360	21	126°
PS3	18	108°
DS	12	72°
Other	9	54°
TOTAL	60	360°

$\times 6 \rightarrow$



Scatter Graphs

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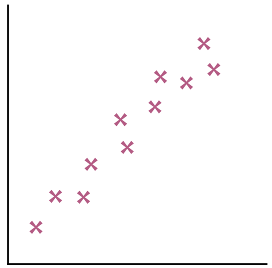
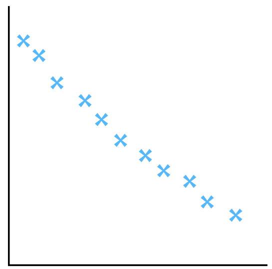
Scatter graphs show the correlation (relationship) between 2 variables.



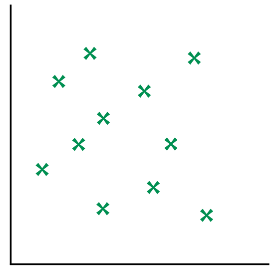
A positive correlation. As the x-value increases, so does the y-value.

This is a moderately strong correlation as the points are reasonably close to a straight line.



A negative correlation. As the x-value increases, the y-values decrease.

This is a strong correlation as the points are very close to a straight line.



No correlation.

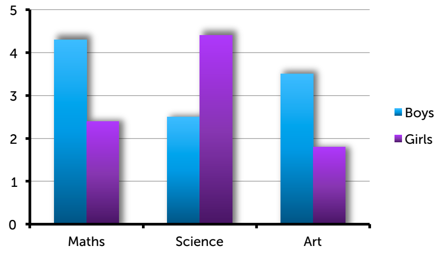
There is no relationship visible.

More tables/charts/graphs

Tally chart/Frequency table – quickly collect data

Eye Colour	Tally	Frequency
Blue	𝍸 II	7
Green	𝍸 𝍸 I	11
Other	𝍸 III	8

Grouped Bar Chart – good for comparing multiple values for 2 or more groups

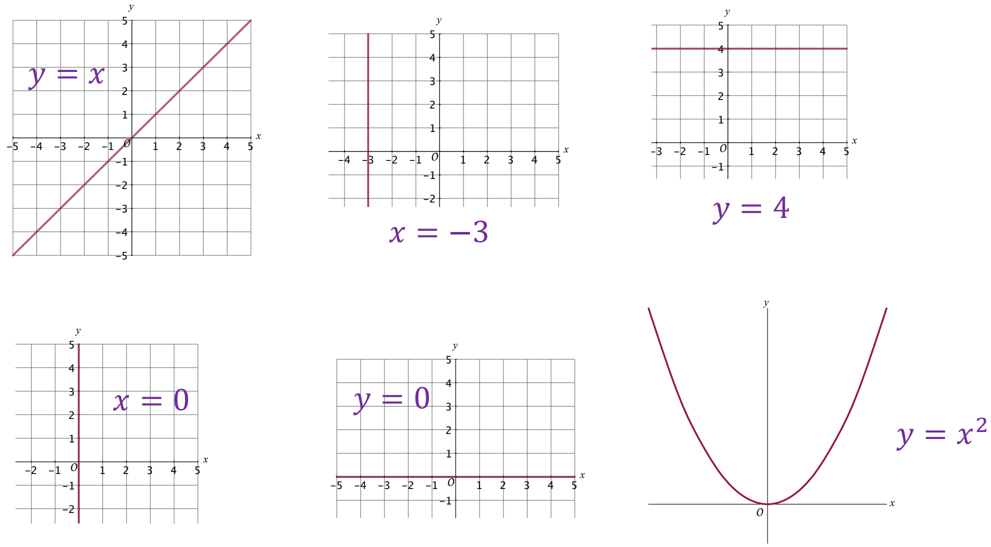


Stem-and-leaf diagram
(don't forget the key!)

1 | 4 6 7 9
2 | 2 5 8
3 | 2 3

Key:
1 | 6
means 16

Some graphs to know...



Probability

Probabilities may be given as fractions, percentages or decimals.
You should only use words to describe a probability when the question clearly states you should.

The total of probabilities for outcomes of any given event should always equal 1.

e.g. The probability a football team wins is 0.3
The probability they draw is 0.1
What is the probability they lose?

Answer: 1 – 0.3 – 0.1 = 0.6

Also, for any event...

$$\text{Probability of an event occurring} = \frac{\text{number of ways event can occur}}{\text{number of possible outcomes}}$$

e.g. The probability of getting an even number on a normal dice is $\frac{3}{6}$
There are 3 even numbers on a dice and 6 sides altogether.