Suggestions on how *not* to mishandle mathematical formulæ

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Abstract

Quite frequently mathematical formulæ in reports, theses or even articles are not written correctly. Indeed, the basic rules for composing the formulæ are almost never explicitly stated. This article provides some suggestions to fill this gap.

1 Introduction

Formulæ are an excellent tool to express (some) mathematical concepts. This statement may seem obvious, but scientists realized it only after a long historical process. Natural language might at first seem easier to use than formulæ. However, scientists know very well that formulæ are an essential tool to communicate clearly, unambiguously and concisely. Formulæ are then part of a language and therefore should follow some rules. Unfortunately formulæ are often written "badly", that is not clearly, ambiguously, and not concisely.

The purpose of this article is to give some indications on how to write, or better on how *not* to write formulæ. In fact, children need to go to school, make mistakes, and be corrected to learn how to write in natural languages. Typically, uneducated people can communicate orally, but make many mistakes in writing. The same applies to mathematical formulæ. Only occasionally do people think about which rules to follow and which errors to avoid. The examples reported in the article are taken from publications and theses. Most suggestions and remarks have a general validity. However, LaTeX users will also find indications on how to implement them.

This article is mostly aimed to authors of technical/scientific documents (engineers, experimental physicists, chemists, and in general anyone who uses mathematics as a tool) and not to mathematicians. In pure mathematics there is much more attention to the proper typographic rendering of mathematical concept, and each field has its own rules (which do not necessarily comply with ISO standards (Gregorio, 2007)).

2 General rule

The general rule is very easy: formulæ must be unambiguous and concise, that is clear and simple. This is (or should be) true for many forms of communication, not only the written ones. When doing science, however, the writer should be even more careful about how to explain concepts in order to make them easy to understand. Further details on how to write mathematical formulæ can be found in the ISO 31 standard (ISO 31/11, 1982; ISO 31/12, 1982), which is discussed in depth in Wikipedia (2008a,b,c,d) and in Beccari (1997).

3 Operations

3.1 Multiplication

One of the most common errors is using the "dot" to indicate multiplication between scalars. For example the following formulæ

$$a \cdot x^2 + b \cdot x + c = 0, \qquad \sigma \cdot \varepsilon = 2 \cdot \alpha$$

should be written as

$$ax^2 + bx + c = 0, \qquad \sigma\varepsilon = 2\alpha.$$

In fact, for the sake of simplicity, the standard multiplication between letters, or between a number and a letter, does not require any symbol. If, on the other hand, the multiplication is between two numbers, the $\times$ or $\cdot$ symbols are required to avoid ambiguity. For example you should write

$$2 \times 3 = 6 \quad \text{and not} \quad 2\,3 = 6.$$

The same symbol should also be used when a line break occurs at a product, as shown in Eq. (1). The asterisk is absolutely not acceptable:

$$a * x^2 + b * x + c = 0, \qquad \sigma * \varepsilon = 2 * \alpha.$$

The dot has to be used to indicate a scalar product between vectors. For this purpose either a thin or a thick dot can be used. The first one is provided by `\cdot`, the second one by `\bcdot`, which is defined as

```
\newcommand{\bcdot}{\boldsymbol{\cdot}}
```

The two commands produce respectively

$$\mathbf{a} \cdot \mathbf{c} = 0 \qquad \text{and} \qquad \mathbf{a} \boldsymbol{\cdot} \mathbf{c} = 0.$$

3.2 Mathematical operators

Mathematical operators, including the differential, must be indicated with upright characters, as in the following examples:

$$\int \sin x \, \mathrm{d}x, \qquad \frac{\mathrm{d}y}{\mathrm{d}x}, \qquad \lim_{x \to 0} \cos x = 1.$$

They are indeed abbreviations of words and the upright font reminds us of this fact and avoids any ambiguity with products. For example, $\tan x$ is undoubtedly the tangent of x, while $tan x$ might be the product of four scalars.

LaTeX and the amsmath package define the most common mathematical operators, including `\lim`, `\sin`, `\min`, etc. In order to define new ones, the amsmath package provides the `\DeclareMathOperator` command, to be used only in the preamble, and the

`\operatorname` that allows one to define an operator directly in the text. For example

$$\operatorname{argmax} f(x)$$

can be obtained in two ways:

- declaring in the preamble

```
\DeclareMathOperator{\argmax}{argmax}
```

  and then using in the text

```
\argmax f(x)
```

- directly using in the text

```
\operatorname{argmax} f(x)
```

Defining an operator is different than using `\mathrm`[1] because the first one also takes care of the space between the operator and its argument. Note for example the difference between

$$\sin x \qquad \text{and} \qquad \sin\! x$$

obtained respectively with

```
\sin x
```

and

```
\mathrm{sin}x
```

Moreover, operators correctly handle superscripts and subscripts as for limits. For example

$$\lim_{x\to\infty} \frac{1}{x}$$

can be easily obtained with

```
\lim_{x\to\infty}\frac{1}{x}
```

A peculiar definition is required to properly write the differential symbol. It is in fact an operator that has a space only on its left. In Beccari (2007b) the following solution is proposed:

```
\newcommand{\ud}{\mathop{}\!\mathrm{d}}
```

It uses an empty operator and eliminates the space on its left with `\!`.[2] Note the difference between

$$\int \sin x \mathrm{d}x \qquad \text{and} \qquad \int \sin x \, \mathrm{d}x,$$

where the differential is obtained respectively with `\mathrm{d}` and `\ud`.

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[1] `\mathrm` can be used to obtain an upright font in mathematical mode. The other methods to use an upright font in mathematical mode are described in par. 5.1.1.

[2] `\!` inserts a negative space as wide as the one that TeX inserts between an operator and a variable (Beccari, 2007a).

3.3   Multi-line equations

Long equations sometimes need to be split over several lines. The general rule is to indicate the operation or relation symbols *only once* at the end of the line for text style formulæ and at the beginning of the line for display style formulæ[3]

$$f = \frac{a}{b}\left[\sin\left(\frac{c}{d}\right) + \tan\left(\frac{c}{d}\right)\right]$$
$$\times \left[1 + \sin\left(\frac{e}{f}\right) - \cos\left(\frac{g}{h}\right)\right]. \quad (1)$$

Operation and relation symbols should always be indicated only once because it is simpler and, more importantly, to avoid ambiguity. If, for example, you write

$$a = b + c + d-$$
$$- e - f,$$

it is not clear which of the following two you mean:

$$a = b + c + \boxed{d - e} - f$$
$$a = b + c + \boxed{d + e} - f.$$

This rule also applies to equalities:

$$a = b + c + d$$
$$= -e - f$$
$$\simeq g.$$

3.3.1   LaTeX commands

While LaTeX automatically splits long text style formulæ, the author has to do this manually with display style equations. The amsmath package provides several environments to split equations, such as split, multline, gather, align, aligned, alignat; refer to the amsmath package documentation (American Mathematical Society, 2002) for the details.

3.3.2   How to handle parentheses

When using the `\left` and `\right` commands to let LaTeX automatically choose the bracket size, problems may arise if the formula is split between `\left` and `\right`. For example the code

```
\begin{multline*}
    f(x,y,z)= (1+x+y-z)\left[\pi\\
        +\sin\left(\frac{a}{b}\right)
        +\cos\left( \frac{c}{d}\right)\right].
\end{multline*}
```

---

[3] Text style formula means a formula that appears inside a line of text (Knuth, 1992), such as $c = a + b$, while display style formula means a formula that occupies a line by itself (Knuth, 1992), such as

$$c = a + b.$$

The first ones can be obtained in LaTeX with the `$...$` command, while the second ones can alternatively be obtained with the `\[...\]` command or the equation environment.

cannot be compiled because LaTeX expects all the `\left` to be closed by the respective `\right` inside each line.

A solution is to close `\left` and `\right` respectively with `\right.` and `\left.`.

```
\begin{multline*}
   f(x,y,z)= (1+x+y-z)\left[\pi\right.\\
      \left.+\sin\left(\frac{a}{b}\right)
      +\cos\left( \frac{c}{d}\right)\right].
\end{multline*}
```

but now the bracket dimension for each couple `\left` and `\right` are computed independently and this can lead to awful results such as

$$f(x, y, z) = (1 + x + y - z) \left[\pi \right.$$
$$\left. + \sin\left(\frac{a}{b}\right) + \cos\left(\frac{c}{d}\right) \right].$$

A possible solution is to find the tallest term, in this case

$$\left(\frac{a}{b}\right),$$

and to insert it inside a `\vphantom` command in the other line

```
\begin{multline*}
   f(x,y,z)= (1+x+y-z)\left[\pi\vphantom{
      \left(\frac{a}{b}\right)}\right.\\
      \left.+\sin\left(\frac{a}{b}\right)
      +\cos\left( \frac{c}{d}\right)\right].
\end{multline*}
```

obtaining

$$f(x, y, z) = (1 + x + y - z) \left[\pi \right.$$
$$\left. + \sin\left(\frac{a}{b}\right) + \cos\left(\frac{c}{d}\right) \right].$$

Another solution is to use fixed height brackets[4] instead of looking for the tallest element. For example the code

```
\begin{multline*}
   f(x,y,z)= (1+x+y-z)\Bigl[\pi\\
      +\sin\Bigl(\frac{a}{b}\Bigr)
      +\cos\Bigl( \frac{c}{d}\Bigr)\Bigr].
\end{multline*}
```

gives

$$f(x, y, z) = (1 + x + y - z) \Big[\pi$$
$$+ \sin\Big(\frac{a}{b}\Big) + \cos\Big(\frac{c}{d}\Big) \Big].$$

Note that this solution is applicable only to cases in which brackets are not taller than `\Bigg`.

In addition to the previous solutions, Michael Downes and Morten Høgholm developed the breqn

package to automatically handle this problem. When the package is loaded, the `\left` and `\right` commands work properly even when the equation is split.

3.4 Punctuation

There are two approaches on how to handle external punctuation, that is, the punctuation in display style formulæ. Some authors think that it should never be used (Beccari, 2007a), others that it is necessary and essential.[5] Whichever of these two conventions you choose, the fundamental thing is to be consistent and always use the same method.

The authors of this article believe that formulæ, both in display and text style, are part of the argumentation and so punctuation should be used to help the reader. An example of good use of punctuation is:

Since
$$a = b$$
and
$$b = c,$$
it is proven that
$$a = c.$$

4 Variables

4.1 Vectors and matrices

4.1.1 Synthetic notation

Vectors are usually indicated with bold face lower case letters[6]

$$3\mathbf{a} + \alpha\mathbf{c} = \mathbf{v}.$$

Other notations such as

$$\vec{a}, \quad \underline{a}, \quad \{a\}, \quad [a]$$

should be avoided.

The same applies to matrices which can be indicated with upper case bold face letters

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

avoiding expressions such as

$$[A]\{x\} = \{b\}.$$

In a linear algebra text, which deals entirely with matrices and vectors, it is perfectly legitimate to get rid of the bold face and simply write

$$Ax = b.$$

---

[4] Fixed height brackets can be obtained with the commands `\big`x, `\Big`x, `\bigg`x, and `\Bigg`x (where x is 'l' for left brackets, 'r' for right brackets, 'm' or nothing for a regular symbol).

[5] Most mathematics books, new and old ones (including books typeset with metal movable type), use, although not always consistently, semicolon, comma, and full stop in display style equations (Bellacchi, 1894; Bertini, 1907; Bianchi, 1899; Bonola, 1906; Burali-Forti, 1904; Caprilli, 1912; Cesaro, 1894; Dini, 1878; Fubini, 1908; Gazzaniga, 1903; Peano, 1887; Sacchi, 1854; Veronese, 1891; Vivanti, 1916).

[6] Upright bold face characters in mathematical mode can be obtained with the `\mathbf` command.

4.1.2　Component notation

In some cases it is necessary to show explicitly vector and matrix components. Books and theses often report expressions like

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}.$$

This notation should be avoided because it uses different brackets for matrices and vectors and also because it uses curly brackets for vectors. Much better is the following:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \tag{2}$$

Although there is no rule or customary practice on the type of brackets to use for vectors and matrices in component notation, it is advisable to:

- use either round or square brackets and avoid all the other types (curly, angle, etc.);
- maintain the same notation for both matrices and vectors[7] throughout the document.

Matrices and vectors should be written in LaTeX with the following environments provided by the amsmath package: pmatrix (for round brackets) and bmatrix (for square brackets). For example Eq. (2) can be obtained with

```
\begin{bmatrix}
    A_{11}&A_{12}&A_{13}\\
    A_{21}&A_{22}&A_{23}\\
    A_{31}&A_{32}&A_{33}\\
\end{bmatrix}
\begin{bmatrix}
    a_1\\ a_2\\ a_3
%%\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
    b_1\\ b_2\\ b_3
\end{bmatrix}
```

When a column vector has to appear in the text in component notation, it is advisable to represent it as a transposed row vector in order not to add too much space between lines. There are at least two notations for row vectors in the text. The first one consists in using the notation for display style vectors. For example you could write $\mathbf{v} = [v_1 \; v_2 \; v_3]^T$ with

```
\mathbf{v} = [v_1\;v_2\;v_3]^T
```

The second one, adopted for example by Strang (2005), is more compact and consists in separating

---

[7] In fact, in component notation, a vector is just a one column (or one row, depending on the convention adopted) matrix.

the component with commas without indicating the transposition, which is implicit. For example you could write $\mathbf{v} = (v_1, v_2, v_3)$ with

```
\mathbf{v} = (v_1,v_2,v_3)
```

In either case, the pmatrix and bmatrix environments should not be used since they would produce too big brackets.

The choice between round and square brackets depends only on personal taste.

4.2　One symbol, one letter

A mathematical symbol is usually indicated by *one* letter, not two or three. If, for example, we want to suggest that the *factor of safety* is equal to three, we should not write

$$FoS = 3$$

because FoS looks like the product between three scalars. It is much better to use a subscript

$$F_s = 3.$$

CG is quite often used to indicate the center of gravity. Wouldn't G be better?

ISO standards admit some exceptions, such as Mach number Ma and Reynolds number Re, which are represented by two letters. In these rare cases in which the italic font is used for a symbol of several letters, the \mathit command should be used to avoid spacing problems between the letters. Note for example the difference between Ma and Ma, obtained respectively with Ma and Ma. In the first case the spacing between M and a might induce one to consider it as a product while in the second case the spacing is correct.

Formulæ should never include whole words such as

$$mass \times acceleration = force.$$

First of all, this should be a vector relation and the formula does not suggest it. Moreover the words should be written in an upright font

$$\text{mass} \times \text{acceleration} = \text{force},$$

in order to have the right spacing between letters. However it is still not as clear and concise as

$$m\mathbf{a} = \mathbf{F}.$$

Another common error consists in using in a formula the same symbols employed in a computer program. For example

$$A\_fl + B\_fl = d\_k\_fl$$

instead of

$$A + B = d.$$

4.3 Superscripts and subscripts

Subscripts should be used only if necessary and, even in these cases, they should not be abbreviations. For example you should not write $\sigma_{\mathrm{nominal}}$ or σ_{nom}, where "nominal" has the regular English meaning; in this case you could simply write σ_{n}. Obviously it is allowed to use subscripts (or superscripts) for indices, as in a third order tensor σ_{ijk}. In this case mathematical italic characters should be used with the regular command `\sigma_{ijk}`.

Subscripts and superscripts follow the same rules that apply for symbols (par. 5.1); they must be in an italic font if representing physical quantities or mathematical variables, in an upright font otherwise. For example you should write A_T if T represents the temperature but A_{T} if T represents a name such as "trajectory". You should write A_i if i represents a summation index.

Using words as subscripts can disfigure even simple formulæ such as

$$T_{wheel,braking} =$$
$$\frac{(T_{engine,supplied} - T_{engine,absorbed}) * C}{N}$$

which should have been written as

$$T_{\mathrm{b}} = \frac{C(T_{\mathrm{s}} - T_{\mathrm{a}})}{N}. \tag{3}$$

Since the "b", "s", and "a" subscripts are abbreviation of respectively "braking", "supplied", and "absorbed", they must be in an upright font according to the previous rule.

4.4 Appropriate notation

Appropriate notation is fundamental to make a scientific text easy to understand. To realize how important the notation is, you can just try to compute a product with Roman numerals

$$\text{MMCDXXVIII} \times \text{XIX} \quad \text{instead of} \quad 2428 \times 19.$$

A common error consists in using different *types* of letters for quantities of the same kind; for example indicating some lengths with Latin letters (l, s, a) and others with Greek letters (α, γ) does not help the reader. Subscripts should be chosen carefully as well. Some books contain figures in which ω_1 is the angular velocity of body 2 and ω_2 that of body 1.

5 Physical and mathematical quantities

5.1 Symbols

ISO standards prescribe the use of italic symbols for all the physical and mathematical quantities that can assume different values; for example the names
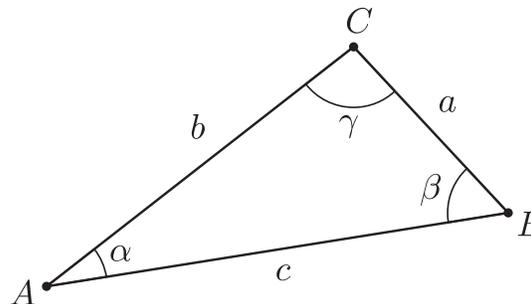
Figure 1: Example of mathematical variables indicated with italic letters.

of geometrical entities[8] (Fig. 1) and all the physics constants whose values are not "constant" because more accurate measures might change it.[9]

All the other mathematical and physical quantities must be written in an upright font. Among these the imaginary unit $2 + 4\mathrm{i}$, the base of natural logarithms $\mathrm{e} = 2.718\ldots$, and pi $\pi = 3.141{,}592\ldots$ (Beccari, 2007a).

As already mentioned in the introduction, "pure" mathematics does not always follow these rules (Gregorio, 2007).

5.1.1 Character types in LaTeX

In mathematical mode, LaTeX typesets letters in italic by default and so they represent non-constant mathematical and physical quantities. For example `$a=b$` gives $a = b$.

In order to write constants, that is upright letters in mathematical mode, the `\mathrm` command can be used. For example `$a=\mathrm{b}$` gives $a = \mathrm{b}$. `\mathrm`, although using upright characters, is still in mathematical mode so it ignores spaces. If spaces are needed, it is convenient to use the `\text` command from amsmath. Note for example the difference between

$$v_{\mathrm{reduced\ section}} \quad \text{and} \quad v_{\mathrm{reducedsection}}$$

obtained respectively with

```
v_\text{reduced section}
```

and

```
v_\mathrm{reduced section}
```

---

[8] Points and angles, being mathematical variables, must be written in italic. Most authors indicate points with Latin upper case letters (A, B, C, etc.), segments with Latin lower case letters (a, b, c, etc.), and angles with Greek lower case letters (α, β, γ, etc.).

[9] For example the charge of the electron e, Planck's constant h, the reduced Planck's constant $\hbar$, and Boltzmann's constant k.

`\mathrm`, as opposed to `\text`, does not let superscripts and subscripts become italic in an italic context. For example

```
\textit{This produces $L_\text{eff}\ne L_\mathrm{
    eff}$}
```

gives

> *This produces* $L_{eff} \neq L_{\mathrm{eff}}$

(Both `\mathrm` and `\text` with their arguments can be used directly as subscripts and superscripts as shown here; ordinarily, such multi-token sequences would need to be enclosed in braces.)

Upright Greek letters are provided by the up-greek package. Note the difference betwen π and π obtained respectively with `\uppi` and `\pi`.

The previous examples are only meant to show how to use the various commands but, as explained in the previous paragraphs, superscripts and subscripts with full words should be avoided.

5.2 Numbers

Numbers must always be in an upright font (123), not in italic (*123*).[10] This is LaTeX's default behavior in mathematical mode, but problems might arise in the text. For this reason it is always advisable to write numbers in mathematical mode even in the text. If, for example, a number appears in an italic sentence, the mathematical mode enforces the upright font. For example

```
walk \emph{at most 2 km} north
```

gives

> walk *at most 2 km* north

instead of the correct form

> walk *at most* 2 *km* north

that can be obtained with

```
walk \emph{at most $2$ km} north
```

Lesina (1986) provides some rules for numbers in the text. They can be summarized in (Gregorio, 2005):

- *Numbers less than twenty*[11] must be in words. For example: the pinion has eleven teeth.
- If a number represents a *precise measure*, it must be in figures. For example: the long side is 1.5 m.

- If a number represents an *approximate measure*, it must be in words, unless the number is too long (when written). In this case it is easy to round it because no one is interested in knowing that a building is one kilometer or 1123 m distant (except in some specific cases). When reporting approximate numbers, the units must be explicitly written as well. You can write then "in about twenty meters" but not "in about 20 m".
- A number must be written in figures if it is coupled with another one that is in figures. For example you can write: there are 10 subjects of the first type and 154 of the second one.
- Never begin a sentence with a number written in figures.

Roman numerals, although quite rare in scientific texts, must always be written in an upright font, like the Arabic numbers. You should then write XVI instead of *XVI*.

Long numbers should have a comma every three digits:[12]

$$125,362$$
$$0.398,276$$
$$12.345,4.$$

In scientific documents the comma is usually substituted by a thin space (`\,`) (Wright, 2008a). Luckily this can be done automatically with the command `\np` of the numprint package. Here are some of the features of this package:

- Addition of a *separator every three digits*. The separator (`,` or `.` or `\,` or `~`) depends on the language in use (defined by the babel package). The English separator is a comma `,` and so `\np{15000000}` gives 15,000,000.
- Substitution of the *decimal symbol*. Independently from the decimal symbol used in the LaTeX code, numprint represents numbers with the one that is appropriate for the language in use: `.` in English and `,` for the other European languages. For example, when writing in English, `\np{3,15}` becomes 3.15.
- *Approximation of decimal numbers* to the desired number of significant figures. If using three significant figures, `\np{2,742647826672}` becomes 2.743.
- Conversion of the `E`, `e`, `D`, and `d` characters into the *exponential format*.[13] For example `\np{1,234E-4}` becomes 1.234×10^{-4}.

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[10] Dates are an exception to this rule and might be in italic if their context is.

[11] Other authors suggest numbers of at most one digit.

[12] Often four-digit numbers are not separated. Thus you would write 1234 but 12,345 (Wright, 2008a).

[13] This function is particularly useful when reporting data produced by software such as FORTRAN, MATLAB, etc.
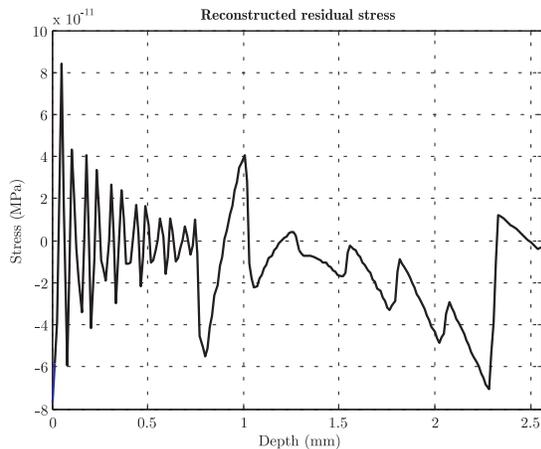
Figure 2: Units enclosed in round brackets.

χ	C_1 (N/rad)	C_2 (N/rad)	a_1 (m)
0	73,000.0	90,000.0	0.912
0.05	70,899.3	89,143.7	0.912
0.10	68,565.1	88,850.5	0.912

Table 1: Units enclosed in round brackets.

- *Automatic addition of zeros*, if necessary. For example `\np{,12}` becomes 0.12.

5.3 Significant figures and scientific notation

Experimental sciences often express evaluations and results with numerical quantities. Some numerical quantities, such as π, e, $\sqrt{2}$, $2/3$, are exact. Quantities deriving directly or indirectly from measures or estimates are, on the contrary, approximate. The representation, both in floating-point and in fixed-point, of approximate quantities is very important because it provides information about the precision with which the quantities are known. For example

$$h = 1.23 \, \text{m} \qquad \text{and} \qquad h = 1.230,00 \, \text{m}$$

might seem similar but have quite different meanings. In the first case h is a length known with a margin of ± 5 mm and then it derives from quite inaccurate measures, in the second case the margin is ± 5 μm and then it derives from very accurate measures.

Therefore, the number of significant digits of a numerical quantity is very important, sometimes even more important than the digits themselves.[14] Unfortunately, numerical quantities are very often represented with too many significant digits, especially when reporting results of calculations.

Scientific notation, which is a way of expressing approximate numbers with integer powers of ten, provides a concise notation and allows to immediately identify the number of significant digits.

5.4 Units

Units (ISO 1000, 1982; Bureau International des Poids et Mesures, 2006) are often made of multiple

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[14] For example, in some contexts, there might be more difference between 2.1 and 2.150,2 than between 2.1 and 2.9.

letters (kg, Pa, mm, MN, rad, etc.) and must follow some rules:

- they must be written with an upright font;
- they cannot be separated by a line break from the numeric quantity to which they refer;
- they cannot be enclosed in square brackets.

Square brackets cannot be used with units because in metrology they mean "unit of" (Beccari, 2007a). For example

$$a = 25 \, \text{m/s}^2 \qquad \text{and} \qquad [a] = \text{m/s}^2$$

can be written, but not

$$a = 25 \, [\text{m/s}^2].$$

On the other hand, round brackets can be used in tables and graphs when units appear next to a symbol of the corresponding physical quantity instead of the numeric value to which they refer (Tab. 1 and Fig. 2).

Some conceptual errors can be added to the previous typographic errors. One of the most common is to indicate the second (unit of time) with "sec" (that means secant in trigonometry) instead of the symbol "s". A good guide on units is the manual of the siunitx package (Wright, 2008b). With this package the user does not have to format units by hand. For example

```
\unit{32.1}{\micro\meter}
```

gives

```
32.1 μm
```

The package writes the units with upright characters and adds an indivisible space between the number and the unit.

Another common error is using the upper case "K" to indicate kilo. In the context of units, "K" can only mean "Kelvin" while kilo is written with the lower case "k". Thus you should write kg, kB, etc. to indicate kilogram, kilobyte, etc. In general the name of the units of the International System have an upper case initial when named after scientists (e.g. N, W, Pa, J), lowercase initials in all other cases (e.g. m, lm, cd). The liter, which can be written both upper and lower case (l, L) in Europe and must be upper case in the USA (L), is the only exception to

this rule (Ceraolo, 2007; Bureau International des Poids et Mesures, 2006). (Also, the official spelling is 'litre', although 'liter' is typical in the USA, as seen in the present article.) When units are spelled out, they must always start with a lowercase letter, e.g. newton, siemens (Ceraolo, 2007; NIST, 2001).

6   Ambiguities

The digit "zero" should not be indicated by $\emptyset$ in handwriting. The context will clearly show that it is not a lower case "o". However, the symbol $\emptyset$ should be used when writing program code by hand because, in this particular case, it would be difficult to distinguish "O" from "0".

With electronic typesetting it is advisable to indicate liters with the capital letter "L" instead of the lower case "l". Although the standards allows both forms (ISO 1000, 1982; Giacomo, 1980; Bureau International des Poids et Mesures, 1979),[15] the lower case letter could be confused with the number one and the upper case "i". Note the similarity between the following: l, 1, I.

7   Conclusions

These notes are meant to contribute to reducing the "illiteracy" with which, sometimes, very interesting and profound mathematical formulæ are written. The importance of clarity and efficacy of communication, especially scientific communication, should never be underestimated.

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[15] The ISO standards allow the use of liters although the International System unit for the volume is m^3 and its multiples (Bureau International des Poids et Mesures, 2006).

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