A Seasonal Box-Jenkins Model for Nigerian Inflation Rate Series

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Abstract

Time series analysis of Nigerian Inflation rate series is done. A seasonal difference and then a non-seasonal one were obtained. The correlogram of the differenced series revealed a seasonal nature. It also revealed a seasonal moving average component and a non-seasonal autoregressive component. $A(5, 1, 0) \times (0, 1, 1)_{12}$ seasonal model was fitted and shown to be adequate.

Keywords: inflation, ARIMA modelling, seasonal models, Nigeria

1. Introduction

A time series is defined as a set of data collected sequentially in time. It has the property that neighbouring values are correlated. This tendency is called autocorrelation. A time series is said to be stationary if it has a constant mean and variance. Moreover the autocorrelation is a function of the lag separating the correlated values and called the autocorrelation function (ACF).

A stationary time series $\{X_t\}$ is said to follow an autoregressive moving average model of orders p and q (designated ARMA(p,q) if it satisfies the following difference

$$X_t + 1X_{t-1} + 2X_{t-2} + ... + pX_{t-p} = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_2\varepsilon_{t-2} + \beta_q\varepsilon_{t-q} \tag{1}$$

or

$$A(L)X_t = B(L)\varepsilon_t \tag{2}$$

where $\{\varepsilon_t\}$ is a sequence of uncorrelated random variables with zero mean and constant variance, called a white noise process, and the $\alpha'_i s$ and $\beta'_j s$ constants;

$$A(L) = 1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_p L^p$$

and

$$B(L) = 1 + \beta_1 L + \beta_2 L^2 + ... + \beta_q L^q$$

and L is the backward shift operator defined by $L^k X_t = X_{t-k}$.

If $p = 0$, model (1) becomes *a moving average model of order q* (designated MA(q)). If, however, $q = 0$ it becomes *an autoregressive process of order p* (designated AR(p)). An AR(p) model of order p may be defined as a model whereby a current value of the time series X_t depends on the immediate past p values: $X_{t-1}, X_{t-2}, ..., X_{t-p}$. On the other hand an MA(q) model of order q is such that the current value X_t is a linear combination of the immediate past values of the white noise process: $\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_{t-q}$. Apart from stationarity, invertibility is another important requirement for a time series. It refers to the property whereby the covariance structure of the series is unique (Priestley, 1981). Moreover it allows for meaningful association of current events with the past history of the series (Box & Jenkins, 1976).

An AR(p) model may be more specifically written as

$$X_t + \alpha_{p1}X_{t-1} + \alpha_{p2}X_{t-2} + ... + \alpha_{pq}X_{t-q} = \varepsilon_t$$

Then the sequence of the last coefficients $\{\alpha_{ii}\}$ is called the partial autocorrelation function (PACF) of $\{X_t\}$. The ACF of an MA(q) cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off

slowly. On the other hand the PACF of an MA(q) model dies off slowly whereas that of an AR(p) model cuts off after lag p. AR and MA models are known to have some duality properties. These include:

1) A finite order AR model is equivalent to an infinite order MA model.

2) A finite order MA model is equivalent to an infinite order AR model.

3) The ACF of an AR model exhibits the same behaviour as the PACF of an MA model.

4) The PACF of an AR model exhibits the same behaviour as the ACF of an MA model.

5) An AR model is always invertible but is stationary if $A(L) = 0$ has roots outside the unit circle.

6) An MA model is always stationary but is invertible if $B(L) = 0$ has roots outside the unit circle.

Parametric parsimony consideration in model building entails preference for the mixed ARMA fit to either the pure AR of the pure MA fit. Stationarity and invertibility conditions for model (1) or (2) are that the equations $A(L) = 0$ and $B(L) = 0$ should have roots outside the unit circle respectively.

Often in practice a time series is non-stationary. Box and Jenkins (1976) proposed that differencing of an appropriate order could render a non-stationary series $\{X_t\}$ stationary. Let the degree of differencing necessary for stationarity be d. Such a series $\{X_t\}$ may be modelled as

$$(1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_p L^p)\nabla^d X_t = B(L)\varepsilon_t \tag{3}$$

where $\nabla = 1 - L$ and in which case

$$A(L) = (1 + \alpha_1 L + \alpha_2 L^2 + ... + \alpha_p L^p)\nabla^d = 0$$

shall have unit roots d times. Then differencing to degree d renders the series stationary. The model (3) is said to be an autoregressive integrated moving average model of orders p, d and q and designated ARIMA(p,d,q).

1.1 Seasonal ARIMA Models

A time series is said to be seasonal of order d if there exists a tendency for the series to exhibit periodic behaviour after every time interval d. Traditional time series methods involve the identification, unscrambling and estimation of the traditional components: secular trend, seasonal component, cyclical component and the irregular movement. For forecasting purpose, they are reintegrated. Such techniques could be quite misleading.

The time series $\{X_t\}$ is said to follow a multiplicative $(p, d, q) \times (P, D, Q)_s$ seasonal ARIMA model if

$$A(L)\Phi(L^s)\nabla^d\nabla_s DX_t = B(L)\Theta(L^s)\varepsilon_t \tag{4}$$

where Φ and Θ are polynomials of order P and Q respectively. That is,

$$\Phi(L^s) = 1 + \phi_1 L^s + ... + \phi_P L^{sP} \tag{5}$$

$$\Theta(L^s) = 1 + \theta_1 L + ... + \theta_Q L_{sQ} \tag{6}$$

where the ϕ_i and θ_j are constants such that the roots of the equations (5) and (6) are all outside the unit circle for stationarity and invertibility respectively. Equation (5) represents the autoregressive operator whereas (6) represents the moving average operator.

Existence of a seasonal nature is often evident from the time plot. Moreover for a seasonal series the ACF or correlogram exhibits a spike at the seasonal lag. Box and Jenkins (1976) and Madsen (2008) are a few authors that have written extensively on such models. A knowledge of the theoretical properties of the models provides basis for their identification and estimation. The purpose of this paper is to fit a seasonal ARIMA model to Nigerian Inflation Rate Series (NINFR). Earlier works on Nigerian inflation include Abidemi and Maliq (2010), Olatunji et al. (2010) and Chiakwelu (2012). Whereas Abidemi and Maliq think that Nigerian inflation rates are stationary Olatunji et al. are of a contrary opinion. The graph of the inflation rates from 1990 to 2010 given by Chiakwelu corroborates the opinion of Olatunji et al. There is, perhaps, no known earlier attempt to model Nigerian inflation rates by a seasonal ARIMA model.

2. Materials and Methods

The data for this work are monthly composite price index series (NCPI) from March 1963 to December 2003 obtainable from the Abstracts of the National Bureau of Statistics of Nigeria. The inflation rate was calculated by

$$NINFR(t) = [NCPT(t) - NCPI(t-1)]/NCPI(t-1)$$

2.1 Determination of the Orders d, D, p, P, q and Q

Seasonal differencing is necessary to remove the seasonal trend. If there is secular trend non-seasonal differencing will be necessary. To avoid undue complexity it has been advised that orders of differencing d and D should add up to at most 2 (i.e. $d + D < 3$). If the ACF of the differenced series has a positive spike at the seasonal lag then a seasonal AR component is suggestive; if it has a negative spike then a seasonal MA term is suggestive. As already mentioned above, an AR(p) model has a PACF that truncates at lag p and an MA(q) has an ACF that truncates at lag q. In practice $\pm 2/\sqrt{n}$ where n is the sample size are the non-significance limits for both functions.

2.2 Model Estimation

The involvement of the white noise terms in an ARIMA model entails a nonlinear iterative process in the estimation of the parameters. An optimization criterion like least error sun of squares, maximum likelihood or maximum entropy is used. An initial estimate is usually used. Each iteration is expected to be an improvement on the last one until the estimate converges to an optimal one. However, for pure AR and a pure MA models linear optimization techniques exist (See for example, Box & Jenkins, 1976; Oyetunji, 1985). There are attempts to adopt linear methods to estimate ARMA models (See for example, Etuk, 1987; 1998). We shall use Eviews software which employs the least squares approach involving nonlinear iterative techniques.

2.3 Diagnostic Checking

The model that is fitted to the data should be tested for goodness-of-fit. We shall do some analysis of the residuals of the model. If the model is correct, the residuals would be uncorrelated and would follow a normal distribution with mean zero and constant variance. The autocorrelations of the residuals should not be significantly different from zero.

3. Results and Discussion

The time plot of the original series in Figure 1 does not show any secular trend. Because of the so many spikes involved seasonality is not obvious. Seasonal (i.e. 12-month) differencing of the series produces a series SDNINFR with no trend and no noticeable seasonality still. See Figure 2 for this time plot. Non-seasonal differencing yields a series DSDNINFR with no trend and no clear seasonality (see Figure 3). Its ACF in Figure 4 has a negative spike at lag 12 revealing a seasonality of lag 12 and a seasonal MA component to the model. The PACF cuts off at lag 5 suggesting the involvement of a non-seasonal AR(5) component. We therefore propose a $(5, 1, 0) \times (0, 1, 1)_{12}$ seasonal model. That means
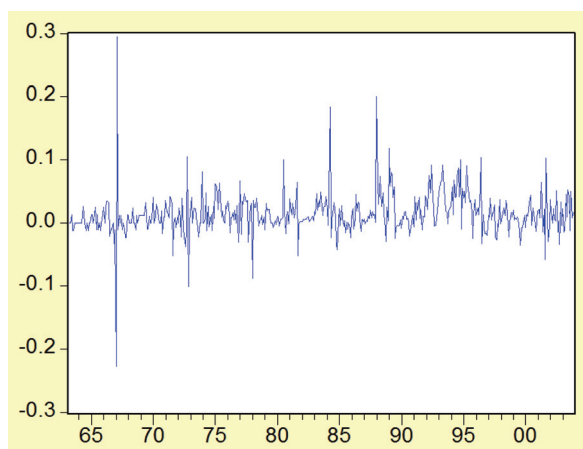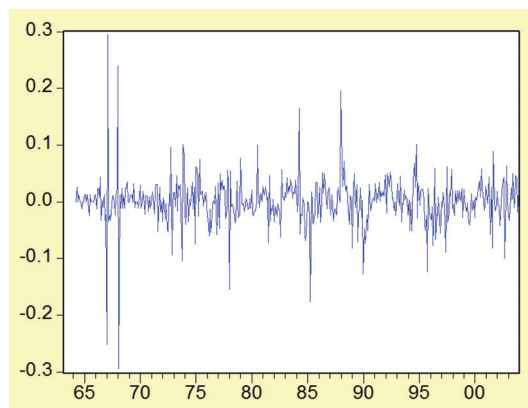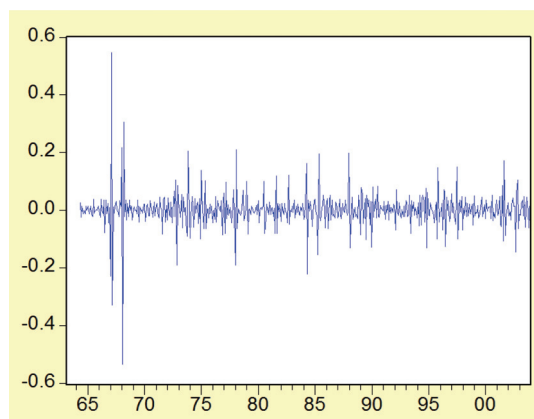

Figure 1. NINFR
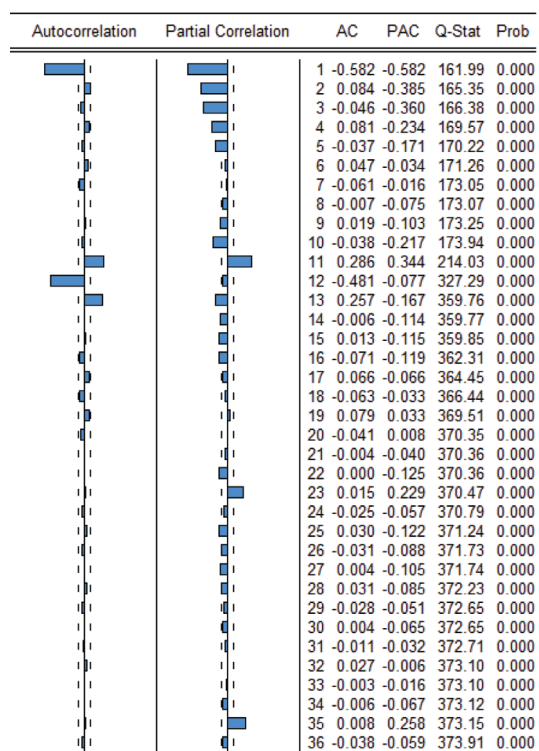
Figure 2. SDNINFR

Figure 3. DSDNINFR

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.582	-0.582	161.99	0.000
		2	0.084	-0.385	165.35	0.000
		3	-0.046	-0.360	166.38	0.000
		4	0.081	-0.234	169.57	0.000
		5	-0.037	-0.171	170.22	0.000
		6	0.047	-0.034	171.26	0.000
		7	-0.061	-0.016	173.05	0.000
		8	-0.007	-0.075	173.07	0.000
		9	0.019	-0.103	173.25	0.000
		10	-0.038	-0.217	173.94	0.000
		11	0.286	0.344	214.03	0.000
		12	-0.481	-0.077	327.29	0.000
		13	0.257	-0.167	359.76	0.000
		14	-0.006	-0.114	359.77	0.000
		15	0.013	-0.115	359.85	0.000
		16	-0.071	-0.119	362.31	0.000
		17	0.066	-0.066	364.45	0.000
		18	-0.063	-0.033	366.44	0.000
		19	0.079	0.033	369.51	0.000
		20	-0.041	0.008	370.35	0.000
		21	-0.004	-0.040	370.36	0.000
		22	0.000	-0.125	370.36	0.000
		23	0.015	0.229	370.47	0.000
		24	-0.025	-0.057	370.79	0.000
		25	0.030	-0.122	371.24	0.000
		26	-0.031	-0.088	371.73	0.000
		27	0.004	-0.105	371.74	0.000
		28	0.031	-0.085	372.23	0.000
		29	-0.028	-0.051	372.65	0.000
		30	0.004	-0.065	372.65	0.000
		31	-0.011	-0.032	372.71	0.000
		32	0.027	-0.006	373.10	0.000
		33	-0.003	-0.016	373.10	0.000
		34	-0.006	-0.067	373.12	0.000
		35	0.008	0.258	373.15	0.000
		36	-0.038	-0.059	373.91	0.000

Figure 4. Correlogram of DSDNINFR

$$X_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \alpha_3 X_{t-3} + \alpha_4 X_{t-4} + \alpha_5 X_{t-5} = \beta_{12}\varepsilon_{t-12} + \varepsilon_t \qquad (7)$$

where X represents DSDNINFR. The estimation of the model summarized in Table 1 yields the model

$$X_t + 1.0162X_{t-1} + 0.8235X_{t-2} + 0.6157X_{t-3} + 0.3357X_{t-4} + 0.1435X_{t-5} + 0.9348\varepsilon_{t-12} = \varepsilon_t$$

$$(\pm 0.0458)(\pm 0.0639)(\pm 0.0688)(\pm 0.0638)(\pm 0.0459)(\pm 0.0174)$$

Table 1. Model estimation dependent variable: DSDNINFR

Method: Least Squares
Date: 01/17/12   Time: 13:08
Sample(adjusted): 1964:10 2003:12
Included observations: 471 after adjusting endpoints
Convergence achieved after 7 iterations
Backcast: OFF (Roots of MA process too large for backcast)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-1.016220	0.045839	-22.16910	0.0000
AR(2)	-0.823456	0.063870	-12.89264	0.0000
AR(3)	-0.615654	0.068778	-8.951339	0.0000
AR(4)	-0.335682	0.063797	-5.261719	0.0000
AR(5)	-0.143493	0.045897	-3.126411	0.0019
MA(12)	-0.934773	0.017405	-53.70707	0.0000
R-squared	0.750042	Mean dependent var		4.47E-05
Adjusted R-squared	0.747355	S.D. dependent var		0.065535
S.E. of regression	0.032940	Akaike info criterion		-3.975574
Sum squared resid	0.504559	Schwarz criterion		-3.922646
Log likelihood	942.2478	F-statistic		279.0632
Durbin-Watson stat	2.017964	Prob(F-statistic)		0.000000
Inverted AR Roots	.21+.67i	.21 -.67i	-.36 -.53i	-.36+.53i
	-.71			
Inverted MA Roots	.99	.86+.50i	.86 -.50i	.50+.86i
	.50 -.86i	.00+.99i	-.00 -.99i	-.50+.86i
	-.50 -.86i	-.86+.50i	-.86 -.50i	-.99

The estimation involved 7 iterations. We note that all coefficients are significantly different from zero, each being larger than twice its standard error. The fact that $R^2 = 0.75$ implies that as much as 75% of the variations in DSDNINFR has been accounted for by the model. There is considerable agreement between the actual and the fitted models as shown in Figure 5. The histogram of the residuals in Figure 6 shows a normal distribution with zero mean. The correlogram of the residuals in Figure 7 further depicts the adequacy of the model since virtually all the autocorrelations are not significantly different from zero.
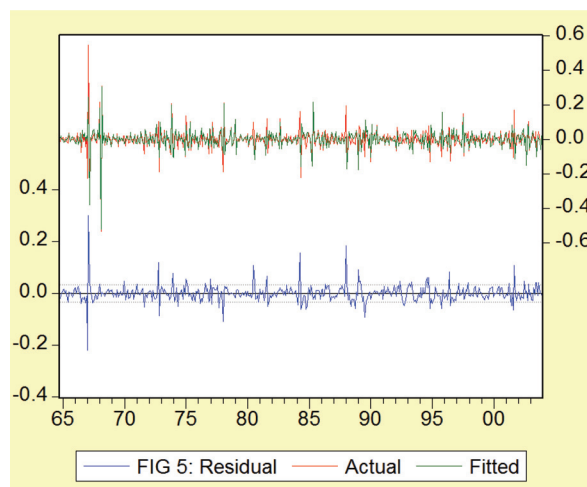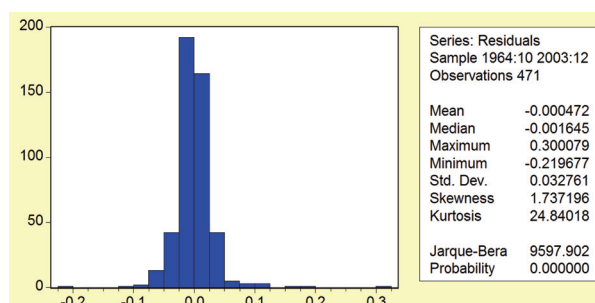

Figure 5. Residual

Figure 6. Histogram of the residuals

Figure 7. Correlogram of the residuals

4. Conclusion

The NINFR series has been shown to be nonstationary and to follow a $(5, 1, 0) \times (0, 1, 1)_{12}$ model. This model has been shown to be highly adequate.

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