Quiz 2: January 27
Time Limit: 40 minutes

(1) (a) (2 points) What is $|U_{12}|$?

4.

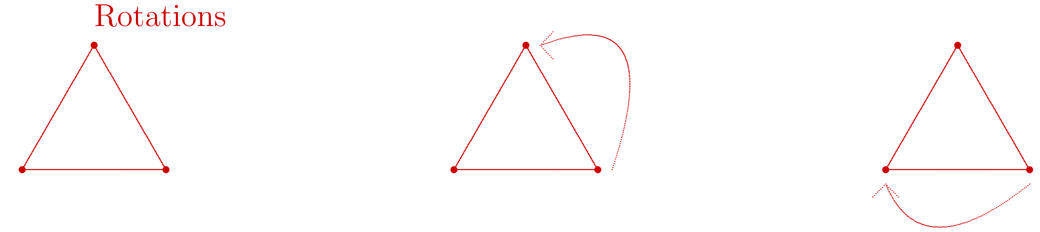
U_{12} is the set of units in $\mathbb{Z}_{12}$, so $U_{12} = \{1, 5, 7, 11\}$.

(b) (3 points) Write out a multiplication table for U_{12}.

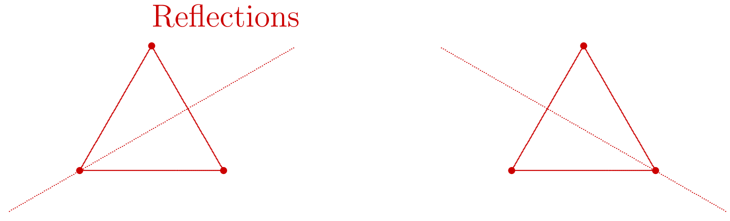
$\cdot$	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

(2) (3 points) Describe each element of D_3.

Rotations



Reflections



(3) (4 points each) Determine if the following algebraic structures are groups. If not, which group axioms fail? If so, prove it.

(a) $(\mathbb{Z}_7, \cdot)$

No.

There is no inverse of 0.

(b) (b) $(\mathbb{Z}, \Box)$, where $a\Box b = a + b - 4$

Yes.

Associativity:

$$(a\Box b)\Box c = (a+b-4)\Box c = (a+b-4)+c-4 = a+b+c-8$$

$$= a+(b+c-4)-4 = a\Box(b+c-4) = a\Box(b\Box c).$$

Closure: For any integers a and b, $a+b-4 \in \mathbb{Z}$.
Identity: The element 4 is the identity since $4\Box a = a\Box 4 = a$ for all $a \in \mathbb{Z}$.
Inverses: For any element a, $8-a$ is its inverse since

$$a\Box(8-a) = (8-a)\Box a = 4$$

and 4 is the identity.

(4) (2 points) Show that the group S_4 is non-abelian.

Let $x, y \in S_4$ be the elements $x = (1234)$ and $y = (123)(4)$. Then

$$xy = (1324) \neq (1342) = yx.$$

Since $xy \neq yx$, S_4 is non-abelian.