# Fluid Mechanics

## What you'll learn:

- What is a fluid?
- How do I know if an object will sink or float?
- Why do divers have specially designed pressure suits when they do deep dives, but don't need them for shallow dives?
- How is the velocity of a flowing liquid related to pressure?
- What is Bernouli's equation and when can it be used?

## Motivation

Any child will be able to tell you that a glass of milk that is spilled on the floor will "go everywhere." The same child will also be able to tell you that a liquid water flows out of the sink and solid water (ice) doesn't flow at all. Fluids include substances such as room temperature water, juice, automobile anti-freeze, or hydrofluoric acid. They can be flowing, like blood in your veins, or stationary, like tea in a tea cup.

## But what is a Fluid?

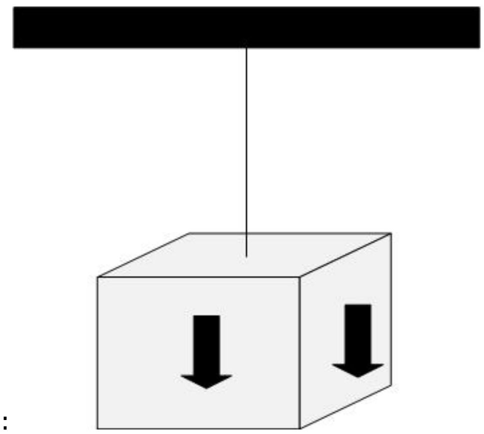
A **Fluid** can easily deform to any shape. It is defined as any substance that deforms continuously under a shear. If you think about a piece of ice (solid water), unless the ice melts, no amount of pushing on the ice will cause it to deform enough to let your finger inside until it reaches its breaking point. Liquid water, on the other hand, deforms easily. If you push your figure into a large glass of water, the water deforms easily to accommodate your figure. In fact, you could push in your figure and your hand into a large glass of any liquid with very little resistance (unless you hit the solid cup).

A liquid can deform like this because of **intermolecular forces**. There are tight bonds holding the molecules of a solid together. This keeps the solid from wanting to change shape (i.e. the molecules don't want to move/ break their bonds). Fluids have weak intermolecular forces, so it takes very little external force to break these bonds and move the molecules. The tight bonds of the molecules are evident in the distribution of molecules (as was discussed in the thermodynamics section of these notes).

**Example:**

If I hold a block of wood up by a string there could be several shearing forces. Only the shear due to gravitational pull is shown in figure 1.

Figure 1:



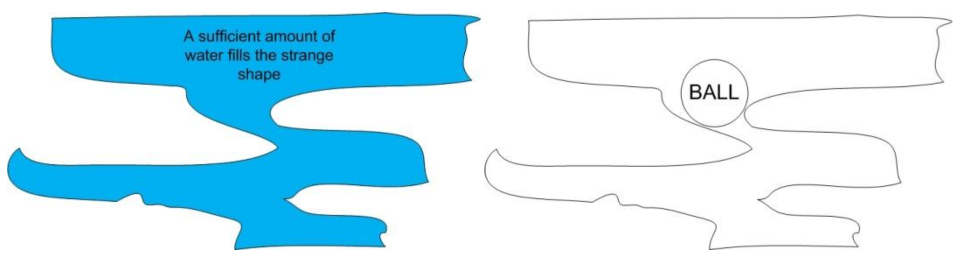
This force is NOT enough to move the molecules and change the shape of the block. It is common knowledge that under normal conditions a block of wood would not start deforming simply because it was hung up. On the other hand, it is not even possible to hang a liquid like this. The shear simply due to gravitational forces would cause the fluid to deform and fall to the floor instantly!

If I pour water from a cup to a bowl the water fills the bowl just as well as it filled the cup. It is a property of liquids that they take the shape of their container, while solids do not.

**Example:**

Water in a strange container will fill the shape of the container. Dropping a ball into the same container does nothing to the shape of the ball and the container is not filled.

Figure 2:




In fact, if you added several balls to the container, even if they were all different sizes, the container would never truly be filled with just balls. The solid balls would not lose their shape and so, air gaps would form in the container.

What are the forces can act on the fluids in containers to make them deform?

- Atmospheric
- Body
- Outside forces (including pistons pushing the liquid, and any other forces).

Some liquids deform more than others under the same shear. You will note that honey pours much slower then juice does. Also, you may notice that hot honey pour faster than cold honey.

Figure 3:

a)  b) 

Where a shows juice pouring into a cup and b shows honey pouring into a bowl. Notice how easily the juice deforms.

It is convenient to have a property to describe the resistance of a fluid to deforming under a given shear. This property is called **viscosity**. Remember: we defined a fluid as anything that continuously deforms under a shear. We never said how much or how easily the fluid would deform. Viscosity is also known as absolute viscosity or dynamic viscosity and is represented by the symbol $\mu$. The relation between viscosity shear force and how fast an object moves is:

$$\tau = \mu \frac{du}{dy}$$

Where $\tau$ is the shear stress (just like in solid mechanics), u is the velocity of the fluid in the x direction and du/dy is the slope of the velocity profile.

### Example:

I have a fluid between 2 plates. I pull the top plate sidewise, but the bottom plate is fixed to the ground. If I know the sheer force applied is 1 N, the plates have a $1 m^2$ area, the plates are .5 m apart, and I am pulling the top plate with a velocity of .2 m/s, what is the viscosity? (You can assume that the fluid that touches the top plate has the same velocity as the plate, and the fluid that touches the bottom plate has the same velocity as the bottom plate. This is called the no slip condition and for all cases where the viscosity is important, we will assume that is assumption holds).

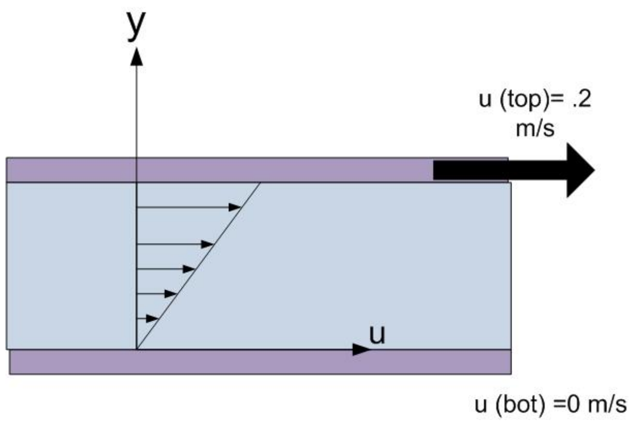



Figure 4:

We can see that the water at the top of the channel has a velocity = .2 m/s and has a velocity = 0 at the bottom. If we assume this profile is linear we can say that du/dy = .2/.5=.4 (1/s).

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{.4\,{}^{1}\!/\!_{s}} = \frac{\frac{1N}{1m^2}}{.4\,{}^{1}\!/\!_{s}} = 2.5\frac{Ns}{m^2}$$

In this, we can see that, if we knew the shear applied for many fluids, we could chart which fluids were more or less viscous.

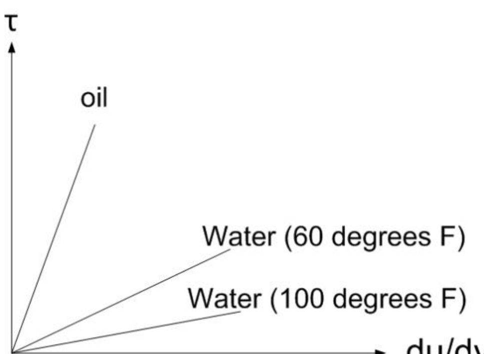



Figure 5:

Sometimes, however, the shear is NOT related to du/dy in a constant manor. These fluids can have an increasing or decreasing viscosity with increasing or decreasing shear (remember the viscosity is the slope in the $\tau$- du/dy graph). These fluids are called **non-Newtonian fluids**. Some examples of these types of fluids are polymers, latex paints, sand-water mixtures, water-cornstarch mixtures, and many others. These materials will not be covered farther in this class.

Viscosity is often given in the form of kinematic viscosity ($\nu$). The kinematic viscosity is equal to $\mu/\rho$ (where $\rho$ is the density). This form of viscosity is convenient for many mathematical relations.

## Statics

We learned from previous lessons that F=PA or that force exerted is equal to the pressure exerted multiplied by the area over which the pressure is being applied. We mentioned before that atmospheric forces were one force that could act on a fluid body. Let's look at a cup of liquid water.

### Example

Do you think there is more pressure from the water at A or B? Why?

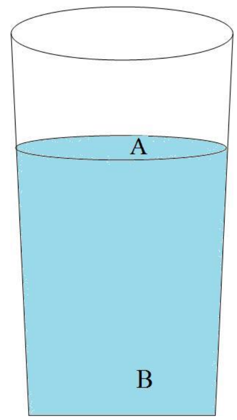


Figure 6:

At point A there is only atmospheric pressure being felt. At point B there is the pressure from the atmosphere plus the pressure from all the water above the point.

What if the cup were filled with mercury? Would point B have more pressure/ less pressure/ or the same pressure as it did when the cup was filled with water?

The pressure that point B feels from the atmosphere will be the same, but the pressure it feels from the liquid will be different. The density of the liquid translates to a pressure due to the weight of the liquid above the point. Mercury is more dense (heavier) then water, so point B will feel more pressure with mercury above it then with water above it.

The pressure due to fluid weight at any point in a static fluid is as follows:

$$P = \rho g h = \gamma h$$

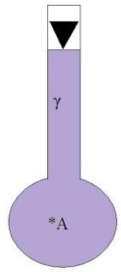
Where P is the pressure, ρ is the dencity of the fluid, g is acceleration due to gravity (9.8 $m/s^2$), and h is the height BELOW the fluid surface. γ is known as the **specific weight** and is equal to ρg. It is the weight per unit volume.

## Manometers

A **manometer** is a device that uses liquid(s) in a column to measure pressure.

### Example

What is $P_A$?




a)

$P_A = P_o + \gamma h$ (where $P_o$ is atmospheric pressure)

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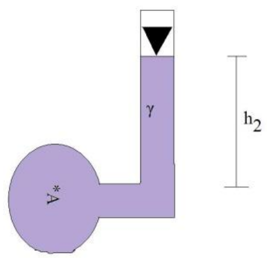


b)

$P_A = P_o + \gamma h_2$

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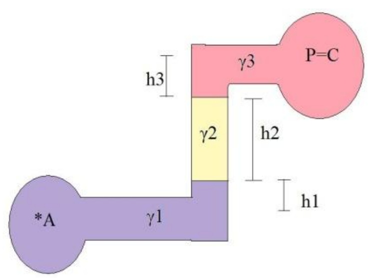
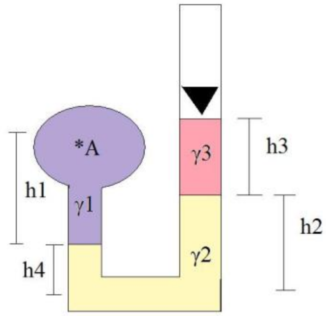



c)

$P_A = C + \gamma 3h3 + \gamma 2h2 + \gamma 1h1$




d)

$P_A = P_o + \gamma 3h3 + \gamma 2(h2 - h4) - \gamma 1h1$

Note: Going deeper increases pressure, going shallower decreases pressure!

## Buoyancy/ Flotation:

Why do some things sink and others float?

**Archimedes principle** states that when a body is completely submerged, or floating, in a fluid the resultant force on the object by the fluid is the buoyant force and that force is equal to the weight of the fluid displaced by the object. If an object is in water the volume of water displaced by the object must be equal to the volume of the object that is submerged. Therefore the buoyant force can be shown to be:

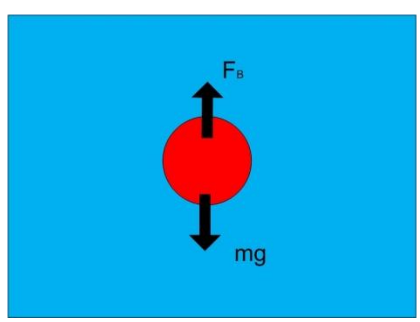
$$F_B = \rho g V$$

Where ρ is the density of the FLUID, g is acceleration due to gravity and V is the volume displaced. Archimedes principle is derived from the static pressure difference that occurs at lower depths. It is a common mistake to include the P=ρgh (static pressure) into calculations when an object is submerged and the buoyant force is already taken into account. Only $F_B$ *OR* the static pressure can be used in a force balance on the object, otherwise the force is double counted!

### Example:

A ball with radius 62 cm is completely submerged in a liquid where γ= 10 kN/m$^3$. The ball has a density of 100 kg/m$^3$. Will the object sink or float or remain steady in the middle of the fluid?

Figure7:



In order to find if the ball will sink or float we assume it is totally emerged and perform a force balance. If the force up is stronger than the force down, the ball will float (i.e. not all of the ball will be in the water, so the real static solution will have a smaller volume in the water.). If the force down is greater than the force up the ball will sink (i.e. the ground is needed to create a larger force upward then just the water can provide). If the force down = force up, the ball is be stationary in the water.

$$F_B = \rho_{liq} g V = \gamma_{liq} V$$
$$V = V_{ball} = 4/3\pi r^3 = 1m^3$$
$$\gamma_{liq} = 10kN/m^3$$
$$F_B = 10 * 10^3 N/m^3 * 1m^3 = 10kN$$
$$Weight = mg = \rho_{ball} g V_{ball} = 100kg/m^3 * 9.8m/s^2 * 1m^3 = 980N$$
$$F_B > Weight$$

Therefore the ball will float. You can find out how much of the ball is in the water by performing a force balance on the ball. Try it yourself! If $F_B$ < weight the object would sink and if $F_B$ = weight the object will hover.

## Fluid Dynamics

Fluids do not always stay still. Static fluid dynamics does not explain what happens to flowing fluids. If a fluid is in motion it is useful to have an equation that relates the pressure to the velocity.

If we assume, within the liquid:

- Viscous effects are negligible (so the "no slip" condition would not hold here)
- The flow is steady (no turbulence, no shock waves, etc.)
- The flow is incompressible (flow traveling well below mach 1)
- We can follow a stream line.
- Then there is a relation that related pressure to velocity call the **Bernoulli Equation**.

The Bernoulli Equation tells what happens to an ideal fluid along a streamline. A **streamline** is a line that is tangential to the velocity vectors at any point throughout the flow. Streamlines can be described as the path that would be traced, in a flow, by particles with negligible mass.

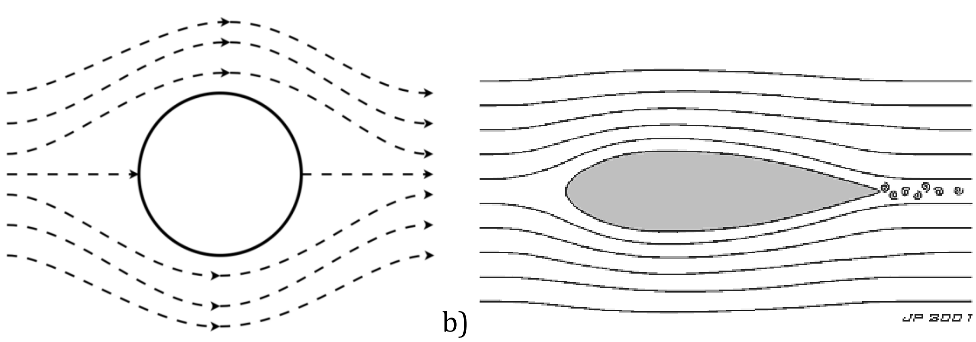


Figure 8: a) b)

In figure 8 streamlines are shown for a fluid moving left to right over a cylinder (a) and an airfoil (b). The swirls behind the airfoil in (b) are from a wake and will not be farther discussed in this class.

The Bernoulli equation was developed by Daniel Bernoulli in 1738. It relates pressure to height and velocity along a streamline. The conditions to which you can use the Bernoulli equation are highly stressed!!! It is important to understand when this equation is useful and when it is not.

**Example:**
Can I use the Bernoulli equation if:

| | |
|---|---|
| Honey flows slowly down a hill? | NO: Honey is too viscous to assume viscous effects are reliable. |
| An airplane travels down the runway? | Probably: Air has a low viscosity and there are plenty of stream lines to follow, but to use the equation the air must be steady and the airplane must be going much slower then mach 1. |
| Water flowing in a pipe with no turbulence? | Yes |
| When I use the no slip condition? | NO: If I use the no slip condition I am assuming a viscosity of the liquid such that is does not want to slip past the wall. It wants to hold onto it, this viscosity would not be negligable. |
| Steam coming from a smoke stack? | Maybe: The steam far from the stack is probably turbulent, but if assumptions can be made that close to the stack the flow is slower, it may be ok. |
| Flow through a hole in a dam? | Yes |

So what is the Bernoulli equation?

$$P + 1/2\rho v^2 + \gamma z = C$$

Here, z is the height ABOVE a reference (not to be confused with h in the fluid statics section of this class, which was height BELOW a reference). This states that the pressure plus a velocity term plus the pressure due to depth is constant. In this we can say that:

$$P_1 + 1/2\rho v_1^2 + \gamma z_1 = P_2 + 1/2\rho v_2^2 + \gamma z_2$$

This shows that is pressure is inversely related to the velocity squared:

$$P \sim \frac{1}{v^2}$$

So the faster the fluid is flowing, the less the pressure is.

In the Bernoulli equation the first term, P, is the actual pressure (**static pressure** defined as the pressure of the fluid if it were not flowing) of the fluid as it flows. In order to measure this pressure one may use a probe that sticks into the liquid. You must use caution, however, because many probes read stagnation pressure. **Stagnation pressure** is the pressure due to stopping the flow. It includes the static pressure and the velocity term! This happens if the probe reads the pressure by causing the flow to stop.

**Example:**

A pitot-static tube (probe) is stuck into a flow in a pipe the probe would stop the flow (remember the stream lines for flow over objects). Assuming you know the velocity of the flow in the pipe and the pressure at some point downstream, what is the pressure that the probe reads?

Can we use Bernoulli? Yes if we assume that the flow in the pipe is from a liquid that has negligible viscous effects, we can assume that the flow is incompressible, and the flow is steady. We know we can draw a streamline, so all conditions are met. Lets also assume we know what kind of fluid is flowing so we can look up any properties.

Draw a streamline from somewhere that all the information is known (point 1), to the probe (point 2).

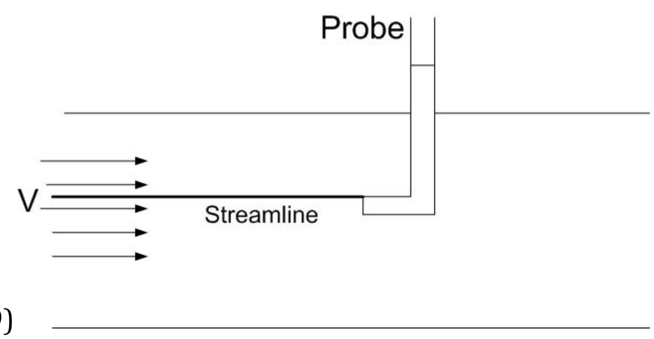



Figure 9)

What information do we know?

$P_1$ , $v_1$, $z_1=z_2$, $v_2=0$

Use Bernoulli equation:

$$P_1 + 1/2\rho v_1^2 = P_2$$

Conclusion: The probe reads $P_1$ plus a velocity term. If we wanted to find just the static pressure we would have to subtract the velocity term from the probe reading.

**Example:**

The same pipe is used with a manometer that does not stick down into the flow. Does this measurement device give the static pressure or the stagnation pressure?

Can we use Bernoulli? Yes if we assume that the flow in the pipe is from a liquid that has negligible viscous effects, we can assume that the flow is incompressible, and the flow is steady. We know we can draw a streamline, so all conditions are met. Lets also assume we know what kind of fluid is flowing so we can look up any properties.

Draw a streamline from somewhere that all the information is known (point 1), to the probe (point 2).

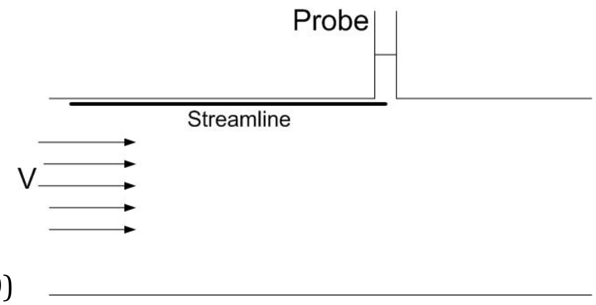



Figure 10)

What information do we know?

$$P_1,\ v_1=v_2,\ z_1=z_2$$

Use Bernoulli equation:

$$P_1 = P_2$$

Conclusion: This is the static pressure (the pressure we knew was in the tube to begin with).

Sometimes there is more than one unknown in the Bernoulli equation. It is convenient to have another equation to fall back on. This equation **is Conservation of Mass**. This is an incredibly important law! A middle school child can usually tell you that mass can't be created or destroyed, but this definition of conservation of mass principle is not the most helpful. Instead we will state that:

Mass flow rate in – Mass flow rate out = Mass accumulation Rate.

In this class we will only look at steady flows so the "rate" portion of the equation is not needed. Instead we can simply say:

Mass in – Mass out = Mass accumulated.

$$\sum \rho A_{in} v_{in} = \sum \rho A_{out} v_{out} + \text{Mass flow rate accumulated}$$

$$\sum \rho V_{in} = \sum \rho V_{out} + \text{Mass accumulation rate}$$

**Example:**

A four branch pipe is shown. The pipe is situated so figure 11 is a top view (i.e. the pipe is all on the same elevation). If all the pipe areas are the same and you know ρ, what is the velocity in exits 2,3, and4? Assume you know v1, P1, P3, and P4. Also, assume that you can use the Bernoulli equation.

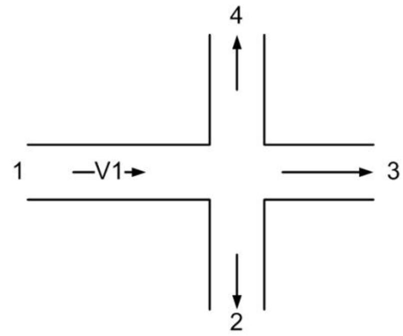



Figure 11)

$$\sum \rho A_{in} v_{in} = \sum \rho A_{out} v_{out}$$

In order to define areas we must define an area of interest (A control volume) where the surfaces of "in" and "out" can be defined. Our control volume will be throughout the pipes as shown:

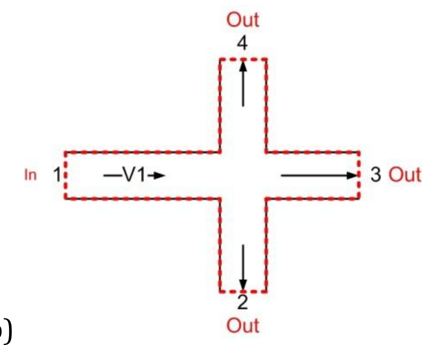



Figure 11b)

Mass $\text{rate}_{in} = \rho A_1 v1$

Mass $\text{rate}_{out}$=Mass $\text{rate}_2$+Mass $\text{rate}_3$+Mass $\text{rate}_4$= $\rho A_2 v2 + \rho A_3 v3 + \rho A_4 v4$

$A_1 = A_2 = A_3 = A_4 = A$

Conservation of mass states:

$\rho A v1 = \rho A(v2+v3+v4)$

$v1 = v2+v3+v4$

Drawing three streamlines:

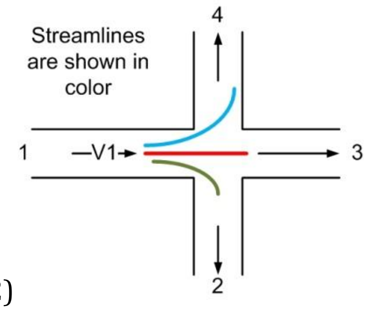



Figure 12)

Using the blue streamline:

$P1 + 1/2\rho v1^2 = P4 + 1/2\rho v4^2$

$v4 = \sqrt{(2/\rho(P1-P4) + v1^2)}$

and using the red line and a similar equation

$P1 + 1/2\rho v1^2 = P3 + 1/2\rho v3^2$

$v3 = \sqrt{(2/\rho(P1-P3) + v1^2)}$

Now that v1, v3, and v4 are known we got back to the conservation of mass equation and find:

$v2 = v1 - v3 - v4$

Putting it all together… here are a few more examples of the Bernoulli equation used with the conservation of mass.

**Example:**

A tank with D=1m has a small hole on its side near the bottom (as shown). The hole has diameter = .1m. There is an input pipe above the tank with liquid water flowing such that the water level does not change in the tank. What is the flow rate provided by the input pipe? Flow rate (Q) is equal to the velocity times the area the fluid flows through (Q=Av).

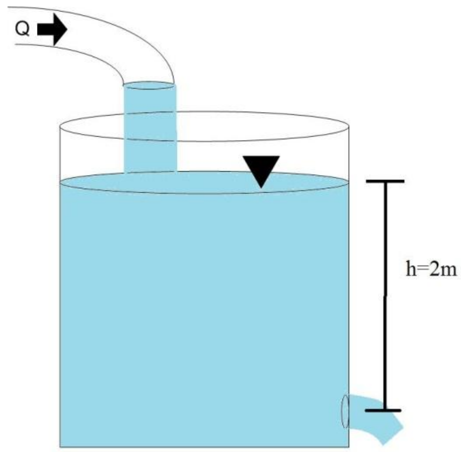



Figure 13

Define area of interest

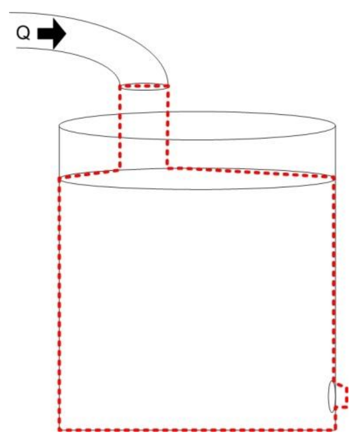



Figure 13 b)

Where in is at the pipe and out is at the hole. Things we know:

- P at top of water (far from the inlet pipe) = $P_o$
- P coming out jet (because it is a "free jet") = $P_o$
- v at the top of the water (far from the inlet) = 0m/s (or close to zero) because the height doesn't change.
- h = 2m (z top = 2m, z bottom = 0)
- D = 1m
- d= .1m
- $\gamma=\rho g$
- $\rho$ is constant

Conservation of mass:

$M\ rate_{in} = M\ rate_{out} = \rho A_{in} v_{in} = \rho A_{out} v_{out}$

$Q = Av$

$Q_{in} = A_{out} v_{out}$

$A_{out} = \pi(.1/2)^2 = .00785\ m^2$

$V_{out} = ?$

Can we use Bernoulli to solve for v?

viscous effects negligible = yes

flow is steady = yes

flow is icompressable = yes

can I find a streamline? = yes

Draw a streamline:

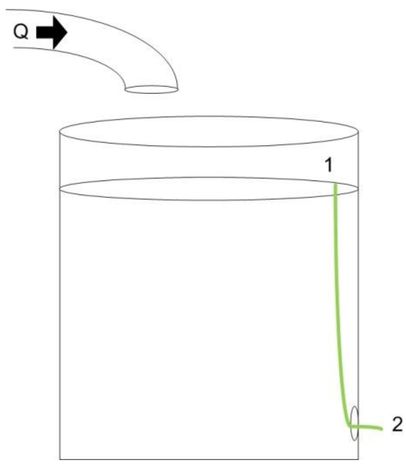



Figure 4)

Using Bernoulli :

$P_o + 1/2\rho v_1^2 + \rho g z_1 = P_o + 1/2\rho v_2^2 + \rho g z_2$

Where $v_{out} = v_2$

$\rho(9.8 m/s^2)(2m) = 1/2\rho v_2^2$

$v_2 = 6.26\ m/s$

Solve for Q

$Q_{in} = A_{out} v_{out} = .0492\ m^3/s$