Multiplication Facts: Passive Storage Or Dynamic Reorganization?

by

Brian Butterworth, Noemi Marchesini *, and Luisa Girelli
Department of Psychology and the Institute of Cognitive Neuroscience,
University College London

To appear in "The development of arithmetical concepts and skills",
edited by
Arthur J. Baroody and Ann Dowker. LEA

Acknowledgments

The research was supported by a grant from the Wellcome Trust (045013) to Butterworth, an ERASMUS Exchange studentship to Marchesini and EC Human Capital Mobility Fellowship (ERBCHBICT941608) to Girelli. We are grateful to the staff and pupils at "Ippolito Nievo" Primary School in Mantua for their participation in this study. We would like to thank Art Baroody and Ann Dowker for their helpful comments on the manuscript. Address for correspondence: Professor Brian Butterworth or Dr Luisa Girelli, Institute of Cognitive Neuroscience, University College London, 17 Queen Square, London WC1N 3AR. (b.butterworth@ucl.ac.uk, l.girelli@ucl.ac.uk)

*Also at Department of General Psychology, University of Padua, Italy.

Introduction

The organisation of arithmetical facts in memory is often regarded as fixed, at least in adults, whether in an array (e.g. Ashcraft & Battaglia, 1978) or in an associative network (e.g Campbell, 1995). Some authors, following Seigler (1988), have proposed an ontogenetic dynamic that shapes the final organisation by reducing the relative strength of associative links between problems (e.g. 3 + 5) and erroneous solutions (7) while strengthening the links to the correct answers (e.g. 8). In this study we raise the possibility that links between problems and **correct** solutions may in certain circumstances be relatively strengthened or relatively weakened. That is to say, the organisation of facts in memory may be **reorganised** so as to favour some representations of correct facts over others. One obvious source of dynamic change is familiarity, or frequency of occurrence, where the more frequently produced or encountered is more strongly represented. But this isn't the only possible source of dynamic change.

Consider addition and multiplication. Both are commutative: 3 + 5=5 + 3 and 3 x 5=5 x 3. Provided one understands this, one doesn't need to store both forms of the commuted pair in memory (cp. Baroody, 1994). One may come to privilege one member of the pair over the other. However, we cannot argue for this on the basis of economy of storage, since the storage capacity of human memory is not known. Nor can we argue on for it on the basis of whether speed of retrieval would be better or worse with one form or two: what you save on search time, you might lose on transforming one form into the other. However, there is indirect evidence that **learning** is more efficient when only one of the commutes is learned. In China and in Iceland, children do not learn the complete multiplication tables. They are taught only half. In China, for example, they learn 3 x 2 as part of the Two Times table, but the Three Times table starts at 3 x 3, so they are never taught the 2 x 3 explicitly. (They also do not learn 1 x anything). There have been many studies demonstrating that Chinese adults are superior on multiplication to Westerners (LeFevre & Liu, 1997). Although studies have not explored this potential causal factor in Chinese superiority, on general learning principles, it strikes us as plausible.

In cases where children learn both of the commuted pairs, as in the rote learning of multiplication tables in the US or in most European schools, does this mean that both forms are stored? Even if both are represented in memory, what determines whether there is privileged access to just one of the forms. On general learning principles, the most practiced or perhaps the first learned, is likely to be the more accessible. Normally, children learn smaller tables before larger tables. Thus, in the US and the UK, 6 x 2 is learned before 2 x 6, since the former is part of the Two Times Table, while the latter is part of the Six Times Table. On the other hand, if there is a privileged form, and it turns out not to be the one favoured by priority or amount of learning, but rather it is favoured by a principle that reflects the meaning of the terms, then this outcome will have a bearing on one of the oldest debates in this whole area: the debate between drill (as represented by Thorndike, 1922) and meaningful learning (Brownell, 1935). Baroody (1985) summarised the evidence as it stood in 1985 thus: "Research tends to support the meaning theory" (p 85). The organization, and, potentially, the re-organization, of these simple number facts is a useful testbed for evaluating this issue again.

In this study we ask whether the form of the commuted pair retrieved from memory is the form learned, or most learned. If not, is there a single privileged form? If there is a privileged form, is it the first learned? We will also ask whether the form retrieved changes in the course of development.

Our measure will be the time taken to solve each form of the commuted pair problem. That is, will it take longer to retrieve 2 x 6 or 6 x 2? We assume that the form more quickly solved is the one more similar to the stored representation. If they are solved equally quickly we infer that both (or, conceivably, neither) forms are in memory.

In this study, our subjects are Italian children from the ages of 8 to 11. Although their schools use rote learning of tables, like other Italian schools, the tables are recited the other way to children in the US and UK. They learn their tables with the table name in the second position: "Once two is two, two twos are four, three twos are six". For Italian children, the table name is in first position. Thus the Two Times table goes "due per uno è due ("two times one is two"), due per due è quattro, due per tre è sei, due per quattro è otto," and so forth. So although they learn both 2 x 6 and 6 x 2, they learn 6 x 2 in the Six Times table many months, often a year, after they have learned 2 x 6 as part of their Two times table. Thus, on the basis of priority of learning and probably total exposure, 2 x 6 should be privileged over 6 x 2.

However, in the present study, we directly tested the contrary hypothesis that, as their arithmetic knowledge increases, both in terms of strengthening of the associations between problems and correct answers and in terms of appreciation of the commutativity law, children re-organise their memory representations to favor problems with the larger operand in first position, regardless of the actual chronological order in which children are taught complements. The preferred problem form, i.e., Nxm (our convention is that Nxm means the larger operand is in the first or left position, while mxN means that it is in the second or right position), would partially result from the application of the repeated addition strategies that, as informally observed by the teachers, in expert children consists in adding the larger operand a number of time indicated by the smaller operand, thus reducing the number of operations to be computed. Previous chronometric studies of children's multiplication skills did not evaluate the extent to which commuted items are differentially treated (e.g., Campbell and Graham, 1985; Siegler, 1988; Svenson, 1985, for related issues in addition).

Method

Primary-school Sample and teaching method

Twenty-eight third-grade children (mean age = 8 years 7 months), thirty-one four-grade children (mean age = 9 years and 5 months) and thirty Fifth-grade children (mean age = 10 years and 6 months) participated in the study. All children were recruited in a primary school in Northern Italy; none of them was diagnosed as learning disabled and they were all introduced and trained with simple multiplication following the same teaching method.

Simple multiplication is only introduced when children are relatively familiar with addition and subtraction, that is by the end of the First-grade or beginning of the Second-grade. The teaching of multiplication generally proceeds from providing children with concrete examples where the computation is part of a familiar situations (e.g., dolls and dresses, Figure 1) to the introduction of more abstract representations such as visualising arrangement or grouping of objects (Figure 2), and then grouping of points (Figure 3). The use of these different type of representations should help the child first, to grasp the meaning of the multiplication itself then to move from the ability to manipulate and represent quantities of objects to the ability to manipulate and represent numbers as abstract concepts. It is only towards the end of the second-grade, beginning of the Third-grade that multiplication facts are introduced starting from the tables of 1, 2 and 3. At this stage, the commutative law is explicitly taught. The others tables are gradually introduced and tested first, by repetition of the table sequence (e.g. 2 x 1, 2 x 2, 2 x 3), then by testing a single table in random order (e.g., 2 x 4, 2 x 7, 2 x 2) and finally by testing all problems. The testing for this study took place in late January.

Notes on Figures 1, 2 and 3.

Ways of illustrating multiplication used in Italian primary schools. They go from familiar concrete objects - girls and dresses in Figure 1 - to more abstract dots in Figure 3.

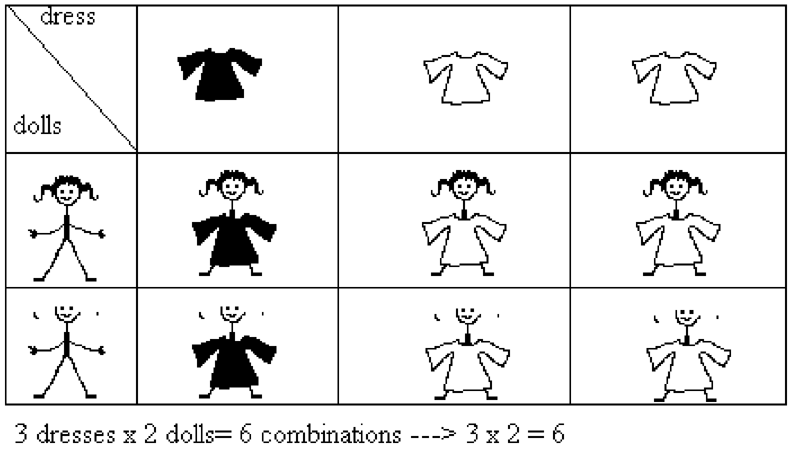

Figure 1.

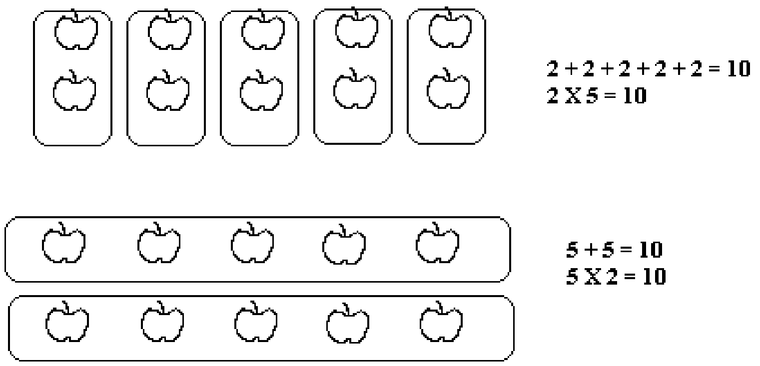

Figure 2.

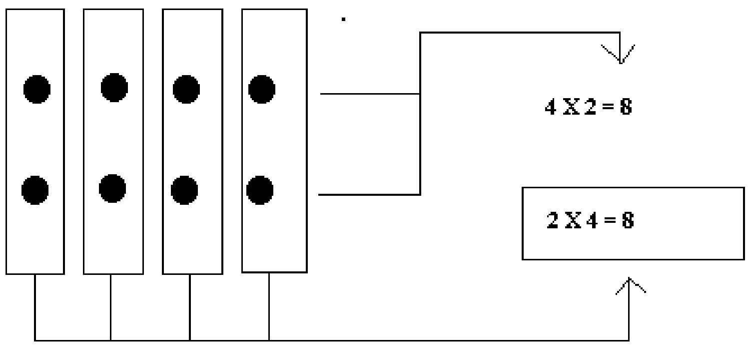

Figure 3.

Material

Because we wished to assess the effects of the factors of age/grade as well as position of the larger number, we selected just those problems that were reasonably familiar to all three Groups. The results of a pilot study indicated that large problems yielded high error rate and were often solved via back-up strategies, rather than retrieval from memory. Thus, just the multiplication problems between 2 x 2 and 6 x 5 were used as stimuli (N=48). Problems were assigned randomly to two different lists, with the only constrain that commuted items were never included in the same list (e.g. 3 x 4 was put in list A, and 4 x 3 in list B) and that each list had half of the problems with the larger operand in first position (e.g., 4 x 2) and the other half with the larger operand in second position (e.g., 2 x 4). The two lists were presented on different days.

Procedure

Each subject was tested individually at school. Stimuli were read aloud to the children and latencies were hand-timed using a stop-watch from the time the experimenter finished to read the problem till the subject began to make a response. Any verbal comment or overt strategies used by the child were recorded, but no attempts was made to distinguish trials where subjects appeared to retrieve answers from memory from those where they may have been using another solution strategy. Consecutive trials were separated by an interval of a few seconds. Before the experiment began, subjects were presented with training trials to familiarise them with the task and the experimental setting. Children were asked to do the task the best they could, but that their performance would not be graded for school in any way.

Results

Within each grade, subjects were divided in high- and low-skilled, based on their performance on the task. Individuals whose median reaction time and error rates were below their age-group's averages were classified as high-skilled; all remaining subjects were classified as low-skilled. Following this criterion, the third-grade included 13 high-skilled children and 15 low-skilled children; the fourth-grade included 12 high-skilled and 19 low-skilled children and the fifth-grade 10 high-skilled and 20 low-skilled children.

Because of marked heterogeneity of variance in the data, a reciprocal transformation of reactions times and an arcsine transformation of error proportions were performed prior to the statistical analyses of the results. Harmonic means and error rates for the different set of problems are reported in Table 1.

	N x m		m x N		Ties	
Grade	RTs	Er	RTs	Er	RTs	Er
Third	3.40	13.5	3.47	13.1	2.15	7.3
Fourth	2.22	9.4	2.50	8.8	1.59	4.8
Fifth	1.90	4.5	2.06	6.1	1.45	0.0

Table 1.
Harmonic means (in sec) and error rates as a
function of Position of the Larger and of Grade level.

Reaction times analysis

Overall, we found a strong correlation between the magnitude of a problem's result (e.g., 20 for 4 x 5) and the median RTs (Problem size effect: Spearman rank correlation $r=0.479$; $p < .01$, $r=0.773$, $p<.0001$, $r=0.764$, $p<.0001$ for Third-, Fourth- and Fifth-grade respectively).[1] These findings are in line with previous studies on the development of multiplication skills (e.g., Campbell and Graham, 1985).

Mean transformed correct RTs were submitted to a first repeated measures ANOVA with grade (Third, Fourth, and Fifth) and skill (High and Low) as between-subject factors and position of the larger operand (Position of Larger: Left and Right) as a within-subject factors. The main effect of grade ($F[2, 85]=44.69$; $Mse=.776$, $p<.0001$) was significant. Post-hoc comparisons indicated that each grade differed from the others (Newman-Keuls; all $p<.05$) with means of 3.43 sec, 2.37 sec and 1.81 sec for Third, Fourth and Fifth grades respectively. Not surprisingly high-skilled subjects were significantly faster than low-skilled subjects (2.10 sec versus 2.74 sec; $F[1, 85]=43.80$; $Mse=.760$, $p<.0001$). More critically, the main effect of Position of Larger was significant ($F[1, 85]=30.63$; $Mse=.049$, $p<.0001$) as well as its interaction with grade ($F[2, 85]=5.96$; $Mse=.010$, $p<.005$). In fact, problems with the larger operand in Left position were answered faster than problems with the larger operand in Right position (2.34 versus 2.55), though this effect was significant in Fourth (2.22 versus 2.50; $F[1, 30]=36.76$, $Mse=.051$, $p<0001$) and Fifth grade only (1.90 versus 2.06; $F[1, 29]=13.84$, $Mse=.023$, $p<001$; Third grade, $F[1, 25)<1$) (Figure 5). No other interaction were significant.

---

[1] These correlations were taken from data on responses to tables from Two through Nine, excluding problems where both numbers are greater than 5.

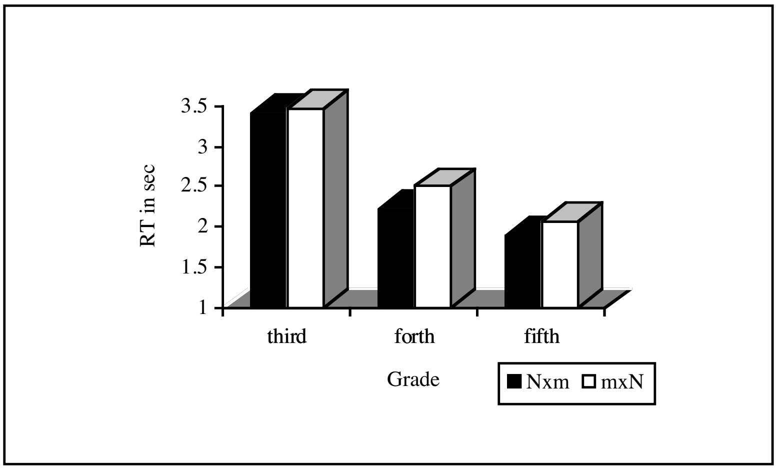

Figure 4 .
RT in seconds as a function of the
Position of the Larger operand and grade level.

Mean transformed correct RTs were then entered in a repeated measures ANOVA with grade (Third, Fourth, and Fifth) and skill (High and Low) as between-subject factors and *Min* (the smaller of the two operands and Position of the Larger operand (Left and Right) as a within-subject factors. The main effect of Grade was significant, $F[2, 85]= 44.28$; $Mse=2.89$, $p <.0001$; post-hoc comparison showed that each grade differed from the others (Newman-Keuls; all $p<.05$). High-skilled subjects were significantly faster than low-skilled subjects ($F[1, 85]=54.05$, $Mse= 3.53$, $p<.0001$). The *Min* factor was significant, $F[3, 255]=115.80$, $Mse=1.319$, $p<.0001$ indicating that overall, reaction time increased with the size of the *Min* (with means of 1.92 sec, 2.73 sec and 3.35 sec for *Mins* of Two, Three and Four respectively) with the exception of min equal Five (2.69 sec). The *MIn* factor interacted significantly with grade ($F[6, 255]= 7.89$, $Mse=.090$, $p<. 0001$). In fact, the effect of the *Min* was greater in older children, though for all subjects the effect was highly significant (all, $p<.0001$) (Figure 5)

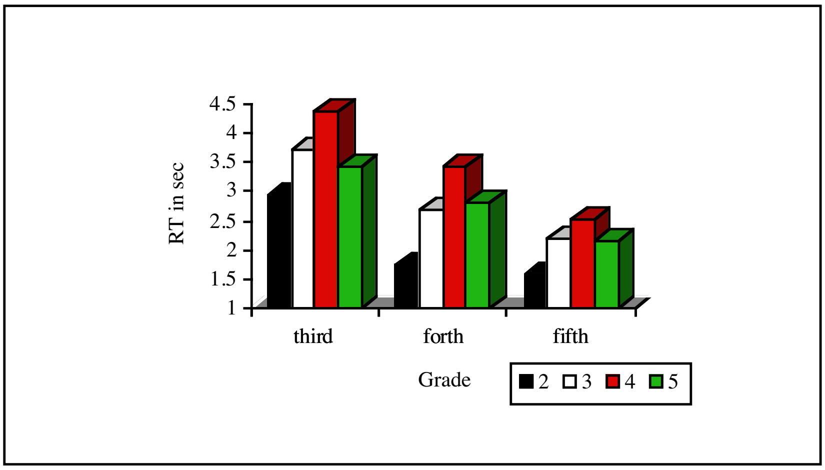

Figure 5.
Effect of *Min* (the smaller of the two operands) at different grade levels.

The main effect of Position of Larger was highly significant (F[1, 85]=25.51; Mse= .145, p<.0001) as well as its interaction with grade (F[2, 85]=7.99; Mse= .046, p<.001) replicating the previous analysis. More interestingly, there was a significant interaction of *Min* x Position of Larger (F[3, 255]=5.86; Mse= .031, p<.005). Decomposition into contrasts revealed that the advantage for the problems with the larger operand in first position decrease with the size of the min. In other words, this advantage was highly significant for problems with min equal two (F[1, 85]=31.84; Mse= .197, p<.0001) and marginally significant for problems with min equal three (F[1, 85]=3.63; Mse= .023, p=.05), but it was negligible (F[1, 85]=2.87; Mse= .018, p=.09) and totally absent (F[1, 85] <1) for min four and five respectively (Figure 6).

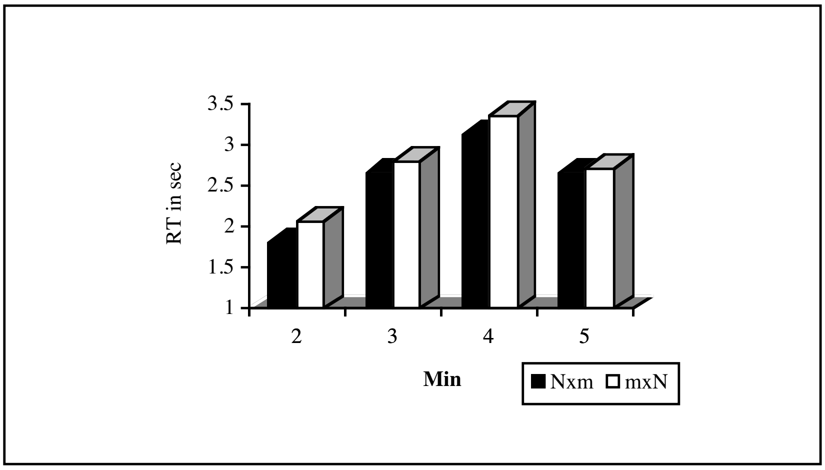

Figure 6.
RT in seconds as a function of the Position of the Larger operandand *Min* (the smaller operand).

These results were replicated in a similar by item ANOVA with *Min* (Two, Three, Four and Five) as between subjects factor and grade (Third, Fourth and Fifth) and Position of Larger (Left and Right) as within subjects factors. All main factors were significant: *Min* (F[3,18] = 11.10, Mse= .286, p<.0005); Grade (F[2, 36] = 180.66, Mse= .434, p<.0001) and Position of Larger (F[1,18]=22.93, Mse= .027, p<.0001. As previously pointed out, the significant interaction between *Min* and Grade (F[6. 36]=8.51, Mse= .020, p<.0001) indicated that the relationship between latencies and *Min* was much steeper for older children than for younger one (Figure 6). Moreover, the effect of the relative order of the operands was again modulated by Grade (F[2, 36]=7.66, Mse= .006, p<.001) and min (F[3,18]=6.57, Mse= .008, p<.005).

Error analysis

Parallel analyses were performed on arcsine transformed error proportions. A first repeated measures ANOVA with grade (Third, Fourth, and Fifth) and skill (High and Low) as between-subject factors and Position of the Larger operand (Left and Right) as a within-subject factors. The main effect of grade (F[2, 85]=11.40; Mse=.129, p<.0001) was significant. Post-hoc comparisons indicated that each grade differ significantly form the others (Newman-Keuls; all p<.05) with means error rates of 13.3% , 9.1% and 5.3% for Third, Fourth and Fifth grades respectively. High-skilled subjects were significantly more accurate than low-skilled subjects (3.7% versus 12.6%; F[1, 85]=36.09; Mse=.407, p<.0001). On the other hand, the relative position of the larger operand did not have any effect on the error rate (F[1, 85]<1), nor did it interact with any of the other effects.

A repeated measures ANOVA with grade (Third, Fourth, and Fifth) and skill (High and Low) as between-subject factors and *Min* (Two, Three, Four and Five) and position of the larger operand (Position of Larger: first and second) as a within-subject factors. Both main effects of Grade F[2, 85]= 11.37; Mse=.615, p <.0001) and skill (F[1, 85]=36.79, Mse= 1.99, p<.0001) was significant, replicating the previous analysis. The *Min* factor was significant (F[3, 255]=23.53, Mse=.515, p<.0001) as well as its interactions with grade (F[6, 255]= 3.67, Mse=.080, p<. 001) and skill (F[3, 255]=11.47, Mse=.251, p<.0001). Though in all children the error rate increased with the magnitude of the min and decreased for min equal five, in Fifth-graders this effect was less pronounced and the error rate was relatively similar across the problems (Figure 7). Similarly, the effect of the min was magnified in low-skilled subjects (F[3, 153]=31.05, Mse=.090, p<.0001) compared to high-skilled subjects (F[3, 96]=3.02, Mse=.026, p<.05).

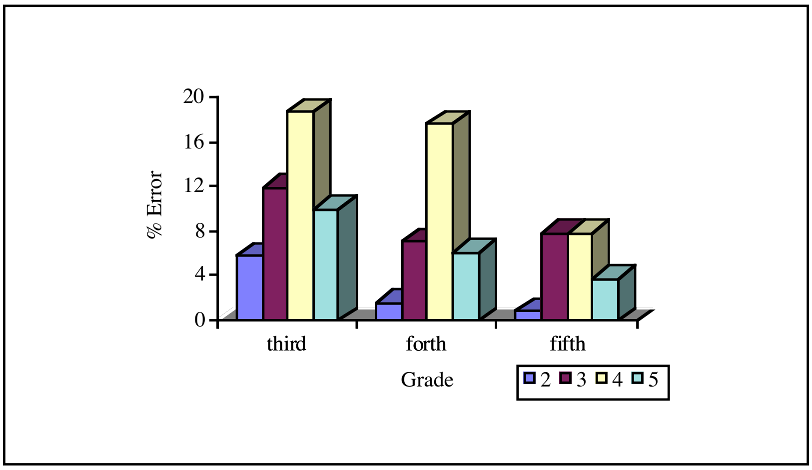

Figure 7.
Error rates according to different
grade levels and *Min* (smaller operand).

Qualitative error analysis

Following previous studies (Campbell and Graham, 1985; Siegler, 1988; McCloskey et al., 1991) errors were classified into four categories:

1. operand errors - any multiple of one of the operands (e.g., 5 x 4 = 24);

2. close miss errors - plus or minus ten percent of the correct result (e.g. 6 x 7= 38);

3. table errors - answer that belongs to a table different from either operands (e.g., 3 x 4 = 25);

4. non-table errors - an answer not included in the any of the tables (e.g., 3 x 4 = 17).

Errors that satisfied more than one of these criteria were assigned to just one category in the above order. (e.g. 6 x 5=31 would have been assigned to close miss not non-table). The distribution of error types is shown in Table 2.

Grade	Type of Errors operand	close-miss	table	non-table	total
THIRD					
high-skilled	65.4	15.4	19.2	0	4.2
low-skilled	71.3	11.2	11.2	6.3	19.8
FOURTH					
high-skilled	84.0	12.0	4.0	0	4.3
low-skilled	68.3	22.1	3.8	5.8	11.4
FIFTH					
high-skilled	87.5	12.5	0	0	1.6
low-skilled	77.5	14.5	6.5	1.5	6.4

Table 2.
Error rates (%) according to grade level and skill.

The overall error rate decreased with the grade level, but the distribution of error types did not change dramatically across the ages. In fact, for Third-graders as for Fourth and Fifth-graders the most frequent errors were operand errors, though the proportions of the other categories seemed to differ across grades. For all grades, close-miss errors were relatively frequent, but the probability of making a table errors and non-table errors decreased with age and with experience. High-skilled subjects were, at any age, more likely to produce an error that was plausible either in terms of magnitude (i.e., close miss) or in terms of table-status (i.e., table vs. non table numbers) than lower- skilled subjects.

The corpus of operand errors was further analysed to disentangle any systematic patterns in the production of an incorrect but related answers. In fact, 62.1% of these errors consisted of correct answers for items that not only shared an operand with the target items but were also close in magnitude with respect to the other operand (e.g., 7 x 9=56, that is the correct response for 7 x 8). This so-called operand distance effect has been already reported both in normal subjects (Campbell and Graham, 1985; Miller, Perlmutter and Keating, 1984) and patients (Sokol et al., 1991). Table 3 illustrates the relative proportions of operand errors in terms of the operand numerical distance between the target and actual responses. Distance “0” refers to the operand errors corresponding to a multiple of both target operands (e.g., 3 x 6 = 12). Interestingly, the proportion of close-operand errors increases with age as the probability of producing a distant-operand error decreases.

Grade	DISTANCE				
	0	X±1	X±2	X±3	X>X±3
THIRD	11.8	55.5	16.8	12.6	3.4
FOURTH	18.5	59.8	15.2	4.3	2.2
FIFTH	13.4	71.2	9.6	5.4	0.0

Table 3.
Percentage of Operand errors according to Grade and distance, where **X** is the distance between target and error operand (e.g for 5x4=24, error operand X±1 the target 5)

Furthermore, within the errors that could be unambiguously classified, the relative proportion of the operand errors corresponding to a multiple of the smaller or of the larger operand as well as of the first or of the second operand was analyzed. Because the probability of producing a multiple of one or other of the two operands is equal, a trend in a specific direction may indicate a preferred order for retrieving the solution and/or of applying a specific strategy (e.g., transforming a multiplication into a repeated addition of the same addend -first/second or smaller/larger operand).

Third-graders were more likely than chance to produce a multiple of the smaller operand (64), X(1)=8.32, p<.01; they were also more likely than chance to produce a multiple of the first operand (59.4), X=4.36, p<.05. This pattern was only marginally significant in the Fourth-graders (First and Smaller, both 60%, p=.07). In the Fifth grade, the observed operand errors were equally likely to correspond to a multiple of either the first or second factor as well as of either the smaller or the larger factor. These results may be interpreted by assuming that younger children, being more familiar with the 2-times and 3-times Tables are more likely to produce as errors a multiple of the smaller operand, though, at the same time, they tend to answer the problem as it is presented, i.e., by activating the table of the first operand. As skills progress, however, children seem to be less tied to the problem presentation format and the production of operand errors is mainly characterized by the numerical closeness to the correct answers.

Discussion

The overall results indicate that children's performance in simple multiplication changes over the course of skill acquisition both quantitatively and, more critically, qualitatively.

It is clear that children become faster and more accurate with increasing skill levels, though from the very beginning both latencies and error profiles are characterized by some standard effects. First, all groups showed a significant problem-size effect: in general RTs increased with the increase of the magnitude of the problem. However, the 5-times table was faster than the problems' numerical size would predict and, interestingly, this was true for all groups. This is an extremely robust effect and it has been so far interpreted by assuming that 5-problems are solved by reference to the rule according to which any multiple of 5 may only end by 5 or 0 (Baroody, 1983, 1984). This rule would constrain the number of candidate answers to this set of problems, reducing the chance to produce errors (Campbell and Graham, 1985). The fact that this rule is appreciated so early in the acquisition process, though not explicitly taught by the teachers, seems by itself to suggest that, the development of arithmetical skill and, specifically, the acquisition of arithmetic tables does not proceed as a rote and meaningless learning process but may benefit and be facilitated by the appreciation of regularities and principles that govern them.

The most critical finding, as predicted by our hypothesis, consists of the effect of the relative order of the operands within a given problem. In Italian schools, children always learn m x N (e.g. 2 x 6) before they learn N x M (6 x 2) bacause the table name is in the first position (unlike the US and UK) and smaller tables are learned before larger tables. Nevertheless, despite earlier and longer exposure to the m x N form, children showed a time advantage in answering the problems with the larger operand in first position. However, this effect emerged only *after* the third-grade. Certainly, there is evidence that by the end of third-grade, children perform as accurately as on unpracticed commuted pairs as on their practiced counterparts (Baroody, forthcoming), and it would be surprising if this understanding were not used to organise multiplication knowledge.

A further result seems to favour the hypothesis of a principled reorganization of memory representations. In fact, the order effect did not hold for all problems to the same extent, but it was function of the size of the min: the relative position of the operands was more critical when the smaller operand was equal to 2 or to 3. These were the problems that were learned earliest in school, and it precisely these that were the most susceptible to reorganisation. These results, and in particular the combined effects of the operand order and the magnitude of the problems, may be considered only indicative given that we tested a subset of problems (i.e., 2 x 2 to 6 x 5). Possibly, re-testing all single-digit combinations may disclose even more clear-cut results, in particular with regards to the effect of the position of the operands in the most difficult problems.

The qualitative analysis of the errors disclosed further intriguing patterns. From the beginning, operand errors were the most frequent type of errors, but their proportion increased as skills progress. (These findings are consistent with those of Lemaire & Siegler, 1995, for changes over six-month period). Interestingly, within the different groups, the absence of non-table errors characterized the performance of high-skilled children, even within the younger group. Thus, the gradual acquisition and refinement of multiplication skills is not simply captured by a faster and more accurate performance, but also by the occurrence of more plausible errors both in terms of numerical closeness and in terms of arithmetical relation, i.e. multiple of one of the two factors, to the correct result.

The child learning multiplication facts is not passive, simply building associative connections between problems and solutions as they are experienced in recitation or in problem presentation. Rather, the facts in memory seem to be reorganised in a principled way that takes account of a growing understanding of the commutativity, and perhaps other properties of multiplication.

References

Ashcraft, M. H. and Battaglia, J. (1978) Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory, 4:* 527-538.

Baroody, A. J. (1983) The development of procedural knowledge: An alternative explanation for chronometric trends of mental arithmetic. *Developmental Psychology, 3*, 225-230

Baroody, A. J. (1984) A reexamination of mental arithmetic models and data: a reply to Ashcraft. *Developmental Review, 4*, 148-156

Baroody, A. J. (1985) Mastery of basic number combinations: Internalization of relationships or facts? *Journal for Research in Mathematical Education*, *16*, 83-98

Baroody, A. J. (1994) An evaluation of the evidence supporting fact-retrieval models. *Learning and Individual Differences*, *6*, 1-36

Baroody, A. J. (forthcoming) The roles of estimation and the commutativity principle in the development of third-graders' mental multiplication.

Brownell, W. A. (1935) Psychological considerations in the learning and the teaching of arithmetic. In *The Teaching of Arithmetic* (10th Yearbook of the National Council of Teachers of Mathematics). New York: Bureau of Publications, Teachers College, Columbia University.

Campbell, J. I .D. (1995) Mechanisms of simple addition and multiplication: A modified network-interference theory and simulation. *Mathematical Cognition, 1,* 121-164.

Campbell, J. I. D and Graham D. J. (1985) Mental multiplication skill: structure, process and acquisition. *Canadian Journal of Psychology, 39:* 338-366.

Graham, D. F. (1987) An associative retrieval model: How children learn to multiply. In J. Sloboda and D. Rogers (Eds), *Cognitive processes in mathematics,.* Oxford, England: Oxford University Press. Ch.7, pp.123-141.

LeFevre, J. & Liu, J. (1997) The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, *3*, 31-62

Lemaire, P. & Siegler, R. S. (1995) Four aspects of strategic change: Contribution s to children's learning of multiplication. *Journal of Experimental Psychology: General, 124 ,* 83-97

Miller, K., Perlmutter, M. and Keating, D. (1984) Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Learning, Memory and Cognition, 10:* 46-60.

Siegler, R.S. (1988). Strategy choice procedures and the development of multiplicaiton skill. *Journal of Experimental Psychology: General, 3, 258-275*

Sokol, S., McCloskey, M., Cohen N. J. and Aliminosa, D. (1991) Cognitive representations and processes in arithmetic: Interference from the performance of brain-damaged subjects. *Journal of Experimental Psychology: Learning, Memory and Cognition, 17:* 355-376.

Svenson, O. (1985) Memory retrieval of answers of simple additions as reflected in response latencies. *Acta Psychologica*, *59*, 285-304

Thorndike, E. L. (1922) *The Psychology of Arithmetic*. New York: Macmillan