Geometry Module

developed by
The Rice University
School Mathematics Project
(RUSMP)

Funding for the *Geometry Module* was provided by the Texas Education Agency and the Texas Higher Education Coordinating Board.

2004

The Texas Education Agency and the Texas Higher Education Coordinating Board
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Introduction

The Rice University School Mathematics Project (RUSMP) developed the *Geometry Module* as a comprehensive teacher training module with funding from the Texas Education Agency and the Texas Higher Education Coordinating Board. The *Geometry Module* effectively assists teachers in developing a deeper understanding of the underlying concepts that support the *Texas Essential Knowledge and Skills* (TEKS) in Geometry and helps teachers develop the pedagogical tools necessary to provide their students the opportunity to meet Texas' challenging state content and student performance standards. The *Geometry Module* also supports related TExES Mathematics Competencies. The rigor of the *Geometry Module* is of sufficient nature as to allow participating teachers who have not yet met the requirements of a "highly qualified" teacher, as defined by the United States NO CHILD LEFT BEHIND ACT of 2001 (NCLB), to progress towards this goal.

Theoretical Framework for the *Geometry Module*

The National Council of Teachers of Mathematics (NCTM) proposed major changes in pre-college mathematics curriculum in its *Standards* (1989, 1991, 1995, 2000). The National Research Council in *Adding It Up: Helping Children Learn Mathematics* (2001) and *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium* (2001) provides research-based recommendations for teaching and learning that support effective mathematics education. This research indicates that active, student-centered mathematical investigations, group cooperation, and alternative assessments are more effective in reaching diverse student populations than the passive, teacher-centered learning methods which have dominated mathematics instruction in the past. The *Geometry Module* materials are consistent with these recommendations.

The *Geometry Module* is based on the van Hiele model of geometric thought. NCTM in its *Standards* (1989), acknowledged the importance of the van Hieles' research.

> Development of geometric ideas progresses through a hierarchy of levels. Students first learn to recognize whole shapes and then to analyze the relevant properties of a shape. Later they can see relationships between shapes and make simple deductions (p. 48).

Traditional geometry curriculum often fails, because there is a mismatch between geometry instruction and a student's van Hiele level. The hierarchy of levels in the van Hiele model consists of (1) the Visual Level, (2) the Descriptive Level, (3) the Relational Level, (4) the Deductive Level, and (5) Rigor. The *Geometry Module* provides van Hiele-based experiences (Crowley, 1987) to move participants through the hierarchy from the Visual Level to Rigor. The *Geometry Module* provides descriptive behavior criteria which identify the different van Hiele levels of student performance, so that participants may identify and select corresponding activities to ensure success for all. Throughout the *Geometry Module*, participants will identify the van Hiele levels within the activities.

Tools for Learning Geometry

The *Geometry Module* utilizes construction tools, manipulatives, and technology: (1) to address various learning styles, (2) to model or represent mathematical concepts, (3) to abstract from the manipulative representations, (4) to construct and explore mathematical properties of geometric objects, (5) to generate authentic data, and most importantly (6) to progress participants through the van Hiele levels. The appropriate use of construction tools, manipulatives, the graphing calculator, *The Geometer's Sketchpad*, and NonEuclid is incorporated into module materials.

RUSMP's Unique Qualifications to Write the *Geometry Module*

RUSMP was established in 1987, with a grant from the National Science Foundation (NSF), in order to provide a bridge between the Rice University mathematics research community and Houston-area mathematics teachers. In addition to the original grant, RUSMP has received funding from a second NSF grant, the United States Department of Education Eisenhower and Teacher Quality Programs, and from corporations, foundations and school districts. The mission of RUSMP is to help teachers and administrators better understand the nature of mathematics, the effective teaching and assessing of mathematics, and the importance of mathematics in today's society. RUSMP's major goal is to enhance the mathematical and pedagogical knowledge of Houston PreK-12 teachers and support them in implementing more effective mathematics programs.

The RUSMP approach is founded on the belief that sustained instructional changes can best be supported through the development of professionalism among teachers and the creation of a network of teachers who have extensive knowledge of both mathematical content and pedagogy. All RUSMP activities are designed to support the development of teachers' professionalism.

Introduction

RUSMP has developed an extensive array of programs and courses available to teachers and administrators. These include long-term, intensive professional development for teachers, day-long workshops, and opportunities for networking across schools and districts. In addition, RUSMP has undertaken several collaborative projects with districts, schools, and other community members in the Houston area. While there is great diversity among the programs and activities offered by RUSMP, they are all anchored by a common curriculum and approach to instruction. The *Geometry Module* is the latest of RUSMP's efforts to improve the teaching of pre-college mathematics.

As a result of RUSMP's eighteen-year partnership with Houston-area school districts to improve mathematics instruction, RUSMP has the knowledge and experience necessary to develop an effective *Geometry Module* that meets the needs of current and future teachers. The *Geometry Module* builds upon the strengths and recommendations of prior curricula that RUSMP has designed and implemented for Houston-area PreK-12 teachers.

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January 28, 2004

Anne Papakonstantinou
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Dear Anne,

It was a pleasure seeing you last week. As we discussed, Key Curriculum Press is pleased to support you in your and your colleagues' efforts in producing geometry curriculum and materials in support of the Department of Higher Education, Participation & Success -- Institution & Educator Initiatives. Accordingly, we extend permission to you to reproduce, in print or electronically, portions of text and diagrams from the following Key Curriculum Press Publications for inclusion in Initiative materials:

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Sincerely,

Kelvin Taylor

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April 27, 2004

Dear Anne:

You are permitted to load *The Geometer's Sketchpad*® software program onto the computer labs in Fort Worth and Houston for the duration of the Geometry Module Workshops. After the completion of the workshops, please unload the programs from all the computers.

Feel free to contact me if you have any questions or concerns.

Thank You,

Lesa Zimmerman
Central Regional Manager
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The Texas Education Agency and The Texas Higher Education Coordinating Board
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Table of Contents

Comprehensive materials list

Consumables

easel paper (several sheets per group of 4)
colored markers
patty paper (several pieces per participant)
graph paper
colored pencils (1 package of assorted colors per group of 4)
centimeter grid paper (several sheets per participant)
small colored dot (1 for demonstration)
transparency sheet (several sheets per group of 4 and 1 for demonstration)
overhead projector pens (1 package of 4 colors per group of 4)
unlined 8.5 in. by 11 in. paper (several sheets per person)
3 in. by 5 in. index cards (1½ card per participant)
11 in. by 17 in. paper (1 per participant)
masking tape (1 roll per group of 4)
cardstock
floral wire (several pieces per participant)
modeling clay
one–inch easel grid paper (1 sheet per group of 4)
spaghetti
clear tape (1 roll per group of 4)
glue (1 bottle or stick per table)
cups (preferably large plastic cups)
geoboard dot paper (several sheets per participant—provided in the Appendix)
3 in. square adhesive notes (2 of different colors for each participant)
paper cone shaped drinking cups
plastic rice
string (1 spool per group of 4)
equilateral triangle paper with side length at least one inch (several sheets per participant provided in the Appendix)

Non-consumables

selection of geometry reference books or textbooks
centimeter ruler (1 per participant)
protractor (1 per participant)
linking cubes (several per participant)
small object such as a color tile
plastic mirror (1 per participant)
compass (1 per participant)
graphing calculator (1 per participant)
scissors (1 pair per participant)
straightedge (1 per participant)
centimeter cubes
geoboard (1 per participant)
centimeter grid transparency (1 for demonstration)

flexible protractor (1 per participant)
globe, beach ball, or Lénárt sphere (1 per group of 4)
transparencies "Constructing a Polygon's Exterior Angles" and "Determining the Sum of a Polygon's Exterior Angles"
wire-frame constructions from Unit 2: *Exploring Prisms*

Technology
PowerPoint presentation: The van Hiele Model of Geometric Thought (or transparencies of Power Point slides)
Flash animation video 3-D.html
The Geometer's Sketchpad with sketches: *Dilation Investigation, Mona Lisa, Golden Construction, Spiral, Trigonometry Ratios, Trigonometry Tracers*
Computer lab and/or computer with projector
NonEuclid (available at http://cs.unm.edu/~joel/NonEuclid/NonEuclid.html)

Required Materials by Activity

Unit 1 - Transformations	
Activity Name	**Materials**
Terms and Definitions	a selection of geometry reference books or textbooks, easel paper, colored markers
What is a Translation?	easel paper, centimeter ruler, colored markers
Reflections	easel paper, centimeter ruler, colored markers, patty paper, graph paper
Theoretical Framework: The van Hiele Model of Geometric Thought	PowerPoint presentation: The van Hiele Model of Geometric Thought or transparencies of PowerPoint slides
Rotations	centimeter ruler, patty paper, protractor, colored pencils, centimeter grid paper, small colored dot, transparency sheets (1 per group of 4), two overhead pens of different colors (for each group)
Composite Transformations	protractor, centimeter ruler, transparency sheets (1 per group of 4), overhead projector pens in at least two different colors
Tessellations	centimeter ruler, patty paper, protractor, colored pencils, unlined 8.5 in. by 11 in. paper, 3 in. by 5 in. index card cut in half, 11 in. by 17 in. sheet of paper, colored markers, masking tape, easel paper
Do You See What I See?	linking cubes, plastic mirrors, a small object such as a color tile for each participant

Unit 2 - Triangles	
Activity Name	**Materials**
Equilateral Triangles	patty paper, straightedge, compass, easel paper, colored markers
Two Congruent Angles	patty paper, straightedge, protractor, compass, easel paper, colored markers
Scalene Triangles	patty paper, centimeter ruler, compass, protractor
The Meeting Place	patty paper, centimeter ruler, compass, calculator

Unit 3 - Quadrilaterals	
Activity Name	**Materials**
Isosceles Right Triangle Reflections	colored pencils, easel paper, colored markers, centimeter ruler, transparency
Scalene Right Triangle Reflections	colored pencils, easel paper, graph paper, colored markers, centimeter ruler

Scalene Acute/Obtuse Triangle Reflections	colored pencils, easel paper, colored markers, centimeter ruler
Rotate a Triangle	easel paper, graph paper, colored markers, patty paper, centimeter ruler
Truncate a Triangle's Vertex	easel paper, graph paper, colored markers, centimeter ruler
Vesica Pisces	compass, easel paper, colored markers, centimeter ruler
Exploring Prisms	cardstock, scissors, floral wire, modeling clay, one-inch grid easel paper, Flash animation video 3-D.html, computer lab and/or computer with projector

Unit 4 - Informal Logic/Deductive Reasoning	
Activity Name	**Materials**
Informal Logic	easel paper, colored markers
Inductive Triangle Congruence	unlined 8.5 in. by 11 in. paper, compass, centimeter ruler, protractor, spaghetti, scissors
Deductive Triangle Congruence	
Quadrilateral Proofs	easel paper, colored markers
Alternate Definitions	easel paper, colored markers
Circle Proofs	

Unit 5 - Area	
Activity Name	**Materials**
What Is Area?	3in. by 5 in. index cards, patty paper, straightedge
Investigating Area Formulas	transparency sheets, colored pencils, glue or clear tape, patty paper, scissors
Area of Trapezoids	patty paper, scissors
Area of Circles	cups (preferably large plastic cups), glue or clear tape, graphing calculator, colored markers, patty paper, scissors
Applying Area Formulas	graphing calculator
What Is Surface Area?	centimeter grid paper, linking cubes, scissors, straightedge, tape
What Is Volume?	centimeter cubes, straightedge, centimeter grid paper, scissors, tape
Net Perspective	paper, scissors, tape, rulers, centimeter grid paper (optional), centimeter cubes
Area Proofs	colored markers, easel paper

Unit 6 - Pythagoras	
Activity Name	**Materials**
Sides of Squares	centimeter grid paper
Squares on the Sides of Acute or Obtuse Triangles	centimeter grid paper, centimeter ruler
Applying Pythagoras, Part I	graphing calculator
Pythagorean Triples	transparencies of the tables for the activity, calculator
Special Right Triangles	geoboard or geoboard dot paper (provided in the appendix), unlined 8.5 in. x 11 in. paper
Distance Formula	centimeter grid paper, centimeter grid transparency, 3 in. square adhesive notes in two colors (one of each color per participant)
Applying Pythagoras, Part II	graphing calculator

Unit 7 - Polygons and Circles	
Activity Name	**Materials**
Diagonals of a Polygon	straightedge, graphing calculator
Interior and Exterior Angles of a Polygon	graphing calculator, straightedge, unlined 8.5 in. by 11 in. paper, scissors, tape, transparencies "Constructing a Polygon's Exterior Angles" and "Determining the Sum of a Polygon's Exterior Angles"
Polygons in Circles	graphing calculator, centimeter ruler
Angles Associated with a Circle	protractor, centimeter ruler
Parts of a Circle	compass, centimeter ruler, graphing calculator, easel paper, colored markers

Unit 8 - Similarity	
Activity Name	**Materials**
Magnification Ratio	graphing calculator, compass, centimeter grid paper, protractor or patty paper, straightedge
What Do You Mean?	compass, centimeter grid paper, patty paper, centimeter ruler
Dilations	compass, *The Geometer's Sketchpad*, *The Geometer's Sketchpad* Sketch: *Dilation Investigation,* centimeter grid paper, straightedge

Similarity and the Golden Ratio	compass, *The Geometer's Sketchpad*, *The Geometer's Sketchpad* Sketches: *Mona Lisa, Golden Construction, Spiral*, graphing calculator, centimeter grid paper, patty paper, straightedge
Trigonometry	cardstock, compass, *The Geometer's Sketchpad*, *The Geometer's Sketchpad* Sketches: *Trigonometry Ratios, Trigonometry Tracers*, graphing calculator, centimeter grid paper, patty paper, protractor, scissors, straightedge
Exploring Pyramids and Cones	wire-frame constructions from Unit 2: *Exploring Prisms*, centimeter ruler, scissors, protractor, patty paper (optional), compass (optional), paper cone-shaped drinking cups, plastic rice, cardstock

Unit 9 - Non-Euclidean Geometries	
Activity Name	**Materials**
When is the Sum of the Measures of the Angles of a Triangle Equal to 180º?	straightedge, compass, patty paper, colored pencils, transparency sheet, scissors, overhead projector pens
Euclid's First Five Postulates in Euclidean Space	straightedge, protractor
Curvature in Different Geometries	
Euclid's First Five Postulates in Elliptic Space	flexible protractor, string, overhead projector pens, globe, beach ball, or Lénárt sphere
Euclid's First Five Postulates in Hyperbolic Space	compass, straightedge, colored pencils, computer lab and/or computer with projector, NonEuclid (available at http://cs.unm.edu/~joel/NonEuclid/NonEuclid.html)
Visualizing Three Different Geometries	Equilateral triangle paper with side length of at least one inch (several sheets per participant—provided in the Appendix), scissors, clear tape

The Texas Education Agency and the Texas Higher Education Coordinating Board *Geometry Module* Suggested Institute Timeline

This suggested timeline assumes 10 days of instruction with 6 hours of instruction per day.

Day 1

Hour 1	Welcome and Pre-Test
Hours 2-5	Unit 1: Introduction and Transformations
Hour 6	*The Geometer's Sketchpad* Unit 1: Introduction to the Program

Day 2

Hours 1-2	Unit 1: Introduction and Transformations (cont.)
Hours 3-5	Unit 2: Triangles
Hour 6	*The Geometer's Sketchpad* Unit 2: Transformations

Day 3

Hours 1-2	Unit 2: Triangles (cont.)
Hours 3-6	Unit 3: Quadrilaterals

Day 4

Hours 1-4	Unit 3: Quadrilaterals (cont.)
Hours 5-6	*The Geometer's Sketchpad* Unit 3: Triangles and Quadrilaterals

Day 5

Hours 1-5	Unit 4: Reasoning
Hour 6	Unit 5: Area

Day 6

Hours 1-5	Unit 5: Area (cont.)
Hour 6	*The Geometer's Sketchpad* Unit 4: Perimeter and Area

Day 7

Hours 1-5	Unit 6: Pythagoras
Hour 6	*The Geometer's Sketchpad* Unit 5: Pythagoras

Day 8

Hours 1-5	Unit 7: Polygons and Circles
Hour 6	*The Geometer's Sketchpad* Unit 6: Polygons and Circles

Day 9

Hours 1-6	Unit 8: Similar Figures and Trigonometry (*The Geometer's Sketchpad* embedded)

Day 10

Hours 1-5	Unit 9: Non-Euclidean Geometries (NonEuclid embedded)
Hour 6	Post-Test and Closing