Fluid Mechanics Qualifying Examination Syllabus

Students planning to take the Fluid Mechanics qualifying examination are expected to demonstrate competency in the following areas of Fluid Mechanics:

1. Differential form of the conservation equations governing fluid flow
   - Kinematics, rate of strain tensor and vorticity
   - Stress tensor and linear momentum equation
   - Constitutive equations and fluid properties (viscosity, surface tension, density, etc.)
   - Mass continuity equation
   - Navier-Stokes equations
2. Incompressible Newtonian flows
   - Classical solutions of the Navier-Stokes equations
   - Laminar and turbulent boundary layers
   - Laminar and turbulent free shear flows
   - Laminar flows in ducts
   - Elements of stability theory
   - Skin friction and drag on streamlined bodies
3. Ideal fluid flows
   - Stream function and velocity potential
   - Circle theorem and method of images
   - Conformal mapping
   - Superposition of sources, sinks, vortices, doublets, and uniform streams
   - Lift and drag on airfoils, Kutta-Jukowski theorum
   - Flows over cylinders, spheres, and bluff bodies
4. Compressible fluid flow
   - Isentropic one-dimensional flow in variable area ducts, nozzles, and diffusers
   - Rankine-Hugoniot jump conditions
   - Stationary normal shocks
   - Fanno flow, Rayleigh flow
   - Oblique shocks, Prandtl-Meyer expansions

It is recommended that before taking the Fluid Mechanics qualifying examination, students take the following courses offered by the MechSE department (or their equivalent at another institution):

ME 410	Intermediate Gas Dynamics
ME 411	Viscous Flow and Heat Transfer
TAM 435	Intermediate Fluid Mechanics

Suggested text books for the above material include:

Compressible Fluid Flow: Patrick H. Oosthuizen and William Carscallen, published by McGraw Hill

Viscous Fluid Flow: Frank M. White, published by McGraw Hill

Fundamental Mechanics of Fluids, 3rd Edition: Iain G. Currie, published by Taylor & Francis

QUALIFYING EXAMINATION
FOR
Fluid Mechanics

Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign

Wednesday, August __, ____
9:00 AM – 12:00 PM

IMPORTANT EXAMINATION INFORMATION

1. Identify your examination and work with your University Identification Number (UIN, I-Card number in blue beginning with 65) on each page. **DO NOT ENTER YOUR NAME ANYWHERE IN THE EXAMINATION.**

2. Choose 3 out of the 4 problems.

3. Each problem counts 10 points.

4. Start each problem in a new examination booklet and write on only the right-hand side (front side) of each sheet.

5. Hand in this problem package with your exam booklets.

Subject: FLUID: Question (1)

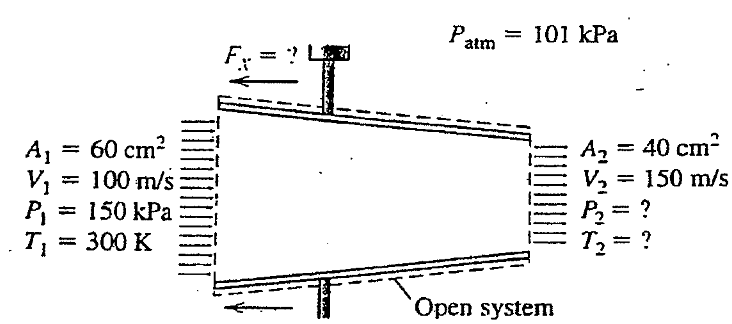
COMPRESSIBLE STEADY FLUID FLOW

Consider air flow in a converging channel with an inlet area of 60 cm2 and an exit area of 40 cm2. At the inlet the average velocity is 100 m/sec, the pressure is 150 kPa, and the temperature is 300 K. [See schematic below. Note we assume adiabatic flow.]
]
The velocity at the exit is 250 m/sec. [We will need to find the exit pressure and the temperature.]

The surrounding atmospheric pressure, is 101 kPa. Treat air as an ideal gas, so that the density = p/RT, and the enthalpy, h = cp˙T. [Recall for air, R =0.287 kJ/kg˙K and assume cp = 1.00 kJ/kg˙K.]

PROBLEM:
Using conservation of mass, energy and momentum, show all work and calculate:

(a) Mass flow rate through the channel (kg/sec);
(b) The exit pressure (kPa);
(c) The exit temperature (K);
(d) The net Force, F required to keep the channel in place (kN).



FLUIDS: Question (2) (P. 1/2)

Poiseuille Flow: Fully Developed Pipe Flow (Intro)

The velocity distribution for steady, laminar ($Re_D <\sim 2100$), fully developed flow throug a circular pipe with diameter D as shown in Figure 7.3 can be determined by using th conservation of mass and linear momentum in cylindrical coordinates $\mathbf{V} = \mathbf{V}(r, \theta, z$ ([redacted]), and the same boundary conditions and assumptions used in th plane Poiseuille flow. Additionally, we assume that the flow is axisymmetric (i.e., the flov is independent of the azimuthal direction θ). Therefore, all quantities that depend on θ are set to zero. This flow is known as the *Hagen-Poiseuille flow* or simply the *Poiseuill flow*.

The equations of continuity and conservation of linear momentum for the two dimensional laminar flow problem for a Newtonian fluid in the cylindrical coordinate shown in Figure 7.3 are

Governing Equations

Continuity

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0.$$

r-momentum

$$\rho\left(v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z}\right) = \rho g_r - \frac{\partial P}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{\partial^2 v_r}{\partial z^2}\right].$$

z-momentum

$$\rho\left(v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z}\right) = \rho g_r - \frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(v_z)\right) + \frac{\partial^2 v_z}{\partial z^2}\right].$$

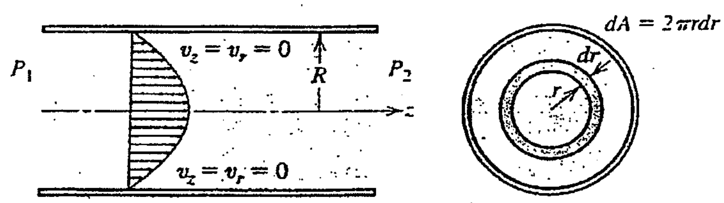


FIGURE 7.3 Fully developed flow in a pipe.

Solution

The fully developed assumption implies that $v_r = v_r(r)$, $v_z = v_z(r)$, and that $\frac{\partial v_r}{\partial z} = \frac{\partial v_z}{\partial z} = 0$. The continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{\partial v_z}{\partial z} = 0,$$

then reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) = 0 \Rightarrow v_r = \text{constant},$$

(cont'd

where, since v_r cannot be a function of z, it must be a constant number. Moreover, since $v_r = 0$ at the pipe wall, the continuity equation is satisfied only when v_r is zero everywhere in the flow.

Again, using the assumption of steady, fully developed flow with negligible body forces, the momentum equations reduce to

$$0 = -\frac{\partial P}{\partial r}.$$

$$0 = -\frac{\partial P}{\partial z} + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(v_z)\right).$$

Upon integration, the momentum equation in the r-direction gives

$$\frac{\partial P}{\partial r} = 0 \Rightarrow P = P(z),$$

indicating that the pressure is only a function of z. Consequently,

$$\frac{\partial P}{\partial z} = \frac{dP}{dz}.$$

As discussed in Section 7.1. the pressure gradient is always a negative number.

Returning to the momentum in the z-direction

$$\underbrace{\mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(v_z)\right)}_{\textit{function of } r} = \underbrace{\frac{dP}{dz}}_{\textit{function of } z}.$$

Since the left-hand side is only a function of r and the right-hand side is only a function of z, the two are equal when they are both constants.

By rearranging.

$$\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(v_z)\right) = r\frac{1}{\mu}\frac{dP}{dz},$$

and by integrating

$$\frac{\partial u}{\partial r} = \frac{1}{\mu}\frac{dP}{dz}\frac{r}{2} + \frac{C_1}{r}. \qquad \text{(A)}$$

WHAT's REQUIRED:

(a) Integrate equation (A) and apply to appropriate Boundary Conditions (showing all work) to solve for the velocity profile as a function of radius and 'other' constant parameters.

(b) Calculate the volumetric flow rate (m3/sec). $\dot{Q} = \int_0^R u(r)\,dA$

(c) Calculate the pressure gradient, dp/dz, if $\dot{Q}$ = 1 m3/sec; R = 1 cm2; and μ = 1.8 E-5 (Nt-sec)/m2.

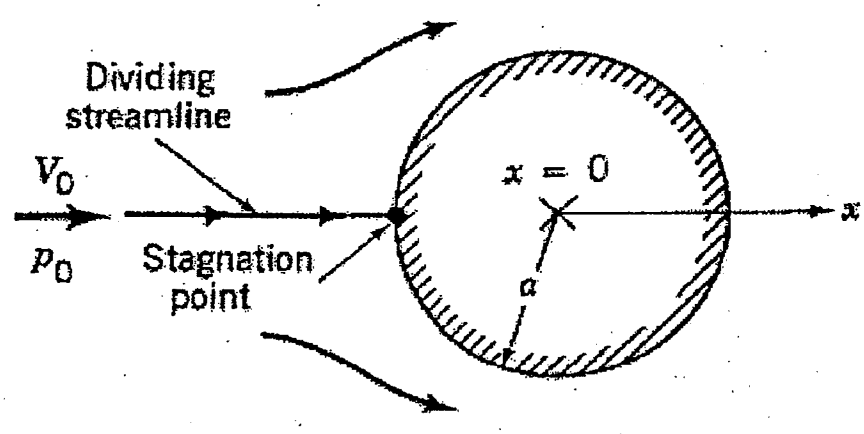
3. Consider steady, incompressible and inviscid flow past a circular cylinder, as shown in the figure below. The fluid velocity along the dividing streamline ($-\infty \leq x \leq -a$) is given by $V=V_0\,(1-a^2/x^2)$, where a is the radius of the cylinder and V_0 is the upstream velocity.

(a) Determine the pressure gradient along the dividing streamline. Note that $\partial P/\partial x = -\rho V \partial V/\partial x$ along that streamline.

(b) If the upstream pressure is P_0, integrate the pressure gradient to obtain the pressure distribution P(x) for $-\infty \leq x \leq -a$.

(c) Show from the result of part (b) that the pressure at the stagnation point is $P(-a) = P_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.

(d) Which flow assumptions (steady, incompressible or inviscid ?) are necessary for the validity of the <u>general</u> Bernoulli equation ?



4. The pump shown in the schematic below adds 25 kW to the water and causes a flow rate of 0.04 m^3/s. Determine the flow rate expected if the pump is removed from the system. The pipe is 30 m in length and of 60 mm internal diameter. The water exits at a 40 mm diameter nozzle. Assume a friction coefficient f =0.016 for either case (with and without the pump) and neglect minor losses.