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#3 Components of a Vector

if
$$V = 34 \text{ m/sec } \angle 48^{\circ}$$

then

 $V_i = 34 \text{ m/sec} \cdot (\cos 48^\circ); \text{ and } V_I = 34 \text{ m/sec} \cdot (\sin 48^\circ)$

#4 Weight = $m \cdot g$

g = 9.81 m/sec² near the surface of the Earth = 9.795 m/sec² in Fort Worth, TX

Density = mass / volume

$$\rho = \frac{m}{V} \left(unit : kg / m^3 \right)$$

#7 Ave speed = distance / time = v = d/t
Ave velocity = displacement / time = v = d/t
Ave acceleration = change in velocity / time

#8 Friction Force

$$\mathbf{F}_{\mathbf{F}} = \mu \cdot \mathbf{F}_{\mathbf{N}}$$

If the object is not moving, you are dealing with static friction and it can have any value from zero up to μ_s $F_{_N}$

If the object is sliding, then you are dealing with kinetic friction and it will be constant and equal to $\mu_{_{\rm K}}\,F_{_{\rm N}}$

#9 Torque

$$\tau = F \cdot L \cdot \sin \theta$$

Where θ is the angle between F and L; unit: Nm

#11 Newton's Second Law

$$F_{net} = \sum F_{Ext} = m \cdot a$$

#12 Work = $\mathbf{F} \cdot \mathbf{D} \cdot \cos \theta$

Where D is the distance moved and θ is the angle between F and the direction of motion,

unit: J

#16 Power = rate of work done

$$Power = \frac{Work}{time}$$
 unit: watt

 $\begin{aligned} & \textbf{Efficiency} = \textbf{Work}_{out} \, / \, \textbf{Energy}_{in} \\ \textbf{Mechanical Advantage} & = \textbf{force out} \, / \, \textbf{force in} \\ & M.A. = F_{out} \, / \, F_{in} \end{aligned}$

#19 Constant-Acceleration Linear Motion

$$\begin{aligned} v &= v_o + a \bullet t & x \\ (x - x_o) &= v_o \bullet t + \frac{1}{2} \bullet a \bullet t^2 & v \\ v^2 &= v_o^2 + 2 \bullet a \bullet (x - x_o) t \\ (x - x_o) &= \frac{1}{2} \bullet (v_o + v) \bullet t & a \\ (x - x_o) &= v \bullet t - \frac{1}{2} \bullet a \bullet t^2 & v_o \end{aligned}$$

#20 Heating a Solid, Liquid or Gas

 $Q = m \cdot c \cdot \Delta T$ (no phase changes!) Q = the heat addedc = specific heat.

 ΔT = temperature change, K

#21 Linear Momentum

momentum = $\mathbf{p} = \mathbf{m} \cdot \mathbf{v} = \text{mass} \cdot \text{velocity}$ momentum is conserved in collisions

#23 Center of Mass – point masses on a line

$$x_{cm} = \Sigma(mx) / M_{total}$$

#25 Angular Speed vs. Linear Speed

Linear speed = $v = r \cdot \omega = r \cdot angular$ speed

#26 Pressure under Water

$$P = \rho \cdot g \cdot h$$

h = depth of water $\rho = density of water$

#28 Universal Gravitation

$$F=Grac{m_1m_2}{r^2}$$
 $_{
m G}=6.67~{
m E-}11~{
m N}~{
m m}^2/{
m kg}^2$

#29 Mechanical Energy

$$PE_{Grav} = P = m \cdot g \cdot h$$

$$KE_{Linear} = K = \frac{1}{2} \cdot m \cdot v^{2}$$

#30 Impulse = Change in Momentum

$$F \cdot \Delta t = \Delta (m \cdot v)$$

#31 Snell's Law

$$n_1 \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$$

Index of Refraction

$$n = c / v$$

 $c = \text{speed of light} = 3 E+8 \text{ m/s}$

#32 Ideal Gas Law

$$P \bullet V = n \bullet R \bullet T$$

n = # of moles of gas R = gas law constant = 8.31 J/K mole.

#34 Periodic Waves

$$v = f \cdot \lambda$$

 $f = 1 / T$ $T = period of wave$

#35 Constant-Acceleration Circular Motion

$$\begin{split} \omega &= \omega_o + \alpha \bullet t & \theta \\ \theta - \theta_o &= \omega_o \bullet t + \frac{1}{2} \bullet \alpha \bullet t^2 & \omega \\ \omega &= \omega_o^2 + 2 \bullet \alpha \bullet (\theta - \theta_o) & t \\ \theta - \theta_o &= \frac{1}{2} \bullet (\omega_o + \omega) \bullet t & \alpha \\ \theta - \theta_o &= \omega \bullet t - \frac{1}{2} \bullet \alpha \bullet t^2 & \omega_o \end{split}$$

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#36 Buoyant Force - Buoyancy

$$\begin{split} F_B = \rho \bullet V \bullet g = m_{Displaced \ fluid} \bullet g = weight_{Displaced \ fluid} \\ \rho = density \ of \ the \ fluid \\ V = volume \ of \ fluid \ displaced \end{split}$$

#37 Ohm's Law

 $V = I \cdot R$ V = voltage applied I = current R = resistance

Resistance of a Wire

 $R = \rho \cdot L / A_x$ $\rho = \text{resistivity of wire material}$ L = length of the wire $A_x = \text{cross-sectional area of the wire}$

#39 Heat of a Phase Change

Q = m•L L = Latent Heat of phase change

#41 Hooke's Law

 $F = k \cdot x$

Potential Energy of a spring $W = \frac{1}{2} \cdot k \cdot x^2 = W$ ork done on spring

#42 Electric Power

 $P = I^2 \cdot R = V^2 / R = I \cdot V$

#44 Speed of a Wave on a String

$$T = \frac{mv^2}{L}$$

T = tension in string m = mass of string L = length of string

#45 Projectile Motion

Horizontal: $x-x_0=v_0 \cdot t + 0$ **Vertical:** $y-y_0=v_0 \cdot t + \frac{1}{2} \cdot a \cdot t^2$

#46 Centripetal Force

 $F = \frac{mv^2}{r} = m\omega^2 r$

#47 Kirchhoff's Laws

Loop Rule: $\Sigma_{\text{Around any loop}} \Delta V_i = 0$ Node Rule: $\Sigma_{\text{at any node}} I_i = 0$

#51 Minimum Speed at the top of a Vertical Circular Loop

 $v = \sqrt{rg}$

#53 Resistor Combinations

SERIES

 $R_{eq} = R_1 + R_2 + R_3 + \dots$ **PARALLEL** $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$

#54 Newton's Second Law and Rotational Inertia

 $\tau = torque = I {\color{red} \bullet} \alpha$ $I = moment \ of \ inertia = m {\color{red} \bullet} r^2 \ (for \ a \ point \ mass)$

#55 Circular Unbanked Tracks

$$\frac{mv^2}{r} = \mu mg$$

(See table in Lesson 58 for I of 3D shapes.)

#56 Continuity of Fluid Flow

 $A_{in} \bullet v_{in} = A_{out} \bullet v_{out}$ A = Areav = velocity

#58 Moment of Inertia cylindrical hoop solid cylinder or disk solid sphere hollow sphere thin rod (center) thin rod (end)

#58 Moment of Inertia - I $m^{\circ}r^{2}$ $m^{\circ}r^{2}$ 1/2 1

#59 Capacitors $O = C \cdot V$

Q = charge on the capacitor C = capacitance of the capacitor V = voltage applied to the capacitor

RC Circuits (Discharging)

$$V_{c} = V_{o} \cdot e^{-t/RC}$$

$$V_{c} - I \cdot R = 0$$

#60 Thermal Expansion

Linear: $\Delta L = L_o \cdot \alpha \cdot \Delta T$ **Volume:** $\Delta V = V_o \cdot \beta \cdot \Delta T$

#61 Bernoulli's Equation

$$\begin{split} P + \rho \bullet g \bullet h + {}^{1}\!\!/_{2} \bullet \rho \bullet v \,\,^{2} &= constant \\ Q_{Volume\;Flow\;Rate} &= A_{1} \bullet v_{1} = A_{2} \bullet v_{2} = constant \end{split}$$

#62 Rotational Kinetic Energy (See LEM, pg 8)

 $KE_{rotational} = \frac{1}{2} \bullet I \bullet \omega^{2} = \frac{1}{2} \bullet I \bullet (v / r)^{2}$ $KE_{rolling w/o slipping} = \frac{1}{2} \bullet m \bullet v^{2} + \frac{1}{2} \bullet I \bullet \omega^{2}$

Angular Momentum = $L = I \cdot \omega = m \cdot v \cdot r \cdot \sin \theta$

Angular Impulse equals CHANGE IN Angular Momentum

 $\Delta L = \tau_{\text{orque}} \bullet \Delta t = \Delta (I \bullet \omega)$

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#63 Period of Simple Harmonic Motion

$$T = 2\pi \sqrt{\frac{m}{k}}$$
 where k = spring constant

$$f = 1 / T = 1 / period$$

#64 Banked Circular Tracks

$$v^2 = r \cdot g \cdot \tan \theta$$

#66 First Law of Thermodynamics

$$\Delta U = Q_{Net} + W_{Net}$$

Change in Internal Energy of a system =

+Net Heat added to the system

+Net Work done on the system

Flow of Heat through a Solid

$$\Delta Q / \Delta t = k \cdot A \cdot \Delta T / L$$

k = thermal conductivity A = area of solid

L = thickness of solid

#68 Potential Energy stored in a Capacitor

$$P={}^{1}\!/_{\!2} \bullet C \bullet V^{2}$$

RC Circuit formula (Charging)

$$V_{c} = V_{cell} \cdot (1 - e^{-t/RC})$$

 $R \cdot C = \tau = time constant$

$$V_{cell}$$
 - $V_{capacitor}$ - $I \cdot R = 0$

#71 Simple Pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$
 and $f = 1/T$

#72 Sinusoidal motion

$$x = A \cdot \cos(\omega \cdot t) = A \cdot \cos(2 \cdot \pi \cdot f \cdot t)$$

 ω = angular frequency

f = frequency

#73 Doppler Effect

$$f' = f \frac{343 \pm \frac{Toward}{Away} v_o}{343 \mp \frac{Toward}{Away} v_s}$$

 v_0 = velocity of observer: v_s = velocity of source

#74 2nd Law of Thermodynamics

The change in internal energy of a system is

 $\Delta U = Q_{Added} + W_{Done\ On} - Q_{lost} - W_{Done\ By}$

Maximum Efficiency of a Heat Engine

(Carnot Cycle) (Temperatures in Kelvin)

$$\%Eff = (1 - \frac{T_c}{T_h}) \cdot 100\%$$

#75 Thin Lens Equation

f = focal length

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{o} + \frac{1}{i}$$
 i = image distance
o = object distance

Magnification

$$M = -D_i / D_o = -i / O = H_i / H_o$$

Helpful reminders for mirrors and lenses

Focal Length of:	positive	negative
mirror	concave	convex

lens converging diverging

Object distance = O all objects Object height = H_O all objects

 $\begin{aligned} & \text{Image distance} = i & \text{real} & \text{virtual} \\ & \text{Image height} = H_i & \text{virtual, upright} & \text{real, inverted} \\ & \text{Magnification} & \text{virtual, upright} & \text{real, inverted} \end{aligned}$

#76 Coulomb's Law

$$F = k \frac{q_1 q_2}{r^2}$$

$$\frac{1}{r} = 9E9 \frac{N \cdot m}{r}$$

$k = \frac{1}{4\pi\varepsilon_o} = 9E9 \frac{N \cdot m^2}{C^2}$

#77 Capacitor Combinations

PARALLEL

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$
SERIES

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \sum_{i=1}^n \frac{1}{C_i}$$

#78 Work done on a gas or by a gas

$$W = P \bullet_{\Delta} V$$

#80 Electric Field around a point charge

$$E = k \frac{q}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_{a}} = 9E9 \frac{N \cdot m^{2}}{C^{2}}$$

#82 Magnetic Field around a wire

$$B = \frac{\mu_o I}{2\pi r}$$

Magnetic Flux

$$\Phi = B \cdot A \cdot \cos \theta$$

Force caused by a magnetic field on a moving charge

$$F = g \cdot v \cdot B \cdot \sin \theta$$

#83 Entropy change at constant T

$$\Delta S = O / T$$

(Phase changes only: melting, boiling, freezing, etc)

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#84 Capacitance of a Capacitor

$$C = \kappa \cdot \epsilon_0 \cdot A / d$$

 κ = dielectric constant

A = area of plates

d = distance between plates

 $\varepsilon_{0} = 8.85 \text{ E}(-12) \text{ F/m}$

#85 Induced Voltage

$$N = \# \text{ of loops}$$

$$Emf = N \frac{\Delta \Phi}{\Delta t}$$

Lenz's Law – induced current flows to create a B-field opposing the change in magnetic flux.

#86 Inductors during an increase in current

$$V_{L} = V_{\text{cell}} \cdot e^{-t/(L/R)}$$

$$I = (V_{cell}/R) \cdot [1 - e^{-t/(L/R)}]$$

 $L / R = \tau = time constant$

#88 Transformers

$$N_1 / N_2 = V_1 / V_2$$

 $I_1 \bullet V_1 = I_2 \bullet V_2$

#89 Decibel Scale

B (Decibel level of sound) = $10 \log (I/I_0)$

I = intensity of sound

 I_0 = intensity of softest audible sound

#92 Poiseuille's Law

$$\Delta P = 8 \bullet \eta \bullet L \bullet Q / (\pi \bullet r^4)$$

 η = coefficient of viscosity

L = length of pipe

r = radius of pipe

Q = flow rate of fluid

Stress and Strain

$$\mathbf{Y}$$
 or \mathbf{S} or $\mathbf{B} = \mathbf{stress} / \mathbf{strain}$

$$stress = F/A$$

Three kinds of strain: unit-less ratios

I. Linear: strain = $\Delta L / L$

II. Shear: strain = $\Delta x / L$

III. Volume: strain = $\Delta V / V$

#93 Postulates of Special Relativity

- **1.** Absolute, uniform motion cannot be detected.
- **2.** No energy or mass transfer can occur at speeds faster than the speed of light.

#94 Lorentz Transformation Factor

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}$$

#95 Relativistic Time Dilation

$$\Delta t = \Delta t_{\alpha} / \beta$$

#96 Relativistic Length Contraction

$$\Delta x = \beta \cdot \Delta x$$

Relativistic Mass Increase

$$m = m_0 / \beta$$

#97 Energy of a Photon or a Particle

$$E = h \cdot f = m \cdot c^2$$

h = Planck's constant = 6.63 E(-34) J sec

f = frequency of the photon

#98 Radioactive Decay Rate Law

$$A = A_0 \cdot e^{-kt} = (1/2^n) \cdot A_0$$
 (after n half-lives)
Where $k = (\ln 2) / \text{half-life}$

#99 Blackbody Radiation and

the Photoelectric Effect

 $E= n \cdot h \cdot f$ where h = Planck's constant

#100 Early Quantum Physics Rutherford-Bohr Hydrogen-like Atoms

$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{n_s^2} - \frac{1}{n^2}\right) meters^{-1}$$

or

$$f = \frac{c}{\lambda} = cR \left(\frac{1}{n_s^2} - \frac{1}{n^2} \right) Hz$$

R = Rydberg's Constant

 $= 1.097373143 E7 m^{-1}$

 n_s = series integer (2 = Balmer)

 $n = an integer > n_s$

Mass-Energy Equivalence

$$m_v = m_o / \beta$$

Total Energy = KE + $m_o c^2 = m_o c^2 / \beta$ Usually written simply as $E = m c^2$

de Broglie Matter Waves

For light:
$$E_p = h \cdot f = h \cdot c / \lambda = p \cdot c$$

Therefore, momentum: $p = h / \lambda$ Similarly for particles, $p = m \cdot v = h / \lambda$, so the matter wave's wavelength must be

$$\lambda = h / m v$$

Energy Released by Nuclear Fission or Fusion Reaction

$$E = \Delta m_0 \cdot c^2$$

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MISCELLANEOUS FORMULAS

Quadratic Formula

if
$$a x^2 + b x + c = 0$$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Definitions

 $\sin \theta = \text{opposite} / \text{hypotenuse}$

 $\cos \theta = \text{adjacent / hypotenuse}$

 $\tan \theta = \text{opposite} / \text{adjacent}$

$$sec \theta = 1 / cos \theta = hyp / adj$$

$$\csc \theta = 1 / \sin \theta = \text{hyp} / \text{opp}$$

 $\cot \theta = 1 / \tan \theta = adj / opp$

Inverse Trigonometric Definitions

$$\theta = \sin^{-1}(\text{opp} / \text{hyp})$$

$$\theta = \sin^{-1} (\text{opp / hyp})$$

$$\theta = \cos^{-1} (\text{adj / hyp})$$

$$\theta = \tan^{-1} (\text{opp / adj})$$

$$\theta = \tan^{-1}(\text{opp} / \text{adj})$$

Law of Sines

$$a / \sin A = b / \sin B = c / \sin C$$

$$\sin A / a = \sin B / b = \sin C / c$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

 $b^2 = c^2 + a^2 - 2 c a \cos B$

$$c^2 = a^2 + b^2 - 2 a b \cos C$$

T-Pots

For the functional form

$$\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$$

You may use "The Product over the Sum" rule.

$$A = \frac{B \cdot C}{B + C}$$

For the Alternate Functional form

$$\frac{1}{A} = \frac{1}{B} - \frac{1}{C}$$

You may substitute T-Pot-d

$$A = \frac{B \cdot C}{C - B} = -\frac{B \cdot C}{B - C}$$

Fundamental SI Units

Unit	Base Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time Electric	second	S
Current Thermodynamic	ampere	A
Temperature Luminous	kelvin	K
Intensity Quantity of	candela	cd
Substance	moles	mol
Plane Angle	radian	rad
Solid Angle	steradian	sr or str

Some Derived SI Units

Symbol/Unit	Quantity	Base Units
C coulomb	Electric Charge	 A•s
F farad	Capacitance	$A^2 \cdot s4/(kg \cdot m^2)$
H henry	Inductance	$kg \cdot m^2 / (A^2 \cdot s^2)$
Hz hertz	Frequency	s ⁻¹
J joule	Energy & Work	$kg \cdot m^2/s^2 = N \cdot m$
N newton	Force	kg•m/s ²
Ω ohm	Elec Resistance	$kg \cdot m^2 / (A^2 \cdot s^2)$
Pa pascal	Pressure	$kg/(m \cdot s^2)$
T tesla	Magnetic Field	$kg/(A \bullet s^2)$
V volt	Elec Potential	$kg \cdot m^2 / (A \cdot s^3)$
W watt	Power	$kg \cdot m^2/s^3$

Non-SI Units

°C	degrees Celsius	Temperature
eV	electron-volt	Energy, Work

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Aa acceleration, Area, A_x=Cross-sectional Area, Amperes, Amplitude of a Wave, Angle,

Bb Magnetic Field, Decibel Level of Sound, Angle,

Cc specific heat, speed of light, Capacitance, Angle, Coulombs, °Celsius, Celsius Degrees, candela,

Dd displacement, differential change in a variable, Distance, Distance Moved, distance,

Ee base of the natural logarithms, charge on the electron, Energy,

Ff Force, frequency of a wave or periodic motion, Farads,

Gg Universal Gravitational Constant, acceleration due to gravity, Gauss, grams, Giga-,

Hh depth of a fluid, height, vertical distance, Henrys, Hz=Hertz,

Ii Current, Moment of Inertia, image distance, Intensity of Sound,

Ji Joules

Kk K or KE = Kinetic Energy, force constant of a spring, thermal conductivity, coulomb's law constant, kg=kilograms, Kelvins, kilo-, rate constant for Radioactive decay =1/τ=ln2 / half-life,

Ll Length, Length of a wire, Latent Heat of Fusion or Vaporization, Angular Momentum, Thickness, Inductance,

Mm mass, Total Mass, meters, milli-, Mega-, m_o=rest mass, mol=moles,

Nn index of refraction, moles of a gas, Newtons, Number of Loops, nano-,

Oo

Pp Power, Pressure of a Gas or Fluid, Potential Energy, momentum, Power, Pa=Pascal,

Qq Heat gained or lost, Maximum Charge on a Capacitor, object distance, Flow Rate,

Rr radius, Ideal Gas Law Constant, Resistance, magnitude or length of a vector, rad=radians

Ss speed, seconds, Entropy, length along an arc,

Tt time, Temperature, Period of a Wave, Tension, Teslas, $t_{1/2}$ =half-life,

Uu Potential Energy, Internal Energy,

Vv velocity, Velocity, Volume of a Gas, velocity of wave, Volume of Fluid Displaced, Voltage, Volts,

Ww weight, Work, Watts, Wb=Weber,

Xx distance, horizontal distance, x-coordinate east-and-west coordinate.

Yy vertical distance, y-coordinate, north-and-south coordinate,

Zz z-coordinate, up-and-down coordinate,

Aα Alpha angular acceleration, coefficient of linear expansion,

Bβ Beta coefficient of volume expansion, Lorentz transformation factor,

Xχ Chi

 $\Delta\delta$ **Delta** Δ =change in a variable,

Eε Epsilon $ε_o$ = permittivity of free space,

Φφ Phi Magnetic Flux, angle,

Γγ Gamma surface tension = F / L, 1 / γ = Lorentz transformation factor,

Hη Eta

It Iota

9φ Theta and Phi lower case alternates.

Κκ Kappa dielectric constant,

Λλ Lambda wavelength of a wave, rate constant for Radioactive decay =1/τ=ln2/half-life,

Mμ Mu friction, $μ_0$ = permeability of free space, micro-,

Nv Nu alternate symbol for frequency,

Oo Omicron

 $\Pi \pi$ Pi 3.1425926536...,

 $\Theta\theta$ Theta angle between two vectors,

Pρ Rho density of a solid or liquid, resistivity,

 $\Sigma \sigma$ Sigma Summation, standard deviation,

Tτ Tau torque, time constant for a exponential processes; eg τ=RC or τ=L/R or τ=1/k=1/λ,

Yv Upsilon

ς w Zeta and Omega lower case alternates

Ωω Omega angular speed or angular velocity, Ohms

Ξξ Xi

Ψψ Psi

Zζ Zeta

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Values of Trigonometric Functions for 1st Quadrant Angles

(simple mostly-rational approximations)

θ	sin θ	cos θ	tan θ
0°	0	1	0
10^{o}	1/6	65/66	11/65
15°	1/4	28/29	29/108
20°	1/3	16/17	17/47
29°	15 ^{1/2} /8	7/8	$15^{1/2}/7$
30°	1/2	$3^{1/2}/2$	1/3 ^{1/2}
37°	3/5	4/5	3/4
42°	2/3	3/4	8/9
45°	21/2/2	$2^{1/2}/2$	1
49°	3/4	2/3	9/8
53°	4/5	3/5	4/3
60	$3^{1/2}/2$	1/2	3 ^{1/2}
61°	7/8	15 ^{1/2} /8	$7/15^{1/2}$
70°	16/17	1/3	47/17
75°	28/29	1/4	108/29
80°	65/66	1/6	65/11
90°	1	0	∞

(Memorize the **Bold** rows for future reference.)

Derivatives of Polynomials

For polynomials, with individual terms of the form Ax^n , we define the derivative of each term as

$$\frac{d}{dx}(Ax^n) = nAx^{n-1}$$

To find the derivative of the polynomial, simply add the derivatives for the individual terms:

$$\frac{d}{dx}(3x^2+6x-3)=6x+6$$

Integrals of Polynomials

For polynomials, with individual terms of the form Ax^n , we define the indefinite integral of each term as

$$\int \left(Ax^n\right) dx = \frac{1}{n+1} Ax^{n+1}$$

To find the indefinite integral of the polynomial, simply add the integrals for the individual terms and the constant of integration, C.

$$\int (6x+6)dx = \left[3x^2 + 6x + C\right]$$

Prefixes

Factor	<u>Prefix</u>	Symbol	Example
10^{18}	exa-	Е	38 Es (Age of the Universe in Seconds)
10^{15}	peta-	P	in seconds)
10 ¹²	tera-	T	0.3 TW (Peak power of a 1 ps pulse from a typical Nd-glass laser)
109	giga-	G	22 G\$ (Size of Bill & Melissa Gates' Trust)
10 ⁶	mega-	M	6.37 Mm (The radius of the Earth)
10 ³	kilo-	k	1 kg (SI unit of mass)
10 ⁻¹	deci-	d	10 cm
10-2	centi-	c	2.54 cm (=1 in)
10-3	milli-	m	1 mm (The smallest division on a meter stick)
10 ⁻⁶	micro-	μ	
10 ⁻⁹	nano-	n	510 nm (Wavelength of green light)
10 ⁻¹²	pico-	p	1 pg (Typical mass of a DNA sample used in genome studies)
10 ⁻¹⁵	femto-	f	studiosj
10 ⁻¹⁸	atto-	a	600 as (Time duration of the shortest laser pulses)

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Linear Equivalent Mass

Rotating systems can be handled using the linear forms of the equations of motion. To do so, however, you must use a mass equivalent to the mass of a non-rotating object. We call this the Linear Equivalent Mass (LEM). (See Example I)

For objects that are both rotating and moving linearly, you must include them twice; once as a linearly moving object (using m) and once more as a rotating object (using LEM). (See Example II)

The LEM of a rotating mass is easily defined in terms of its moment of inertia, I.

$$LEM = I/r^2$$

For example, using a standard table of Moments of Inertia, we can calculate the LEM of simple objects rotating on axes through their centers of mass:

	<u> </u>	<u>LEM</u>
Cylindrical hoop	mr^2	m
Solid disk	$^{1}/_{2}$ mr 2	½m
Hollow sphere	$\frac{2}{5}$ mr ²	² / ₅ m
Solid sphere	$^{2}/_{3}$ mr 2	²⁄₃m

Example I

A flywheel, M = 4.80 kg and r = 0.44 m, is wrapped with a string. A hanging mass, m, is attached to the end of the string.

When the hanging mass is released, it accelerates downward at 1.00 m/s². Find the hanging mass.

To handle this problem using the linear form of Newton's Second Law of Motion, all we have to do is use the LEM of the flywheel. We will assume, here, that it can be treated as a uniform solid disk. The only external force on this system is the weight of the hanging mass. The mass of the system consists of the hanging mass plus the linear equivalent mass of the fly-wheel. From Newton's 2nd Law we have

$$\begin{split} F = ma, & therefore, & mg = \left[m + (LEM = \frac{1}{2}M)\right] a \\ & mg = \left[m + \frac{1}{2}M\right] a \\ & (mg - ma) = \frac{1}{2}M \ a \\ & m(g - a) = \frac{1}{2}Ma \\ & m = \frac{1}{2} \cdot M \cdot a \ / \ (g - a) \\ & m = \frac{1}{2} \cdot 4.8 \cdot 1.00 \ / \ (9.81 - 1) \\ & m = 0.27 \ kg \\ & If \ a = g/2 = 4.905 \ m/s^2, & m = 2.4 \ kg \\ & If \ a = \frac{3}{4}g = 7.3575 \ m/s^2, & m = 7.2 \ kg \end{split}$$

Note, too, that we do not need to know the radius unless the angular acceleration of the fly-wheel is requested. If you need α , and you have r, then $\alpha = a/r$.

Example II

Find the kinetic energy of a disk, m = 6.7 kg, that is moving at 3.2 m/s while rolling without slipping along a flat, horizontal surface. ($I_{DISK} = \frac{1}{2}mr^2$; LEM = $\frac{1}{2}m$)

The total kinetic energy consists of the linear kinetic energy, $K_L = \frac{1}{2}mv^2$, plus the rotational kinetic energy, $K_R = \frac{1}{2}(I)(\omega)^2 = \frac{1}{2}(I)(v/r)^2 = \frac{1}{2}(I/r^2)v^2 = \frac{1}{2}(LEM)v^2$.

KE =
$$\frac{1}{2}$$
mv² + $\frac{1}{2}$ •(LEM= $\frac{1}{2}$ m)•v²
KE = $\frac{1}{2}$ •6.7•3.2² + $\frac{1}{2}$ •($\frac{1}{2}$ •6.7)•3.2²
KE = 34.304 + 17.152 = 51 J

Final Note:

This method of incorporating rotating objects into the linear equations of motion works in every situation I've tried; even very complex problems. Work your problem the classic way and this way to compare the two. Once you've verified that the LEM method works for a particular type of problem, you can confidently use it for solving any other problem of the same type.