
Fluid Mechanics - Hydrostatics

AP Physics B

States of Matter

Before we begin to understand the nature of a Fluid we must understand the nature of all the states of matter:

The 3 primary states of matter

- - **solid** - Definite shape and volume.
- - **liquid** - Takes the shape of its container, yet has a definite volume.
- - **gas** - Takes the shape and volume of its container.

Special "states

- - **Plasma, Bose-Einstein Condensate**
-

Density

The 3 primary states have a distinct **density**, which is defined as ***mass per unit of volume***.

Density is represented by the Greek letter, "RHO", ρ

$$\rho_{(density)} = \frac{M}{V} = \frac{kg}{m^3}$$
$$\rho_{(units)} = \frac{kg}{m^3}$$

What is a Fluid?

By definition, a **fluid** is any material that is unable to withstand a static shear stress. Unlike an elastic solid which responds to a shear stress with a recoverable deformation, a fluid responds with an irrecoverable flow.

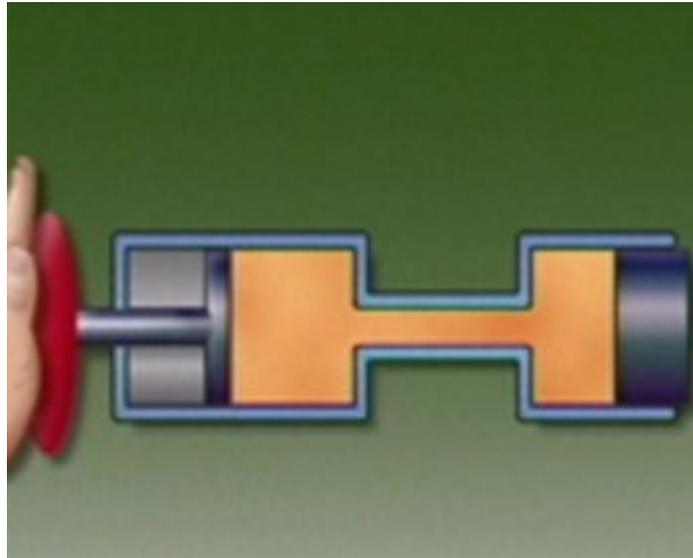
Examples of fluids include gases and liquids.



Why fluids are useful in physics?

Typically, liquids are considered to be **incompressible**.

That is once you place a liquid in a sealed container you can DO WORK on the FLUID as if it were an object. The **PRESSURE** you apply is transmitted throughout the liquid and over the entire length of the fluid itself.



Pressure

One of most important applications of a fluid is
it's **pressure- defined as a Force per unit
Area**

$$P = \frac{F}{A} = \frac{N}{m^2}$$
$$P_{(units)} = \frac{N}{m^2}$$

Example

A water bed is 2.0 m on a side and 30.0 cm deep.

(a) Find its weight if the density of water is 1000 kg/m³.

(b) Find the pressure that the water bed exerts on the floor. Assume that the entire lower surface of the bed makes contact with the floor.

$$a) \quad V = 2 * 2 * 0.30 = \mathbf{1.2 \text{ m}^3}$$

$$\rho = \frac{m}{V} \rightarrow 1000 = \frac{m}{V} = \mathbf{1200 \text{ kg}}$$

$$W = mg = \mathbf{11760 \text{ N}}$$

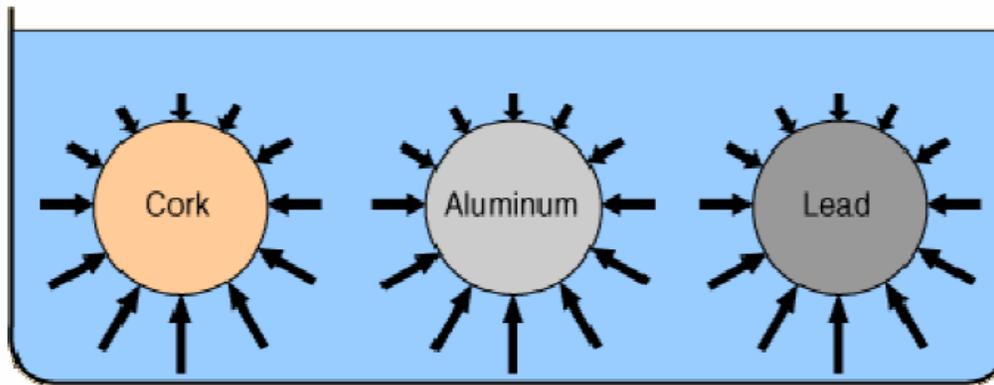
$$b) \quad P = \frac{F}{A} = \frac{mg}{A} = \frac{11760 \text{ N}}{4 \text{ m}^2} = \mathbf{2940 \text{ N/m}^2}$$

Hydrostatic Pressure

Suppose a Fluid (such as a liquid) is at REST, we call this
HYDROSTATIC PRESSURE

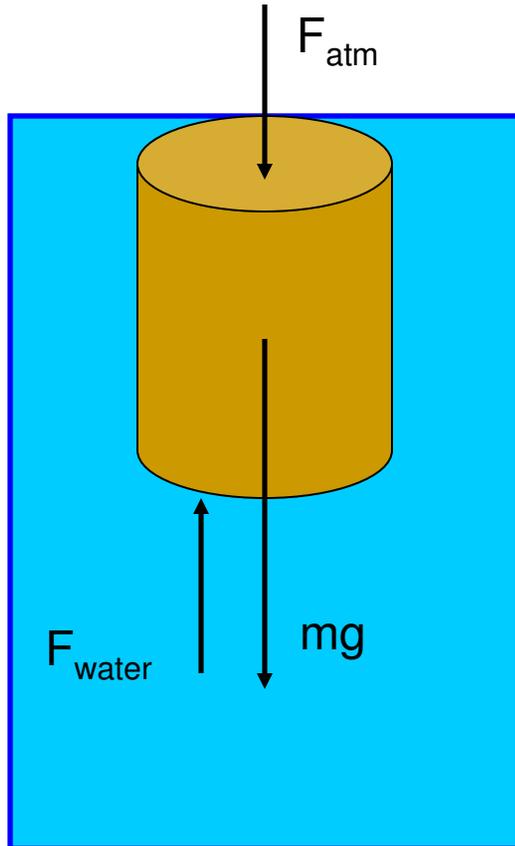
Two important points

- A fluid will exert a pressure in all directions
- A fluid will exert a pressure perpendicular to any surface it compacts



Notice that the arrows on TOP of the objects are smaller than at the BOTTOM. This is because pressure is greatly affected by the DEPTH of the object. Since the bottom of each object is deeper than the top the pressure is greater at the bottom.

Pressure vs. Depth



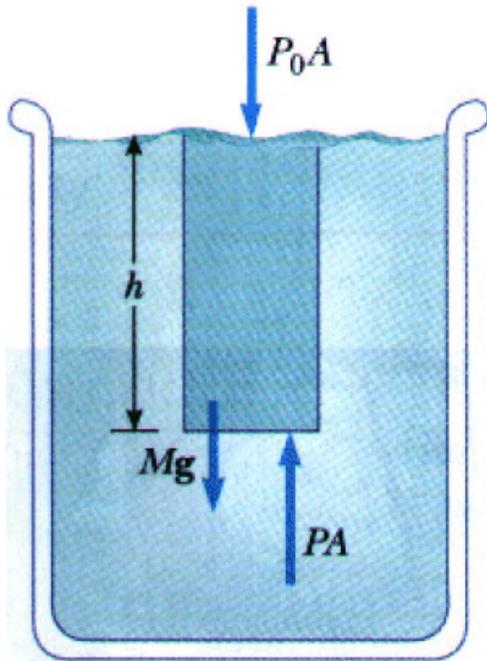
Suppose we had an object submerged in water with the top part touching the atmosphere. If we were to draw an FBD for this object we would have three forces

1. The weight of the object
2. The force of the atmosphere pressing down
3. The force of the water pressing up

$$F_{\text{water}} = F_{\text{atm}} + mg$$

Pressure vs. Depth

But recall, pressure is force per unit area. So if we solve for force we can insert our new equation in.



$$P = \frac{F}{A} \quad F_{water} = F_{atm} + mg$$

$$PA = P_o A + mg$$

$$\rho = \frac{m}{V} \rightarrow m = \rho V$$

$$PA = P_o A + \rho V g$$

$$V = Ah$$

$$PA = P_o A + \rho Ahg$$

$$P = P_o + \rho gh$$

Note: The initial pressure in this case is atmospheric pressure, which is a CONSTANT.

$$P_o = 1 \times 10^5 \text{ N/m}^2$$

A closer look at Pressure vs. Depth

$$P = P_o + \rho gh$$

Depth below surface

Initial Pressure – May or MAY NOT be atmospheric pressure

ABSOLUTE PRESSURE

$$\Delta P = \rho gh$$

Gauge Pressure = CHANGE in pressure or the DIFFERENCE in the initial and absolute pressure

Example

- a) Calculate the absolute pressure at an ocean depth of 1000 m. Assume that the density of water is 1000 kg/m^3 and that $P_o = 1.01 \times 10^5 \text{ Pa}$ (N/m^2).
- b) Calculate the total force exerted on the outside of a 30.0 cm diameter circular submarine window at this depth.

$$P = P_o + \rho gh$$

$$P = 1 \times 10^5 + (1000)(9.8)(1000)$$

$$P = \mathbf{9.9 \times 10^6 \text{ N/m}^2}$$

$$P = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{F}{\pi (0.15)^2} =$$

$$\mathbf{7.0 \times 10^5 \text{ N}}$$

A closed system

If you take a liquid and place it in a system that is CLOSED like plumbing for example or a car's brake line, the PRESSURE is the same everywhere.

Since this is true, if you apply a force at one part of the system the pressure is the same at the other end of the system. The force, on the other hand MAY or MAY NOT equal the initial force applied. It depends on the AREA.

You can take advantage of the fact that the pressure is the same in a closed system as it has MANY applications.

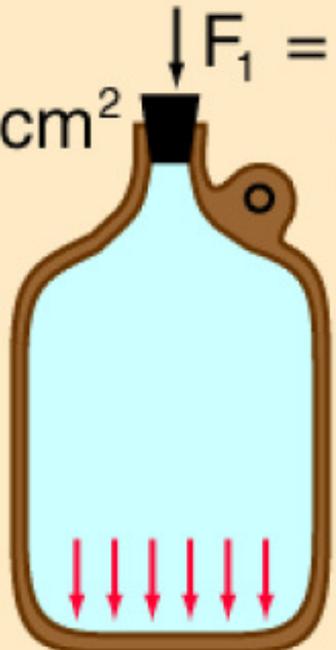
The idea behind this is called PASCAL'S PRINCIPLE

$$P_1 = P_2$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Pascal's Principle

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$



$A_1 = 5 \text{ cm}^2$

$F_1 = 10 \text{ N}$ Applied force to the stopper

$P_1 = \frac{10 \text{ N}}{5 \text{ cm}^2} = 2 \text{ N/cm}^2$

Like a liquid lever, changing areas in an enclosed fluid permit multiplication of force

Pressure is transmitted undiminished in an enclosed static fluid.

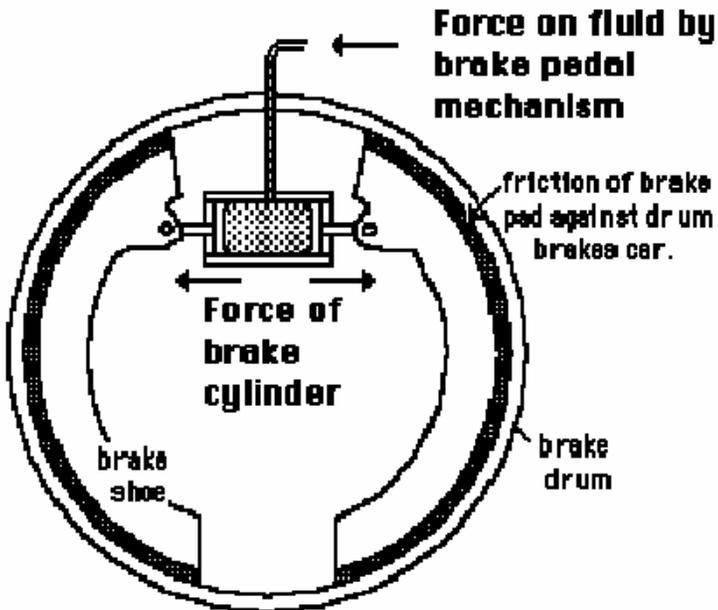
$$F_2 = P_2 A_2 = (2 \text{ N/cm}^2)(500 \text{ cm}^2) = 1000 \text{ N!!}$$

plus the force from the weight of the liquid.

Resulting force on bottom of jug.

$$A_2 = 500 \text{ cm}^2$$

Another Example - Brakes



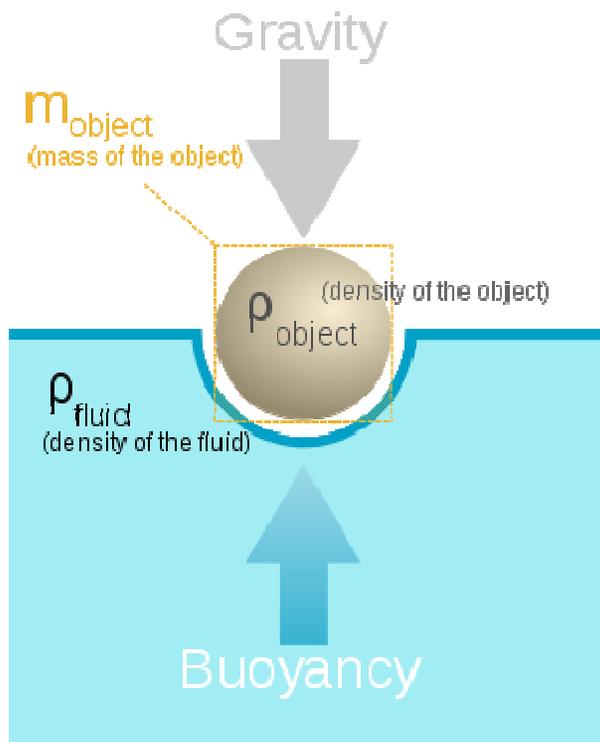
In the case of a car's brake pads, you have a small initial force applied by you on the brake pedal. This transfers via a brake line, which had a small cylindrical area. The brake fluid then enters a chamber with more AREA allowing a LARGE FORCE to be applied on the brake shoes, which in turn slow the car down.

$$P_1 = P_2$$

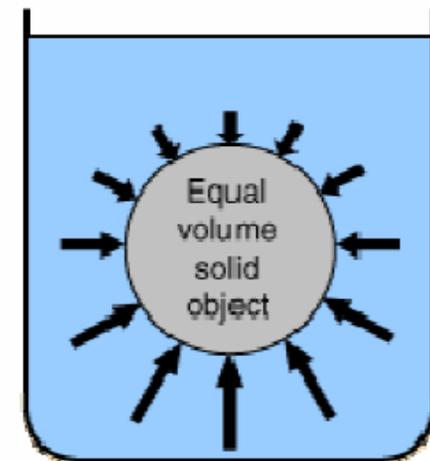
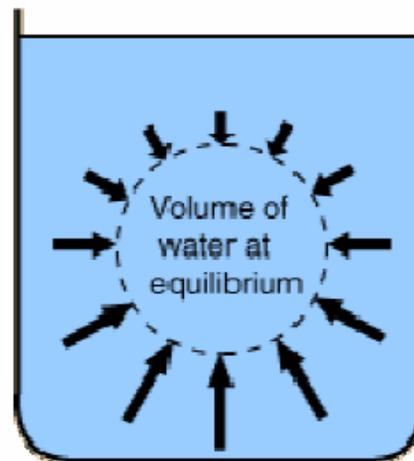
$$\frac{F_{\text{brake pedal}}}{A_{\text{brake pedal}}} = \frac{F_{\text{brake pad / shoe}}}{A_{\text{brake pad / shoe}}}$$

Buoyancy

When an object is immersed in a fluid, such as a liquid, it is buoyed UPWARD by a force called the BUOYANT FORCE.

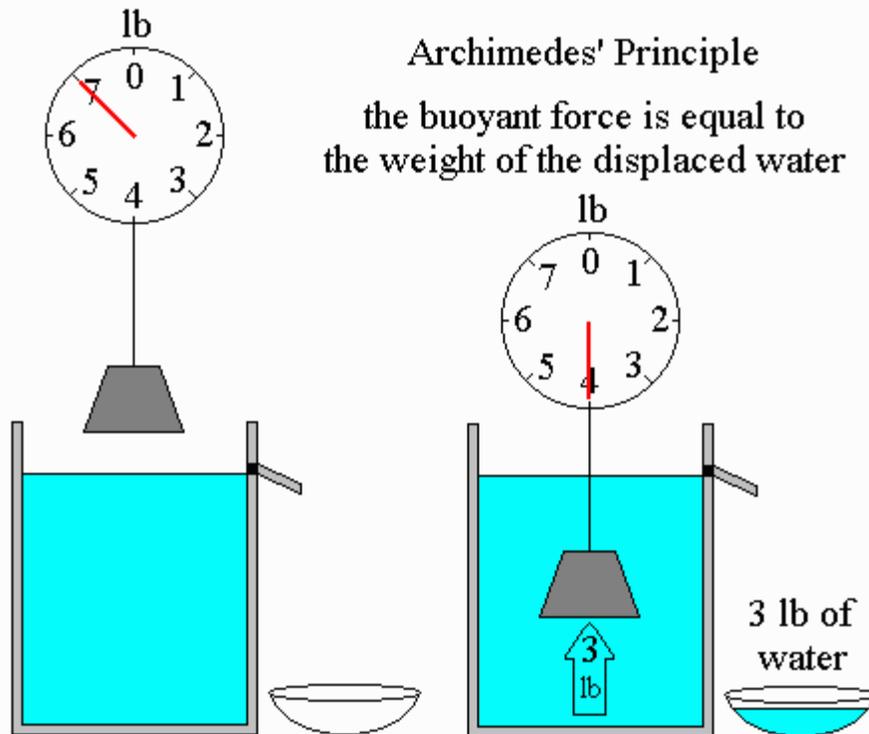


When the object is placed in fluid it DISPLACES a certain amount of fluid. If the object is completely submerged, the VOLUME of the OBJECT is EQUAL to the VOLUME of FLUID it displaces.



Archimedes's Principle

"An object is buoyed up by a force equal to the weight of the fluid displaced."



In the figure, we see that the difference between the weight in AIR and the weight in WATER is 3 lbs. This is the buoyant force that acts upward to cancel out part of the force. If you were to weight the water displaced it also would weigh 3 lbs.

Archimedes's Principle

$$F_B = (mg)_{FLUID} \quad m = \rho V$$

$$F_B = (\rho V g)_{Fluid}$$

$$V_{object} = V_{Fluid}$$

Example

A bargain hunter purchases a "gold" crown at a flea market. After she gets home, she hangs it from a scale and finds its weight in air to be 7.84 N. She then weighs the crown while it is immersed in water (density of water is 1000 kg/m³) and now the scale reads 6.86 N. Is the crown made of pure gold if the density of gold is 19.3 x 10³ kg/m³?

$$F_{object(air)} - F_{object(water)} = F_{buoyant}$$

$$7.84 - 6.86 = F_B = 0.98 \text{ N}$$

$$F_B = (mg)_{fluid} = \rho_{fluid} V_{fluid} g$$

NO! This is NOT gold as 8000 < 19300

$$V_{fluid} = 0.0001 \text{ m}^3$$

$$V_{object} = 0.0001 \text{ m}^3$$

$$mass_{object} = 0.80 \text{ kg}$$

$$\rho_{object} = \frac{m_{object}}{V_{object}} = 8000 \text{ kg/m}^3$$