Space System Design

MAE 342, Princeton University
Robert Stengel

- **Some themes for this term**
  - Deep space missions
  - Planetary defense
  - Collaborative learning

Preliminaries

- *Office Hours*
  - *Tues - Thurs, 1:30-3pm*
- *Assistants in Instruction:*
  - *Will Coogan*
  - *Yibin Zhang*
    - *Office hours: TBD*
    - *Precepts, tutorials: TBD*
- *MATLAB/SimuLink, STK-AGI, CREO*
- *Course Home Page, Syllabus, and Links*
    - *www.princeton.edu/~stengel/MAE342.html*
- *Wednesday afternoon "Lab Sessions" following regular class: TBD*

- **GRADING**
  - Class participation: 10%
  - Assignments: 45%
  - Term Paper: 45%

Text and References

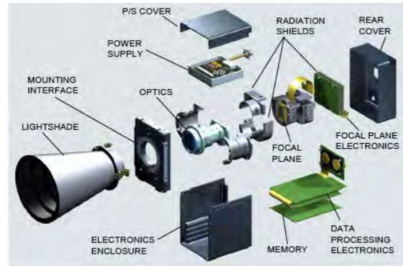
- Principal textbook:
- *Spacecraft Systems Engineering*, Fortescue, Stark, and Swinerd, J. Wiley & Sons, 2011
- Supplemental references
  - *Space Mission Analysis and Design*, Wertz et al, Microcosm Press, 2011
  - *Fundamentals of Space Systems*, V. L. Pisacane, Oxford University Press, 2005
  - Various technical reports and papers (e.g., NASA and AIAA pubs)
  - Books on reserve at Engineering Library (paper and on-line)
  - Web pages
    - http://blackboard.princeton.edu/
    - http://www.princeton.edu/~stengel/MAE342.html

First Half of the Term

- Overview and Preliminaries
- Orbital Mechanics
- Planetary Defense
- Spacecraft Guidance
- Spacecraft Environment
- Chemical/Nuclear Propulsion Systems
- Electric Propulsion Systems
- Launch Vehicles
- Spacecraft Structures
- Spacecraft Configurations
- Spacecraft Dynamics
- Spacecraft Control

Second Half of the Term

- System Engineering & Integration
- Sensors & Actuators
- Electrical Power Systems
- Thermal Control
- Telecommunications
- Telemetry, Command, Data Handling & Processing
- Spacecraft Mechanisms
- Electromagnetic Compatibility
- Space Robotics
- Human Factors of Spaceflight
- Product Assurance
- Ground Segment



Electronic Devices in Class

- *Silence all cellphones and computer alarms*
- *Don't check e-mail or send text, tweets, etc.*
- *If you must make a call or send a message, you may leave the room to do so*
- *Tablets/laptops for class-related material ONLY*

Collaborative Learning

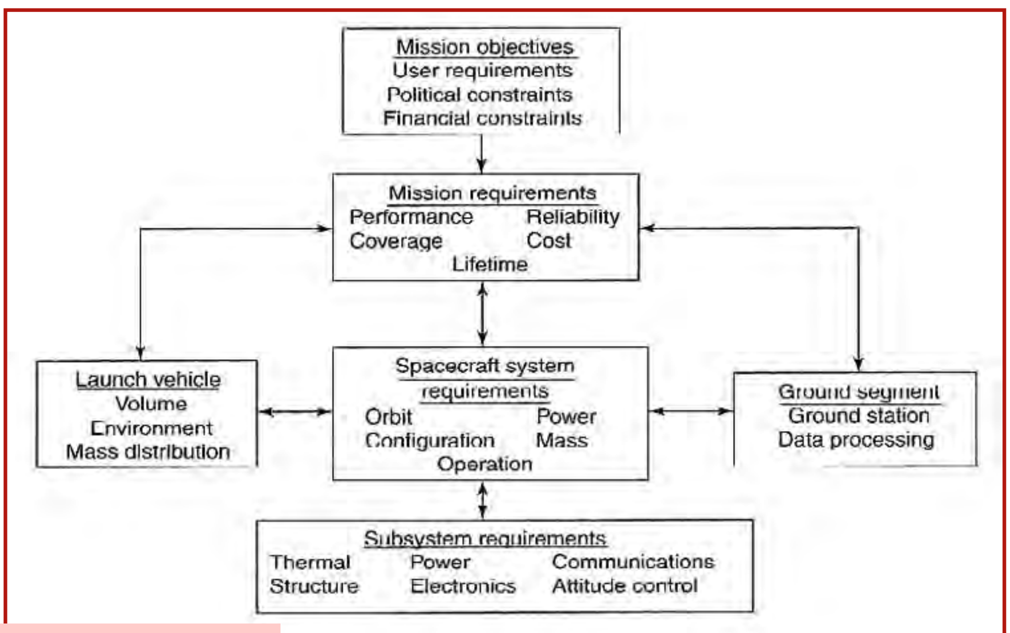
- Significant student participation in most classes, Q&A
- Slides will be available before each class
- Discussion of slides by students
- Randomly assigned teams for assignments during first half
- Project-oriented teams during second half
- Single grade for each team

Written Assignment Reporting Format

- Assignments evolve toward Technical Reports
  - Abstract
  - Introduction
  - Methods and Results
  - Conclusion
  - References
- Write-ups should present explanations, not just numbers, graphs, or computer code
- Orderliness and neatness count
- Don't forget your name, date, and assignment title and number

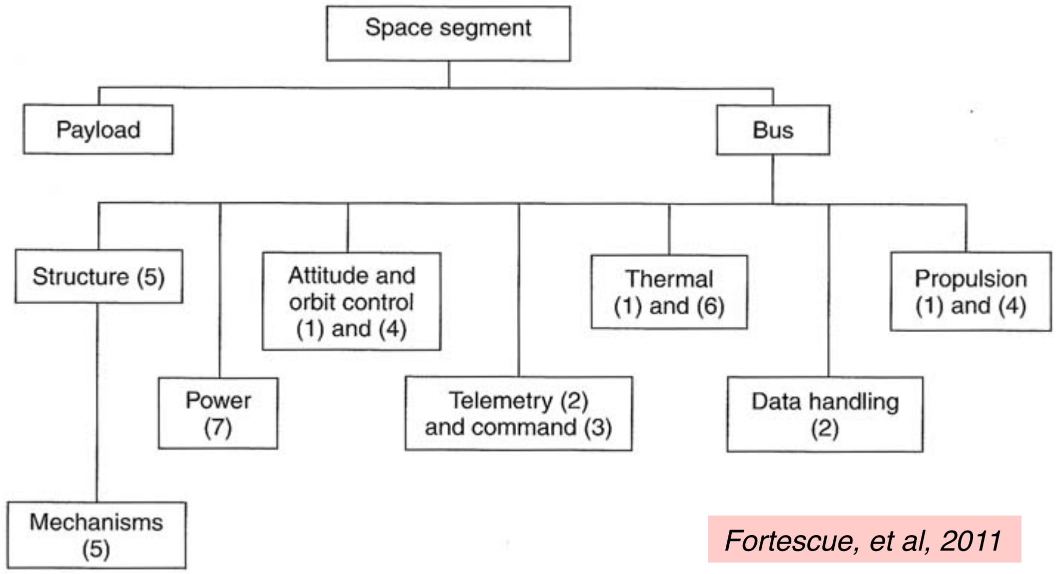
Overview

Spacecraft Mission Objectives and Requirements



Fortescue, et al, 2011

Spacecraft Subsystems



Fortescue, et al, 2011

Functional Requirements of Spacecraft Subsystems

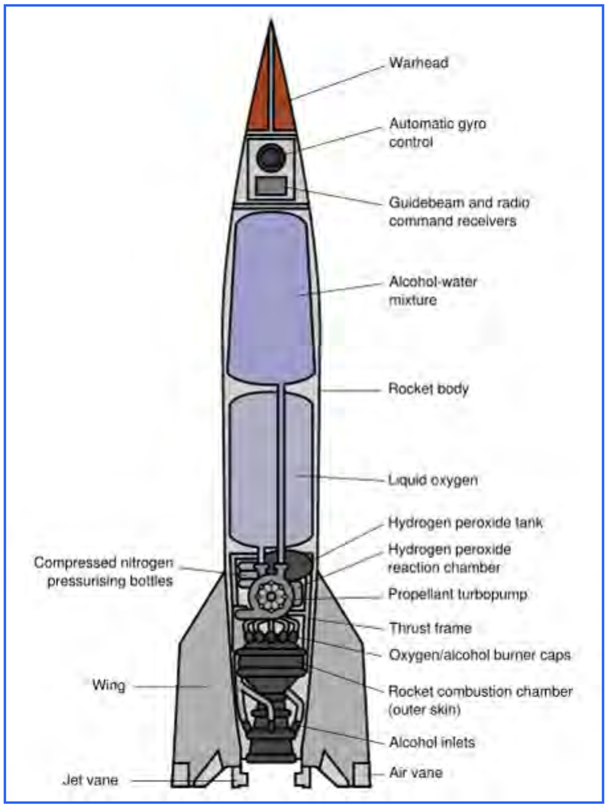
1. **Desired dynamic state (attitude, position, velocity) must be maintained**
2. **Desired orbits for the mission must be maintained**
3. **Payload must be operable**
4. **Payload must be held together and mounted on the spacecraft structure**
5. **Payload must operate reliably over some specified period**
6. **Adequate power must be provided**
7. **Operation should be largely autonomous, but …**
   - a) **Data must be communicated to the ground**
   - b) **Ground control must be maintained**

Brief History of Spaceflight

Early History of Space Systems

- **1926-1945: Goddard rockets; V-2 and its precursors**
  - Development of rocket technology
  - Development of guidance and control systems
- **1945-1949: Learning from the V-2; Altitude sounding**
  - V-2/WAC-Corporal to 250-mi altitude (Project Bumper)
  - Development of military missiles

German A-3



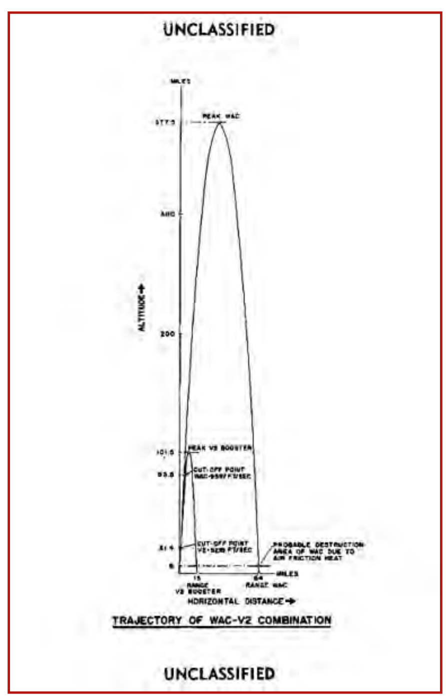
German A-4 (V-2) Rocket

Wernher Von Braun

- Liquid-fuel rocket
- 6,084 built; 1000+ test flights; 3,225 launched in combat
- Gyroscopes and accelerometer for guidance
- Air and jet vanes for pitch, yaw, and roll control torques
- Aft tail for aerodynamic stability

Project Bumper (V-2/WAC Corporal)

- 8 flights, 4 failures; Mach 9, 400-km apogee
- Engineering development
- High-altitude photography
- Atmospheric temperature profile
- Cosmic radiation

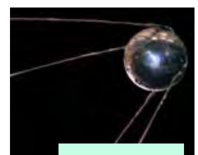


Post-WWII History of Space Systems

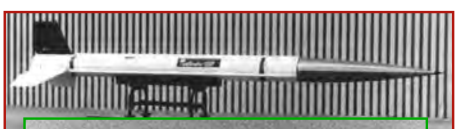
- **1945-1957: Payload design; animals in space**
  - Sounding rockets
  - Aerobee, Viking ➔ Vanguard
  - IRBMs and ICBMs received major emphasis in US and USSR

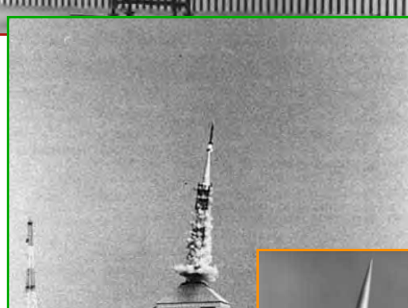
- **1957-1961: Unmanned satellites; animals in orbit; manned spaceflight about the Earth**
  - 1957: Sputnik 1
  - 1958: Van Allen belts (Explorer 1); NACA -> NASA
  - 1959: Luna 1-3
  - 1961: Gagarin orbit; Ham and Shepard sub-orbit; Enos orbit


Aerobee


Sputnik 1

Sounding Rockets

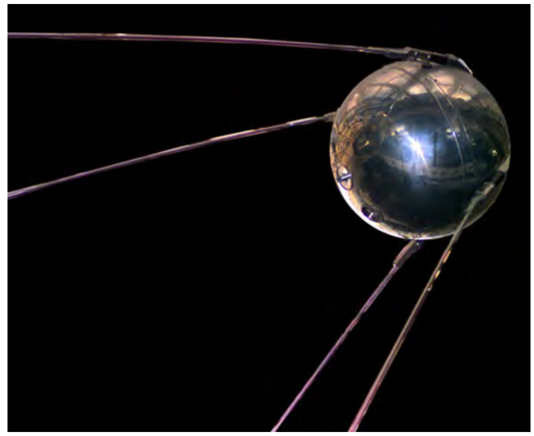




- Several minutes of high-altitude flight
  - "Weightlessness"
  - "Above the atmosphere"
  - Near-vacuum
  - High-altitude measurements
- Simplicity, low cost
- Recoverable payloads
- Wide range of flight conditions

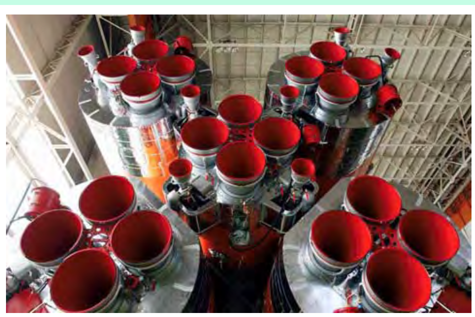
Sputnik 1
(October 4, 1957)

- 84 kg, 58-cm diameter
- 96-min, elliptical orbit
- 1,440 orbits
- Measurements
  - Gravity
  - Ionospheric effects
  - Internal temperature and pressure
  - Micrometeoroid detection



R-7 (Semyorka) Launch Vehicle
(October 4, 1957)

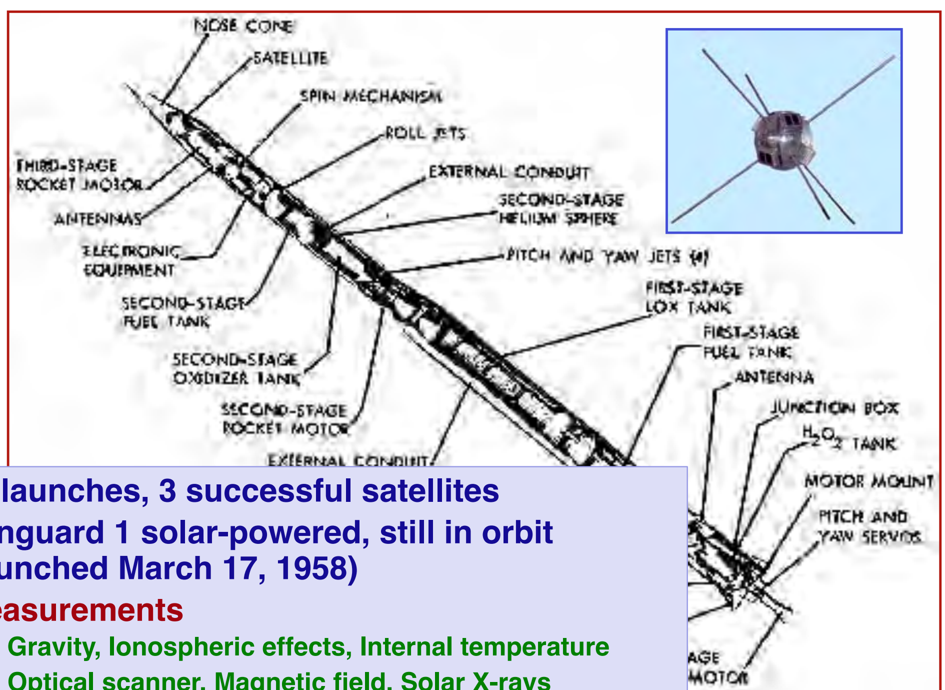
- R-7 (Semyorka) launch vehicle
  - 1-1/2-stage ICBM
  - 4 strap-on booster rockets
  - 1 core-stage rocket
  - Liquid oxygen and kerosene
- Lift-off thrust: 3.9 MN
- Gross weight: 267 metric tons



Project Vanguard (1957-1959)

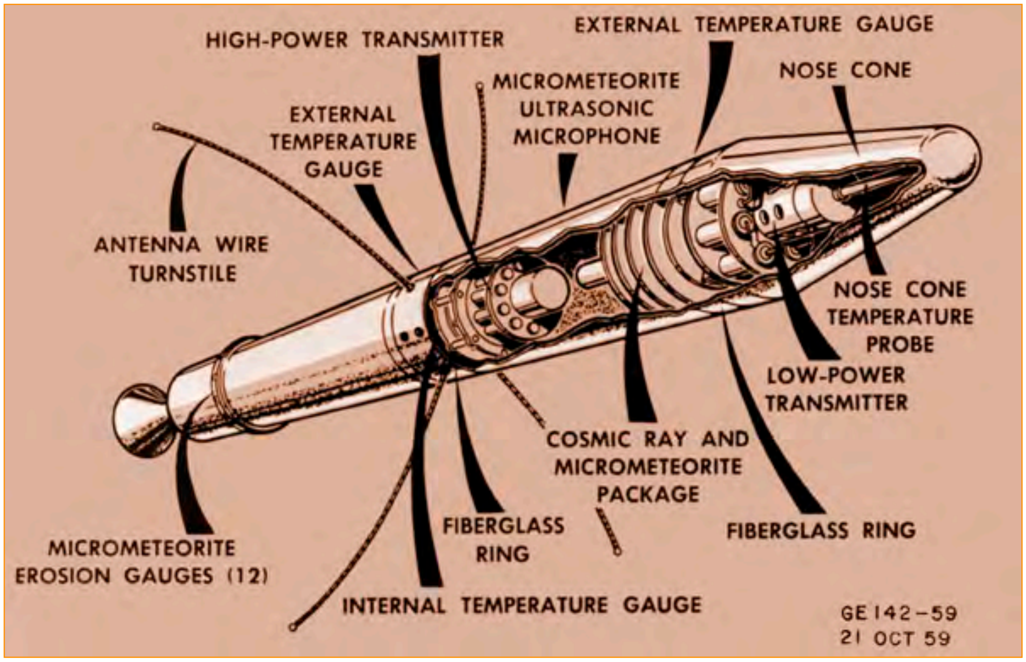
- **3 stages**
  - **1st-stage based on Viking; gimballed motor for control**
  - **2nd-stage based on Aerobee; reaction-control thrusters**
  - **Solid-fuel 3rd stage; spin stabilized**

Project Vanguard (1957-1959)



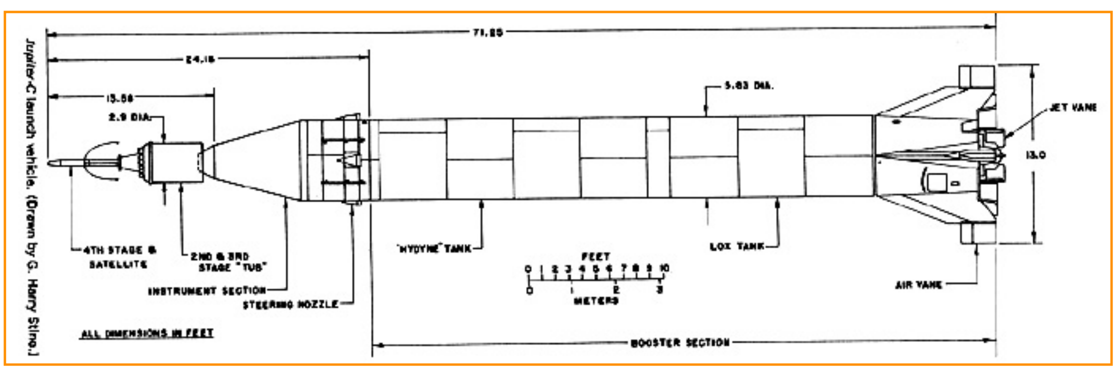
- **11 launches, 3 successful satellites**
- **Vanguard 1 solar-powered, still in orbit (launched March 17, 1958)**
- **Measurements**
  - **Gravity, Ionospheric effects, Internal temperature**
  - **Optical scanner, Magnetic field, Solar X-rays**
  - **Micrometeorites**

Explorer 1
(January 31, 1958)



Juno 1 Launched Explorer 1
(January 31, 1958)

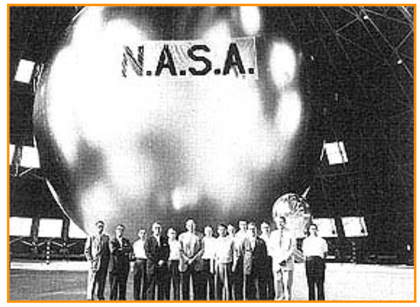
- Juno lineage from V-2
  - Jupiter
  - Redstone

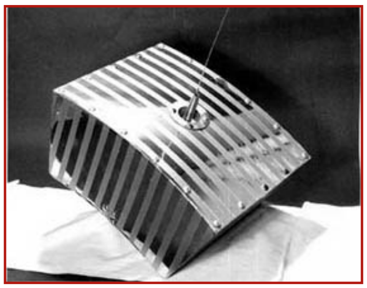


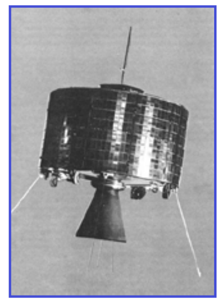
Jupiter-C launch vehicle. (Drawn by G. Harry Stine.)

Earth Satellite "Firsts"

- **Communications satellites**
  - 1960: Echo 1
  - 1961: First amateur radio satellite (OSCAR 1)
  - 1962: Telstar 1
  - 1963: Geosynchronous satellite (Syncom 1)

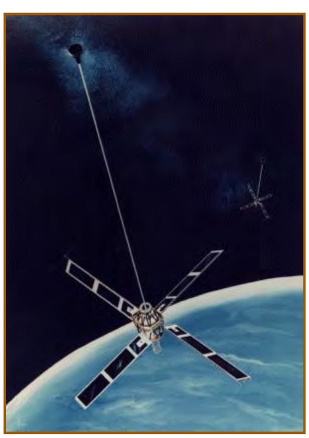


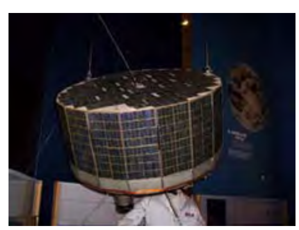




Earth Satellite "Firsts"

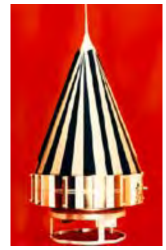
- **1960: Weather satellite:** TIROS-1
- **1972: Earth observation satellite:** Landsat 1 (ERTS 1)
- **1962: Navigation satellite:** Transit
- **1962: Astronomical satellite:** Ariel (UK/US)

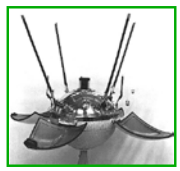




Lunar Probe “Firsts”

- **1959:**
  - –Lunar flyby (Luna 1, Pioneer 4)
  - –Lunar impact (Luna 2)
  - –Pictures of “The Far Side” (Luna 3)
- **1966:**
  - –Lunar soft landing (Luna 9, Surveyor 1)
  - –Lunar orbit (Lunar Orbiter 1)



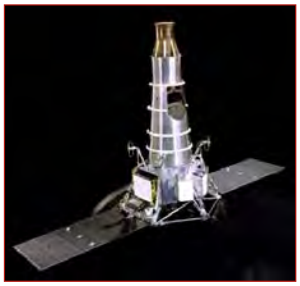


Typical Lunar Probe Instrumentation, 1959

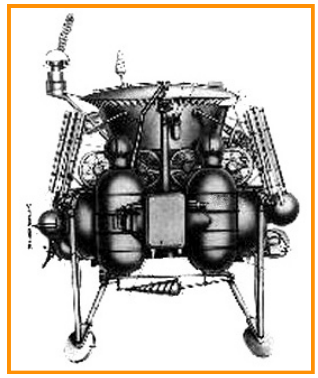
Radios for telemetry and command
Cosmic ray counter
Magnetometers
Temperature
Pressure
Micrometeorite sensors
Sodium cloud release

Sodium cloud release at high altitude, Wallops Island, 2009
https://www.youtube.com/watch?v=lb45uBaj2Mc

Lunar Probe "Firsts"

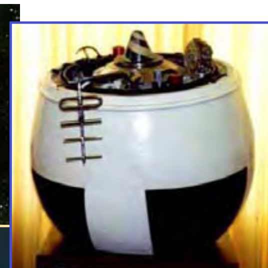


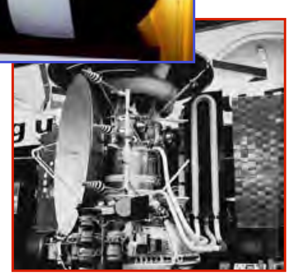
- **1967:**
  - **High-resolution photos (Ranger 7)**
- **1969:** ***Apollo 11***
- **1970:**
  - **Robotic sample return (Luna 16)**
  - **Robotic lunar rover (Luna 17)**



Inner-Planet Probe "Firsts"

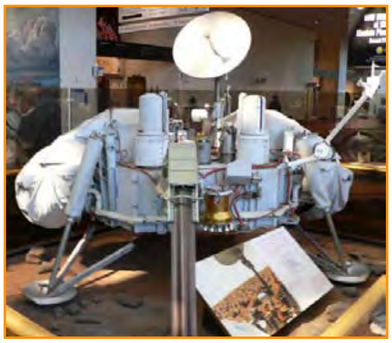
- **1962: Venus flyby (Mariner 2)**
- **1964: Mars flyby (Mariner 4)**
- **1970: Venus lander (Venera 8)**
- **1971: Mars orbit (Mars 2 Orbiter)**



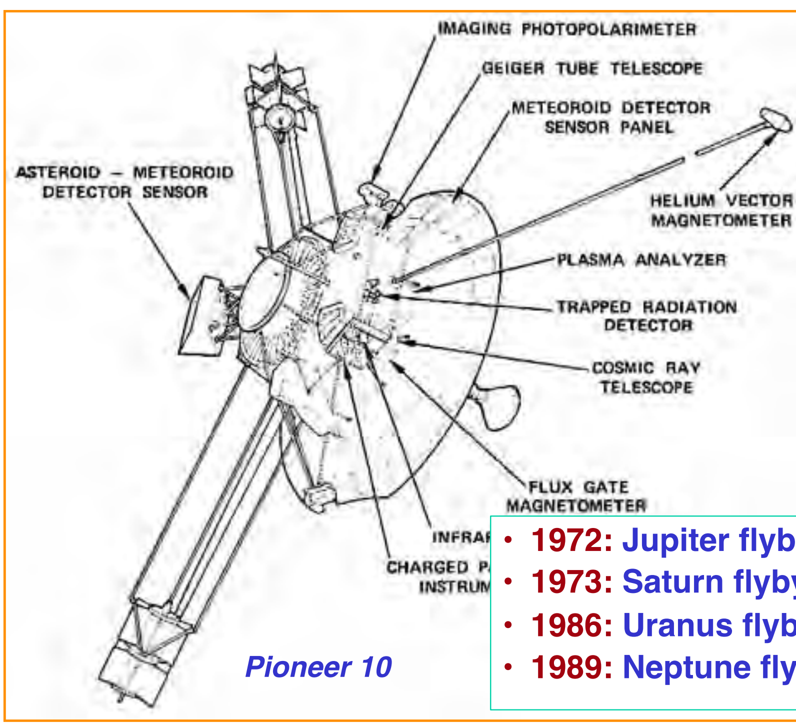


Inner-Planet Probe "Firsts"

- **1973:** Mercury flyby (Mariner 10)
- **1975:** Mars landing (Viking 1)
- **1978:** Venus orbit (Pioneer Venus 1)



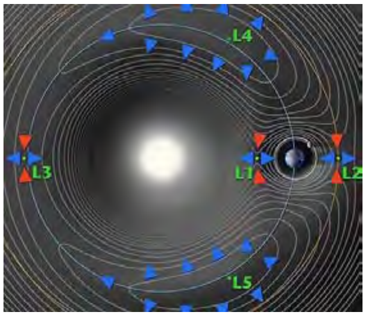
Outer-Planet Probe "Firsts"



Pioneer 10

Voyager 2

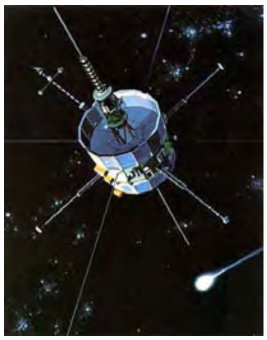
- **1972:** Jupiter flyby (Pioneer 10)
- **1973:** Saturn flyby (Pioneer 11)
- **1986:** Uranus flyby (Voyager 2)
- **1989:** Neptune flyby (Voyager 2)

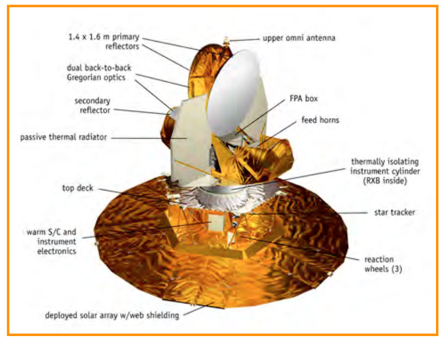


Lagrange-Point "Firsts"

- 5 equilibrium points in a rotating 2-body system, e.g.,
  - Sun-Earth
  - Earth-Moon

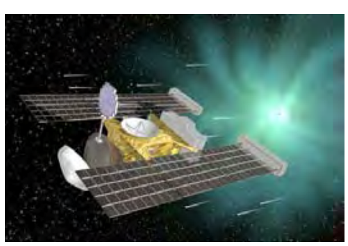
- **1978:** Solar observatory at L1 (ISEE-3); later rendezvoused with a comet as ICE (1983)
- **2001:** Astronomical observatory at L2 (Wilkinson Microwave Anisotropy Probe)

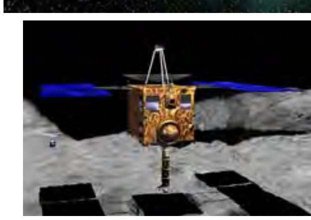




Comet and Asteroid Rendezvous "Firsts"

- **1999:** Comet sample return (Stardust)
- **2005:** Asteroid landing (Muses-C/Hayabusa)
- **2014:** Comet 67P Flyby/Landing (Rosetta/Philae)





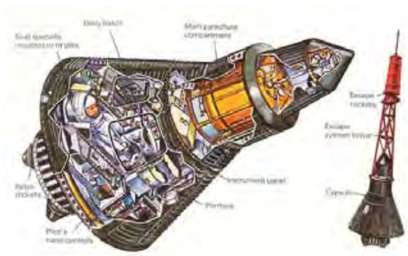
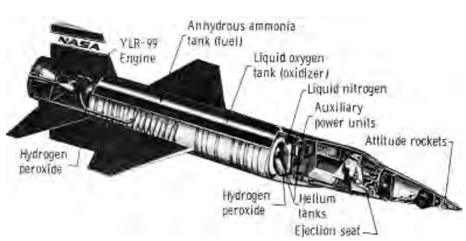
Manned Spacecraft Launch Vehicles

- **1961: Gagarin orbit (Vostok); Shepard sub-orbit (Mercury/Redstone)**
- **1962: Glenn orbit (Mercury/ Atlas)**
- **1964: USSR 3-person crew in orbit (Voshkod)**
- **1965: US 2-person crew in orbit (Gemini/Titan II)**



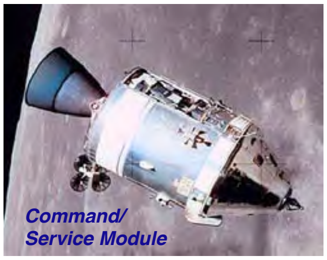

Manned Spacecraft

- **1961: Gagarin orbit (Vostok); Shepard sub-orbit (Mercury/Redstone)**
- **1962: Glenn orbit (Mercury/Atlas)**
- **1963: X-15 rocket plane reaches 100-km altitude**
- **1964: USSR 3-person crew in orbit (Voshkod)**
- **1965: US 2-person crew in orbit (Gemini/Titan II)**

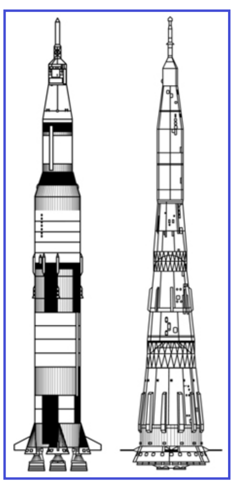
Manned Flight to the Moon

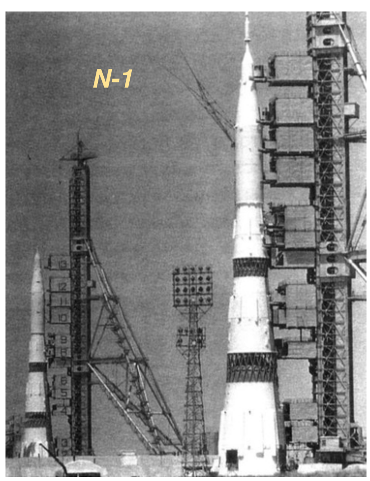
- **1961-1972: Apollo Program**
  - 6 lunar landings and returns

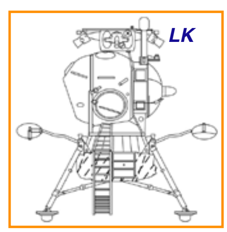


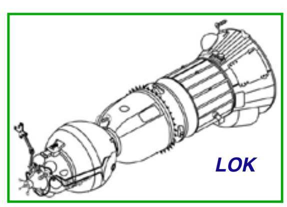
Manned Flight to the Moon

- **1961-1974: Soviet Lunar Program**
  - 4 launches (unmanned), none successful



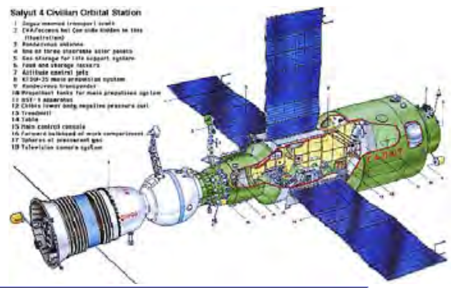


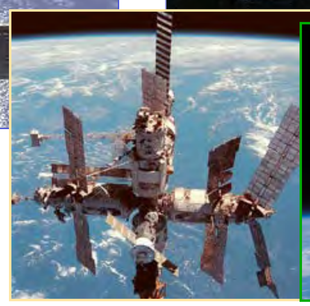


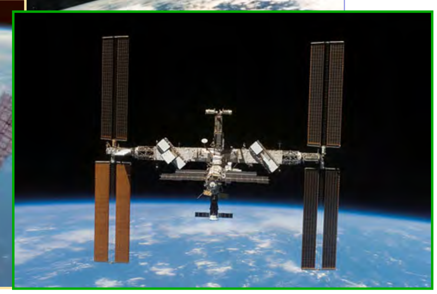


Space Stations

- **1971-1982: Salyut**
- **1973-1974: Skylab**
- **1975: Soyuz-Apollo docking**
- **1986-2001: Mir**
- **1998-present: International Space Station**

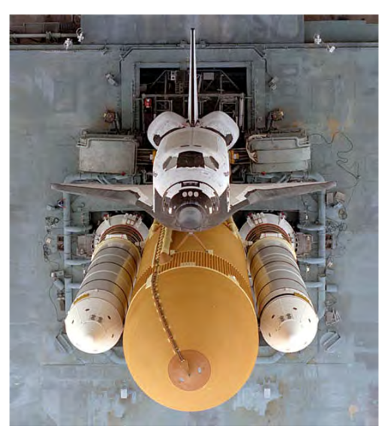






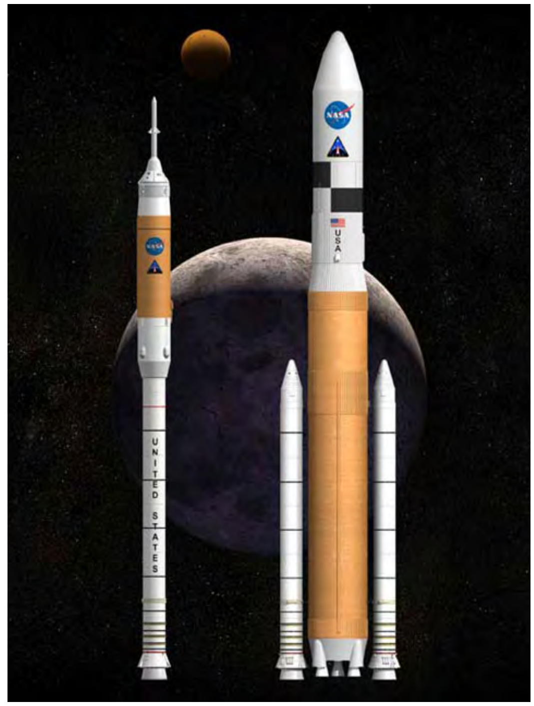
Space Shuttle

1981-2011:
- 5-7 astronauts
- 50,000-lb payload
- 135 missions flown
- 5 operational vehicles; 2 destroyed
  - 1986: *Challenger* accident
  - 2003: *Columbia* accident

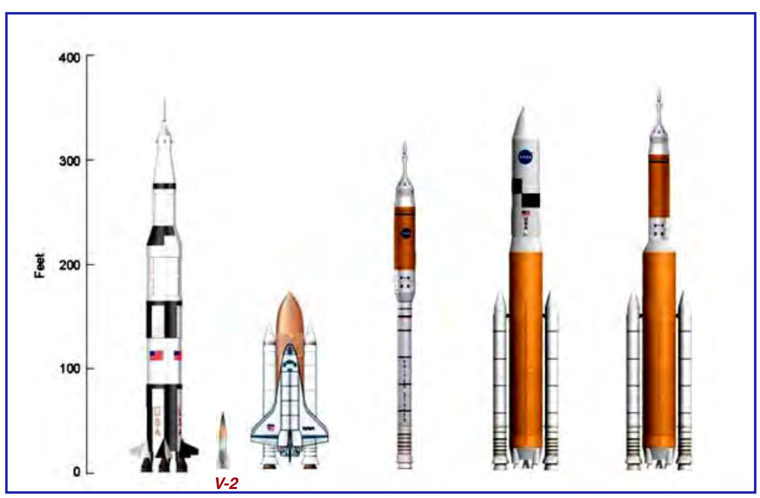


Project Constellation

- Orion: Crew Spacecraft
- Ares 1: Crew Launch Vehicle
  - First (only) unmanned launch: 2009
  - 5-segment Shuttle Solid-Rocket Booster
- Ares 5: Cargo Launch Vehicle
- Manned return to the Moon
- Project cancelled in 2011
- Development of Orion continues
- Ares 5 morphed into the Space Launch System (SLS)



Saturn V - Space Shuttle – Ares - SLS Size Comparison



Planetary Defense Term Project

- Design of spacecraft to protect against asteroid/ comet impact that would extinguish life on Earth
- Detection, characterization, intercept, and deflection of a "Doomsday Rock"
- *Design Teams* to address distant and near-Earth intercepts
- Single final report written "with one voice" by the class as a *Working Group*

Assignment #1 Report on the Book, *Project Icarus*

- Teams will discuss segments of the 1979 book during Lab Session, Feb 10th, including the following:
  - *Overview*
  - *Main points*
  - *Conclusions to be drawn*
- *Team members TBD*

Next Time:
Orbital Mechanics

Supplemental Material

Math Review

- Scalars and Vectors
- Sums and Multiplication
- Inner Product
- Derivatives and Integrals

Scalars and Vectors

- **Scalar**: usually lower case: *a*, *b*, *c*, …, *x*, *y*, *z*
- **Vector**: usually bold or with underbar: **x** or $\underline{x}$
    - Ordered set
    - Column of scalars
    - Dimension = *n* x 1

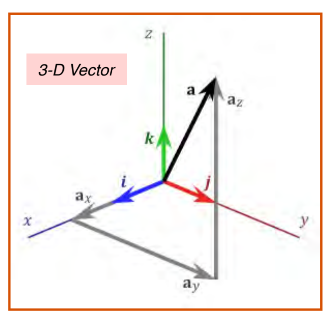
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad ; \quad \mathbf{y} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

3 x 1 4 x 1

Transpose: interchange rows and columns

$$\mathbf{x}^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$$

1 x 3



Multiplication of Vector by Scalar

Multiplication of vector by scalar is associative, commutative, and distributive

$$a\mathbf{x} = \mathbf{x}a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}$$

$$a(\mathbf{x}+\mathbf{y}) = (\mathbf{x}+\mathbf{y})a = (a\mathbf{x}+a\mathbf{y})$$

$\dim(\mathbf{x}) = \dim(\mathbf{y})$

$$a\mathbf{x}^T = \begin{bmatrix} ax_1 & ax_2 & ax_3 \end{bmatrix}$$

- Could we add $(\mathbf{x}+a)$?
- Only if $\dim(\mathbf{x}) = (1 \times 1)$

MATLAB allows it as an "overloaded function" *https://en.wikipedia.org/wiki/Function_overloading*

Addition

Conformable vectors and matrices are added term by term

$$\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix} \quad ; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\mathbf{x}+\mathbf{z} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$$

Inner (Dot) Product

Inner (dot) product of vectors produces a scalar result

$$\mathbf{x}^T\mathbf{x} = \mathbf{x} \bullet \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(1 \times m)(m \times 1) = (1 \times 1)$$

$$= (x_1^2 + x_2^2 + x_3^2)$$

Derivatives and Integrals of Vectors

Derivatives and integrals of vectors are vectors of derivatives and integrals

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{bmatrix}$$

$$\int \mathbf{x}\,dt = \begin{bmatrix} \int x_1\,dt \\ \int x_2\,dt \\ \int x_3\,dt \end{bmatrix}$$

$$\mathbf{x}(t) = \begin{bmatrix} 7 \\ 8t \\ 9t^2 \end{bmatrix}; \quad \frac{d\mathbf{x}(t)}{dt} = \begin{bmatrix} 0 \\ 8 \\ 18t \end{bmatrix}$$

$$\mathbf{x}(t) = \begin{bmatrix} 7 \\ 8t \\ 9t^2 \end{bmatrix}; \quad \int \mathbf{x}(t)\,dt = \begin{bmatrix} 7t + x_1(0) \\ 8t^2/2 + x_2(0) \\ 9t^3/3 + x_3(0) \end{bmatrix}$$

MATLAB Code for Math Review

```
%   MAE 345 Lecture 1 Math Review
%   Rob Stengel

    clear
    disp('   ')
    disp('=====================================')
    disp('>>>MAE 345 Lecture 1 Math Review<<<')
    disp('=====================================')
    disp('   ')
    disp(['Date and Time are ', num2str(datestr(now))]);
    disp('   ')

%   Scalars and Vectors
    a   =   4               %   Scalar
    x   =   [1; 2; 3]       %   Column Vector
    y   =   [4; 5; 6; 7]    %   Column Vector

%   Vector Transpose
    xT  =   x'
    yT  =   y'

%   Multiplication by Scalar
    w   =   a * x
    v   =   x * a
    wT  =   a * xT
```


MATLAB Code for Math Review

```
%   Vector Addition
    zz  =   [8; 9; 10]
    u   =   x + zz

%   Inner (Dot) Product
    zzz =   x' * x

%   Symbolic Toolbox
    disp(' ')
    disp('Symbolic Toolbox')
    disp(' ')
    syms x y z z1 z2 z3 z4

    y   =   x * x       %   Define Function
    z   =   diff(y)     %   Differentiate Function
    z1  =   int(y)      %   Integrate Function

    z2  =   [x; y; z]   %   Column Vector

    z3  =   diff(z2)    %   Derivative of Column Vector
    z4  =   int(z2)     %   Integral of Column Vector
```

MATLAB Command Window Output for Math Review

```
=======================================
>>>MAE 345 Lecture 1 Math Review<<<
=======================================
Date and Time are 24-May-2013 12:31:13

a =  4

x =
     1
     2
     3

y =
     4
     5
     6
     7

xT = 1     2     3

yT = 4     5     6     7
```

```
w =
     4
     8
    12

v =
     4
     8
    12

wT = 4     8     12

zz =
     8
     9
    10

u =
     9
    11
    13

zzz = 14
```

```
Symbolic Toolbox

y = x^2

z = 2*x

z1 = x^3/3

z2 =
  x
 x^2
 2*x

z3 =
  1
 2*x
  2

z4 =
 x^2/2
 x^3/3
  x^2
```


Math Review

- *Matrix and Transpose*
- *Sums and Multiplication*
- *Matrix Products*
- *Identity Matrix*
- *Matrix Inverse*
- *Transformations*

Matrix and Transpose

- **Matrix**:
  - Usually bold capital or capital: **F** or F
  - Dimension = (*m* x *n*)

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}$$

4 x 3

- **Transpose**:
  - Interchange rows and columns

$$\mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}$$

3 x 4

Matrix Products

Matrix-vector product transforms one vector into another

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 + cx_3 \\ dx_1 + ex_2 + fx_3 \\ gx_1 + hx_2 + kx_3 \\ lx_1 + mx_2 + nx_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

Matrix-matrix product produces a new matrix

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}; \quad \mathbf{AB} = \begin{bmatrix} (a_1b_1 + a_2b_3) & (a_1b_2 + a_2b_4) \\ (a_3b_1 + a_4b_3) & (a_3b_2 + a_4b_4) \end{bmatrix}$$

$$(n \times m) = (n \times l)(l \times m)$$

Numerical Example 1

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(n \times 1) = (n \times m)(m \times 1)$$

$$= \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Numerical Example 2

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad ; \quad \mathbf{B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} (5+14) & (6+16) \\ (15+28) & (18+32) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\mathbf{x}_A = \mathbf{A}\mathbf{x}_B \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B$$

$$\mathbf{x}_B = \mathbf{B}\mathbf{x}_o \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o$$

$$\mathbf{x}_A = \mathbf{A}\mathbf{x}_B = \mathbf{AB}\mathbf{x}_o \quad ; \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_A = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_o$$

Square Matrix Identity and Inverse

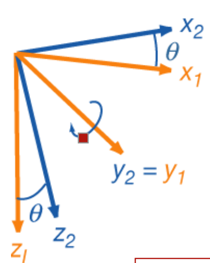
- **Identity matrix: no change when it multiplies a conformable vector or matrix**

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{I}\mathbf{x}$$

- **A non-singular square matrix multiplied by its inverse forms an identity matrix**

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$



Matrix Inverse Example

Transformation

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_2 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_1$$

Inverse Transformation

$$\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_2$$

Consequently, …

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1 = \mathbf{A}\mathbf{A}^{-1}\mathbf{x}_2 = \mathbf{x}_2$$

Computation of (*n* x *n*) Matrix Inverse

$$\mathbf{y} = \mathbf{A}\mathbf{x}; \quad \mathbf{x} = \mathbf{A}^{-1}\mathbf{y}$$
$$\dim(\mathbf{x}) = \dim(\mathbf{y}) = (n \times 1); \quad \dim(\mathbf{A}) = (n \times n)$$

$$[\mathbf{A}]^{-1} = \frac{\mathrm{Adj}(\mathbf{A})}{|\mathbf{A}|} = \frac{\mathrm{Adj}(\mathbf{A})}{\det \mathbf{A}} \quad \frac{(n \times n)}{(1 \times 1)}$$
$$= \frac{\mathbf{C}^T}{\det \mathbf{A}}; \quad \mathbf{C} = \textit{matrix of cofactors}$$

*Cofactors are <u>signed minors</u> of **A***

*ij^{th} minor of **A** is the <u>determinant</u> of **A** with the i^{th} row and j^{th} column removed*

MATLAB Code for Math Review

Use of Symbolic Variables

```
%   MAE 345 Lecture 2 Math Review
%   Rob Stengel

    clear
    disp('   ')
    disp('=====================================')
    disp('>>>MAE 345 Lecture 2 Math Review<<<')
    disp('=====================================')
    disp('   ')
    disp(['Date and Time are ', num2str(datestr(now))]);
    disp('   ')

%   Matrix
    syms A AT a b c d e f g h k l m n
    A   =   [a b c;d e f;g h k;l m n]   %   Matrix
    AT  =   A'                          %   Matrix Transpose

%   Matrix-Vector Product
    syms x x1 x2 x3 y1 y2 y3 y4
    x   =   [x1;x2;x3]
    y   =   [y1;y2;y3;y4]
    y   =   A * x
```


MATLAB Code for Math Review

```
%   Matrix-Matrix Product
    syms A a1 a2 a3 a4 B b1 b2 b3 b4 AB
    A   =   [a1 a2;a3 a4]
    B   =   [b1 b2;b3 b4]
    AB  =   A * B

%   Example 1
    syms A
    A   =   [2 4 6;3 -5 7;4 1 8;-9 -6 -3]
    y   =   A * x

%   Example 2
    A   =   [1 2;3 4]
    B   =   [5 6;7 8]
    AB  =   A * B

    syms xA xB x0
    x0  =   [x1;x2]
    xA  =   A * xB
    xB  =   B * x0
    xA  =   A * B * x0
```

MATLAB Code for Math Review

```
%   Matrix Identity and Inverse
    I3      =   eye(3)
    x       =   I3 * x
    syms A Ainv
    A       =   [a b c;d e f;g h k]
    Ainv    =   inv(A)
    I3      =   simplify(A * Ainv)
    I3      =   simplify(Ainv * A)

%   Matrix Inverse Example
    syms A Th cTh sTh Ainv
    A       =   [cTh 0 sTh;0 1 0;-sTh 0 cTh]
    Ainv    =   inv(A)
    detA    =   det(A)

    cTh     =   cos(Th)
    sTh     =   sin(Th)
    Th      =   pi / 4
    syms A Ainv
    A       =   [cos(Th) 0 sin(Th);0 1 0;-sin(Th) 0 cos(Th)]
    Ainv    =   inv(A)

%   Consequently, ...
    I3      =   A * Ainv

%   Computation of (n x n) Inverse
    detA    =   det(A)
    AdjA    =   Ainv * detA
```


MATLAB Command Window Output for Math Review

```
======================================
>>>MAE 345 Lecture 2 Math Review<<<
======================================

Date and Time are 03-Sep-2013 13:49:40

A =
[ a, b, c]
[ d, e, f]
[ g, h, k]
[ l, m, n]

AT =
[ conj(a), conj(d), conj(g), conj(l)]
[ conj(b), conj(e), conj(h), conj(m)]
[ conj(c), conj(f), conj(k), conj(n)]

x =
 x1
 x2
 x3

y =
 y1
 y2
 y3
 y4

y =
 a*x1 + b*x2 + c*x3
 d*x1 + e*x2 + f*x3
 g*x1 + h*x2 + k*x3
 l*x1 + m*x2 + n*x3
```

```
A =
[ a1, a2]
[ a3, a4]

B =
[ b1, b2]
[ b3, b4]

AB =
[ a1*b1 + a2*b3, a1*b2 + a2*b4]
[ a3*b1 + a4*b3, a3*b2 + a4*b4]

A =  2     4     6
     3    -5     7
     4     1     8
    -9    -6    -3

y = 2*x1 + 4*x2 + 6*x3
    3*x1 - 5*x2 + 7*x3
      4*x1 + x2 + 8*x3
  - 9*x1 - 6*x2 - 3*x3

A =  1     2
     3     4

B =  5     6
     7     8

AB = 19    22
     43    50
```

```
x0 =
 x1
 x2

xA =
[   xB, 2*xB]
[ 3*xB, 4*xB]

xB =
 5*x1 + 6*x2
 7*x1 + 8*x2

xA =
 19*x1 + 22*x2
 43*x1 + 50*x2

I3 =
     1     0     0
     0     1     0
     0     0     1

x =
 x1
 x2
 x3
```

MATLAB Command Window
Output for Math Review

```
A =
[ a, b, c]
[ d, e, f]
[ g, h, k]

Ainv =
[  (f*h - e*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k), -(c*h - b*k)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k), -(b*f - c*e)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]
[ -(f*g - d*k)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k),  (c*g - a*k)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k),  (a*f - c*d)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]
[ -(d*h - e*g)/(a*f*h - b*f*g - c*d*h + c*e*g - a*e*k +
b*d*k),  (a*h - b*g)/(a*f*h - b*f*g - c*d*h + c*e*g -
a*e*k + b*d*k), -(a*e - b*d)/(a*f*h - b*f*g - c*d*h +
c*e*g - a*e*k + b*d*k)]

I3 =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]

I3 =
[ 1, 0, 0]
[ 0, 1, 0]
[ 0, 0, 1]
```

```
A = [  cTh, 0, sTh]
     [    0, 1,   0]
   [ -sTh, 0, cTh]

Ainv =
[ cTh/(cTh^2 + sTh^2), 0, -sTh/(cTh^2 + sTh^2)]
[                   0, 1,                    0]
[ sTh/(cTh^2 + sTh^2), 0,  cTh/(cTh^2 + sTh^2)]

detA = cTh^2 + sTh^2

cTh = cos(Th)

sTh = sin(Th)

Th = 0.7854

A = 0.7071         0    0.7071
         0    1.0000         0
   -0.7071         0    0.7071

Ainv = 0.7071         0   -0.7071
            0    1.0000         0
       0.7071         0    0.7071

I3 = 1     0     0
     0     1     0
     0     0     1

detA = 1

AdjA = 0.7071         0   -0.7071
            0    1.0000         0
       0.7071         0    0.7071
```