

# An Introduction to Mathematics and Scientific Document Preparation using LaTeX

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documents. There is so much more that can be done using LaTeX so this document is not meant to be comprehensive and answer all questions you may have about using LaTeX. But, it should get you started on creating professional quality documents using this powerful tool.

## 1 Introduction

This document provides an introduction to LaTeX, a bundle of software programs to produce professional quality mathematics and scientific documents. To generate a LaTeX document you will need to create a text file with the extension **.tex**. To compile the .tex file and produce a LaTeX document, view it, and print it will require installation of three components on your computer:

1. The **MiKTeX** files that form the foundation for production LaTeX documents. This requires  $\approx 200$  mb on a hard drive.
2. The **WinEdt** interface which lets you create the LaTeX file. It includes many point-and-click option to reduce the amount of typing and having to remember LaTeX commands. WinEdt processes the LaTeX file to produce a viewable .dvi document. WinEdt can also convert the .dvi file to a .pdf file or a .ps (postscript) file. It creates a log file that indicates if any errors were found.
3. The **GSView** (Ghostview) and **Ghostscript** packages that let you preview the document.

All three components can be downloaded from the following websites:

- For WinEdt go to <http://www.winedt.com/download.html>  
The most recent version is WinEdt 5.5. Click on one of the CTAN (Comprehensive TeX Archive Network) Servers.
- For the MiKTeX files, go to <http://www.winedt.com/download.html> (the same as above). Click on the link to MiKTeX's Home Page. Then select MiKTeX 2.7 and then download the "Basic MiKTeX 2.7" Installer
- There are many websites to get Ghostview and Ghostscript. For example, go to <http://www.seas.ucla.edu/~ee5cta/ghostView/> and follow the directions.

Or, you can get a copy of each from me on a CD (PC version only), or I can transfer them to a flash drive for you.

## 2 The Preamble

Every LaTeX document begins with a Preamble. Note that most LaTeX commands begin with a backslash \ followed by the command word. The **Preamble** contains

- A **\documentclass** statement that includes options for the font size and the document style. A university (such as Montana State University) will provide a .sty style file that will format a dissertation to meet university requirements. Often a journal will provide a .sty style file that will format a manuscript for that journal. We will only consider the **article** style which is the standard style and included with MiKTeX..

- A **\pagestyle** statement. Two commonly-used page styles options **plain** (the default which includes a page number) and **empty** which is the same as "plain" but with no page numbers.
- There are several ways to modify the page margins. I will show you the 4 optional statements I use. Each begins with a **\setlength** command followed by the property to modify, and then a value. I use inches (in), but you can also use centimeters (cm). The default page has a top margin of  $\approx 1.75$  inches, left margin of  $\approx 1.625$  inches, visible text width of  $\approx 5.25$  inches, and visible text height of  $\approx 7.25$  inches.
- Optional definitions using the **\def** command. These are shortcuts for commands or expressions that you expect to use many times in the text. Instead of typing 'John J. Borkowski' or 'Montana State University' in the text, I can just type \jjb or \msu, respectively.

### 2.1 Preamble Example

This is part of the Preamble I used to create this document (excluding the comments to the right).

---

```

\documentclass [12pt] {article}      <-- Font size=12 pt, Document type=article
\usepackage{graphicx}               <-- Include when the document has graphics
\pagestyle{plain}
\setlength{\topmargin}{-.7in}       <-- Reduces the top margin by .7 inches
\setlength{\textheight}{9.5in}     <-- Makes the visible text height 9.5
inches
\setlength{\oddsidemargin}{-.3in}   <-- Reduces the left margin by .3 inches
\setlength{\textwidth}{7in}         <-- Makes the visible text width 7 inches
\def\jjb{John J. Borkowski }        <-- Shortcut for 'John J. Borkowski'
\def\msu{Montana State University } <-- Shortcut for 'Montana State
University'

```

---

## 3 The Body of the Document

After the Preamble comes the body of the document. It begins with a **\begin{document}** statement and ends with a **\end{document}** statement. In-between these two statements is your LaTeX document. The **\tableofcontents** statement is optional. It produces a Table of Contents based on the numbered sections and subsections you created in your document.

---

[The Preamble]

```

\begin{document}
\tableofcontents

```

(Enter the LaTeX document here)

```

\end{document}

```

---

If you plan to use numbered sections and subsections in your manuscript (as I have done in this document), this is easily done by using the following commands:

```
\section{ } \label{ }
\subsection{ } \label{ }
\subsubsection{ } \label{ }
```

At the begin of a section or subsection, enter one of these section commands. Include the section title in the braces { }. Then, enter a label of choice for this section or subsection in the braces following the \label statement.

Sections are numbered 1., 2., .... Subsections are numbered within sections as 1.1, 1.2, ..., 2.1, 2.2, .... Subsubsections are numbered within subsections. For example, 1.1.1, 1.1.2, ... within subsection 1.1. To this point in this document I have used the following to create sections 1, 2 and 3, and subsections 2.1 and 3.1. Note how these appear in the table of contents.

---

```
\section{Introduction} \label{s:intro}
:
\section{The Preamble} \label{s:preamb}
:
\subsection{Preamble Example} \label{ss:preambex}
:
\section{The Body of the Document} \label{s:body}
:
\subsection{Creating Numbered Sections and Subsections} \label{ss:numsec}
```

What is the purpose of the \label? First, the use of labels is optional. It is, however, good practice to do so. Why? The label is used when we want to cite the section or subsection number in the body of the document. To do this, type \ref{ } in the text and enter the section or subsection label in the { }. For example:

---

If I wanted to reference the Preamble by section number, I would enter Section \ref{s:preamb}. To reference subsection Creating Numbered Sections and Subsections, I would enter Section \ref{ss:numsec}.

.....

If I wanted to reference the Preamble by section number, I would enter Section 2. To reference subsection Creating Numbered Sections and Subsections, I would enter Section 3.1.

---

Another reason to use section statements is that LaTeX will automatically renumber the sections and subsections if you include or exclude a section or subsection. Thus, you never need to enter a specific number in the text.

Note that in my labels I used s: for sections and ss: for subsections. This is not necessary. The reason I do this is to remind me later that I am referencing a section or subsection. As you will see later, we can label tables, figures, and equations the same way. In those cases I will use t: in table labels, f: in figure labels, and eq: in equation labels.

The default line spacing is single space. You can change the line spacing anytime in a document by using a \renewcommand{\baselinestretch} followed by a multiplier in { }. Here are examples:

---

```
\renewcommand{\baselinestretch}{2} <-- Double spacing
\renewcommand{\baselinestretch}{1.5} <-- 1.5 spacing
\renewcommand{\baselinestretch}{.9} <-- .9 spacing
\renewcommand{\baselinestretch}{1} <-- Single spacing
```

---

For example, if you want the entire document to be double-spaced or 1.5 spaced, then place a \renewcommand{\baselinestretch}{2} or \renewcommand{\baselinestretch}{1.5} command after the \begin{document} statement.

### 3.3 Paragraphs and Line Justification

A new paragraph is created by just having a blank line in the text. By default, the first line of a paragraph is indented and each line is both left and right justified. If you do not want a new paragraph to be indented, just type a \noindent command at the beginning of the new paragraph. There are three other common line justifications: center, justified (flush) left only, and justified (flush) right only. Use the following pairs of commands to change the justification.

```
\begin{center} \end{center}
\begin{flushleft} \end{flushleft}
\begin{flushright} \end{flushright}
```

Consider the following example and the output that follows:

---

```
\begin{center} Here is an example of centering a couple of sentences. Once a
line reaches a certain length a new line is created and it is centered. This
continues until you leave the centering mode. Note how the three lines are
centered.
\end{center}
\begin{flushleft} Here is an example of justifying a couple of sentences with
the left margin only. This is also called ragged right format. Once a line
reaches a certain length a new line is created and it is justified left only.
This continues until you leave the flushleft mode.
\end{flushleft}
\begin{flushright} Here is an example of justifying a couple of sentences
with the right margin only. This is also called ragged left format. Once a
line reaches a certain length a new line is created and it is justified
right only. This continues until you leave the flushright mode.
\end{flushright}
```

Note there is a blank line above this line which creates a new paragraph.

```
\noindent If you do not want a new paragraph to be indented, just type a
\textbackslash noindent command at the beginning of the new paragraph.
```

Here is an example of centering a couple of sentences. Once a line reaches a certain length a new line is created and it is centered. This continues until you leave the centering mode. Note how the three lines are centered.

Here is an example of justifying a couple of sentences with the left margin only. This is also called ragged right format. Once a line reaches a certain length a new line is created and it is justified left only. This continues until you leave the flushleft mode.

Here is an example of justifying a couple of sentences with the right margin only. This is also called ragged left format. Once a line reaches a certain length a new line is created and it is justified right only. This continues until you leave the flushright mode.

Note there is a blank line above this line which creates a new paragraph. If you do not want a new paragraph to be indented, just type a `\noindent` command at the beginning of the new paragraph.

3.4 Page Breaks and Line Breaks

- For a page break, just type `\newpage` at the point in the text you want to go to a new page.
- There are two types of line break: justified and unjustified.
  - For a justified line break, enter `\linebreak`.
  - For an unjustified line break, enter `\newline`.

For example:

Suppose I want to insert a line break right here. \linebreak What would happen?

Suppose I want to insert a line break right here. \newline What would happen?

.....

Suppose I want to insert a line break right here.

What would happen?

Suppose I want to insert a line break right here.

What would happen?

4 Modifying the Text

4.1 Font Size and Font Type

The default font size is called `\normalsize`. If you want to increase or decrease the font size you use one of the following commands: `\huge`, `\LARGE`, `\Large`, `\large`, `\normalsize`, `\small`, `\footnotesize`, `\scriptsize`, `\tiny` inside a set of braces `{ }`. Here is a table containing examples of the various font sizes.

<code>{\LARGE Really Large}</code>	Really Large
<code>{\Large Somewhat Large}</code>	Somewhat Large
<code>{\large Big, but not too big}</code>	Big, but not too big
<code>{\normalsize The default size}</code>	The default size
<code>{\small A little smaller than the default}</code>	A little smaller than the default
<code>{\footnotesize Even smaller yet}</code>	Even smaller yet
<code>{\scriptsize Quite small}</code>	Quite small
<code>{\tiny Can you read this?}</code>	Can you read this?

There are many options for the font type. The default is **roman**. The following table give examples of common font types:

LaTeX Command	Alternate Command	LaTeX Output
<code>\textbf{boldface John }</code>	<code>{\bf boldface John}</code>	<b>boldface John</b>
<code>\textit{italics John }</code>	<code>{\it italics John}</code>	<i>italics John</i>
<code>\textrm{roman John }</code>	<code>{\rm roman John}</code>	roman John
<code>\textsl{slanted John }</code>	<code>{\sl slanted John}</code>	<i>slanted John</i>
<code>\texttt{typewriter John }</code>	<code>{\tt typewriter John}</code>	<b>typewriter John</b>
<code>\textsc{small caps John }</code>	<code>{\sc small caps John}</code>	SMALL CAPS JOHN
<code>\textsf{sans serif John }</code>	<code>{\sf sans serif John}</code>	sans serif John
<code>\textem{emphasis John }</code>	<code>{\em emphasis John}</code>	<i>emphasis John</i>

An Example: You can also combine font size and font type. For example consider the following LaTeX code and the corresponding LaTeX output:

{\bf\LARGE If you want to} change the {\it\Large font size and font type} all {\rm\large you} do is {\sf\huge combine} a {\tt\footnotesize font type option} with a {\sf\large font size option}.

.....

If you want to change the *font size and font type* all you do is **combine** a font type option with a font size option.

4.2 Including Quotations

For a simple quotation, start the quotation with “ , then enter the quotation, and end the quotation with ”. Note these are two individual single left quote (‘) and two individual right quote (’) symbols. We do not use the double quote (”) symbol. For a longer quotation to be separated from the rest of a paragraph, we use `\begin{quote}` and `\end{quote}` with the quote entered in between. Here is an example. Note also that I changed the font size to `\scriptsize`. Remember to include `}` at the end to indicate you want `\scriptsize` to terminate.

statistical theory of classical sampling plans should have a copy of {\it Sampling Techniques} by Cochran.’’ In the 1953 edition of this text, Cochran makes the following comments about advantages of systematic sampling:

```
\begin{quote}
```

Intuitively, systematic sampling seems likely to be more precise than simple random sampling. In effect, it stratifies the population into [{\it N}] strata, which consist of the first [{\it L}] units, the second [{\it L}] units, and so on. We might therefore expect the systematic sample to be about as precise as the corresponding stratified random sample with one unit per stratum. The difference is that with the systematic sample the units all occur at the same relative position in the stratum, whereas with the stratified random sample the position in the stratum is determined separately by randomization within each stratum. The systematic sample is spread more evenly over the population, and this fact has sometimes made systematic sampling considerably more precise than stratified random sampling.

```
\end{quote}
```

.....  
 John J. Borkowski of Montana State University says “I recommend that anyone interested in learning about the statistical theory of classical sampling plans should have a copy of *Sampling Techniques* by Cochran.” In the 1953 edition of this text, Cochran makes the following comments about advantages of systematic sampling:

Intuitively, systematic sampling seems likely to be more precise than simple random sampling. In effect, it stratifies the population into [*N*] strata, which consist of the first [*L*] units, the second [*L*] units, and so on. We might therefore expect the systematic sample to be about as precise as the corresponding stratified random sample with one unit per stratum. The difference is that with the systematic sample the units all occur at the same relative position in the stratum, whereas with the stratified random sample the position in the stratum is determined separately by randomization within each stratum. The systematic sample is spread more evenly over the population, and this fact has sometimes made systematic sampling considerably more precise than stratified random sampling.

### 4.3 Including Text from an External Source

In your document, you may want to include text from some other source. For example you may want to include output from a statistical software package. To include the text, we use `\begin{verbatim}` and `\end{verbatim}` and insert the text in-between. The text will appear exactly as it appears. It will not wrap the end of lines. For example, I want to include SAS code and output into a document. Here is an example of how to do it. Note also that I changed the font size to `\scriptsize`.

```
{\scriptsize
\begin{verbatim}
*****
*** Sleep deprivation example: Contrasts and Multiple Comparison Test ***
*****
DATA in;
  DO hours = 12 to 30 by 6;
  DO rep = 1 to 8;
```

```
LINES;
20 20 17 19 20 19 21 19      21 20 21 22 20 20 23 19
25 23 22 23 21 22 22 23      26 27 24 27 25 28 26 27

PROC GLM DATA=in;
  CLASS hours;
  MODEL time = hours / SS3 SOLUTION;
TITLE 'SLEEP DEPRIVATION EXAMPLE';
RUN;

=====

                                SLEEP DEPRIVATION EXAMPLE

Dependent Variable: TIME

Source                                DF              Sum of
                                Squares              Mean
                                Square              F Value              Pr > F

Model                                3              213.25000
Error                                28              42.75000
Corrected Total                      31              256.00000

                                R-Square              C.V.              Root MSE              TIME Mean
                                0.833008              5.553401              1.2356              22.250

Source                                DF              Type III SS              Mean Square              F Value              Pr > F
HOURS                                3              213.25000              71.08333              46.56              0.0001

                                T for H0:              Pr > |T|              Std Error of
Parameter                                Estimate              Parameter=0              Estimate

INTERCEPT              26.25000000 B              60.09              0.0001              0.43686178
HOURS              12              -6.87500000 B              -11.13              0.0001              0.61781585
                   18              -5.50000000 B              -8.90              0.0001              0.61781585
                   24              -3.62500000 B              -5.87              0.0001              0.61781585
                   30              0.00000000 B              .              .              .
\end{verbatim} }
```

Below is what would appear in the LaTeX output:

```
*****;
*** Sleep deprivation example: Contrasts and Multiple Comparison Test ***;
*****;
DATA in;
  DO hours = 12 to 30 by 6;
  DO rep = 1 to 8;
    INPUT time @@; OUTPUT;
  END; END;
LINES;
20 20 17 19 20 19 21 19      21 20 21 22 20 20 23 19
25 23 22 23 21 22 22 23      26 27 24 27 25 28 26 27
```

TITLE 'SLEEP DEPRIVATION EXAMPLE';  
RUN;

=====

SLEEP DEPRIVATION EXAMPLE

Dependent Variable: TIME

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	213.25000	71.08333	46.56	0.0001
Error	28	42.75000	1.52679		
Corrected Total	31	256.00000			

R-Square	C.V.	Root MSE	TIME Mean
0.833008	5.553401	1.2356	22.250

Source	DF	Type III SS	Mean Square	F Value	Pr > F
HOURS	3	213.25000	71.08333	46.56	0.0001

Parameter	Estimate	T for H0: Parameter=0	Pr >  T	Std Error of Estimate
INTERCEPT	26.25000000	B 60.09	0.0001	0.43686178
HOURS 12	-6.87500000	B -11.13	0.0001	0.61781585
18	-5.50000000	B -8.90	0.0001	0.61781585
24	-3.62500000	B -5.87	0.0001	0.61781585
30	0.00000000	B .	.	.

4.4 Inserting Blank Spaces

If you want to insert a blank space, insert ~ where you want a space. If you want to insert a blank space of a fixed length, you can use the \hspace{} for a horizontal space anywhere or use \vspace{} for a vertical space at the end of a paragraph. Inside {} you enter the size of the space you want. Use in for inches and cm for centimeters. For example:

For example, if you type jo~hn bo~~r~kow~s~k~i what happens?

What if I decide I want a one inch blank space \hspace{1in} on this line?

\noindent What happens \hspace{4cm} if I do this? \vspace{.5in}

Note how this line is 1/2 inch lower. What if I type \msu? Hint: look at the definition in the Preamble.

.....

For example, if you type jo hn bo r kow s k i what happens?

What if I decide I want a one inch blank space on this line?

What happens if I do this?

.....

Note how this line is 1/2 inch lower. What if I type Montana State University ? Hint: look at the definition in the Preamble.

.....

- To underline and overline text requires two different approaches.
- To underline words use \underline{}. Enter the text you want underlined inside {}.
  - To overline words use \$\overline{\rm }\$. Enter the text you want overlined after \rm. The dollar sign (\$) corresponds to “math mode”. We will discuss math mode later in great detail.

For example, if I want to underline my name I would type \underline{John J. Borkowski}. But, if I want to overline Montana State University, I would type \$\overline{\rm Montana State University}\$. Or, I could use the shortcut definitions in the Preamble and type \underline{\jjb} and \$\overline{\rm \msu}\$.

.....

For example, if I want to underline my name I would type John J. Borkowski. But, if I want to overline Montana State University, I would type MontanaStateUniversity. Or, I could use the shortcut definitions in the Preamble and type John J. Borkowski and MontanaStateUniversity.

.....

4.6 Skipping Lines

A simple way to skip lines at the end of a paragraph is to use \bigkip for a large vertical space, \medskip for a medium vertical space, and \smallkip for a small vertical space. You can also use the \vspace command discussed earlier. Here is an example:

When I write a letter of recommendation (as well as other correspondence), I often use a big line space between paragraphs. Here is an example. \bigskip

If you want a slightly smaller space between paragraphs then you should consider using a medium space. Note the slightly smaller vertical space. \medskip

But, if a medium size space is still too much, then you should try a small space between paragraphs. The spacing to the next line is smaller. \smallskip

You have just seen the three types of line skipping options in LaTeX.

.....

When I write a letter of recommendation (as well as other correspondence), I often use a big line space between paragraphs. Here is an example.

If you want a slightly smaller space between paragraphs then you should consider using a medium space. Note the slightly smaller vertical space.

But, if a medium size space is still too much, then you should try a small space between paragraphs. The spacing to the next line is smaller.

You have just seen the three types of line skipping options in LaTeX.

Because certain symbols in LaTeX serve specific functions (such as `&` in tables, `%` for comment lines, `\` to start commands or `\\` to end lines in tables), they will not appear if you just type that symbol. To have that symbol appear, you have to use the LaTeX form. The following table lists some of the most common symbols that are reserved within LaTeX and how we can get them to appear as text.

Insert	Output	Insert	Output
<code>\textbackslash</code>	<code>\</code>	<code>\\$</code>	<code>\$</code>
<code>\textasciitilde</code>	<code>~</code>	<code>\#</code>	<code>#</code>
<code>\textasciicircum</code>	<code>^</code>	<code>\{</code>	<code>{</code>
<code>\%</code>	<code>%</code>	<code>\}</code>	<code>}</code>

If you expect that you will need to have many backslashes appear in your document and you do not want to type `\textbackslash` every time, then I suggest you include a short definition for `\textbackslash` in the Preamble. This is what I did for this document. In the Preamble, I included `\def\tbs{\textbackslash}`. Then, every time I want a `\` to appear, I just type `\tbs`.

## 5 Itemized Lists

### 5.1 Bulleted Lists

To form a bulleted list, use `\begin{itemize}` and `\end{itemize}`. The list of items is included in between with each item beginning with `\item`. For example, the following is from my vita:

```
{\footnotesize
\begin{itemize}
\item Nguyen, N.-K. and Borkowski, J.J. (2008) ‘‘New 3-Level Response Surface
Designs Constructed From Incomplete Block Designs’’, {\it Journal of
Statistical Planning and Inference}, {\bf 138}: 294-305.
\item Borkowski, J.J. and Piepel, G.F. ‘‘Space-Filling Designs for
Highly-Constrained Mixture Experiments’’ accepted for publication in
{\it Journal of Quality Technology}, June 2008.
\item Borkowski, J.J., Invited Discussant of ‘‘Response Surface
Design Evaluation and Comparison’’ by Anderson-Cook, C., Borror, C.,
and Montgomery, D., to appear in {\it Journal of Statistical
Planning and Inference}.
\item Borkowski, J.J. (2008) ‘‘Center Points’’, {\it Encyclopedia of
Statistics in Quality and Reliability}, Editors: Ruggeri,F., Kenett,R., and
Faltin,F.W., John Wiley \& Sons Ltd., Chichester, UK, 289-292.
\item Borkowski, J.J. (2008) ‘‘Optimal Linear-Quadratic (LQ) Model Designs’’,
{\it Thailand Statistician}, {\bf 6}: 47-64.
\item Sharp, J.L., Borkowski, J.J., Schmoyer, D., Daly, D.S., Purvine, S.,
Cannon, W., and Hurst,. G.B., ‘‘Statistically Appraising Process
Quality of Affinity-Isolation Experiments’’, to appear in {\it Journal
of Computational Statistics and Data Analysis}.
\end{itemize} }
```

- Borkowski, J.J. and Piepel, G.F. ‘‘Space-Filling Designs for Highly-Constrained Mixture Experiments’’ accepted for publication in *Journal of Quality Technology*, June 2008.
- Borkowski, J.J., Invited Discussant of ‘‘Response Surface Design Evaluation and Comparison’’ by Anderson-Cook, C., Borror, C., and Montgomery, D., to appear in *Journal of Statistical Planning and Inference*.
- Borkowski, J.J. (2008) ‘‘Center Points’’, *Encyclopedia of Statistics in Quality and Reliability*, Editors: Ruggeri,F., Kenett,R., and Faltin,F.W., John Wiley & Sons Ltd., Chichester, UK, 289-292.
- Borkowski, J.J. (2008) ‘‘Optimal Linear-Quadratic (LQ) Model Designs’’, *Thailand Statistician*, **6**: 47-64.
- Sharp, J.L., Borkowski, J.J., Schmoyer, D., Daly, D.S., Purvine, S., Cannon, W., and Hurst,. G.B., ‘‘Statistically Appraising Process Quality of Affinity-Isolation Experiments’’, to appear in *Journal of Computational Statistics and Data Analysis*.

### 5.2 Numbered Lists

To form a numbered list, use `\begin{enumerate}` and `\end{enumerate}`. The list of items is included in between with each item beginning with `\item`. For example, consider the previous list, but now the items are numbered:

```
{\footnotesize
\begin{enumerate}
\item Nguyen, N.-K. and Borkowski, J.J. (2008) ‘‘New 3-Level Response Surface
Designs Constructed From Incomplete Block Designs’’, {\it Journal of
Statistical Planning and Inference}, {\bf 138}: 294-305.
\item Borkowski, J.J. and Piepel, G.F. ‘‘Space-Filling Designs for
Highly-Constrained Mixture Experiments’’ accepted for publication in
{\it Journal of Quality Technology}, June 2008.
\item Borkowski, J.J., Invited Discussant of ‘‘Response Surface
Design Evaluation and Comparison’’ by Anderson-Cook, C., Borror, C.,
and Montgomery, D., to appear in {\it Journal of Statistical
Planning and Inference}.
\item Borkowski, J.J. (2008) ‘‘Center Points’’, {\it Encyclopedia of
Statistics in Quality and Reliability}, Editors: Ruggeri,F., Kenett,R., and
Faltin,F.W., John Wiley \& Sons Ltd., Chichester, UK, 289-292.
\item Borkowski, J.J. (2008) ‘‘Optimal Linear-Quadratic (LQ) Model Designs’’,
{\it Thailand Statistician}, {\bf 6}: 47-64.
\item Sharp, J.L., Borkowski, J.J., Schmoyer, D., Daly, D.S., Purvine, S.,
Cannon, W., and Hurst,. G.B., ‘‘Statistically Appraising Process
Quality of Affinity-Isolation Experiments’’, to appear in {\it Journal
of Computational Statistics and Data Analysis}.
\end{enumerate} }
```

```
.....
1. Nguyen, N.-K. and Borkowski, J.J. (2008) ‘‘New 3-Level Response Surface Designs Constructed From Incomplete Block Designs’’, Journal of Statistical Planning and Inference, 138: 294-305.
2. Borkowski, J.J. and Piepel, G.F. ‘‘Space-Filling Designs for Highly-Constrained Mixture Experiments’’ accepted for publication in Journal of Quality Technology, June 2008.
```

4. Borkowski, J.J. (2008) “Center Points”, *Encyclopedia of Statistics in Quality and Reliability*, Editors: Ruggeri, F., Kenett, R., and Faltin, F.W., John Wiley & Sons Ltd., Chichester, UK, 289-292.
5. Borkowski, J.J. (2008) “Optimal Linear-Quadratic (LQ) Model Designs”, *Thailand Statistician*, **6**: 47-64.
6. Sharp, J.L., Borkowski, J.J., Schmoyer, D., Daly, D.S., Purvine, S., Cannon, W., and Hurst, G.B., “Statistically Appraising Process Quality of Affinity-Isolation Experiments”, to appear in *Journal of Computational Statistics and Data Analysis*.

### 5.3 Nested Lists

You can nest lists within any other item in a list. You can also mix bulleted with numbered lists. You can also change the default bullets in bulleted lists to any symbol or text that you would like instead. Just type `\item[ ]` and enter whatever you want between the brackets. Example:

```
{\footnotesize
\noindent {\bf 2008 Addresses/Presentations/Seminars:}
\begin{enumerate}
\item “Variance Dispersion Graphs for Mixture Experiments”
  \begin{itemize}
    \item National Tsing Hua University, Taiwan, August 2008.
    \item National Institute of Development Administration (NIDA), Thailand,
August 2008.
    \item Kasetsart University, Thailand, August 2008.
    \item King Mongkut University of Technology North Bangkok (KMUTNB),
Thailand, August 2008.
  \end{itemize}
\item “Using a Genetic Algorithm (GA) to Generate Small Exact Response
Surface Designs”
  \begin{itemize}
    \item[A1.] Academia Sinica, Taiwan, August 2008.
    \item[A2.] National Cheng Chi University, Taiwan, August 2008.
  \end{itemize}
\item “Evaluating and Comparing Properties of Response Surface Designs”,
  \begin{enumerate}
    \item National Central University, Taiwan, August 2008.
    \item National Institute of Development Administration (NIDA), Thailand,
August 2008.
    \item Kasetsart University, Thailand, August 2008.
  \end{enumerate}
\item “Considerations for Publishing in International Journals”, King
Mongkut University of Technology North Bangkok (KMUTNB), Thailand, August
2008.
\item “Directions of Research in Statistics and Applied Statistics: Physical
Sciences and Engineering”, 2008 Conference on Statistics and its
Applications, Pattaya Cholburee, Thailand, May 2008.
\end{enumerate} }
```

1. “Variance Dispersion Graphs for Mixture Experiments”
  - National Tsing Hua University, Taiwan, August 2008.
  - National Institute of Development Administration (NIDA), Thailand, August 2008.
  - Kasetsart University, Thailand, August 2008.
  - King Mongkut University of Technology North Bangkok (KMUTNB), Thailand, August 2008.
2. “Using a Genetic Algorithm (GA) to Generate Small Exact Response Surface Designs”
  - A1. Academia Sinica, Taiwan, August 2008.
  - A2. National Cheng Chi University, Taiwan, August 2008.
3. “Evaluating and Comparing Properties of Response Surface Designs”,
  - (a) National Central University, Taiwan, August 2008.
  - (b) National Institute of Development Administration (NIDA), Thailand, August 2008.
  - (c) Kasetsart University, Thailand, August 2008.
4. “Considerations for Publishing in International Journals”, King Mongkut University of Technology North Bangkok (KMUTNB), Thailand, August 2008.
5. “Directions of Research in Statistics and Applied Statistics: Physical Sciences and Engineering”, 2008 Conference on Statistics and its Applications, Pattaya Cholburee, Thailand, May 2008.

## 6 Tables and Figures

### 6.1 Making Tables

In this section, we will review how to use the tabular environment to make tables. Creating a table starts with `\begin{tabular}` and ends with `\end{tabular}` statements. The general format for making a table in LaTeX is the following:

```
\begin{tabular}{column info} options
  first row specifications \\ options
  second row specifications \\ options
  :           :
  last row specifications \\ options
\end{tabular}
```

- Suppose your table has  $C$  columns. Then the **{column info}** contains
  - A string of  $C$  letters: **c**, **l**, or **r**. Use **c** if you want that column centered, use **l** if you want that column left-justified, or use **r** if you want that column right-justified.
  - Vertical bars **|** between consecutive letters if you want a vertical line to appear in the table.

For example, `\begin{tabular}{|cr|lc|}` will produce a table with 4 columns with vertical lines before the first column, between columns 2 and 3, and after the last column. The first and last columns will be centered, the second column is right-justified, and the third column is left-justified.



backslashes `\\` indicate a line break. Consider the following table. Note that I centered that table on the page using `\begin{center}` and `\end{center}`.

---

```
\begin{center}
\begin{tabular}{|c|rlc|c|}
Name & Exam 1 & Exam 2 & Exam 3 & {\bf Grade} \\
John & 100 & 99 & 100 & {\bf A+} \\
Mary & 95 & 87.5 & 91 & {\bf A} \\
Dumbo & 34 & 8 & 24.5 & {\bf F} \\
\end{tabular}
\end{center}
```

---

Name	Exam 1	Exam 2	Exam 3	Grade
John	100	99	100	A+
Mary	95	87.5	91	A
Dumbo	34	8	24.5	F

---

- Finally, if you want to include horizontal lines or line segments between rows of the table, these can be defined as **options** at the end of each line. The `\hline` option will draw a horizontal line across all columns. The `\cline{c1-c2}` option will draw a horizontal line segment from column `c1` to column `c2`. You can use multiple `\cline` options at the end of a line. For example `\cline{1-2}` `\cline{4-5}` will draw a line segment from column 1 to column 2 and from column 4 to column 5.
- Let us modify the previous table with line options.

---

```
\begin{center}
\begin{tabular}{|c|rlc|c|} \hline
Name & Exam 1 & Exam 2 & Exam 3 & {\bf Grade} \\
John & 100 & 99 & 100 & {\bf A+} \\
Mary & 95 & 87.5 & 91 & {\bf A} \\
Dumbo & 34 & 8 & 24.5 & {\bf F} \\
\end{tabular}
\end{center}
```

---

Name	Exam 1	Exam 2	Exam 3	Grade
John	100	99	100	A+
Mary	95	87.5	91	A
Dumbo	34	8	24.5	F

---

If you want a table entry to extend across multiple columns, use the `\multicolumn` statement which has the form

`\multicolumn{number of columns}{position}{column entry}`

- number of columns** is the number of columns you want the entry to extend to.
- position** is either **c** for centered, **r** for right-justified, or **l** for left-justified. If you want vertical lines to appear, include `|` around position letter `c`, `r`, or `l`.
- column entry** is what you want to appear across the columns.

You can insert a blank line in the table, if you use `\multicolumn{C}{c}{}`  where `C` is the number of columns in the table and nothing is entered in the braces `{}`.

**Warning:** You must remove extra `&` symbols from a line when you use `\multicolumn` statements.

Let us modify the previous table again by adding information above the table and centering all columns. Note the removal of `&` symbols.

---

```
\begin{center}
\begin{tabular}{|c|ccc|c|} \hline
& \multicolumn{3}{|c|}{Exam} & \\
Name & 1 & 2 & 3 & {\bf Grade} \\
John & 100 & 99 & 100 & {\bf A+} \\
Mary & 95 & 87.5 & 91 & {\bf A} \\
Dumbo & 34 & 8 & 24.5 & {\bf F} \\
\end{tabular}
\multicolumn{5}{c}{} \\
\multicolumn{5}{l}{\footnotesize\bf Here is a footnote for the table.}
\end{tabular}
\end{center}
```

---

Name	Exam			Grade
	1	2	3	
John	100	99	100	A+
Mary	95	87.5	91	A
Dumbo	34	8	24.5	F

Here is a footnote for the table.

---

### 6.1.2 Modifying Spacing Between Columns

If you want to increase or decrease the spacing between columns, use the `\tabcolsep=` statement just before the `\begin{tabular}` statement.

- Enter the amount of spacing you want after the `=` sign. The units of measurements can be **cm** (centimeters), **in** (inches), or **pt** (points).

```

\begin{center} \tabcolsep=20pt
\begin{tabular}{|c|ccc|c|} \hline
& \multicolumn{3}{|c|}{Exam} & \\
Name & 1 & 2 & 3 & {\bf Grade} \\ \hline
John & 100 & 99 & 100 & {\bf A+} \\ \hline
Mary & 95 & 87.5 & 91 & {\bf A} \\ \hline
\end{tabular}
\end{center}

```

---

Name	Exam			Grade
	1	2	3	
John	100	99	100	A+
Mary	95	87.5	91	A

### 6.1.3 Tables of Tables

To make a table of tables, just treat each table as a row and column entry in a larger table. For example, the following code produces a  $2 \times 2$  table of tables.

```

\begin{center}
\begin{tabular}{cc}
\begin{tabular}{|cc|} \hline
2 & 10 \\ \hline
555 & 2210 \\ \hline
\end{tabular}
&
\begin{tabular}{|c|c|c|} \hline
1.98 & 2.29 & 0.10 \\ \hline
5.55 & 7.51 & 2.21 \\ \hline
\end{tabular}
\\ \hline
\begin{tabular}{|c|c|} \hline
2000 & 1000 \\ \hline
5000 & 8000 \\ \hline
\end{tabular}
&
\begin{tabular}{|c|c|c|} \hline
128 & 296 & 101 \\ \hline
775 & 375 & 222 \\ \hline
\end{tabular}
\\ \hline
\end{tabular}
\end{center}

```

2	10
555	2210

1.98	2.29	0.10
5.55	7.51	2.21

2000	1000
5000	8000

128	296	101
775	375	222

To import graphics you need to include the `\usepackage{graphicx}` statement in the preamble. The easiest and best way to include a figure is to save it as or convert it to an **encapsulated Postscript** file also known as a **.eps** file.

Before I discuss how to create a .eps file, let us assume that I have one. To include it in the LaTeX document, use the `\includegraphics[scale=]{xxx.eps}` statement

- The `[scale = ]` is used if you want to resize your figure. For example `[scale = .5]` would decrease the figure size by 50%, while `[scale = 1.25]` would increase the figure size by 25%. If you want the figure size to be unchanged, delete the `[scale = ]` option or use `[scale = 1]`.
- `xxx` is the .eps filename in which the figure is saved.

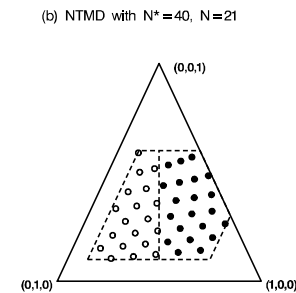
For example suppose I have a figure saved in the file *t3mcc.eps*. Then the statements

```

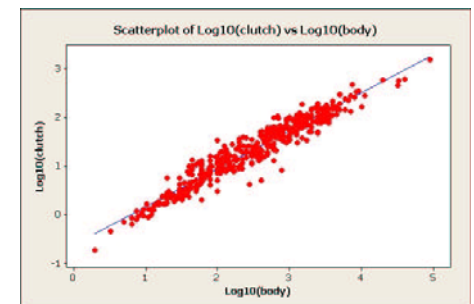
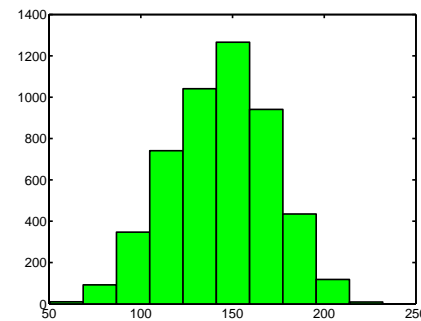
\begin{center}
\includegraphics[scale=.9]{t3mcc.eps}
\end{center}

```

would produce the following:



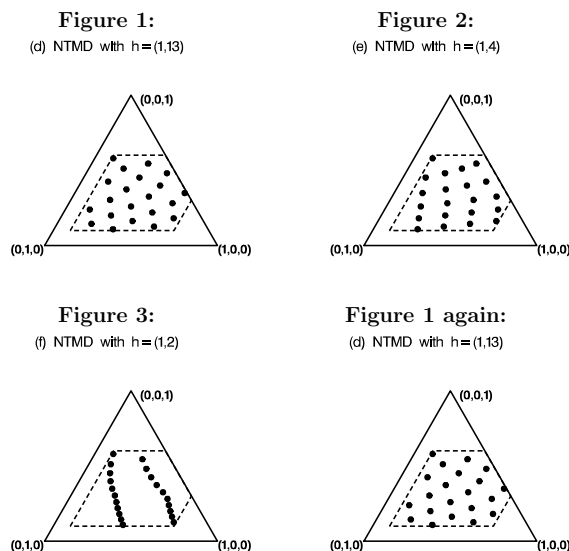
Here are graphs created from Matlab and from Minitab:



```

\begin{center}
\begin{tabular}{cc}
{\bf Figure 1:} & {\bf Figure 2:} \\
\includegraphics[scale=.9]{t13.eps} & \includegraphics[scale=.9]{t4.eps} \\
{\bf Figure 3:} & {\bf Figure 1 again:} \\
\includegraphics[scale=.9]{t2.eps} & \includegraphics[scale=.9]{t13.eps}
\end{tabular}
\end{center}

```



You can also include photographs:



## 6.2.1 Creating a .eps File

The first thing to check after creating a figure with a software package is to check the **Save As** or **Export** options and see if .eps is an option. If so, then just save it as a .eps file.

Most software packages do not have a .eps file option. But, they will let you save it in some other common format such as .pdf, .jpg, .tif, or .gif. Choose .pdf if it is an option. If not, select any of the other formats.

Next use *Adobe Acrobat Professional* to convert the file.

(I) If it is a .pdf file, open it in *Adobe Acrobat Professional*. Then

**File** → **Export** → **Postscript** → **Encapsulated Postscript**

and then save it in the same directory as the LaTeX .tex file. It can now be imported into LaTeX.

(II) If it is not a .pdf file, convert it to a .pdf file in *Adobe Acrobat Professional* by:

**File** → **Create PDF** → **From File**

Then open the file you want to convert to pdf (e.g., dino.tif). The figure will now appear. Now save it as a .eps file:

**File** → **Export** → **Postscript** → **Encapsulated Postscript**

and then save it in the same directory as the LaTeX .tex file. It can now be imported into LaTeX.

## 6.3 Creating .eps Files Directly within SAS

The statistical software package SAS lets you save graphics as .eps files so no conversion is necessary. This is done by including **FILENAME** and **GOPTIONS** statements at the beginning of your SAS program. In the following example, all upper case words are SAS commands, and all lower case words are your options.

```
FILENAME grafout 'c:\papers\mix06\t3mcc.eps';
```

```
GOPTIONS GSFNAME= grafout GSFMODE= REPLACE
         RESET = GLOBAL  DEVICE = PSEPSF
         VSIZE = 2.75in  HSIZE = 2.75in ;
```

- In the **FILENAME** statement, you enter a temporary name for the figure created by SAS, and then indicate a directory with a file name. In this example, the temporary name is “grafout”. Once the figure is created, it will be written to a file “t3mcc.eps” in directory “c:\papers\mix06”.
- In the **GOPTIONS** statement, you use the same temporary name that appears in the **FILENAME** statement. By using **DEVICE = PSEPSF**, SAS will save the figure as a .eps file. You can also specify the dimensions of the figure to be saved.

avizdat is another software package that lets you save a figure it creates as an .eps file. From the figure window, all you do is

**File** → **Export** → **Save as type:**

and then save as a .eps file. You have a choice between a standard black and white .eps file or, if you have color graphics, then choose the color .eps option.

## 6.5 Numbered Tables

If you are preparing a document with only a few tables and you want the tables to be numbered, you can always include lines at the top of the table using \multicolumn statements and enter the table number and caption in these statements. For example

---

```
\begin{center}
\begin{tabular}{|c|ccc|c|}
\multicolumn{5}{c}{\bf Table 1: Class Grades for Stat 1000} \\
\multicolumn{5}{c}{} \\ \hline
Name & Exam 1 & Exam 2 & Exam 3 & {\bf Grade} \\ \hline
John & 100 & 99 & 100 & {\bf A+} \\
Mary & 95 & 87.5 & 91 & {\bf A} \\
Dumbo & 34 & 8 & 24.5 & {\bf F} \\ \hline
\end{tabular}
\end{center}
```

---

Table 1: Class Grades for Stat 1000

Name	Exam 1	Exam 2	Exam 3	Grade
John	100	99	100	A+
Mary	95	87.5	91	A
Dumbo	34	8	24.5	F

---

If you do this, you must realize that if you decide to insert a new table, you will have to change the table number on every table that follows it in the document and also make changes to all references to table numbers in the text of the document. To avoid this situation LaTeX will automatically renumber tables if you use

- `\begin{table}[option] \caption{ } \label{ }` before the `\begin{center} \begin{tabular}` statements.
  - The option indicates where you want the table to appear on the page. Use `[t]` for top of page, `[b]` for bottom of page, `[h]` for here on the page, or `[p]` to put the table on a separate page. Or, you can use the default settings in LaTeX by not including an option.
  - Inside `{ }` of the caption you include a caption for the table.
  - Inside `{ }` of the label you include a label name for the table.
- `\end{table}` after the `\end{tabular} \end{center}` statements.

Name	Exam 1	Exam 2	Exam 3	Grade
Jack	85	82	90	B+
Jill	97	88	92	A

The use of the `\table` commands will be extremely useful when preparing a document with many tables and many revisions (such as a dissertation or a research paper). Here is an example:

```
{\footnotesize
\begin{table}[t] \label{t:gr222}
\caption{Class Grades for Stat 222}
\begin{center}
\begin{tabular}{|c|ccc|c|} \hline
Name & Exam 1 & Exam 2 & Exam 3 & {\bf Grade} \\ \hline
Jack & 85 & 82 & 90 & {\bf B+} \\
Jill & 97 & 88 & 92 & {\bf A} \\ \hline
\end{tabular}
\end{center}
\end{table}
```

```
\begin{table}[h]
\caption{Class Grades for Stat 1000} \label{t:gr1000}
\begin{center}
\begin{tabular}{|c|ccc|c|} \hline
Name & Exam 1 & Exam 2 & Exam 3 & {\bf Grade} \\ \hline
John & 100 & 99 & 100 & {\bf A+} \\
Mary & 95 & 87.5 & 91 & {\bf A} \\
Dumbo & 34 & 8 & 24.5 & {\bf F} \\ \hline
\end{tabular}
\end{center}
\end{table} }
```

---

Table 2: Class Grades for Stat 1000

Name	Exam 1	Exam 2	Exam 3	Grade
John	100	99	100	A+
Mary	95	87.5	91	A
Dumbo	34	8	24.5	F

What is the purpose of the `\label`? The label is used when we want to cite the table in the body of the document. To do this, type `\ref{ }` in the text and enter the table label in the `{ }`. For example:

miserably.

will produce the following:

.....  
In Table 1, we see that Jill has performed very well in Stat 222. On the other hand, from Table 2 we see that Dumbo is failing miserably.  
.....

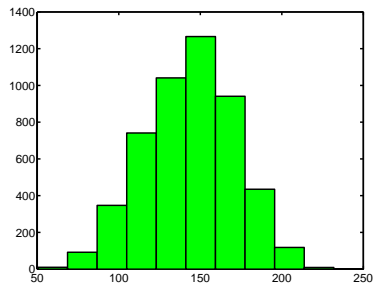
6.6 Numbered Figures

All of the comments regarding numbered tables also apply to numbered figures. The only difference is that we use `\begin{figure}` and `\end{figure}` instead of `\begin{table}` and `\end{table}`. For example

```
\begin{figure}[h]
\caption{Histogram Created by Minitab} \label{f:minihist}
\begin{center}
\includegraphics[scale=.4]{histog.eps}
\end{center}
\end{figure}
```

would produce the following figure:

Figure 1: Histogram Created by Minitab



I could then use the label ‘f:minihist’ to reference the figure. For example,

In Figure \ref{f:minihist}, we see that the shape of the histogram is consistent with a random sample taken from normal distribution.

will produce the following:

.....  
In Figure 1, we see that the shape of the histogram is consistent with a random sample taken from normal distribution.  
.....

7.1 Math Fonts

LaTeX was designed for mathematical word processing (something that is a nightmare in Word). To enter mathematical content you will need to enter a **Math Mode**. This can be done several ways and will be discussed later. For now, I will only show you the unnumbered equation format:

```
\[ \]
```

with the mathematical content in-between the brackets. When entering any math mode, the output will automatically be converted to appear as mathematical fonts.

The first thing you need to be familiar with are the types of mathematical fonts available in LaTeX. There are, however, hundreds of mathematical fonts available in LaTeX. To show you them all here would take too long to prepare. All you have to do is use the Toolbar in WidEdt. By clicking on the  $\Sigma$  icon the following options now appear:

Math	Greek	Symbols
International	Typeface	Functions(x) ...
{ } ...	<>= ...	+/- ...
→ ...	AMS	AMS =<> ...
AMS NOT =<> ...		

Now just click on one of these options to see what symbols are available. To insert any symbol just click on the symbol. For example if you click on **Greek**, the entire Greek alphabet appears. If I then click on  $\alpha$ , the LaTeX command `\alpha` will now appear at the cursor in the document. Suppose I insert various fonts and type in the following equation:

```
\[ 2\alpha + 9\beta - 6\gamma = \Delta(x) \]
```

This will produce

$$2\alpha + 9\beta - 6\gamma = \Delta(x)$$

You will soon remember the LaTeX code for many symbols. You can then just type code directly into the document without having to use the Toolbar.

7.2 Blank Spaces in Math Mode

To insert blank spaces while in math mode, enter one of the following:

Insert	Result
<code>\;</code>	big space
<code>\:</code>	medium space
<code>\,</code>	thin space
<code>\!</code>	thin negative space

7.3 Subscripts and Superscripts

Many mathematical expression requires subscripts and superscripts. For example, what do you do if you want  $2x_1x_2 + z_\alpha^2 = y^{-3}$  to appear in your document?

- For a **subscript**, enter (underscore) `_{ }` after the symbol you want it to appear. Inside `{ }` enter the subscript.

- To include both a superscript and a subscript, just concatenate both after the symbol.

For example,

```
\[ 2x_{1}x_{2} + z^{2}_{\alpha} = y^{-3} \]
```

will produce

$$2x_1x_2 + z_\alpha^2 = y^{-3}$$

If the subscript or superscript is only one character, you do not need the braces. For example,

```
x^{2}      is the same as x^2
y_{6}      is the same as y_6
z_{1}^{9}  is the same as z_1^9
```

You will also use subscripts and superscripts to attach bounds or indices to many mathematical symbols. For example:

```
\[ \int_{0}^{\infty} e^{-x} dx = 1 \]
\[ \sum_{i=1}^n i \; ; \; = \; ; \; ; \; n(n+1)/2 \]
\[ \prod_{j=1}^n j = n! \]
```

$$\int_0^\infty e^{-x} dx = 1$$

$$\sum_{i=1}^n i = n(n+1)/2$$

$$\prod_{j=1}^n j = n!$$

Note that I included 2 blank spaces around  $=$  in the second equation.

## 7.4 Fractions

Suppose that in the previous example, I wanted  $n(n-1)/2$  to appear as the fraction  $\frac{n(n-1)}{2}$ . To do this use `\frac{ }{ }`. Enter the numerator in the first set of braces and the denominator in the second set of braces. For example

```
\[ \sum_{i=1}^n i \; ; \; = \; ; \; ; \; \frac{n(n+1)}{2} \]
```

would produce the following:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

You have several options when inserting equations in your document. You can enter equations individually or as an array, and you can have equations numbered or unnumbered.

### 7.5.1 Unnumbered Equations

I have already shown you how to generate an unnumbered equation by inserting the equation between `\[` and `\]`. This will put the equation on a separate line.

If you want to include an equation as part of a line of text, insert the equation between two dollar signs `$` `$`.

For example, suppose I type `$\sum_{i=1}^n i \; ; \; = \; ; \; \frac{n(n+1)}{2}$` in the middle of a line, what would appear?

.....  
For example, suppose I type  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  in the middle of a line, what would appear?

Note how the limits on the summation now appear to the right of the summation symbol (and not above and below it) and the fraction is squeezed into the line spacing. If you want to make it appear just like it would using `\[ \]`, use the `\displaystyle` option after the first `$`. For example:

Suppose I type `$\displaystyle \sum_{i=1}^n i \; ; \; = \; ; \; \frac{n(n+1)}{2}$` in the middle of a line, what would appear?

.....  
Suppose I typed  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  in the middle of a line, what would appear?

Because I use `\displaystyle` very often, I placed the following shortcut definition in the Preamble:

```
\def\ds{\displaystyle }
```

Then, every time I want to use `\displaystyle`, all I have to do is type `\ds`. For example:

Suppose I type `$\ds \sum_{i=1}^n i \; ; \; = \; ; \; \frac{n(n+1)}{2}$` in the middle of a line, what would appear?

.....  
Suppose I typed  $\sum_{i=1}^n i = \frac{n(n+1)}{2}$  in the middle of a line, what would appear?

In research manuscripts, important equations that will be referenced later in the document are numbered. To produce a numbered equation, type

```
\begin{equation} \label{ } \end{equation}
```

and enter the equation after the `\begin{equation}` statement or after the `\label{ }` option. And (like you did for section, table, and figure labels) enter a label of choice. I use `eq:` to start my equation labels. Suppose I use the three equations we had in Section 7.5.1 but now I want them numbered. Here is what you do:

```
\begin{equation}
\int_{-\infty}^{\infty} e^{-x} dx = 1 \label{eq:intexp}
\end{equation}
\begin{equation}
\sum_{i=1}^n i \; ; \; = \; ; \; ; \; n(n+1)/2 \label{eq:sumint}
\end{equation}
\begin{equation}
\prod_{j=1}^n j = n! \label{eq:nfact}
\end{equation}
```

$$\int_0^{\infty} e^{-x} dx = 1 \quad (1)$$

$$\sum_{i=1}^n i = n(n+1)/2 \quad (2)$$

$$\prod_{j=1}^n j = n! \quad (3)$$

### 7.5.3 Referencing Equation Numbers

To reference an equation by its number, you do exactly the same thing you did when referencing a table or figure. That is you use `\ref{ }` with the equation label in the braces `{ }`. Here is an example of how to reference these three equations:

---

Equation (`\ref{eq:intexp}`) contains a integral of an exponential function.  
Equations (`\ref{eq:sumint}`) and (`\ref{eq:nfact}`) involve a summation and  
a repeated product, respectively.

.....  
Equation (1) contains a integral of an exponential function. Equations (2) and (3) involve a  
summation and a repeated product, respectively.

---

If you want to increase the size of parentheses, brackets, braces, or absolute value bars to match the size of an mathematical expression: replace `( )` with `\left( \right)`, replace `[ ]` with `\left[ \right]`, replace `\{ \}` with `\left\{ \right\}`, and replace `| |` with `\left| \right|` and `\right|`. For example,

```
\[ \sum_{i=1}^n i \; ; \; = \; ; \; ; \; \left( \frac{n(n+1)}{2} \right) \]
\[ \sum_{i=1}^n i^2 \; ; \; = \; ; \; ; \; \left[ \frac{(2n+1)(n+1)n}{6} \right] \]
\[ \left\{ \sum_{j=1}^k \left| \frac{X_j}{Y_j} \right| \right\}
```

would produce the following:

$$\sum_{i=1}^n i = \left( \frac{n(n+1)}{2} \right)$$

$$\sum_{i=1}^n i^2 = \left[ \frac{(2n+1)(n+1)n}{6} \right]$$

$$\left\{ \sum_{j=1}^k \left| \frac{X_j}{Y_j} \right| \right\}$$

## 7.7 Equation Arrays

Many times you will want to present a sequence of equations or inequalities (as you would do in a mathematical proof). You could create the sequence by repeatedly using `\[ \]` or `\begin{equation} \end{equation}` commands, but the alignment of the `=`, `<`, `>`, etc. will not match. To have the equations align properly, use an equation array. The format is

```
\begin{eqnarray}
& \&\& \text{(options)} \\
& \&\& \text{(options)} \\
& : : : \\
& \&\& \text{(options)}
\end{eqnarray}
```

Each equation or inequality would be entered in a `& \& [options]` statement as follows:

- For any equation or inequality, the `& \&` are used to separate the left-hand side from the right-hand side with the relationship symbol (such as `=`, `>`, `<`, `≤`, `≥`) in between `& \&`.
- The first option for any line in the equation array is `\nonumber`.
  - If you do not include `\nonumber`, then that line will be numbered.
  - If you do include `\nonumber`, then that line will not be numbered.
- The second option is `\label{ }`. The format and usage is exactly like it is for a single equation. You can reference that specific line of the equation array using a `\ref{ }` command. Note that you cannot use a label if you used `\nonumber` for that line.

Here is an example:

```
\begin{eqnarray}
S_1 & = & x_1+x_2+ \ldots + x_n \label{eq:s1} \\
S_2 & = & x_1x_2+x_1x_3+\cdots x_{n-1}x_n \label{eq:s2} \\
S_3 & = & x_1x_2x_3+x_1x_2x_4+\cdots x_{n-2}x_{n-1}x_n \label{eq:s3}
\end{eqnarray}
```

---


$$S_1 = x_1 + x_2 + \dots + x_n \quad (4)$$

$$S_2 = x_1x_2 + x_1x_3 + \dots x_{n-1}x_n \quad (5)$$

$$S_3 = x_1x_2x_3 + x_1x_2x_4 + \dots x_{n-2}x_{n-1}x_n \quad (6)$$


---

Now I can reference each of the equations by number. Example:

`$$S_1$` in (`\ref{eq:s1}`) is a sum of `$n$` variables.  
`$$S_2$` in (`\ref{eq:s2}`) is the sum of all pairwise products, while  
`$$S_3$` in (`\ref{eq:s3}`) is the sum of all three-way products.

This would produce:

`.....`  
`S_1` in (4) is a sum of `n` variables. `S_2` in (5) is the sum of all pairwise products, while `S_3` in (6) is the sum of all three-way products.  
`.....`

Here is an example using the `\nonumber` option:

---

```
\begin{eqnarray}
R\left[\sum_{j=1}^n x_j S_i^{(j)}\right] \right] \\
& = & \sum_{j=1}^n R[x_j S_i^{(j)}] \nonumber \\
& = & \sum_{j=1}^n i S_i \\
\end{eqnarray}
```

---


$$\begin{aligned} R \left[ \sum_{j=1}^n x_j S_i^{(j)} \right] &= \sum_{j=1}^n R[x_j S_i^{(j)}] \\ &= \sum_{j=1}^n i S_i^{(j)} \\ &= i S_i \end{aligned} \quad (7)$$


---

If you do now want to number any of the equations in an equation array, there is a quick way to do so without using the `\nonumber` option on each line. Just include an asterisk in the first and last equation array commands as follows:

```
& & \\
: : : \\
& & \\
\end{eqnarray}
```

For example:

---

```
\begin{eqnarray*}
S_1 & = & x_1+x_2+ \ldots + x_n \\
S_2 & = & x_1x_2+x_1x_3+\cdots x_{n-1}x_n \\
S_3 & = & x_1x_2x_3+x_1x_2x_4+\cdots x_{n-2}x_{n-1}x_n
\end{eqnarray*}
```

---


$$\begin{aligned} S_1 &= x_1 + x_2 + \dots + x_n \\ S_2 &= x_1x_2 + x_1x_3 + \dots x_{n-1}x_n \\ S_3 &= x_1x_2x_3 + x_1x_2x_4 + \dots x_{n-2}x_{n-1}x_n \end{aligned}$$


---

## 7.8 Matrices

To create matrices in math mode, you create a table as you would in Section 6.1 except you replace `\begin{tabular}` and `\end{tabular}` with `\begin{array}` and `\end{array}`. Here is an example:

---

```
\left[ \begin{array}{cccc}
1 & x_{11} & x_{12} & x_{13} \\
1 & x_{21} & x_{22} & x_{23} \\
1 & x_{31} & x_{32} & x_{33} \\
1 & x_{41} & x_{42} & x_{43}
\end{array} \right] \times
```

---


$$\begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} \\ 1 & x_{21} & x_{22} & x_{23} \\ 1 & x_{31} & x_{32} & x_{33} \\ 1 & x_{41} & x_{42} & x_{43} \end{bmatrix} \times \begin{bmatrix} \Lambda_1 & 0 & 0 & 0 \\ 0 & \Lambda_2 & 0 & 0 \\ 0 & 0 & \Lambda_3 & 0 \\ 0 & 0 & 0 & \Lambda_4 \end{bmatrix}$$


---



To automatically number theorems, corollaries, and lemmas, you must include the following in the Preamble:

```
\newtheorem{theorem}{Theorem}[ ]
\newtheorem{corollary}{Corollary}[ ]
\newtheorem{lemma}{Lemma}[ ]
```

If you want a theorem, corollary, or lemma number to be indexed by a section number, enter [section]. If you want a theorem, corollary, or lemma number to be indexed by a section.subsection number, enter [subsection]. If you just want the item numbered sequentially but not by any section or subsection, remove the [ ].

In this document, I used:

```
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[section]
\newtheorem{lemma}{Lemma}[subsection]
```

Then use the following with the theorem, corollary, or lemma entered in-between. Also enter a label name as you have done previously for section, tables, figures, and equations.

```
\begin{theorem} \label{ } \end{theorem}
\begin{corollary} \label{ } \end{corollary}
\begin{lemma} \label{ } \end{lemma}
```

Here is an example of each type. Note the theorem and corollary are numbered first by section, while the lemma is numbered by section.subsection numbers.

---

```
\begin{theorem} \label{thm:coup}
The probability  $P(X_{M,n}=i)$  of collection
exactly  $i$  coupon types ( $i=1,\ldots,n$ ) in sample of size
 $M$  given  $n$  possible coupon types is
\[ P(X_{M,n}=i) = \frac{1}{n^M} \sum_{i=1}^n \binom{n}{i} (-1)^{i-1} \]
\end{theorem}

\begin{corollary} \label{cor:coup}
As a direct result of Theorem \ref{thm:coup},
\[ E(X_{M,n}) = \sum_{i=1}^n \frac{1}{n^i} \sum_{j=1}^i \binom{n}{j} (-1)^{j-1} \]
\end{corollary}

\begin{lemma} \label{lem:coup}
Let  $T$  be the sample on which all  $n$  coupon types are collected
\underline{for the first time}. Therefore,
\[ E(T) = \sum_{t=1}^n \frac{1}{n^t} \sum_{i=t}^n \binom{n}{i} (-1)^{i-t} \]
\end{lemma}
```

.....

$$P(X_{M,n} = i) = \frac{\binom{n}{i} \alpha_{M,i}}{n^M}$$

**Corollary 7.1** *As a direct result of Theorem 7.1,*

$$E(X_{M,n}) = \sum_{i=1}^n i \frac{\binom{n}{i} \alpha_{M,i}}{n^M}$$

**Lemma 7.9.1** *Let  $T$  be the sample on which all  $n$  coupon types are collected for the first time. Therefore,*

$$E(T) = \lim_{M \rightarrow \infty} \sum_{t=1}^M t \frac{\alpha_{t-1,n-1}}{n^{t-1}}$$


---

## 7.10 Maple-to-LaTeX Conversion

For those of you who use Maple, you can save equations, expressions, matrices, etc. in your session window as text files that include LaTeX commands. In Maple there is an export as Latex option:

**File  $\rightarrow$  Export As  $\rightarrow$  LaTeX**

and then save it as a .tex file. You will need to do some minor editing of this .tex file and remove unwanted lines. For example, if you entered

```
px:= (1+2*x-3*x^2)^3;
expand(px);
```

you would see  $1 + 6x + 3x^2 - 28x^3 - 9x^4 + 54x^5 - 27x^6$  on your monitor. If I then exported the session as a LaTeX file, the file would include the following:

```
\[ {\it px} := (1 + 2\,x - 3\,x^2)^3 \]
\[ 1 + 6\,x + 3\,x^2 - 28\,x^3 - 9\,x^4 + 54\,x^5 - 27\,x^6 \]
```

which (in math mode) would produce

$$px := (1 + 2x - 3x^2)^3$$


$$1 + 6x + 3x^2 - 28x^3 - 9x^4 + 54x^5 - 27x^6$$

## 8 Some Comments on Using WinEdt

### 8.1 Compiling and Debugging a .tex File

To compile a .tex file in WinEdt, click on the brown bear head (it will say TeXify if you place the cursor over it). If there are no errors, a .dvi window will appear with the LaTeX document for you to view. If you have numbered equations, it is best to compile it a second time so that references to numbered objects are correct.

If you have errors (which you will always have the first time), a screen will appear indicating where the error was detected and information about the type of error. Many times the errors are basic (missing }, missing \end statement, etc.). Just go that line in the document and make the correction and compile again.

If you want to convert the compiled LaTeX document as a .pdf file, just click on the  icon. It will automatically create a .pdf file.

### 8.3 Quick Table and Figure Templates

To create a template for a table with  $n$  rows and  $m$  columns in WinEdt, do the following: Click on **Insert**  $\longrightarrow$  **Tabular ( $n \times m$ )**. You will then be prompted for the number of rows and columns of your table. If you enter 4 for rows and 6 for columns, the following would be inserted into the document (which you then can edit). You can delete the comment line.

```
\begin{tabular}{[*|*|*|*|*|*]}
\hline
% after \: \hline or \cline[col1-col2] \cline[col3-col4] ...
* & * & * & * & * & * & * & \\
* & * & * & * & * & * & * & \\
* & * & * & * & * & * & * & \\
* & * & * & * & * & * & * & \\
\hline
\end{tabular}
```

To quickly insert a figure in WinEdt do the following: **Insert**  $\longrightarrow$  **Objects**  $\longrightarrow$  **Figure**. The following LaTeX code will be inserted into the document (which you then can edit). You can delete the comment line. Note also the caption and label statements appear at the end. The caption and labels can either appear just after the `\begin{figure}` statement or just before the `\end{figure}` statement.

```
\begin{figure}
% Requires \usepackage{graphicx}
\includegraphics[width=*]{*}\
\caption{*}\label{*}
\end{figure}
```

## 9 Compiling Multiple .tex Files

When you are working with a large document that may have many sections, it is advisable to save each section in a separate .tex file. You can then compile the entire document using `\include { }` statements. Inside the braces `{ }` you would enter the .tex filename but without the .tex extension.

The following .tex file is what I used to generate my ST 815 course notes.

```
\pagestyle{plain}
\setlength{\topmargin}{-.7in}
\setlength{\textheight}{9.5in}
\setlength{\oddsidemargin}{-.2in}
\setlength{\textwidth}{6.75in}
%\includeonly{11dist,12adapt}
\begin{document}
\tableofcontents
\include{1intro}
\include{2srs}
\include{3hhht}
\include{4ratreg}
\include{5strat}
\include{6clus}
\include{7multi}
\include{8jack}
\include{9boot}
\include{10detect}
\include{11dist}
\include{12adapt}
\end{document}
```

I had 12 LaTeX files named `lintro.tex`, `2srs.tex`, ... , `12adapt.tex`. Compiling this file would generate the entire set of course notes. If you want a table of contents for your document, include a `\tableofcontents` statement immediately following the `\begin{document}` statement. The table of contents will include section and subsection titles and their initial page numbers.

There is a nice option and that is the `\includeonly{ }` statement. Inside the `{ }` enter .tex files corresponding to the specific sections of the document you would like to compile and view. You should only use an `\includeonly` statement after the enter document has been compiled at least once. This will keep the numbering of all previous items intact. The `\includeonly` statement has been commented out with `%` so it would not be compiled.

## 10 Special Sections in a Document

## 10.1 Including a Cover Page

To create a cover page for a document, the general format is to include the following after the `\begin{document}` statement:

```
\begin{document}
\title{ }
\author{    \\\  \\\  \\\ ...
\and      \\\  \\\  \\\ ... }
\date{}
\maketitle
```

- Enter the first author after `\author{}`. Then between each `\\` enter author information such as professional title, affiliation, etc.
- If there is a second author, enter `\and` after you have entered all information about the first author. Then, once again, between each `\\` enter author information.
- If there are more than two authors, use additional `\and` statements.
- Enter the document date in the `{ }` in the `\date` statement.
- Enter `\maketitle`

To create the title page for this document, I used the following statements. Note that the table of contents statements is placed after making the title.

```
\begin{document}
\title{An Introduction to Mathematics and Scientific Document Preparation
using LaTeX}
\author{John J. Borkowski \\ Professor of Statistics \\
Department of Mathematical Sciences \\ Montana State University \\
www.math.montana.edu/~textasciitilde jobo}
\date{September 2008}
\maketitle
\tableofcontents
```

### 10.2 Including an Abstract

To create a document abstract, enter the abstract between `\begin{abstract}` and `\end{abstract}` statements. For this document I entered the following right after the cover page and `\tableofcontents` statements shown in Section 10.1. For more details, see Section 12.

```
\begin{abstract}
This document is an introduction to using LaTeX to prepare mathematical
and scientific documents. There is so much more that can be done using
LaTeX so this document is not meant to be comprehensive and answer all
questions you may have about using LaTeX. But, it should get you started
on creating professional quality documents using this powerful tool.
\end{abstract}
```

### 10.3 Creating a Numbered Bibliography

In many professional documents you will cite references from journals, texts, websites, etc. To create a bibliography (set of references) that can be cited within the document, the easiest way is to use BibTex which can be compiled within WidEdt.

BibTex reads a text file with extension **.bib**. The .bib file contains a list of references (often called the bibliographic database) that can be used with any .tex document. That is, you can just keep adding references to a .bib file and LaTeX will just include those references cited in the .tex file you compile and ignore all of the other references in the bibliographic database.

compile BibTex      compile BibTex      compile BibTex again

To compile BibTex, click on “Bib” icon in the toolbar in WidEdt. To compile LaTeX, click on the “brown bear head” icon discussed in Section 8.1. Sometimes you will have to run this sequence two or three times if you made changes to the .bib file and included additional citations or changed citations in the .tex document.

The format for creating references in the .bib file is

```
@ type { label ,
        field 1 = "field 1 content" ,
        field 2 = "field 2 content" ,
        :
        :
}
```

- For *type*, enter the reference type.
- For *label*, enter the label you want associated with that reference. It serves exactly the same purpose as all of the labels you have seen before.
- For a specific reference type, there will be required fields such as title, date, author, etc. These are summarized in Table 3.
- In the double quotations “ ” enter what you want to appear for that particular field.

Table 3: The Common Reference Types

Reference Type	Required Fields	Optional Fields
article	author, title, journal name, year	volume, number, note, pages, month
book	author or editor, title, publisher, year	volume or number, series, note, address, edition, month
inbook	author or editor, title, publisher, chapter and/or pages, year	volume or number, series, type, address, edition, month, note
inproceedings	author, title, book title, year	editor, volume or number, series, pages, note, month, organization, publisher, address
manual	author or key, title,	author, organization, address, edition, month, year, note
mastersthesis	author, title, school, year	type, address, month, note
misc	author or key, school, year	type, author, title, month, howpublished, year, note
phdthesis	author, title, school, year	type, address, month, note
proceedings	title, year	editor, volume or number, series, month, organization, publisher, address, note
techreport	author, title, institution, year	type, number, note address, month
unpublished	author, title, note	month, year

Here is a small subset of entries in the .bib file for my dissertation. The file is jjb.bib.

```
@Article( art:kw60 ,
  author= "J. Kiefer and J. Wolfowitz" ,
  title = "The Equivalence of Two Extremum Problems" ,
  journal="Canadian Journal of Mathematics" ,
  year = "1960",
  volume= "12" ,
  pages = "363-366" )

@Book ( bk:rh85 ,
  author= {Ronald .R. Hocking},
  title = {The Analysis of Linear Models},
  publisher={Brooks/Cole Publishing Co. },
  address={Monterey, CA},
  year = {1985})

@phdthesis (phd:jl72 ,
  author= "James M. Lucas" ,
  title = "The Optimum Design of Industrial Experiments",
  school= "Texas A&M University" ,
  year = "1972" ,
  month = "May" )

@techreport ( tr:bj90 ,
  author= "George Box and Stephen Jones ",
  title = "Designing Products that are Robust to the Environment",
  institution ="Center for Quality and Productivity Improvement,
University of Wisconsin ",
  year = "1990 ",
  type = "Technical Report ")
```

A **References** section is automatically created with the cited list of references. See the very end of this document for the References section containing these five cited references. To cite a reference in the .bib file, use `\cite{ }` and enter the citation label in the braces `{ }`. For example:

---

The 1960 article by Kiefer and Wolfowitz `\cite{art:kw60}` was a very important publication in optimal design theory. When I was a PhD student, the professor who taught Linear Models used the textbook by Hocking `\cite{bk:rh85}`. You may also want to cite a PhD dissertation like the one by Lucas `\cite{phd:jl72}` (who was my PhD advisor). Technical reports are also a good source of research results like Box and Jones `\cite{tr:bj90}`.

---

.....

The 1960 article by Kiefer and Wolfowitz [3] was a very important publication in optimal design theory. When I was a PhD student, the professor who taught Linear Models used the textbook by Hocking [2]. You may also want to cite a PhD dissertation like the one by Lucas [4] (who was my PhD advisor). Technical reports are also a good source of research results like Box and Jones [1].

---

contain too many references, you can just type the author's name and date (such as Borkowski (2008)) in the text and create a section at the end of the article what contains a list of references (as you would in Word).

For a good summary of creating a bibliography and many of the options to modify how citations appear, I recommend Harvey Greenberg's introduction to LaTeX at <http://samizdat.mines.edu/latex/>.

## 11 Web Resources

There is a huge number of resources on the internet for LaTeX. Do a Google search with keywords "LaTeX introduction" and you will get numerous links. Here are three that I checked out and liked. I am sure there are many other good sources also.

<http://samizdat.mines.edu/latex/>  
<http://www.maths.tcd.ie/~dwilkins/LaTeXPrimer/>  
<http://www.math.harvard.edu/texman/>

## 12 Final Comments

I thought it would be worthwhile to show you the actual Preamble I used to create this document, and the beginning and end of the document:

```
\documentclass [12pt] {article}
\usepackage{graphicx}
\pagestyle{plain}
\newtheorem{theorem}{Theorem}[section]
\newtheorem{corollary}{Corollary}[section]
\newtheorem{lemma}{Lemma}[subsection]
\setlength{\topmargin}{-.7in}
\setlength{\textheight}{9.5in}
\setlength{\oddsidemargin}{-.3in}
\setlength{\textwidth}{7in}
\def\jjb{John J. Borkowski }
\def\msu{Montana State University }
\def\tbs{\textbackslash}
\def\ds{\displaystyle }
\begin{document}
\title{An Introduction to Mathematics and Scientific Document Preparation
using LaTeX}
\author{John J. Borkowski \\\ Professor of Statistics \\\ Department of
Mathematical Sciences \\\ Montana State University \\\
www.math.montana.edu/\textasciitilde jobo}
\date{September 2008}
\maketitle
\tableofcontents
\newpage
```

```
\end{abstract}

\section{Introduction}\label{s:intro}
:   :   :
:   :   :
\bibliographystyle{plain}
\bibliography{jjb}
\end{document}
```

References

[1] George Box and Stephen Jones. Designing products that are robust to the environment. Technical report, Center for Quality and Productivity Improvement, University of Wisconsin, 1990.

[2] Ronald .R. Hocking. *The Analysis of Linear Models*. Brooks/Cole Publishing Co., Monterey, CA, 1985.

[3] J. Kiefer and J. Wolfowitz. The equivalence of two extremum problems. *Canadian Journal of Mathematics*, 12:363–366, 1960.

[4] James M. Lucas. *The Optimum Design of Industrial Experiments*. PhD thesis, Texas A&M University, May 1972.