

THEORY FOR PRESSURE DROP IN A PULSE-JET CLEANED FABRIC FILTER

DAVID LEITH and MICHAEL J. ELLENBECKER

Department of Environmental Health Sciences, Harvard School of Public Health,
665 Huntington Avenue, Boston, MA 02115, U.S.A.

(First received 3 May 1979)

Abstract – A theory based on Darcy's law has been derived which relates pressure drop in a pulse-jet cleaned fabric filter to characteristics of the filter and dust. Assumptions made are that total pressure drop is the sum of pressure drops across the clean bag and dust deposit, and that the fraction of dust removed from a bag by a cleaning pulse is proportional to the reverse pressure drop across it at cleaning. The theory allows prediction of pressure drop under stable or variable operating conditions including dust concentration, filtration velocity, pulse pressure and pulse frequency, and can be used to identify operating conditions which cause pressure drop to increase without limit. Agreement between the model and experimental data is reasonably good.

NOMENCLATURE

| | |
|------------------|--|
| A | fabric area, m^2 |
| a | see Equation (12) |
| b | see Equation (13) |
| c | see Equation (14) |
| c_i | dust inlet concentration, $kg\ m^{-3}$ |
| F | Frasier permeability ($m\ s^{-1}$ at 125 Pa) |
| $F(N)$ | see Equation (17) |
| F_s | force acting to separate dust deposit from fabric, N |
| K_1 | fabric resistance, $Pa\ s\ m^{-1}$ |
| K_2 | specific resistance of dust deposit, s^{-1} |
| K_3 | constant, see Equation (8) |
| K_4 | constant, see Equation (8) |
| k | proportionality constant, see Equation (3) |
| N | number of filtration cycles since operating conditions change |
| P | pulse pressure, kPa |
| P_s | maximum static pressure developed inside bag as result of cleaning pulse, Pa |
| q | see Equation (15) |
| t | time between pulses to each bag, s |
| t_1 | time during which Δp , acts, s |
| t_2 | time during which F_s , acts, s |
| v | superficial filtration velocity, $m\ s^{-1}$ |
| v^* | maximum velocity achieved by fabric and dust during cleaning, $m\ s^{-1}$ |
| w | areal density of dust deposit, $kg\ m^{-2}$ |
| w_b | areal density of fabric, $kg\ m^{-2}$ |
| w_i | dust deposit areal density at $N=0$, when operating conditions change, $kg\ m^{-2}$ |
| w_0 | dust areal density added during one filtration cycle, $kg\ m^{-2}$ |
| Δp | pressure drop across bag and dust deposit, Pa |
| Δp_i | pressure drop at $N=0$, when operating conditions change, Pa |
| Δp_L | minimum Δp_i if stable operation is to be attained, Pa |
| $\Delta \bar{p}$ | difference between static pressure inside and outside the bag during cleaning, $P_s - \Delta p$, Pa |
| ϵ | fraction of the dust deposit removed from a fabric |

N.B. 100 kPa = 1 bar = 14.5 psig.

INTRODUCTION

Pressure drop through gas cleaning equipment is an important component of system operating cost. For this reason, knowledge of the factors that affect pressure drop and methods for pressure drop prediction are important. The characteristics of gas flow through a porous medium, fundamental to modeling pressure drop in a fabric filter, have been studied by Carman (1937), Williams *et al.* (1940), Stephan *et al.* (1960), Pich (1969), and Rudnick (1978a). Much of this work concerns flow through a particle deposit with known properties such as pore size and porosity. With this information, dust deposit specific resistance, K_2 , the pressure drop per unit velocity and per unit deposit mass, can be predicted. Rudnick (1978a, b) has shown that K_2 can be estimated using an adaptation of the Happel (1958) unit cell model. This approach gives much better agreement with data for the usual case where dust deposit porosity is greater than about 0.8 than does the frequently used Kozeny and Carman model described by Billings and Wilder (1970).

Before these theories can be applied to a practical fabric filter, it is necessary to consider the pressure drop contribution of the dust-conditioned fabric, and to account for the effect of incomplete fabric cleaning. Models by Davis *et al.* (1976) and Robinson *et al.* (1967) proposed that virtually all dust is removed from a woven fabric cleaned by shaking or reverse air. However, experimental work by Dennis *et al.* (1975, 1978) demonstrated that virtually all dust is removed only from some portions of woven fabrics cleaned by these methods, and that other portions are apparently not cleaned at all. As cleaning intensity increases, the ratio of cleaned to uncleaned areas increases. Dennis *et*

al. (1977) described a model which utilizes these ideas for predicting pressure drop in a filter using woven cloth cleaned by shaking or reverse air.

The fabric characteristics and cleaning method for a pulse-jet filter with felt bags cleaned on-line are considerably different from those of a shaker cleaned filter, so that carryover of the model proposed by Dennis *et al.* is inappropriate. For a filter with woven bags cleaned off-line by shaking, Dennis and Wilder (1975) showed that more than half the dust is normally removed. For a pulse-jet filter with felt bags cleaned on-line, Ellenbecker (1979), Ellenbecker and Leith (1978), and Leith *et al.* (1977) showed that 1% or less of the dust on a bag may be removed per cleaning pulse, and this percentage may decrease as filtration velocity increases. For a shaker cleaned filter, Dennis *et al.* (1977, 1978) have established that dust separates from the fabric in patches. For a pulse-jet filter, Ellenbecker (1979) found that the distribution of dust on the bags after cleaning is almost uniform. Off-line shaker cleaning allows time for the separated dust to fall to the hopper before filtration resumes, whereas when a pulse-jet filter is cleaned on-line in a fraction of a second, separated dust has little time to fall to the hopper. Leith *et al.* (1977) found that much of the separated dust may redeposit on neighboring bags or on the pulsed bag. Bag spacing, dust type, pulse characteristics, bag tension, housing design and gas flow pattern within the housing may all affect dust redeposition in a pulse-jet filter, according to Dennis and Wilder (1975) and Leith *et al.* (1978).

Empirical models have been used by Dennis and Silverman (1962) and by Leith and First (1977) to describe pressure drop (Δp). The latter model, developed for fly ash collected on polyester bags for a range of filtration velocities, v , pulse pressures, P , and areal dust densities added during a filtration cycle, w_0 , is:

$$\Delta p \propto \frac{v^{2.34} w_0^{0.45}}{P^{1.38}}, \quad (1)$$

in which:

$$w_0 = c_i v t, \quad (2)$$

Here, c_i is inlet dust concentration and t is time between pulses to each bag. The assumption is made in (2) that filter efficiency is 100%.

This paper presents a theoretical treatment of pressure drop in a pulse-jet filter. In the development, some assumptions must be made which should be reexamined as data become available.

DUST SEPARATION FROM FABRIC

First, consider the forces which bind the dust deposit to the fabric. If a certain separation force per unit area, F_s/A , were applied to this dust deposit in the appropriate direction, a certain fraction, ϵ , would be separated. Figure 1 shows a hypothetical relationship

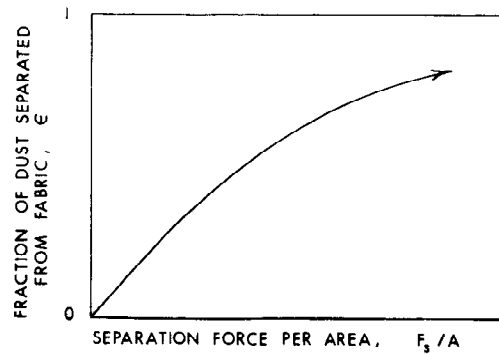


Fig. 1. Hypothetical relationship between fraction of dust deposit removed from fabric, ϵ , and area-specific separation force, F_s/A .

between ϵ and F_s/A . The exact shape of this curve is unknown, but it should pass through the origin as no dust will be separated if no force is applied. The value of ϵ should approach unity as an asymptote, because the effect of additional force is likely to diminish after most of the dust has been removed. Similar curves, which show filter capacity (dust mass removed by cleaning) vs bag acceleration during the cleaning process, were developed by Dennis and Wilder (1975) for woven fabrics cleaned by shaking.

For pulse-jet cleaned filters, experimental measurements by Ellenbecker (1979), Ellenbecker and Leith (1978) and Leith *et al.* (1977) demonstrate that ϵ is less than a few percent. For lack of better information, ϵ may be assumed proportional to F_s/A :

$$\epsilon = k F_s/A. \quad (3)$$

Next, consider the interaction of separating forces with the dust deposit and fabric. The net force per unit fabric area during cleaning, which drives the fabric and dust away from the cage that supports them during filtration, will be called the reverse pressure drop, Δp_r ; it is the difference between the static pressure developed by the pulse within the bag, P_s , and the operating pressure drop across the bag at the time the pulse begins, Δp :

$$\Delta p_r = P_s - \Delta p. \quad (4)$$

If the jet pulse develops a maximum pressure less than the pressure drop across the bag ($P_s < \Delta p$) the bag will not expand, no reverse air will flow through it, and no cleaning will take place.

As cleaning begins, fabric and dust receive an impulse which drives both away from the wire cage and both attain a certain momentum. Division of the impulse/momentum equation by bag area gives,

$$\Delta p_r t_1 = v^*(w_b + w). \quad (5)$$

Here, t_1 is the time over which Δp_r acts, v^* is the outward velocity attained, w_b and w are the areal density of the clean bag and the dust deposit, respectively.

The area specific force, F_s/A , which acts (over time

t_2) to separate dust from the fabric can be found from the impulse necessary to stop the outward momentum of the dust deposit:

$$\frac{F_s t_2}{A} = v^* w. \quad (6)$$

Substitution of (4) to (6) into (3) yields an expression for the fraction of dust which is separated from the fabric by a cleaning pulse:

$$\varepsilon = \frac{(P_s - \Delta p)t_1 k}{t_2} - \frac{v^* w_b k}{t_2} \quad (7)$$

or

$$\varepsilon = K_3(P_s - \Delta p) - K_4. \quad (8)$$

Constant K_3 expresses the effectiveness with which reverse pressure drop interacts with the fabric to remove dust; K_4 expresses the effect of reverse fabric motion. Strangert (1978) reports that increased reverse pressure drop increases cleaning effectiveness.

Dust separated from the fabric must fall to the hopper to be removed from the system. However, some separated dust redeposits on the fabric before reaching the hopper, and causes actual fractional separation to be less than that predicted by (3). Redeposition may cause the effective value of constant K_3 in a full-size filter to depend upon factors that affect redeposition such as filtration velocity, housing design, or other variables. The present model does not account for redeposition; further work on this subject is planned.

AREAL DENSITY OF DUST DEPOSIT

Models for the pressure drop across a woven fabric generally assume that total drop equals the sum of losses across the conditioned fabric, $K_1 v$, and across the dust deposit, $K_2 v w$:

$$\Delta p = K_1 v + K_2 v w. \quad (9)$$

K_1 is the resistance of the fabric after it has been operated for some time and is well conditioned with dust, K_2 is the specific resistance of the dust deposit, and w is the areal density of the dust deposit on the bags.

A mass balance for the dust per unit area on the fabric can be written for one filtration cycle, dN , which includes by definition exactly one cleaning pulse:

Dust mass per unit bag area added to the bag per cycle — Dust mass per unit bag area removed from the bag per cycle
 Dust mass per unit bag = area accumulated on the bag per cycle,

$$w_0 - w\varepsilon = \frac{dw}{dN}. \quad (10)$$

Substitution of (8) and (9) into (10) gives:

$$\frac{dw}{dN} = (K_2 K_3 v) w^2 + (K_1 K_3 v - K_3 P_s + K_4) w + w_0. \quad (11)$$

This can be solved after the following substitutions are made:

$$a = -w_0, \quad (12)$$

$$b = K_3 P_s - K_1 K_3 v - K_4, \quad (13)$$

$$c = -K_2 K_3 v, \quad (14)$$

$$q = (K_3 P_s - K_1 K_3 v - K_4)^2 - 4w_0 K_2 K_3 v. \quad (15)$$

The solution to (11) is:

$$w = \frac{b - \sqrt{q} - (b + \sqrt{q})F(N)}{2c(F(N) - 1)}, \quad (16)$$

in which:

$$F(N) = \left(\frac{2cw_i + b - \sqrt{q}}{2cw_i + b + \sqrt{q}} \right) \exp(-N\sqrt{q}). \quad (17)$$

Here w_i is the initial areal density, the areal density of the dust deposit at $N = 0$, the beginning of the first filtration cycle when operating conditions are changed. Equation (16) gives the areal density of the dust deposit at any time, that is, at any number of filtration cycles, N , since operating conditions are changed and cycle counting begins.

EQUILIBRIUM

If operating conditions are constant, (16) can be used to determine the value of areal dust density that the filter will achieve at equilibrium ($N \rightarrow \infty$). In this case, (17) shows that $F(N) \rightarrow 0$ and (16) becomes:

$$w = \frac{-b + \sqrt{q}}{2c}, \quad (18)$$

or

$$w = \frac{P_s - K_4/K_3 - K_1 v}{2K_2 v} - \frac{\sqrt{(P_s - K_4/K_3 - K_1 v)^2 - 4w_0 v K_2/K_3}}{2K_2 v}. \quad (19)$$

Substitution of (19) into (9) gives equilibrium pressure drop:

$$\Delta p = \frac{P_s - K_4/K_3 + K_1 v}{2} - \frac{\sqrt{(P_s - K_4/K_3 - K_1 v)^2 - 4w_0 v K_2/K_3}}{2}. \quad (20)$$

INSTABILITY

Bakke (1974) has established that under certain operating conditions pressure drop across a pulse-jet filter does not stabilize but increases without limit. Mathematically, this is equivalent to a situation in

which $F(N) = 0.1$, as can be seen by inspection of (16). Solution of (17) for this condition gives the following criterion, which must be satisfied for stable filter operation:

$$2cw_i + b + \sqrt{q} > 0, \quad (21)$$

or

$$P_s + \sqrt{(P_s - K_4/K_3 - K_1v)^2 - 4w_0vK_2/K_3} - 2K_2vw_i - K_1v - K_4/K_3 > 0. \quad (22)$$

The square root term within (20) contains terms related to the characteristics of the filter, K_1 – K_4 , and to the conditions under which it is operated, P_s , v , and w_0 , and must be real if stable filter operation is to be possible:

$$(P_s - K_4/K_3 - K_1v)^2 - 4w_0vK_2/K_3 > 0. \quad (23)$$

If (23) is not met, the characteristics of the filter or its operating conditions must be changed.

The balance of (22) contains variable w_i , the areal density of the dust deposit at time zero. If this term is too large, that is, if too much dust is on the bags, the stability criterion given by (22) will not be met and pressure drop will increase without limit. However, for the same filter and operating conditions, stability may be achieved if the initial areal density is sufficiently low. This has practical implications for fabric filter operation. Before operating conditions such as filtration velocity, v , dust characteristics, K_2 , or pulse pressure, P_s , are changed, the bags should be pulsed repeatedly to remove as much dust as possible and decrease w_i . This will give the best opportunity for the system to achieve stable operation under the new operating conditions.

Equation (22) can be rewritten:

$$\Delta p_i < \Delta p_L, \quad (24)$$

where

$$\Delta p_L = K_1v + K_2vw_i, \quad (25)$$

and

$$\Delta p_L = \frac{P_s - K_4/K_3 + K_1v}{2} + \frac{\sqrt{(P_s - K_4/K_3 - K_1v)^2 - 4w_0vK_2/K_3}}{2}. \quad (26)$$

The actual operating pressure drop, Δp_i , immediately after operating conditions change must not exceed a certain limiting pressure drop, Δp_L , defined by Equation (26). If Δp_i is higher than Δp_L , then the system is unstable and pressure drop will increase without limit. If, however, Δp_i is lower than this quantity, then the system will be stable and approach pressure drop equilibrium.

For example, a filter might have a stable, equilibrium pressure drop of 1000 Pa under a particular set of operating conditions. Pulse pressure might then be reduced, causing a reduction in P_s , and in Δp_L as

predicted by (26). In this case, the initial pressure drop under the new conditions Δp_i (here equal to 1000 Pa) may now be greater than Δp_L , and violate the stability condition set by (24). If this occurs, pressure drop would rise from its initial value of 1000 Pa and increase without limit. However, if the reduction in P_s had been less, the value for Δp_L calculated in (26) might still be satisfied. In that case, pressure drop would still rise from its initial value of 1000 Pa, but would eventually come to equilibrium.

Figure 2 is a plot of pressure drop, Δp , as given by (20) against w_0 , the dust areal density added during a filtration cycle for typical operating conditions. The figure shows that as more dust is fed to the bags between pulses, either by increasing inlet dust concentration or by increasing the time between cleaning pulses, equilibrium pressure drop increases as well. At the maximum value of w_0 for which stable filter operation is possible, the value of $d(\Delta p)/dw_0$ becomes infinite; that is, a slight increase in w_0 causes equilibrium pressure drop to increase at an infinite rate. This point is at the edge of the unstable region defined by (23).

Also shown in Fig. 2 is a plot of Δp_L , the maximum initial pressure drop for which the filter will ultimately achieve stable operation, given by (26). Comparison of (26) and (20) for equilibrium pressure drop shows that they are identical except for the sign on the square root term.

The arrows shown in Fig. 2 represent the direction in which pressure drop will proceed over time and pulses at constant w_0 , starting at any point on the figure. The arrows were determined by evaluating the derivative of areal density with respect to number of pulses at

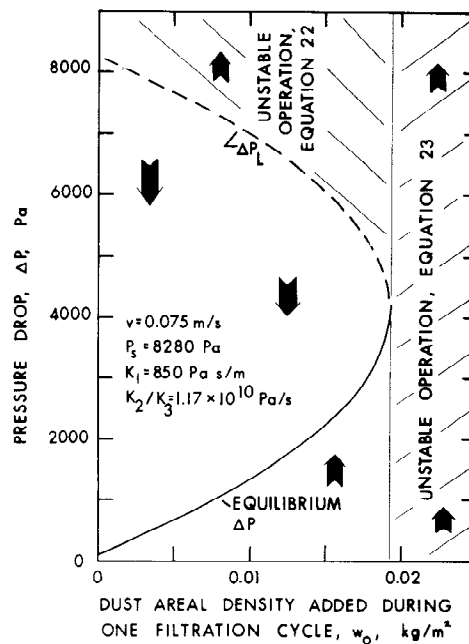


Fig. 2. Pressure drop, Δp , vs dust areal density added during one filtration cycle, w_0 .

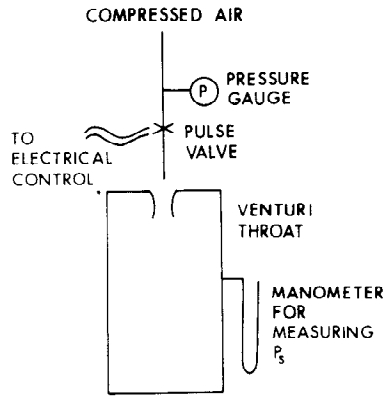


Fig. 3. Apparatus used to determine relationship between pulse pressure, P , and induced static pressure, P_s .

various points. For all initial conditions leading to pressure drops within the parabola shown in Fig. 2, pressure drop will decrease to reach equilibrium. For all initial conditions leading to pressure drops outside the parabola, pressure drop will increase either until it intersects the parabola and reaches equilibrium, or else without limit.

COMPARISON WITH DATA

The static pressure increase within a bag during cleaning, P_s , is related to the rate at which air is forced backwards through the bag during cleaning. For a jet pump, this flowrate depends in part upon jet pressure. The relationship between pulse pressure and static pressure developed, P_s , was determined from a 6.4 mm ($\frac{1}{4}$ ") jet discharging into a standard pulse-jet venturi using the apparatus shown in Fig. 3. Compressed air was injected continuously and the no-flow static pressure across the venturi, P_s , measured. Results of these tests for several pulse pressures are given in Fig. 4, and are described by:

$$P_s(\text{Pa}) = 164P(\text{kPa})^{0.6}. \quad (27)$$

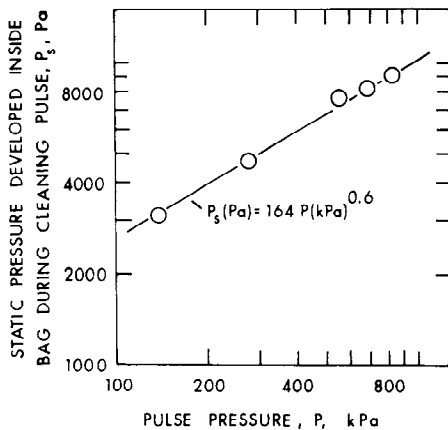


Fig. 4. Static pressure induced, P_s , vs pulse pressure, P .

Bakke (1974) reports a value of P_s about 10% lower than that given in (27) for a pulse pressure of 620 kPa.

Equation (20) for equilibrium pressure drop requires values for operating variables w_0 , v , and P_s as well as for constants K_1 , K_2/K_3 , and K_4/K_3 . Ellenbecker's (1979) data on the pressure drop characteristics of a three bag pulse-jet filter operated at four filtration velocities from 50 to 125 mm s^{-1} have been reported and discussed in detail elsewhere, and are summarized in Table 1. The test dust was fly ash collected electrostatically at a coal-fired utility boiler.

Examination of (7) and (8) shows that:

$$K_4/K_3 = v^*w_b/t_i. \quad (28)$$

Dennis and Wilder (1975) determined that the maximum outward velocity, v^* , attained by one polyester felt bag laden with fly ash is 1.4 m s^{-1} , and that the time necessary to reach this velocity was about 5.5 ms. If the clean fabric has an areal density of 0.54 kg m^{-2} , the value of K_4/K_3 from (28) is 137 Pa, less than 2% of P_s from which it is subtracted in (20) to calculate equilibrium pressure drop. In this case, K_4/K_3 may be neglected, making subsequent analysis much simpler. The size of the error introduced by this assumption is not well established, but may be small unless filter operation is nearly unstable.

If K_4K_3 is neglected, then (20) for equilibrium pressure drop involves as constants only K_1 and the ratio K_2/K_3 . Lacking other data, the resistance of the conditioned fabric, K_1 , might be approximated from the inverse of clean fabric Frasier permeability, F :

$$K_1(\text{Pa s m}^{-1}) = \frac{124}{F(\text{m s}^{-1} \text{ at } 125 \text{ Pa})}. \quad (29)$$

The consequence of this assumption is to incorporate all pressure drop beyond that across the clean fabric into the term for resistance across the dust deposit. Although this approximation may not be realistic for conditioned fabrics which retain an appreciable amount of dust, it does allow the analysis to proceed when data on the actual resistance of the conditioned fabric are unavailable.

If K_1 is known or assumed, and if equilibrium pressure drop is known for one set of stable operating conditions, then K_2/K_3 can be determined and pressure drop under any other stable operating conditions can be found using (20). Values of K_2/K_3 were determined in this way for pressure drop at each velocity listed in Table 1. The values of K_2/K_3 were found to be relatively constant over the range of filtration velocities considered.

Figure 5 is a plot of pressure drop against filtration velocity for the data in Table 1. The line is based on Equation (20), with K_2/K_3 equal to its average value for these data, and reflects well the trend of pressure drop vs velocity. This analysis gives no indication whether agreement between theory and data will be as

Table 1. Experimental conditions for pulse-jet filter tests

| | | | |
|---------------------------------|---|--------------------------------|---------------------------------------|
| Bags | | | |
| Number: | 3 | | |
| Size: | 114 mm dia. and 2.44 m long | | |
| Fabric: | Polyester felt, no surface treatment | | |
| Weight: | 0.54 kg m^{-2} | | |
| Permeability: | 0.15 m s^{-1} at 125 Pa | | |
| K_1 : | 850 Pa s m^{-1} (new fabric) | | |
| Pulses | | | |
| Frequency: | Once per minute to each bag | | |
| Pulse pressure, P : | 690 kPa | | |
| Induced pressure, P_s : | 8280 Pa | | |
| Dust | | | |
| Type: | Fly ash precipitated from coal-fired utility boiler | | |
| Count median diameter: | $0.30 \mu\text{m}$ | | |
| Geometric standard deviation: | 2.7 | | |
| w_0 : | $3.1 \times 10^{-3} \text{ kg m}^{-2}$ | | |
| <hr/> | | | |
| Pressure drop | | | |
| Velocity, mm s^{-1} | ΔP , Pa | K_2/K_3 , Pa s^{-1} | K_4/K_3 , Pa |
| 50 | 260 | 1.13×10^{10} | Assumed equal to zero, all conditions |
| 75 | 390 | 1.13×10^{10} | |
| 100 | 615 | 1.31×10^{10} | |
| 125 | 790 | 1.32×10^{10} | |
| Average: | | 1.17×10^{10} | 0 |

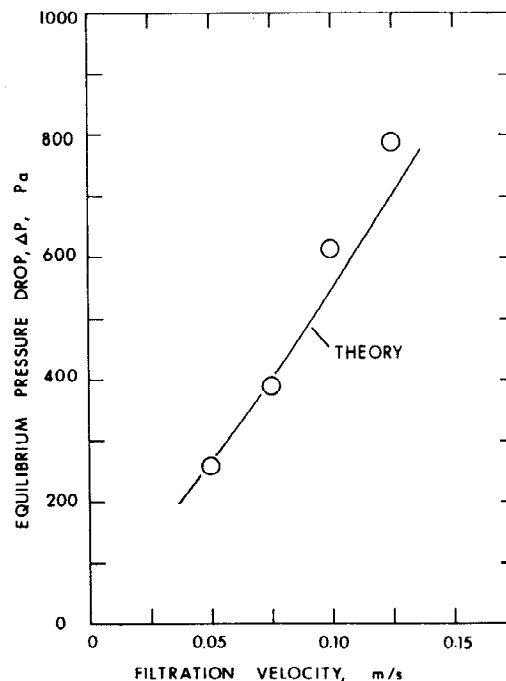
good for variations in variables other than filtration velocity.

The constants listed in Table 1 are specific for the apparatus and experimental conditions used in their determination. Changes in pulse pressure or changes in amount of dust fed to the bags between pulses were not made in these experiments. However, (20) can be used to estimate the effect on pressure drop of variations in these variables. Results are plotted in Fig. 6 and 7, pressure drop vs pulse pressure and w_0 , respectively. In Fig. 6, pulse pressure was converted to P_s using (27) before the theoretical pressure drop line was plotted.

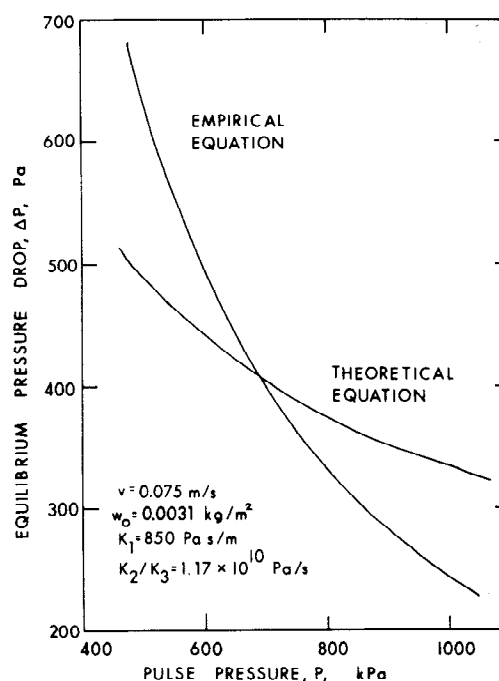
Equation (1) relates pressure drop to these same variables empirically and was established for fly ash collected on polyester felt bags. However, the dust and bags used to develop the constants used in the theoretical analysis were different. The slopes of the pressure drop vs pulse pressure and pressure drop vs w_0 relationships predicted by the empirical equation are also given in Figs 6 and 7. The lines given by the theory and the empirical results are in general agreement. A more complete test of the theory presented here awaits development of more data.

EXAMPLE CALCULATION

The concepts and equations described above can be illustrated using an example calculation. It is proposed

Fig. 5. Equilibrium pressure drop, Δp , vs filtration velocity, v .

to reduce the amount of compressed air used in cleaning a pulse-jet filter by extending the interval from 60 to 120 s between pulses to each bag. Will filter operation be stable under the operating conditions proposed? If so, what will be the effect of the proposed change on filter pressure drop? Current filter operating conditions are given in Table 2.

Fig. 6. Equilibrium pressure drop, Δp , vs pulse pressure, P .

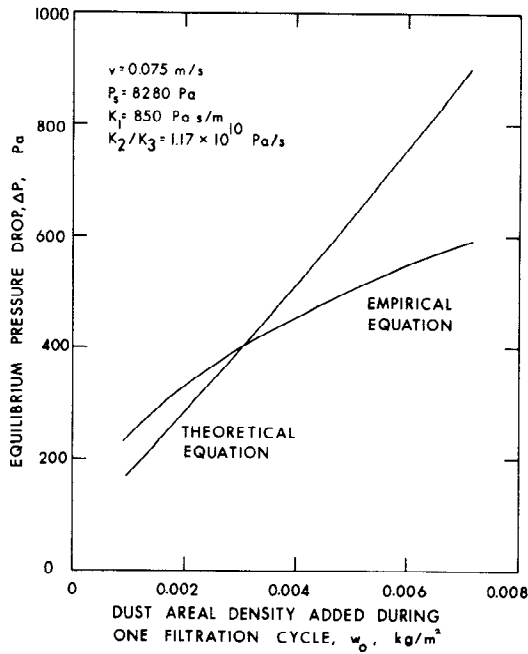


Fig. 7. Equilibrium pressure drop, Δp , vs dust areal density added during one filtration cycle, w_0 .

Table 2. Filter operating conditions: example calculation

| | |
|----------------------------------|---|
| Fabric permeability: | 0.177 m s^{-1} at 125 Pa |
| Pulse frequency: | Currently once per 60 s per bag; proposed once per 120 s |
| Pulse pressure: | 690 kPa (100 psig) |
| Inlet dust concentration: | 2 g m^{-3} ($2 \times 10^{-3} \text{ kg m}^{-3}$) |
| Superficial filtration velocity: | 50 mm s^{-1} (0.05 m s^{-1}) |
| Equilibrium pressure drop: | 1000 Pa currently |

Equation (26) can be used to determine whether filter operation under the proposed operating conditions will be stable; if stable operation is achieved (20) can be used to determine equilibrium pressure drop. Both (26) and (20) contain three unknown constants: K_1 , K_4/K_3 , and K_2/K_3 , and to evaluate operation at the new conditions it is necessary to know or estimate values for each. For lack of better information, K_1 can be estimated from the permeability of the new fabric bags, listed in Table 2, using (29):

$$K_1 (\text{Pa s}^{-1} \text{ m}^{-1}) = \frac{124}{0.177 \text{ m s}^{-1} \text{ at } 125 \text{ Pa}} = 700 \text{ Pa s m}^{-1}. \quad (30)$$

There is no apparent way to evaluate constant K_4/K_3 . However, as discussed above, this constant may be negligibly small compared to values from which it is subtracted under normal pulse-jet operating conditions. If so, it can be neglected; the value of K_4/K_3 will be assumed equal to zero here.

With values for constants K_1 and K_4/K_3 in hand, (20) can be used to evaluate the remaining constant, K_2/K_3 , using the data for the operation of the filter under its current, stable operating conditions. To do this it is necessary to have values for P_s , the static pressure developed within the bag by a cleaning pulse, and for w_0 , the dust areal density added to each bag during one filtration cycle. A value for P_s can be found from pulse pressure, P , using (27):

$$P_s (\text{Pa}) = 164(690 \text{ kPa})^{0.6} = 8280 \text{ Pa}. \quad (31)$$

A value for w_0 can be found from (2) for the current filter operating conditions:

$$w_0|_{\text{current}} = c_i v t = (2 \times 10^{-3})(0.05)(60) = 6 \times 10^{-3} \text{ kg m}^{-2}. \quad (32)$$

Substitution of the current pressure drop value, assumptions about K_1 and K_4/K_3 , and results from (31) and (32) into (20) allows evaluation of constant K_2/K_3 for this pulse-jet filter:

$$1000 = \frac{8280 - 0 + 700(0.05)}{2} - \frac{\sqrt{[(8280 - 0 - 700(0.05))^2 - 4(6 \times 10^{-3})(0.05)K_2/K_3]}}{2}, \quad (33)$$

from which:

$$K_2/K_3 = 2.34 \times 10^{10} \text{ Pa s}^{-1}. \quad (34)$$

The proposed operating conditions can now be checked to be sure they will not cause pressure drop to increase without limit. First, the value for w_0 associated with the proposed 120 s (instead of 60 s) interval between cleaning pulses to each bag can be found from (2):

$$w_0|_{\text{proposed}} = c_i v t = (2 \times 10^{-3})(0.05)(120) = 1.2 \times 10^{-2} \text{ kg m}^{-2}. \quad (35)$$

The limiting initial pressure drop can be found from (26):

$$\Delta p_L = \frac{8280 - 0 + 700(0.05)}{2} + \frac{\sqrt{[(8280 - 0 - 700(0.05))^2 - 4(1.2 \times 10^{-2})(0.05)(2.34 \times 10^{10})]}}{2}, \quad (36)$$

from which:

$$\Delta p_L = 5850 \text{ Pa}. \quad (37)$$

The actual operating pressure drop immediately after operating conditions change, Δp_i , is 1000 Pa. For equilibrium to be established under the new operating conditions (24) must be satisfied:

$$\Delta p_i < \Delta p_L, \quad (24)$$

or in this case:

$$1000 \text{ Pa} < 5880 \text{ Pa}. \quad (38)$$

Because (38) holds, equilibrium will be achieved under the proposed operating conditions.

Next, (20) can be used to estimate pressure drop under these conditions. Substitution of the values for all constants and proposed w_0 into (20) yields:

$$\Delta p|_{\text{proposed}} = \frac{8280 - 0 + 700(0.05)}{2} - \frac{\sqrt{[(8280 - 0 - 700(0.05))^2 - 4(1.2 \times 10^{-2})(0.05)(2.34 \times 10^{10})]}}{2}, \quad (39)$$

from which:

$$\Delta p|_{\text{proposed}} = 2440 \text{ Pa.} \quad (40)$$

In this way the equilibrium pressure drop associated with the new filter operating conditions can be evaluated. Doubling the time between pulses should cause pressure drop to increase by 144%. Similarly, effects on filter operation of proposed changes in filtration velocity, pulse pressure, or inlet dust concentration could also be evaluated. However, the effect of a proposed change to different bags, or to collection of a different dust cannot at present be evaluated using this procedure because values for the constants used in these calculations may depend on characteristics of the fabric or dust used.

SUMMARY

A theory for pressure drop in pulse-jet cleaned filters has been presented. Assumptions made are that total pressure drop is the sum of drops across the conditioned fabric and the dust deposit, that the fraction of dust separated from a bag depends upon the separation force applied, and that this force can be found from impulse and momentum considerations as the bag and dust deposit move away from their supporting cage during a cleaning pulse.

The theory allows prediction of operating conditions under which filter operation will become unstable and cause pressure drop to increase without limit. With simplifying assumptions, the theory can be used to interpret pressure drop data or to predict the equilibrium pressure drop of a pulse-jet cleaned filter operated under any conditions.

Acknowledgement – This work was supported by U.S. Environmental Protection Agency Grant 804-700, under the direction of Dr James H. Turner.

REFERENCES

- Bakke E. (1974) Optimizing filtration parameters. *J. Air Poll. Control Assoc.* **24**, 1150–1154.
- Billings C. E. and Wilder J. (1970) Handbook of fabric filter technology. Volume I. Fabric filter systems study. NTIS Publication Number PB-200-648, Springfield, Va.
- Carman P. C. (1937) Fluid flow through granular beds. *Trans. Inst. Chem. Eng.* **15**, 150–166.
- Davis W. T., LaRosa P. J. and Noll K. E. (1976) The generation and evaluation of fabric filter performance curves from pilot plant data. *Filtr. Sep.* **13**, 555–560.
- Dennis R., Cass R. W., Cooper D. W., Hall R. R., Hampl V., Klemm H. A., Langley E. and Stern R. W. (1977) Filtration model for coal fly ash with glass fabrics. EPA Report EPA 600/7-77-084, NTIS, Springfield, VA.
- Dennis R., Cass R. W. and Hall R. R. (1978) Dust dislodgement from woven fabrics vs filter performance. *J. Air Poll. Control Assoc.* **28**, 47–52.
- Dennis R. and Silverman L. (1962) Fabric filter cleaning by intermittent reverse air pulse. *ASHRAE J.* **4**, 43–52.
- Dennis R. and Wilder J. (1975) Fabric filter cleaning studies. EPA Report EPA 650/2-75-009, NTIS, Springfield, VA.
- Ellenbecker M. J. (1979) Pressure drop in a pulse-jet fabric filter. Sc.D. Thesis, Harvard School of Public Health, 665 Huntington Ave, Boston, MA.
- Ellenbecker M. J. and Leith D. (1979) Dust deposit profiles in a high velocity pulse-jet fabric filter. *J. Air Poll. Control Assoc.* **29**, 1236–1241.
- Happel J. (1958) Viscous flow in multiparticle systems: slow motion of fluids relative to beds of spherical particles. *AIChE J.* **4**, 197–201.
- Leith D. and First M. W. (1977) Pressure drop in a pulse-jet fabric filter. *Filtr. Sep.* **14**, 473–480.
- Leith D., First M. W. and Feldman H. (1977) Performance of a pulse-jet filter at high filtration velocity II. Filter cake redeposition. *J. Air Poll. Control Assoc.* **27**, 636–642.
- Leith D., Gibson D. D. and First M. W. (1978) Performance of top and bottom inlet pulse-jet fabric filters. *J. Air Poll. Control Assoc.* **28**, 696–698.
- Pich J. (1969) The pressure drop in fabric filters in molecular flow. *Staub Reinhalt. Luft* **29**, 10–11.
- Robinson J. W., Harrington R. E. and Spaite P. W. (1967) A new method for analysis of multicompartimented fabric filtration. *Atmospheric Environment* **1**, 499–508.
- Rudnick S. N. (1978b) Fundamental factors governing specific resistance of filter dust cakes. Sc.D. Thesis, Harvard School of Public Health, 665 Huntington Ave. Boston, MA.
- Rudnick S. N. and First M. W. (1978) Specific resistance (K_2) of filter dust cakes: comparison of theory and experiments. 3rd Symposium on Fabric Filters for Particle Collection EPA Report EPA-600/7-78-087, NTIS, Springfield, VA.
- Stephan D. G., Walsh G. W. and Herrick R. A. (1960) Concepts in fabric air filtration. *Am. Ind. Hyg. Assoc. J.* **21**, 1–14.
- Strangert S. (1978) Predicting performance of bag filters. *Filtr. Sep.* **15**, 42–48.
- Williams C. E., Hatch T. and Greenburg L. (1940) Determination of cloth area for industrial filters. *Heat. Piping Air Cond.* **12**, 259–263.