LOGICAL ABSTRACTION AND LOGICAL OBJECTS

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Abstract: In this talk, I shall critically discuss some issues related to Frege?s notion of logical object and his paradigms of second-order abstraction principles: Hume?s Principle and especially Axiom V. A schema for a Fregean abstraction principle can be stated as follows: $Q(\alpha) = Q(\beta) \leftrightarrow R_{eq}(\alpha, \beta)$. Here "Q" is a singular term-forming operator, α and β are free variables of the appropriate type, ranging over the members of a given domain, and "R_{eq}" is the sign for an equivalence relation holding between the values of α and β. I call a Fregean abstraction principle *logical* if the equivalence relation, denoted on its right-hand side, can be defined in second-order or higher-order logic. Frege's principal motive for introducing extensions of concepts (and relations) into his logical theory was to define all numbers as logical objects of a fundamental and irreducible kind in order to ensure that we have the right cognitive access to them qua logical objects via Axiom V. However, reducibility to extensions cannot be the ultimate criterion for Frege of what is to be regarded as a logical object.

In *Grundlagen*, Frege suggests the following strategy for the envisaged introduction of the real and complex numbers as logical objects: it should proceed along the lines of his introduction of the cardinals, namely by starting with a tentative contextual definition of a suitable number operator in terms of an abstraction principle whose right-hand side (a second-order or higher-order equivalence relation) was couched in purely logical terms, and by finally defining these numbers as equivalence classes of the relevant equivalence relation. I consider Frege's tentative introduction of the cardinals via Hume?s Principle and the referential indeterminacy of the cardinality operator to which the so called Julius Caesar problem gives rise as well as his final explicit definition of that operator only very briefly—more details in this respect are provided in my second lecture. However, I shall take a closer look at his projected introduction of the "higher" numbers with special emphasis on the Caesar problem. In this connection, I also analyze his awkward remark towards the end of *Grundlagen* that he attaches no decisive importance to introducing extensions of concepts at all. The remark is flatly at odds with what he does and says elsewhere in *Grundlagen*.

When in *Grundgesetze* Frege introduces what he takes to be the prototype of a logical object in his foundational project ? namely courses-of-values of first-level functions which include extensions of first-level concepts and of first-level relations as special cases—by means of a semantical stipulation later to be enshrined in his Basic Law V, he faces a variant of his old problem of referential indeterminacy from *Grundlagen*, now clad in formal garb. I shall critically discuss several key issues of this problem. I argue, among other things, that while in *Grundlagen* Frege tacitly considers the domain of the first-order variables to be all- encompassing, in *Grundgesetze* there is a serious conflict between (a) his practice of taking the first-order domain again to be all-inclusive when he elucidates and defines first-level functions and (b) the assumption

underlying his proof of referentiality in §31 that the domain is restricted to the objects whose existence is required by the axioms of his logical system: the two truth-values and the courses-of-values. Without this assumption the proof would not even get off the ground. Another point argued for is that Frege's attempt of fixing the references of course-of-values terms (completely) in a piecemeal fashion is bound to fail, if the first-order domain comprises all the objects there are. Moreover, the legitimacy of the first crucial step in this procedure, namely the identification of the True and the False with their unit classes, may be doubted, although the special permutation argument, on which it rests, appears to be sound. If time allows, I shall conclude by explaining why regarding the Caesar problem for cardinal numbers and its alleged solution the situation in *Grundgesetze* differs significantly from that in *Grundlagen*.