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Infinite Series Forms of Double Integrals

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Abstract This paper studies six types of double integrals and uses Maple for verification. The infinite series forms of these double integrals can be obtained using Taylor series expansions and integration term by term theorem. In addition, some examples are used to demonstrate the calculations.

Keywords: *double integrals, infinite series forms, Taylor series expansions, integration term by term theorem, Maple*

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1. Introduction

In calculus and engineering mathematics, there are many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we study the following six types of double integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\cos(n\theta+r\sin\theta)drd\theta\,, \tag{1}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\sin(n\theta+r\sin\theta)drd\theta\,, \tag{2}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ -\sin n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta\,, \tag{3}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\sin n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ +\cos n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta\,, \tag{4}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ +\sin n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta\,, \tag{5}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\sin n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ -\cos n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta\,, \tag{6}$$

where $r_1, r_2, \theta_1, \theta_2$ are any real numbers, and n is any positive integer. We can obtain the infinite series forms of these double integrals using Taylor series expansions and integration term by term theorem; these are the major results of this paper (i.e., Theorems 1-3). Adams et al. [1], Nyblom [2], and Oster [3] provided some techniques to solve the integral problems. Yu [4-29], Yu and B. -H. Chen [30], Yu and Sheu [31], and T. -J. Chen and Yu [32,33,34] used complex power series method, integration term by term theorem, differentiation with respect to a parameter, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula to solve some types of integral problems. In this paper, three examples are used to demonstrate the proposed calculations, and the manual calculations are verified using Maple.

2. Main Results

Some formulas and theorems used in this paper are introduced below.

2.1. Euler's Formula

$e^{ix}=\cos x+i\sin x$, where $i=\sqrt{-1}$, and x is any real number.

2.2. DeMoivre's Formula

$(\cos x+i\sin x)^m=\cos mx+i\sin mx$, where m is any integer, and x is any real number.

The followings are the Taylor series expansions of some analytic functions.

2.3. $e^z=\sum_{k=0}^{\infty}\frac{1}{k!}z^k$, where z is any Complex Number

2.4. $\sin z=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+1}$, where z is any Complex Number

2.5. $\cos z=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}z^{2k}$, where z is any Complex Number

The following two formulas can be found in [[35], p 62]

2.6. $\sin(a+ib)=\sin a\cosh b+i\cos a\sinh b$**, where** a,b **are any real Numbers**

2.7. $\cos(a+ib)=\cos a\cosh b-i\sin a\sinh b$ **, where** a,b **are any real Numbers**

An important theorem used in this study is introduced below, which can be found in [[36], p 269].

2.8. Integration Term by Term Theorem

Suppose that $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

Firstly, we determine the infinite series forms of the double integrals (1) and (2).

Theorem 1. Suppose that $r_1,r_2,\theta_1,\theta_2$ are real numbers, and n is a positive integer. Then the double integrals:

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\cos(n\theta+r\sin\theta)drd\theta = \sum_{k=0}^{\infty}\frac{1}{k!(k+1)(k+n)}(r_2^{k+1}-r_1^{k+1})[\sin(k+n)\theta_2-\sin(k+n)\theta_1], \quad (7)$$

and

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\sin(n\theta+r\sin\theta)drd\theta = \sum_{k=0}^{\infty}\frac{-1}{k!(k+1)(k+n)}(r_2^{k+1}-r_1^{k+1})[\cos(k+n)\theta_2-\cos(k+n)\theta_1] \quad (8)$$

Proof By Formula 2.3, we have:

$$z^n e^z = \sum_{k=0}^{\infty}\frac{1}{k!}z^{k+n}. \quad (9)$$

Let $z=re^{i\theta}$, where r,θ are any real numbers, then:

$$(re^{i\theta})^n e^{re^{i\theta}} = \sum_{k=0}^{\infty}\frac{1}{k!}(re^{i\theta})^{k+n}. \quad (10)$$

By Euler's formula and DeMoivre's formula, we obtain:

$$e^{in\theta}e^{re^{i\theta}} = \sum_{k=0}^{\infty}\frac{1}{k!}r^k e^{i(k+n)\theta}. \quad (11)$$

It follows that:

$$e^{r\cos\theta}\cdot e^{i(n\theta+r\sin\theta)} = \sum_{k=0}^{\infty}\frac{1}{k!}r^k e^{i(k+n)\theta}. \quad (12)$$

Using the equality of real parts of both sides of Eq. (12) yields:

$$e^{r\cos\theta}\cdot\cos(n\theta+r\sin\theta) = \sum_{k=0}^{\infty}\frac{1}{k!}r^k\cos[(k+n)\theta] \quad (13)$$

Therefore,

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\cos(n\theta+r\sin\theta)drd\theta$$
$$=\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\sum_{k=0}^{\infty}\frac{1}{k!}r^k\cos[(k+n)\theta]drd\theta$$
$$=\sum_{k=0}^{\infty}\frac{1}{k!}\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} r^k\cos[(k+n)\theta]drd\theta$$
(by integration term by term theorem)
$$=\sum_{k=0}^{\infty}\frac{1}{k!(k+1)(k+n)}(r_2^{k+1}-r_1^{k+1})[\sin(k+n)\theta_2-\sin(k+n)\theta_1]$$

On the other hand, by the equality of imaginary parts of both sides of Eq. (12) yields:

$$e^{r\cos\theta}\cdot\sin(n\theta+r\sin\theta) = \sum_{k=0}^{\infty}\frac{1}{k!}r^k\sin[(k+n)\theta]. \quad (14)$$

Using integration term by term theorem, we can easily obtain:

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} e^{r\cos\theta}\cdot\sin(n\theta+r\sin\theta)drd\theta = \sum_{k=0}^{\infty}\frac{-1}{k!(k+1)(k+n)}(r_2^{k+1}-r_1^{k+1})[\cos(k+n)\theta_2-\cos(k+n)\theta_1]$$

Next, the infinite series forms of the double integrals (3) and (4) can be obtained below.

Theorem 2. If the assumptions are the same as Theorem 1, then the double integrals:

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ -\sin n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(2k+2)(2k+n+1)}(r_2^{2k+2}-r_1^{2k+2})[\sin(2k+n+1)\theta_2-\sin(2k+n+1)\theta_1] \quad (15)$$

and

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\sin n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ +\cos n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k+1)!(2k+2)(2k+n+1)}(r_2^{2k+2}-r_1^{2k+2})[\cos(2k+n+1)\theta_2-\cos(2k+n+1)\theta_1] \quad (16)$$

Proof Using Formula 2.4 yields:

$$z^n\sin z = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+n+1}. \quad (17)$$

Let $z=re^{i\theta}$, where r,θ are any real numbers, then:

$$(re^{i\theta})^n\sin(re^{i\theta}) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}(re^{i\theta})^{2k+n+1}. \quad (18)$$

By Euler's formula, DeMoivre's formula, and Formula 2.6, we obtain:

$$(\cos n\theta + i\sin n\theta)\cdot\begin{bmatrix}\sin(r\cos\theta)\cosh(r\sin\theta)\\ +i\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}r^{2k+1}e^{i(2k+n+1)\theta}. \quad (19)$$

Using the equality of real parts of both sides of Eq. (19) yields:

$$\begin{aligned}&\cos n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ &-\sin n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\\ &=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}r^{2k+1}\cos[(2k+n+1)\theta]\end{aligned}. \quad (20)$$

Thus,

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ -\sin n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta$$

$$=\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}r^{2k+1}\cos[(2k+n+1)\theta]drd\theta$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}r^{2k+1}\cos[(2k+n+1)\theta]drd\theta$$

(by integration term by term theorem)

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!(2k+2)(2k+n+1)}(r_2^{2k+2}-r_1^{2k+2})[\sin(2k+n+1)\theta_2-\sin(2k+n+1)\theta_1]$$

On the other hand, using the equality of imaginary parts of both sides of Eq. (19) yields:

$$\begin{aligned}&\sin n\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ &+\cos n\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\\ &=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}r^{2k+1}\sin[(2k+n+1)\theta]\end{aligned}. \quad (21)$$

By integration term by term theorem, we obtain Eq. (16).

Finally, we determine the infinite series forms of the double integrals (5) and (6).

Theorem 3 If the assumptions are the same as Theorem 1, then the double integrals:

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ +\sin n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(2k+1)(2k+n)}(r_2^{2k+1}-r_1^{2k+1})[\sin(2k+n)\theta_2-\sin(2k+n)\theta_1] \quad (22)$$

and

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\sin n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ -\cos n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k)!(2k+1)(2k+n)}(r_2^{2k+1}-r_1^{2k+1})[\cos(2k+n)\theta_2-\cos(2k+n)\theta_1] \quad (23)$$

Proof By Formula 2.5, we have:

$$z^n\cos z=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}z^{2k+n}. \quad (24)$$

Let $z=re^{i\theta}$, where r,θ are any real numbers, then:

$$(re^{i\theta})^n\cos(re^{i\theta})=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}(re^{i\theta})^{2k+n}. \quad (25)$$

Using Formula 2.7 yields:

$$(\cos n\theta + i\sin n\theta)\cdot\begin{bmatrix}\cos(r\cos\theta)\cosh(r\sin\theta)\\ -i\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix} = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}r^{2k}e^{i(2k+n)\theta}. \quad (26)$$

By the equality of real parts of both sides of Eq. (26), we have:

$$\begin{aligned}&\cos n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ &+\sin n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\\ &=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}r^{2k}\cos[(2k+n)\theta]\end{aligned}. \quad (27)$$

Hence,

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\begin{bmatrix}\cos n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ +\sin n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta$$

$$=\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}r^{2k}\cos[(2k+n)\theta]drd\theta$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}r^{2k}\cos[(2k+n)\theta]drd\theta.$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!(2k+1)(2k+n)}(r_2^{2k+1}-r_1^{2k+1})[\sin(2k+n)\theta_2-\sin(2k+n)\theta_1]$$

On the other hand, the equality of imaginary parts of both sides of Eq. (26) implies that:

$$\begin{aligned}&\sin n\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ &-\cos n\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\\ &=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k)!}r^{2k}\sin[(2k+n)\theta]\end{aligned}. \quad (28)$$

Thus, using integration term by term theorem yields Eq. (23) holds.

3. Examples

In the following, for the six types of double integrals in this study, we provide three examples and use Theorems 1-3 to determine their infinite series forms. In addition, Maple is used to calculate the approximations of some double integrals and their solutions for verifying our answers.

Example 1. By Eq. (7), we obtain:

$$\int_{\pi/6}^{\pi/3}\int_2^3 e^{r\cos\theta}\cdot\cos(4\theta+r\sin\theta)drd\theta = \sum_{k=0}^{\infty}\frac{1}{k!(k+1)(k+4)}(3^{k+1}-2^{k+1})\left[\sin\frac{(k+4)\pi}{3}-\sin\frac{(k+4)\pi}{6}\right] \quad (29)$$

Next, we use Maple to verify the correctness of Eq. (29).

```
>evalf(Doubleint(exp(r*cos(theta))*cos(4*theta+r*sin(theta)),r=2..3,theta=Pi/6..Pi/3),18);
-0.109048195978862494
>evalf(sum(1/(k!*(k+1)*(k+4))*(3^(k+1)-2^(k+1))*(sin((k+4)*Pi/3)- sin((k+4)*Pi/6)), k=0..infinity), 18);
-0.10904819597886250
```

Also using Eq. (8) yields:

$$\int_{\pi/8}^{\pi/4}\int_{3}^{5} e^{r\cos\theta}\cdot\sin(6\theta+r\sin\theta)drd\theta = \sum_{k=0}^{\infty}\frac{-1}{k!(k+1)(k+6)}(5^{k+1}-3^{k+1})\left[\cos\frac{(k+6)\pi}{4}-\cos\frac{(k+6)\pi}{8}\right] \tag{30}$$

```
>evalf(Doubleint(exp(r*cos(theta))*sin(6*theta+r*sin(theta)), r=3..5,theta=Pi/8..Pi/4),18);
-8.99753972102267964
>evalf(sum(-1/(k!*(k+1)*(k+6))*(5^(k+1)-3^(k+1))*(cos((k+6)*Pi/4)-cos((k+6)*Pi/8)),k=0..infinity), 18);
-8.99753972102267965
```

Example 2. Using Eq. (15) yields:

$$\int_{\pi/6}^{\pi/4}\int_{2}^{5}\begin{bmatrix}\cos 2\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ -\sin 2\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{(2k+1)!(2k+2)(2k+3)}(5^{2k+2}-2^{2k+2})\left[\sin\frac{(2k+3)\pi}{4}-\sin\frac{(2k+3)\pi}{6}\right] \tag{31}$$

We also use Maple to verify the correctness of Eq. (31).

```
>evalf(Doubleint(cos(2*theta)*sin(r*cos(theta))*cosh(r*sin(theta))- sin(2*theta)* cos(r*cos(theta))* sinh(r*sin(theta)), r=2..5, theta=Pi/6..Pi/4), 18);
2.95966791987337516
>evalf(sum((-1)^k/((2*k+1)!* (2*k+2)* (2*k+3))* (5^(2*k+2)-2^(2*k+2))* (sin((2*k+3)*Pi/4)-sin((2*k+3)*Pi/6)), k=0.infinity), 18);
2.95966791987337516
```

On the other hand, Eq. (16) implies that:

$$\int_{\pi/12}^{\pi/6}\int_{4}^{7}\begin{bmatrix}\sin 3\theta\cdot\sin(r\cos\theta)\cosh(r\sin\theta)\\ +\cos 3\theta\cdot\cos(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k+1)!(2k+2)(2k+4)}(7^{2k+2}-4^{2k+2})\left[\cos\frac{(k+2)\pi}{3}-\cos\frac{(k+2)\pi}{6}\right] \tag{32}$$

```
>evalf(Doubleint(sin(3*theta)*sin(r*cos(theta))*cosh(r*sin(theta))+cos(3*theta)*cos(r*cos(theta))*sinh(r*sin(theta)), r=4..7,theta=Pi/12..Pi/6),18);
-1.64755306054358357
>evalf(sum((-1)^(k+1)/((2*k+1)!*(2*k+2)*(2*k+4))*(7^(2*k+2)-4^(2*k+2))*(cos((k+2)*Pi/3)-cos((k+2)*Pi/6)), k=0. infinity), 18);
-1.64755306054358358
```

Example 3. By Eq. (22) we have:

$$\int_{2\pi/3}^{5\pi/6}\int_{3}^{8}\begin{bmatrix}\cos 5\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ +\sin 5\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k}}{(2k)!(2k+1)(2k+5)}(8^{2k+1}-3^{2k+1})\left[\sin\frac{(10k+25)\pi}{6}-\sin\frac{(4k+10)\pi}{3}\right] \tag{33}$$

Using Maple to verify the correctness of Eq. (33) as follows:

```
>evalf(Doubleint(cos(5*theta)*cos(r*cos(theta))*cosh(r*sin(theta))+sin(5*theta)*sin(r*cos(theta))*sinh(r*sin(theta)),r=3..8,theta=2*Pi/3..5*Pi/6),18);
-42.8603724218153132
>evalf(sum((-1)^k/((2*k)!* (2*k+1)*(2*k+5))*(8^(2*k+1)- 3^(2*k+1))*(sin((10*k+25)*Pi/6)-sin((4*k+10)*Pi/3)), k=0. infinity), 18);
-42.8603721218153128
```

In addition, using Eq. (23) yields:

$$\int_{3\pi/8}^{5\pi/8}\int_{2}^{6}\begin{bmatrix}\sin 3\theta\cdot\cos(r\cos\theta)\cosh(r\sin\theta)\\ -\cos 3\theta\cdot\sin(r\cos\theta)\sinh(r\sin\theta)\end{bmatrix}drd\theta = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{(2k)!(2k+1)(2k+3)}(6^{2k+1}-2^{2k+1})\left[\cos\frac{(10k+15)\pi}{8}-\cos\frac{(6k+9)\pi}{8}\right] \tag{34}$$

```
>evalf(Doubleint(sin(3*theta)*cos(r*cos(theta))*cosh(r*sin(theta))-cos(3*theta)* sin(r*cos(theta))* sinh(r*sin(theta)), r=2..6,theta=3*Pi/8..5*Pi/8),18);
-11.5671734819804792
>evalf(sum((-1)^(k+1)/((2*k)!* (2*k+1)*(2*k+3))*(6^(2*k+1)-2^(2*k+1))*(cos((10*k+15)*Pi/8)-cos((6*k+9)*Pi/8)), k=0. infinity), 18);
-11.5671734819804793
```

4. Conclusion

In this paper, we use Taylor series expansions and integration term by term theorem to solve some types of double integrals. In fact, the applications of the two methods are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and use Maple to verify our answers.

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