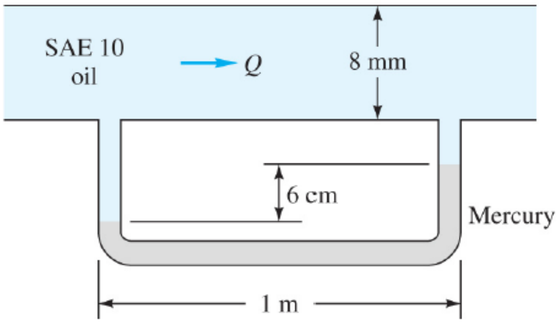
1. (**P4.86 in the book**) SAE 10 oil at 20°C flows between parallel plates 8 mm apart, as shown in Figure. A mercury monometer, with wall pressure taps 1 m apart, registers a 6-cm height, as shown. Estimate the flow rate of oil for this condition.



Solution: Assuming laminar flow, this geometry fits Eqs. (4.143, 144) of the text:

$$V_{avg} = \frac{2}{3}u_{max} = \left(\frac{dp}{dx}\right)\frac{h^2}{3\mu}, \quad \text{where } h = \text{plate half-width} = 4\,\text{mm}$$

For SAE 10W oil, take $\rho = 870$ kg/m^3 and $\mu = 0.104$ kg/m·s. The manometer reads

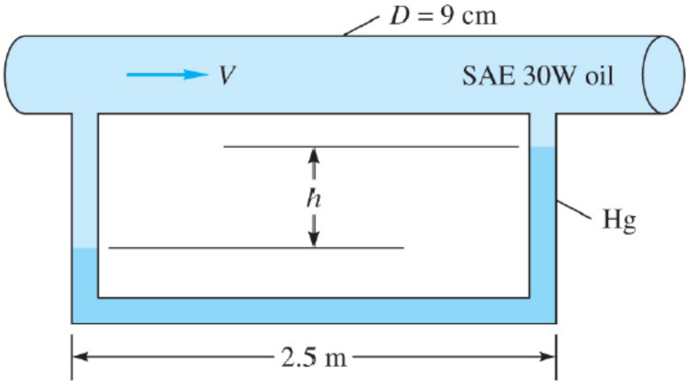
$$\Delta p = (\rho Hg - \rho oil)g\Delta h = (13550 - 870)(9.81)(0.06) \approx 7463 \text{ Pa} \quad \text{for } \Delta x = L = 1\text{m}$$

Then $$V = \frac{\Delta p}{\Delta x}\frac{h^2}{3\mu} = \left(\frac{7463 \text{ Pa}}{1 \text{ m}}\right)\frac{(0.004)^2}{3(0.104)} \approx 0.383 \frac{\text{m}}{\text{s}}$$

The flow rate per unit width is $Q = VA = (0.383)(0.008) \approx \mathbf{0.00306 \frac{m^3}{s \cdot m}}$ *Ans.*

NOTE: The Reynolds number, based upon plate half-width, is 16, *laminar*.

2. (**P4.87 in the book**) SAE 30 W oil at 20°C flows through the 9-cm-diameter pipe in Figure below at an average velocity of 4.3 m/s. (a) Verify that the flow is laminar, i.e, the Reynolds number $Re_d < 2300$. (b) Determine the volume flow rate in m^3/h. (c) Calculate the expected reading h of the mercury monometer, in cm.



Solution: (a) Check the Reynolds number. For SAE 30W oil, from Appendix A.3, $\rho = 891$ kg/m^3 and $\mu = 0.29$ kg/(m·s). Then

$$Re_d = \rho Vd/\mu = (891 \text{ kg/m}^3)(4.3 \text{ m/s})(0.09 \text{ m})/[0.29 \text{ kg/(m·s)}] = 1190 < 2000 \textbf{ Laminar} \quad \textit{Ans. (a)}$$

(b) With average velocity known, the volume flow follows easily:

$$Q = AV = [(\pi/4)(0.09 \text{ m})^2](4.3 \text{ m/s})(3600 \text{ s/h}) = \mathbf{98.5\ m^3/h} \quad \textit{Ans. (b)}$$

(c) The manometer measures the pressure drop over a 2.5 m length of pipe. From Eq. (4.147),

$$V = 4.3\ \frac{m}{s} = \frac{\Delta p}{L}\frac{R^2}{8\mu} = \frac{\Delta p}{2.5\ m}\frac{(0.045\ m)^2}{8(0.29\ kg/m\cdot s)}, \quad \textit{solve for}\ \ \Delta p = 12320\ Pa$$

$$\Delta p_{mano} = 12320 = (\rho_{merc} - \rho_{oil})gh = (13550 - 891)(9.81)h, \quad \text{Solve } \mathbf{h = 0.099\ m} \quad \textit{Ans. (c)}$$

3. (**P4.90 in the book**) It is desired to pump ethanol at 20oC through 25 m of straight smooth tubing under laminar-flow conditions, $\mathrm{Re}_d = \frac{\rho V d}{\mu} < 2300$. The available pressure drop is 10 kPa. (a) What is the maximum possible mass flow, in kg/h. (b) What is the appropriate diameter?

Solution: For ethanol at 20°C, ρ = 789 kg/m^3 and μ = 0.0012 kg/m-s. From Eq. (4.138),

$$Q_{\text{laminar}} = \frac{\pi R^4}{8\mu}\frac{\Delta p}{L} = \frac{\pi d^4}{128\mu}\frac{\Delta p}{L}$$

Clearly, flow increases with diameter, so maximum mass flow requires the maximum diameter consistent with the maximum Reynolds number. The Reynolds number may be written out:

$$\mathrm{Re}_d = \frac{\rho V d}{\mu} = \frac{4\rho Q}{\pi d\mu} = \left(\frac{4\rho}{\pi d\mu}\right)\left(\frac{\pi d^4}{128\mu}\frac{\Delta p}{L}\right) = \frac{\rho d^3 \Delta p}{32\mu^2 L} < 2300$$

$$\text{Or: } d^3 = \frac{2300(32)\mu^2 L}{\rho \Delta p} = \frac{2300(32)(0.0012)^2(25)}{(789)(10{,}000)} = 3.36E-7\,m^3$$

$$\textit{Solve for } \quad d_{\max} = 0.00695\,m \approx \mathbf{7} \;\; mm \qquad Ans.(b)$$

The maximum mass flow is

$$\dot{m}_{\max} = \rho Q_{\max} = \left(789\frac{kg}{m^3}\right)\left[\frac{\pi(0.00695m)^4}{128(0.0012\,kg/m\bullet s)}\right]\left(\frac{10{,}000Pa}{25\,m}\right) = 0.0151\frac{kg}{s} = \mathbf{54}\frac{kg}{h} \quad Ans.(a)$$

Light liquids like ethanol stay laminar only for tiny diameters. To work the same problem with, say, SAE 30W oil, μ = 0.29 kg/m-s, would result in $d_{\max}$ = 26 cm, or 37 times larger. The maximum oil mass flow would be nearly nine thousand times larger.

4. (**P4.93 in the book**) A number of straight 25-cm-long microtubes of diameter d are bundled together into a "honeycomb" whose total cross-sectional area is 0.0006 m^2. The pressure drop from the entrance to exit is 1.5 kPa. It is desired that the total volume flow rate be 1 m^3/h of water at 20°C. (a) What is the appropriate microtube diameter? (b) How many microtubes are in the bundle? (c) What is the Reynolds number of each microtube?

Solution: For water at 20°C, ρ = 998 kg/m^3 and μ = 0.001 kg/m·s. Each microtube of diameter D sees the same pressure drop. If there are N tubes,

$$Q = \frac{1}{3600}\;\frac{m^3}{s} = NQ_{tube} = N\frac{\pi D^4 \Delta p}{128\,\mu L} = N\frac{\pi D^4 (1500\,Pa)}{128(0.001\,kg/m\cdot s)(0.25\,m)} = 1.47E5\,N\;D^4$$

At the same time, $N = A_{bundle}/A_{tube} = \dfrac{0.0006\,m^2}{(\pi/4)D^2}$

Combine to find $D^2 = 2.47E{-}6\;m^2$ or **D = 0.00157 m** *and* **N = 310** *Ans.*(a, b)

With D known, compute $V = Q/A_{bundle} = Q_{tube}/A_{tube}$ = 0.462 m/s and

$Re_D = \rho VD/\mu$ = (998)(0.462)(0.00157)/(0.001) = **724** (laminar) *Ans.* (c)

5. (**P4.94 in the book**) A long solid cylinder rotates steadily in a very viscous fluid, as in Fig. P4.94. Assuming laminar flow, solve the Navier-Stokes equation in polar coordinates to determine the resulting velocity distribution. The fluid is at rest far from the cylinder. [*Hint:* the cylinder does not induce any radial motion.]

Solution: We already have the useful hint that $v_r = 0$. Continuity then tells us that $(1/r)\partial v_\theta/\partial\theta = 0$, hence v_θ does not vary with θ. Navier-Stokes then yields the flow. From Eq. D.6, the tangential momentum relation, with $\partial p/\partial\theta = 0$ and $v_\theta = f(r)$, we obtain Eq. (4.139):

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) = \frac{v_\theta}{r^2}, \quad \text{Solution:} \quad v_\theta = C_1 r + \frac{C_2}{r}$$

$$\text{As } r \to \infty, \; v_\theta \to 0, \quad \text{hence } C_1 = 0$$

$$\text{At } r = R, \; v_\theta = \Omega R = \frac{C_2}{R}; \; C_2 = \Omega R^2; \; \text{Finally}, \; v_\theta = \frac{\Omega R^2}{r} \quad \textit{Ans.}$$

Rotating a cylinder in a large expanse of fluid sets up (eventually) a potential vortex flow.