SCIE1110 Exercises

Exercise 1.
Draw the addition and multiplication tables for base 5.

Exercise 2.
Convert the number 2503 to bases $2, 3, 5, 9, 12, 25, 27$.

Exercise 3.
Convert numbers between different bases (no subscript means base 10).

1. $2038040 =?_{[60]}$.
2. $2038040 =?_{[100]}$.
3. $2038040_{[9]} =?$.
4. $2038040_{[9]} =?_{[3]}$.
5. $2038040_{[9]} =?_{[5]}$.
6. $2038040_{[9]} =?_{[25]}$.
7. $2, 3, 80, 40_{[144]} =?$.
8. $2, 3, 80, 40_{[144]} =?_{[12]}$.
9. $2038040_{[16]} =?_{[2]}$.
10. $2038040_{[16]} =?_{[8]}$.

Exercise 4.
Do the arithmetic calculations in base 2 (subscript [2] omitted). For the division, find the quotient and remainder.

1. $1010011 + 10101$.
2. $1010011 - 10101$.
3. 1010011×10101.
4. $1010011 \div 10101$.
5. $10101 + 101 - 1001$.
6. $10101 \times 101 \div 1001$.

Exercise 5. Do the arithmetic calculations in base 16.

1. $5AB + E07 - C5$.
2. $5AB \times E07 \div C5$.

Exercise 6.
Draw the addition and multiplication tables for $\mathbb{Z}_5$. Compared with the addition and multiplication tables for base 5, what do you find?

Exercise 7.

Let a and b be integers. Explain that $10a + b$ is divisible by 7 if and only if $a - 2b$ is divisible by 7.

Exercise 8.

Find the rule for divisibility by 21.

Exercise 9.

For natural numbers expressed in base 5, find the rule for divisibility by 3.

Exercise 10.

We express numbers in base 11, and denote 10 by T.

1. Calculate $235 + T3T - 1T9$ and $3T7 \times 203$.
2. Show that a number $\cdots N_3N_2N_1N_0$ in base 11 is divisible by T if and only if $N_1 + N_1 + N_2 + N_3 + \cdots$ is divisible by T.
3. Determine the divisibility of $2T2T\cdots 2T$ ($2T$ repeated n times) by T.

Exercise 11.

How many multiplications are there in a Chinese multiplication table? Given the sexagesimal system, how big should the Babylonian multiplication table be?

Exercise 12.

In the sexagesimal system used by the Babylonians, there is no equivalent to the decimal point. This is analogous to expressing $\sqrt{2}$ as $1414\cdots$, omitting the dot.

The following numbers are expressed in our usual decimal notation. Which ones will the Babylonians express in the same way.

$$2.25, \quad 13.5, \quad 135, \quad 225.$$

Exercise 13.

In Babylonian notation, the population of Hong Kong is $36, 6$. Given that the population is about 7.8 million, what is the exact value of $36, 6$ in decimal expression?

Exercise 14.

Express 1 as a sum of three distinct fractions: $1 = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$. Then use this to explain that the fractional expression used by Egyptians is not unique[1].

Exercise 15.

Find the age of Diophantus from the following epigram by Metrodorus:

This tomb holds Diophantus. Ah, how great a marvel! The tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.

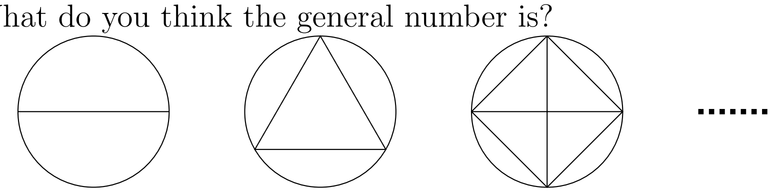
Exercise 16.

What is wrong with the following argument?

1. No dog has 5 legs.
2. A dog has 4 more legs than no dog.
3. A dog has 9 legs.

Exercise 17.

Take n evenly spaced out points on the circle and connect all the possible straight lines between them. The number of regions you get are $2, 4, 8, \ldots$. What do you think the general number is?



Exercise 18.

[1]This leads to interesting question of "best Egyptian fractional expression". Please check out http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html.

In a party, two people are (mutual) friends if they met before, and they are (mutual) strangers if they meet for the first time. Show that in a party of at least 6 people, there are at least 3 people who are either mutual friends to each other or mutual strangers to each other.

[There are obvious extensions you can explore.]

Exercise 19.

Show that the odd harmonic series $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$ diverges.

Exercise 20.

What is the condition that an $m \times n$ grid can be tiled by 2×2 squares (dominos)?

Exercise 21.

Prove that an $m \times n$ grid can be tiled by 2×3 dominos if and only if mn is divisible by 6. How many tilings can you have?

Exercise 22.

Suppose $F(n) = 2F(n-1) + F(n-2) - 2F(n-3)$, $F(0) = 0$, $F(1) = 1$, $F(2) = 2$. Find the general formula for $F(n)$.