About the Minimal Resolution of Space-Time Grains in Experimental Quantum Gravity

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Abstract

We critically analyse and compare various recent thought experiments, performed by Amelino-Camelia, Ng et al., Baez et al., Adler et al., and ourselves, concerning the (thought)experimental accessibility of the Planck scale by space-time measurements. We show that a closer inspection of the working of the measuring devices, by taking their microscopic quantum many-body nature in due account, leads to deeper insights concerning the extreme limits of the precision of space-time measurements. Among other things, we show how certain constraints like e.g. the Schwarzschild constraint can be circumvented and that quantum fluctuations being present in the measuring devices can be reduced by designing more intelligent measuring instruments. Consequences for various phenomenological quantum gravity models are discussed.

1 Introduction

According to general folklore, originally mostly based on simple dimensional considerations or qualitative reasoning and later supported by certain gedanken experiments combined with a couple of fundamental assumptions (to mention but a few sources see e.g. [1], [2], or the discussions in [3],[4]), there existed widespread agreement that fundamental lower limits to space-time measurements and resolution are roughly given by the respective Planck values, e.g. the Planck length $l_P = (G\hbar/c^3)^{1/2}$. One must however concede that in practice it seems to be presently impossible to come near these values in real experiments. Therefore, most of the work is rather of a thought-experimental character.

More recently it has been argued by various groups that these alluded fundamental bounds are in fact much larger and are perhaps just at the brink of becoming observable by using the most recent class of gravity-wave interferometers, more specifically, by observing the effects of (geometric) vacuum fluctuations in e.g. length measurements. As far as we can see, this particular field started more or less with the two papers [5] and [6], a precursor, having however a slightly different focus, being [7].

What is most puzzling is the claim of the authors that (lower bounds to the) uncertainty of length measurement turn out to be proportional to the square root or a simple fractional power of the length (or distance), l, to be measured, i.e. the fundamental uncertainty in length measurement (or rather the respective lower bound) seems to increase with l! This is, at first glance, quite unusual and perhaps even counterintuitive. As it would represent quite a departure from (perhaps too naive?) general accepted wisdom if these arguments turn out to be correct, it is of tantamount importance to scrutinize the correctness of the arguments being advanced in favor of this opinion. We note in passing that we are quite sympathetic in general to such an enterprise of developing a, so to speak, semi-classical quantum gravity phenomenology.

In contrast to earlier work, this more recent line of arguments is based on a paper by Wigner and Salecker ([8]), in which distance measurements in general relativity are analysed if effects of the Heisenberg uncertainty relation are included (as a typical ingredient of quantum theory). Postponing these more technical points to the following section, we continue with a brief discussion of the historical evolution of the field.

While papers on this topic, essentially repeating the original arguments, continue to appear up to quite recent times (see e.g. [9],[10],[11],[12],[13]), one should note that there have been a couple of contributions which pro-

vided arguments against the claimed inevitability of the fundamental measurement limitations of Jack Ng, van Dam and Amelino-Camelia, in particular concerning the strange dependence on the (macroscopic) distance, l, to be measured (see [14],[15],[16]). While the (technical) details of the arguments given in the three papers are certainly slightly different and vary with respect to the tightness and conclusiveness of the steps in the respective lines of reasoning, the overall focus of the papers is similar in spirit (see below).

Remark: One should note that we were not aware of the two earlier papers when we prepared our own contribution [16]!

As far as we can see, the original authors only reacted (in a quite negative way) to the first paper [14] (see [17] and [18]). Apparently they considered the topic to be then settled and, to our knowledge, did not even mention the later (and more elaborate) accounts in [15] and [16] (cf. e.g. the recent [13]). Therefore we feel obliged to give a considerably more careful and detailed account in the present work of our counter arguments. This holds the more so as we feel the whole matter is of extreme importance (both experimentally and theoretically).

Before we proceed with the more technical analysis, some general remarks concerning the whole field and the logical structure of the various arguments seem to be in order. Both the original analysis of e.g. Ng et al. or Amelino-Camelia and our own contribution ([16]) actually consist of roughly two parts (which are of course related). For one, the Salecker-Wigner thought experiment, for another, a semi-phenomenological theory of low-energy quantum gravity or *space-time foam*. We recently developed for example certain aspects of such a theory in [19], based on the *holographic* principle. The second half of [16] also deals with this special topic and consequences drawn by e.g. Ng et al. Furthermore, in [20] we developed a theory of random metric spaces and applied it to models of quantum space-time. We agree with Amelino-Camelia that this is a very important and desirable enterprise (cf. the abstract of [17]). It is however disputable and in fact a different question if the claims concerning fundamental bounds derived from the Salecker-Wigner thought experiment are really correct. We think, our counter arguments have been too quickly brushed under the carpet and that the situation is in fact considerably more subtle. In the present paper we will only deal with this thought experiment and its implications in order to keep the investigation within reasonable size. We plan to treat the question of stochastic fluctuations of space-time in a forthcoming paper.

We close this introduction with the mentioning of two particular points which should be given a closer inspection as they are in fact crucial for the logical coherence of the arguments being advanced in favor of the various points of view. First, in these types of thought experiments where extreme limit situations are studied (concerning the very possibility of the experimental realisability), it is important to check how large or small certain constants or parameters really are, to what extent they can be freely chosen (e.g. in cases where they are assumed to approach zero or infinity), or, on the other hand, whether there exist practical or fundamental constraints. A typical case in point is the habit to tacitly replace an estimate containing the relational symbols > or < by \gtrsim or \lesssim and then proceed by assuming without a more detailed discussion that the upper or lower bound can actually be reached in practice while a closer technical inspection would rather show that this is not! possible and that the relation is more adequately described by the symbol \gg or «.

This problem becomes for example aparent if one replaces the only approximatily correct continum models (e.g. elastic rods) describing the devices, typically used in this context, by their more reliable microscopic counterparts, based on the laws of many-body quantum theory (cf. our sections 3 and 4). It is easy to make adhoc assumptions about the possible physical parameters of these devices as long as one does not go into their microscopic and quantum mechanical details.

Second, and this concerns the second part of the usual argumentation, that is, the relation of the Salecker-Wigner thought experiment to the claimed fundamentality of the measurement bounds: It is clear that the technical (thought) experiments alone are not sufficient to support this claim. Both mentioned groups evidently seem to be intrigued by the functional form of the terms $l^{1/2}$ or $l^{1/3}$, which apparently remind them of versions of Brownian motion models (see e.g. section 4 of [10]). As to the occurrence of these terms one should say that the Salecker-Wigner experiment alone does by no means suggest such a deeper connection to quantum gravity effects. It is evident that these terms occur in certain expressions because of the quantum-uncertainty induced movement of clock and/or mirror. This is, in a sense, a quite trivial effect and does not seem to have anything to do with Planck fluctuations as long as one does not argue that quantum theory is a large scale consequence of quantum gravity (which may in fact be the case).

Be that as it may, the original authors argue that the fundamental character of length fluctuations derives from the cooperative and correlated behavior of the individual grains of space-time. The above power laws would then suggest a rather mild form of correlation. In the second part of our [16] we scrutinized these ideas and came to a different conclusion, that is, the *holographic principle* and other observations rather suggest an extremely strong form of, what we called, *anticorrelation* of the fluctuations of the hypothetical individual grains of space-time. This means, the individual fluctuations have the tendency of cooperating in such a way that the total fluctuation in macroscopic regions remains small, i.e. just the opposite of a central-limit or Brownian motion behavior. We later discussed this peculiar fluctuation structure in greater detail in [19] (see also [20]).

That is, the real question in this context is the following, and this goes beyond the question, mainly adressed in the present paper, which primarily deals with the Salecker-Wigner thought experiment:

Observation 1.1 Are the geometric fluctuations in the quantum vacuum near the Planck scale only weakly correlated, as suggested by the (Brownian-motion) results of Amelino-Camelia or Ng et al., or are they strongly anticorrelated as suggested by our own findings? The latter possibility would entail that e.g. length fluctuations are essentially independent of the length to be measured.

2 A Brief Review of the Salecker-Wigner-Amelino-Camelia-Ng-van Dam Thought Experiment

In this section we will be very brief, only emphasizing certain more relevant aspects, as the topic has meanwhile been described repeatedly (apart from the original sources in e.g. [15] or [16]). We begin our analysis with a general remark.

Remark: One should keep in mind that various of the more technical and practical problems belonging to the special field of length and/or time metrology, are not discussed in the cited papers and also not in the following as this would become a quite cumbersome enterprise. To this belongs for example the problem of the exact determination of the arrival time of light pulses or individual photons, the inescapable microscopical roughness of the surface of mirrors which neccessitates the use of light with wave lengths which are sufficiently long so as to average over this roughness of surfaces etc. In the following we rather try to concentrate on the more fundamental problems. However, we show that such a fundamental analysis may nevertheless lead to interesting technical suggestions (see section 4.3). The original Salecker-Wigner thought experiment deals (among other things) with the quantum-uncertainty of length measurements in a gravitational field. As has been rightly emphasized in [7], its main concern was rather the construction of tight nets of coordinate lines in general relativity if quantum effects are included. Therefore their reliance on freely falling clocks, mirrors etc. was quite reasonable, as this is natural in this context.

Note that the definition of (true) spatial distance in e.g. a static gravitational field is not completely trivial but nevertheless straightforward (for a clear account see for example [22], the respective formulas can also be looked up in [16]). In the Salecker-Wigner approach a light pulse is sent from a small freely falling apparatus which also contains a clock towards an also freely falling mirror where it is reflected. The distance can then be inferred from the total arrival time needed, i.e.

$$2l = t \cdot c \tag{1}$$

with t the elapsed time.

If the quantum nature of clock and mirror is taken into account, their positions at the respective arrival times of the light pulse are uncertain by an amount

$$\delta l + \delta v \cdot t = \delta l + \hbar/m\delta l \cdot l/c = \delta l + \hbar l/mc\delta l \tag{2}$$

with

$$\delta l \cdot \delta p \ge (1/2)\hbar$$
 , $\delta v = \delta p/m$ (3)

m being the mass of clock or mirror, δl the original position uncertainty or, rather, the position uncertainty after the emission of a light pulse; for convenience we discuss only the case of the clock. This yields a minimal uncertainty

$$\delta l_{min} = (\hbar l/mc)^{1/2} = l_c^{1/2} \cdot l^{1/2}$$
(4)

with l_c the Compton wave length of clock or mirror.

One can now try to minimize the uncertainty in length or position measurement by making m as large as possible. There are, obviously, practical limits, for example if one wants to create a dense coordinate net as e.g. in [8] or [7]. But there exists also a fundamental limit given by the *Schwarzschild-bound* as has been exploited in [6] or [5]. One should note that such Schwarzschild-type arguments were of course already used in the past in related contexts. For one, the uncertainty in position imparts also a fluctuation in the gravitational field and the metric tensor ([6]). Furthermore, huge masses lead to a macroscopic distortion of the gravitational field in the large. This, however, represents in our view rather a correction and not! an uncertainty and can be incorporated by a rigorous distance calculation as e.g. described in [22]. Really crucial seems to be, at first glance, the Schwarzschild constraint.

If the geometric size of the clock-lightgun system (or mirror) is given by s, a horizon will form around the clock (or mirror) if

$$m \ge m_s := const \cdot (c^2 s/G) = const \cdot \hbar/c \cdot s/l_p^2$$
 (5)

for some constant of order one. By inserting this estimate in the preceding expression, Amelino-Camelia derives a lower bound on the uncertainty, δl , of the form

$$\delta l \ge const \cdot (l_n^2/s)^{1/2} \cdot l^{1/2} \tag{6}$$

More specifically, one exploits the estimate in the opposite direction in order that the measurement device is able to function as expected.

Remark: We will later comment on the relation of the size of the clock, s, to the length, l, to be measured. Frequently the size is assumed to be very small. This, however, does not seem to be necessary in our view.

Jack Ng and van Dam ([5]) get a slightly different estimate by using a *light clock*. Again they make an estimate of the size of the light clock which is in our view overly restrictive. In their example, photons bounces back and forth in a cavity of size b. They correctly argue that the smallest time interval one can resolve with this clock is of order $t_b = b/c$. This induces a length uncertainty of order $\delta l \gtrsim b$. They then however assume that the size of the clock apparatus is also of size b, i.e. that the whole mass, m, of the clock is squeezed into this (small) region of size b. We do not see that this restrictive assumption is really necessary. We think, one can envisage a massive but extended clock system containing e.g. a small cavity of size b while the size of the whole system, s, is considerably larger (see below). They now conclude that it follows

$$\delta l \ge b \ge l_s := const. \cdot Gm/c^2 \tag{7}$$

i.e. with the rhs the corresponding Schwarzschild radius. They then get

$$\delta l \ge const \cdot l_p^{2/3} \cdot l^{1/3} \tag{8}$$

Both estimates show the (at first glance) strange dependence of δl on the length, l, to be measured. But in our view this is only the consequence of the particular experimental set-up of the Salecker-Wigner experiment with its

freely falling objects. It remains to be shown that it is of a more fundamental significance.

While our technical analysis will start with the following section, we want in this section to comment on a simple example Ng et al. give in order to corroborate their strange result. Furthermore it supports our suspicion that the possibility of strong anticorrelations in this context is obviously not seriously taken into account by some authors (whereas this possibility is sometimes mentioned in passing). The reason is presumably that it seems to be not so easy to imagine physical mechanisms which produce such strong effects; see however the second part of [16] and the detailed analysis in [19].

We begin with some, as we hope, clarifying remarks for readers not so familiar with solid state physics. Both real and harmonic crystals, which we discuss in more detail in the next section, are not stable in one dimension. But as the periodicity in the harmonic model is put in by hand via an explicitly given lattice constant, a, in the 1-dim. case, the unstable behavior is reflected by the divergence of the fluctuations of the atomic positions around their equilibrium values when the particle number, N, goes to infinity. That is, with $u_i := x_i - x_{i,0}$ we have in one dimension for non-vanishing temperature:

$$\langle u_i^2 \rangle^{1/2} \sim N^{1/2}$$
 (9)

with N the number of atoms in the chain. By the same token, with $l = N \cdot a$ the average distance between, say, the atoms at positions $x_{0,0} = 0$ and $x_{N,0} = N \cdot a$, the respective distance fluctuation is

$$\delta l \lesssim \langle u_0^2 \rangle^{1/2} + \langle u_N^2 \rangle^{1/2} \tag{10}$$

(apart from small possible boundary corrections which depend on the boundary conditions being used).

In [5] Ng and van Dam argue that length fluctuations being proportional to some simple fractional power of the length to be measured are typical and natural and give the following example. They mention a one-dimensional chain of N ions connected by springs (an example, they attribute to Wigner) in the high-temperature limit. With b the lattice constant and $\delta x_i := x_i - x_{i-1}$ they argue that

$$\langle (x_N - x_0)^2 \rangle^{1/2} = \delta l \sim (N \cdot \langle \delta x_i^2 \rangle)^{1/2} = l^{1/2} \cdot (\langle \delta x_i^2 \rangle / b)^{1/2}$$
 (11)

with $l = N \cdot b$ and $\langle \delta x_i^2 \rangle$ being independent of the position *i* (modulo certain boundary conditions).

One should say this is a fairly unsurprising observation and does by no means corroborate their general claim. For one, for very high temperatures the individual atoms fluctuate almost independently relative to each other. The above result is then nothing but the well-known *central limit theorem*. But even for ordinary (non-vanishing) temperatures we get a similar result after some calculations (see the next section). On the other hand, in the following sections we mainly deal with the nature of *zero-point motions* at temperature zero. For a one-dimensional harmonic crystal at zero temperature we then get

$$\delta l \sim (\ln N)^{1/2} \tag{12}$$

On the other hand, we will see in the next section that in higher dimensions atomic fluctuations remain small and finite and, by the same token, fluctuations in distances. Therefore the one-dimensional harmonic chain is rather exceptional and does not represent the typical case.

The scenarios, described by the above cited authors rather prevail in gases or other disordered or weakly correlated systems. We remind the reader of the observation at the end of the introduction. In this context we again want to mention our results in the second part of [16] where we already discussed in some detail the harmonic crystal and showed that it is exactly an example from the realm of ordinary physics displaying these strong anticorrelations we mentioned above. In this sense, we think, nothing really follows from this example.

We conclude this section with some brief remarks concerning the different scenarios employed by Amelino-Camelia, Ng et al., Adler et al., Baez et al. or by ourselves, because this gives the motivation for our detailed investigation into the behavior of real quantum solids in this field of quantum gravity research. Most of the devices employed are made of such stuff and we think, the at best approximately correct continuum models do perhaps not! give the correct results in these extreme (high-precision) situations.

The original authors essentially used these devices in the way Salecker-Wigner used them, i.e. clocks, lightguns and mirrors are designed and treated as freely falling, relatively small (but possibly heavy) objects, the microscopic structure of which does not play a crucial role and is more or less neglected. In [14] the clock is assumed to be somehow bound in a harmonic containing potential, the physical nature of which is not openly indicated in detail. Its main purpose is to keep the clock (or mirror) from wandering away under its original momentum uncertainty.

Baez et al. give a more detailed account by assuming the clock being fixed at the end of a (long) rod and then estimate its length oscillations within the framework of continuum mechanics. In [16] we fixed both clock/lightgun and mirror on a solid understructure, the behavior of which we treated in a microscopic quantum mechanical way. Now the relative mean-distance is fixed, because neither clock nor mirror can wander away, but what is still present (as in the other treatments) are the unavoidable quantum fluctuations of (in our case) the individual atomes of the crystal lattice. What is however now avoided to an at least large extent are the strong constraints, resulting from the Schwarzschild bound, which entered in the treatment of the original authors because they assumed that the respective devices are quite small.

Most of the authors seem to follow the original idea of independently located devices in space (i.e. clock, mirror, other objects). They then automatically have to struggle with the quantum-uncertainty induced movement of the objects which introduces these funny terms, we have discussed above. On the other hand, Ng et al. and Amelino-Camelia invoke the possible usefulness of gravity-wave interferometers. But as far as we have understood the subject matter, these are very large and massive complex devices with most of the equipment sitting on a rigid extended under-construction. There exist of course certain parts which are suspended or are able to oscillate, but nevertheless there relative mean distances and positions are essentially fixed (see e.g. [21]). That is, in our view, these constructions seem to resemble rather the experimental set-up we are suggesting, i.e. clock and mirror being parts of a more or less rigid and complex measuring device.

While we have the impression that for example Amelino-Camelia seems to have the opinion that the presence of such extended bodies will distort the gravitational field in a perhaps uncontrollable way, we think, these perturbative effects can be incorporated in the calculations. Anyhow, we have not found a really convincing argument in favor of this pessimistic opinion. This holds the more so as ultimately most of our equipment happens to be fixed to our planet earth or to some other huge body and this is certainly the case for the mentioned interferometers. Furthermore, after all we are in fact interested in matters of principle and not in numerical details. That means, the minute fluctuations of space-time are expected to occur not only in free space but as well in solids and other equipment (remember the solid cylinders of the first gravity-wave experiments).

In [16] we assumed clock (or mirror) to be fixed to the rigid understructure by means of a trap (as in [14]), which was implemented by some oscillator potential. We meanwhile think this additional source of uncertainty is not really necessary. In the following we prefer to regard the clock (and mirror) as being integrated parts of the rigid body itself. The task is then to carefully check the various occurring physical parameters of the different models, employed in this context, as to the possibility of choosing them in such a way so that the final length fluctuations we are interested in become as small as possible.

In this respect two, at least in our view, different questions have to be dealt with. First, the claim of Amelino-Camelia and Ng et al. that length fluctuations really have an intrinsic and unavoidable dependence on the length to be measured, i.e.

$$\delta l \gtrsim l^{1/2} \quad \text{or} \quad \delta l \gtrsim l^{1/3}$$
 (13)

for length scales which can in principle be experimentally observed. Second, good quantitative estimates of the numerical degree of length uncertainty in the different experimental setups. We emphasize this latter point as we suspect that the existing estimates are not very reliable and that certain assumptions are perhaps too optimistic.

3 Solid State Physics meets Quantum Gravity

In the various thought experiments which have been introduced in the field we are discussing, equipment has been employed which is in the last analysis of the nature of quantum many-body systems. As we are, a fortiori, employing this measurement equipment in very extreme situations, we think it is reasonable to take its microscopic many-body nature really into account and not simply regard the measuring devices as being essentially classical or structureless objects.

Take for example the one-dimensional rod introduced in [15], the behavior of which is discussed, at least in the first steps, within the framework of classical continuum mechanics. Its length is denoted by x, the velocity of sound by c_s , the elastic modulus by Y, its mass by m. With $\rho = x/m$ the mass density in one dimension, the general formula for c_s is

$$c_s = (Y/\rho)^{1/2} = (Y \cdot x/m)^{1/2}$$
(14)

With the apriori bound (c being the velocity of light)

$$c_s \le c \tag{15}$$

and the heuristic association of the rod with a harmonic oscillator having spring constant

$$k = 2Y/x \tag{16}$$

Baez et al. finally get the formula for the zero point length fluctuation of the rod via the associated harmonic oscillator model.

$$\Delta x \gtrsim (\hbar x/mc)^{1/2} \tag{17}$$

with, at first glance, an explicit dependence of Δx on x while written in the form

$$\Delta x \gtrsim (\hbar/\rho c)^{1/2} \tag{18}$$

one sees that the length fluctuation, calculated in some oscillator ground mode is actually independent of the length of the rod (at least as long as the *Schwarzschild constraint* has not been introduced).

Remark: We surmise that the authors used some form of Hooke's law in this derivation. One should note that the correct form of Hooke's law reads

$$\Delta l/l = Y^{-1} \cdot F/A = Y^{-1} \cdot \sigma \tag{19}$$

with F the applied force, A the area of the cross section and σ the tension in the rod. The other variant one frequently finds in the literature

$$\Delta l = k^{-1} \cdot F \tag{20}$$

has the disadvantage of hiding the explicit dependence of k on the length of the rod, i.e. we have

$$k = Y/l \cdot A \tag{21}$$

We will show in the following that this (heuristic) fluctuation result, based to a large part on classical physics, coincides with the rigorous microscopic result (for zero temperature!) apart from an (inessential) factor of the form $(\ln N)^{1/2}$ with N the number of atoms. Note again that in a strict sense the one-dimensional harmonic crystal is not stable in the limit $N \to \infty$. That means, the fluctuations of the individual atoms diverge in this limit (but in an extremely slow manner in the case T = 0). We will come back to the approach of Baez et al. in the following section in connection with the Schwarzschild-constraint and the *Hoop-conjecture*. We will then see more clearly the possible weaknesses of such continuum models.

As a typical candidate for a true many-body system serving as a model for the possible macroscopic measuring devices we will employ in the following we now discuss the harmonic crystal and the position fluctuations of its atoms. We will later see that it may be advantageous to also use equipment which is not entirely made of this rigid crystallic structure but contains also parts which are capable of absorbing and damping various sources of external and internal noise. To begin with, we assume the crystal to be cooled down to T = 0. The qualitatively different behavior for non-vanishing T will be discussed at the end of this section. This means, that only the so-called *zero-point motion* of the atoms is taken into account. Various aspects of this model are e.g. discussed in [23] or [24], see also [25], the classical source being [26]. But as the results are frequently widely scattered in these books and as the quantum fluctuation results we are interested in, are either not explicitly given or difficult to find, we will provide the necessary formulas in the following. One should furthermore note that we are sometimes cavalier concerning (in this context) uninteresting prefactors of order one.

In the following discussion we assume that the crystal as a whole is fixed in the respective reference system or, rather, we consider it relative to its center-of-mass system. Put differently, we neglect, for the time being, the purely translatory mode. So, let \mathbf{u}_i be the momentary elongation of the *i*-th atom from its equilibrium position and N the number of constituent atoms. We then have in the quantum case

$$N^{-1} \cdot \sum_{i=1}^{N} < \mathbf{u}_{i}^{2} >= N^{-1} \cdot \sum_{\mathbf{k},s} \hbar/(2M_{0}\,\omega_{\mathbf{k},s})$$
(22)

with the brackets in our present context (T = 0) denoting quantum averages. For non-vanishing temperatures the formula has a slightly different form. The sum on the rhs runs over the first *Brillouin zone* of the crystal and over the different possible *phonon branches* $\omega_{\mathbf{k},s}$. M_0 is the mass of the lattice atoms. For $\mathbf{k} \to 0$ we have

$$\omega_{\mathbf{k},s} \sim c_s(\mathbf{k}/|\mathbf{k}|) \cdot \mathbf{k} \tag{23}$$

with c_s the (in general) branch and direction dependent velocity of sound.

To evaluate the sum we make some (harmless) approximations. We restrict ourselves to a single phonon branch, extend the linear dispersion law of the (accoustic) phonon branch up to the boundary of the Brillouin zone, assume c_s to be independent of the direction and replace the first Brillouin zone by the so-called Debeye sphere. Furthermore, in space dimension greater than one we replace the sum by an integral, using the conversion factor (with *a* some lattice constant)

$$\sum_{\mathbf{k}} = Na^3 / (2\pi)^3 \int \tag{24}$$

We finally get in three or two dimensions

Observation 3.1 For the fluctuation of the position of the lattice atoms in a harmonic crystal at T = 0, i.e. only zero-point fluctuations being taken into account, we get 1. 2,3-dim.:

$$\Delta u_i \approx const \cdot (\hbar a/c_s M_0)^{1/2} \tag{25}$$

2. 1-dim.:

$$\Delta u_i \approx const \cdot (\hbar a/c_s M_0)^{1/2} \cdot (\ln N)^{1/2}$$
(26)

with const being of order one. This implies that the fluctuations of the distance between two arbitrary lattice sites is of the same order.

Proof: The three and two dimensional case follows from our above formulas. The one-dimensional case has to be treated slightly differently. It is more appropriate to directly calculate the discrete sum

$$(\hbar/M_0) \cdot N^{-1} \sum_{Brill.zone} (c_s \cdot |\mathbf{k}|)^{-1}$$
(27)

as the integral version is singular at $\mathbf{k} = 0$. This yields

$$\Delta u_i^2 \approx (\hbar a/c_s M_0) \cdot N^{-1} \cdot \sum_{l=1}^N (2\pi l/Na))^{-1} \approx const \cdot (\hbar a/c_s M_0) \cdot \ln N \quad (28)$$

Remark: In [16] we got already similar results (for space dimension greater than one) with the help of a slightly different reasoning. Our main goal in [16] was however to show how natural strong anticorrelations among individual position fluctuations are already in ordinary physics and that the standard Brownian-motion type results or variants thereof, which all are somehow inspired by the central limit behavior, and which are typically invoked in this context, cannot always be expected.

It is now important to investigate the range within which the occurring physical parameters can be chosen and, furthermore, if they can be independently chosen. Note in this context that in the harmonic crystal model the lattice constant is put in by hand. In the true many-body situation the periodicity of certain states has in principle to be calculated (which is quite difficult). Furthermore, the lattice constant is expected to change if M_0 or e.g. the temperature is varied.

In a first step one can try to make c_s as large as possible. We evidently have an apriori upper bound

$$c_s \le c \tag{29}$$

with c the velocity of light, which is however quite crude as typical values for c_s are of order $10^3 [m]/[s]$. As to the lattice constant a, it seems to be

difficult in ordinary matter to have it smaller than the average distance in dense nuclear matter (e.g. neutron stars), i.e. we may assume

$$a \ge a_0 \approx 10^{-15} [m]$$
 (30)

There exists another relation between c_s , M_0 , a and the coupling constant, α , of the harmonic oscillator potential between neighboring atoms

$$c_s = (\alpha/M_0)^{1/2} \cdot a \tag{31}$$

which, when choosing the extreme values $c_s = c$ and $a = a_0$, yields a relation between α and M_0 .

We see that the following estimates hold.

Observation 3.2 In ordinary matter the expression $(\hbar a/M_0 c_s)^{1/2}$ is lower bounded by

$$(\hbar a/M_0 c_s)^{1/2} \gtrsim l_c (M_0)^{1/2} a^{1/2} \gtrsim l_c (M_0)^{1/2} a_0^{1/2}$$
(32)

with $l_c(M_0)$ the Compton wavelength of the lattice atoms.

It is instructive to calculate this bound numerically. With $M_0 \approx 10^{-25} [kg]$ for ordinary atoms, we get

$$l_c (M_0)^{1/2} \cdot a_0^{1/2} \approx 10^{-16} [m]$$
(33)

On the other hand, for ordinary matter with $a \approx 10^{-10} [m]$ and $c_s \approx 10^3 [m]/[s]$ we get

$$(\hbar a/M_0 c_s)^{1/2} \approx 10^{-11} [m]$$
 (34)

Conclusion 3.3 A reasonable lower bound for the length fluctuations in three, two, or one space dimensions is (with $(\ln N)^{1/2} = O(1)$)

$$\Delta u_i \gtrsim 10^{-16} [m] \tag{35}$$

This lower bound on Δu_i is obviously still far above the Planck scale, but it is a reliable value as long as we do not take special measures. One should compare this bound with the bound of Baez et al. for the one-dimensional elastic rod. One should note that in [15] only a single ground frequency was used in the idealized model. In our microscopic rigorous approach we integrated over all occurring phonon frequencies. The effect is however only an extra numerical prefactor. **Observation 3.4** Relating the estimate in the approach of Baez et al. with our rigorous microscopic calculation we get (without the additional Schwarzschild constraint)

$$\Delta x \gtrsim (\hbar x/m c)^{1/2} \gtrsim (\hbar N \cdot a_0/N \cdot M_0 c)^{1/2} = l_c (M_0)^{1/2} \cdot a_0^{1/2}$$
(36)

i.e., the two estimates give roughly the same value. It is however crucial that the rhs of the above estimate shows that the length fluctuation is completely independent of the parameters x and/or m. So there seems to be no room left to make Δx small by choosing x or m appropriately (see the next section).

Before we proceed we mention the corresponding results for non-vanishing temperature. In this case we have

$$N^{-1} \cdot \sum_{i} \langle u_i^2 \rangle = N^{-1} \cdot \sum_{k} (\hbar/2M_0 \,\omega_k) \,\cot(\beta \,\hbar \,\omega_k/2) \tag{37}$$

with β the inverse temperature. For small **k** we get

$$\cot(\beta \,\hbar \,\omega_k/2) \approx 2/\beta \,\hbar \,c_s \cdot |\mathbf{k}|^{-1} \tag{38}$$

and

$$N^{-1} \cdot \sum_{i} \langle u_{i}^{2} \rangle = N^{-1} \cdot \sum_{k} (\beta M_{0} c_{s})^{-1} \cdot |\mathbf{k}|^{-2}$$
(39)

Conclusion 3.5 For $T \neq 0$ we get the estimate

$$\Delta u_i \approx (\beta \, M_0 \, c_s)^{-1} \cdot a \tag{40}$$

in three or two dimensions and

$$\Delta u_i \approx (\beta \, M_0 \, c_s)^{-1} \cdot a \cdot N^{1/2} \tag{41}$$

in one dimension

To sum up what we have learned in this section; we have seen that, due to the atomic structure of ordinary matter, it is rather academic to make incompatible assumptions in certain thought experiments as to various occurring physical parameters of objects or equipment to be employed in some of the arguments. It is for example problematic to assume that very small but sufficiently heavy objects do actually exist. It may turn out that in the far future some exotic matter may be found having such properties but at the moment it seems to be difficult to pack ordinary matter denser than with interatomic distance $a_0 \approx 10^{-15} [m]$. If we assume that a typical atomic mass is of order $M_0 \approx 10^{-25} [kg]$, we have the following constraint. **Observation 3.6** For the size, s, and mass, m, of a typical object in our discussion we have the following relation (with N the number of atomic constituents):

1. If the object is essentially three-dimensional we have

$$s \gtrsim N^{1/3} a_0 \quad , \quad m = N \cdot M_0$$
 (42)

or

$$s \gtrsim (m/M_0)^{1/3} a_0$$
 (43)

2. In two dimensions we get

$$s \gtrsim (m/M_0)^{1/2} a_0$$
 (44)

3. In one dimension for a rod-like shape we get

$$s \gtrsim N a_0 \quad , \quad m = N \cdot M_0 \tag{45}$$

and

$$s \gtrsim (m/M_0) a_0 \tag{46}$$

These bounds will have certain consequences for the discussion in the following section.

4 Commentary on the various Thought Experiments

In the light of our previous observations we will comment on the thought experiments by Baez et al. and Ng et al and will compare them with our own approach.

4.1 The Modified Thought Experiment of Baez et al. and the Hoop Conjecture

In [15] the Schwarzschild constraint is used for an essentially one-dimensional rod of length x by invoking the so-called *Hoop conjecture* (see e.g. [27] or [1]). If m is the mass of the rod it roughly says that a horizon will form around the rod if

$$m \ge m_s = const \cdot c^2 x/G \tag{47}$$

with some constant of order one.

To make the length fluctuation of the rod as small as possible Baez et al. chose the length of the rod to be roughly the size of the corresponding Schwarzschild radius or, rather, a little bit larger, that is

$$x \approx l_s = \operatorname{const} m \, G/c^2 \tag{48}$$

Then the product $x^{1/2} l_c^{1/2}$, occurring in their derivation, would become approximately

$$l_s^{1/2} \, l_c^{1/2} = l_p \tag{49}$$

and they finally concluded (by incorporating the additional uncertainty of the center of mass of the rod and by choosing m arbitrarily large)

$$\Delta x \gtrsim l_p \tag{50}$$

We already derived in the preceding section a lower bound on Δx for the particular experimental set-up used by Baez et al and which is completely independent of x and/or m but is much larger than the Planck length, that is

$$\Delta x \gtrsim l_c (M_0)^{1/2} \cdot a_0^{1/2} \tag{51}$$

This implies, that something must be wrong in the reasoning of Baez et al. We will show now that it is not possible for the length of an essentially onedimensional rod made from ordinary matter to come near the Scharzschild radius of the rod. At the end of the preceding section we got a relation between mass and length of a one-dimensional rod

$$N \cdot a = l \gtrsim m/M_0 \cdot a_0 = N \cdot a_0 \tag{52}$$

with a the real lattice constant, a_0 its minimal value. This implies

$$l/l_s = a c^2 / (M_0 G) \gtrsim a_0 c^2 / (M_0 G)$$
(53)

With our standard assumptions $M_0 \approx 10^{-25} [kg], a_0 \approx 10^{-15} [m]$ we get

$$M_0 G/c^2 = l_s(M_0) \approx 10^{-53} [m]$$
 and hence $l/l_s \gtrsim 10^{38}$ (54)

Observation 4.1 For ordinary matter the linear extension of e.g. a rod exceeds its Schwarzschild radius by a factor of $\gtrsim 10^{38}$. One should note that this holds for the rather extreme parameter, a_0 , we have chosen. For a more realistic parameter the factor happens to be even larger.

Conclusion 4.2 For the above term $x^{1/2} \cdot l_c^{1/2}$ it holds

$$\Delta x \gtrsim x^{1/2} \cdot l_c^{1/2} \gtrsim 10^{19} l_s^{1/2} \cdot l_c^{1/2} = 10^{19} \, l_p \tag{55}$$

That is, there seems to be no chance that the length fluctuations of a onedimensional rod really come near the respective Planck value.

This is a case in point for what we said in the introduction about estimates using frequently the symbol \gtrsim where rather the symbol \gg would be appropriate.

On the other hand, for three or two dimensions we got lower bounds at the end of the preceding section of the kind (s being the linear extension of the object)

$$s \gtrsim (m/M_0)^{1/3} \cdot a_0 \quad , \quad s \gtrsim (m/M_0)^{1/2} \cdot a_0$$
 (56)

yielding

$$s/l_s \gtrsim const \, (c^2/M_0^{1/3} \, G) \cdot a_0 \cdot m^{-2/3} \quad , \quad s/l_s \gtrsim const \, (c^2/M_0^{1/2} \, G) \cdot a_0 \cdot m^{-1/2}$$
(57)

respectively. Setting $s/l_s = 1$ on the lhs yields:

Conclusion 4.3 While it seems to be impossible to confine a rod-like object of ordinary (atomic) matter within its Schwarzschild sphere or to come at least near this goal, this can be achieved in two or three space dimensions for sufficiently large mass. For three dimensions the mass scale such that $s = l_s$ is $m = m_s \approx 10^{30} [kg]$ which is approximately the mass of the sun. The Schwarzschild radius is $s \approx l_s \approx 10^3 [km]$. This result holds for the assumed extreme limit value a_0 we have chosen as a lower bound. Note that for a fixed value of the parameter a both l_s and m_s are also fixed by the above formulas. Furthermore, for an $a > a_0$ both m_s and l_s become also larger.

Remark: Note that in general, by neglecting the atomic microscopic structure of matter, we can calculate for each given s the Schwarzschild mass, m_s , so that $s = l_s$. In the above calculations we assumed that the average density or, put differently, the average interatomic distance, a, is fixed or even minimal. i.e. $a = a_0$. Then we get another relation between size and mass and the identity $s = l_s$ can only hold for a single mass value.

4.2 A Comparism of the Ng-van Dam Thought Experiment with our Approach

The approach of Ng et al. (see section 2) is based on the Salecker-Wigner method of freely falling (small) clocks and mirrors. In a first step one gets

$$\delta l \gtrsim l^{1/2} \cdot l_c^{1/2} \tag{58}$$

In a second step they add the certainly correct assumption that the size, s, of the clock has to be larger than its own Schwarzschild radius. They then make however an additional assumption which in our view is too restrictive. For the time measurement they choose a so-called *light-clock* in which a photon bounces between the mirrored walls of a cavity. This is also reasonable as c represents a limiting velocity and a large velocity makes the period of the clock short which, by the same token, is responsible for a technical lower bound on the uncertainty of length measurement, i.e. we have

$$\delta l_{tech.} \gtrsim b$$
 (59)

with b the diameter of the cavity. They then however make the assumption that the size of the clock is roughly of the order of the diameter of the cavity, b. They hence get

$$\delta l \gtrsim b \approx s \gtrsim l_s = const \cdot G \, m/c^2 \tag{60}$$

with m the mass of the clock (or mirror). That is, they assume that the whole mass of the clock is concentrated within a sphere roughly of the size b. Combining the two estimates they finally get

$$\delta l \gtrsim l^{1/3} \, l_p^{2/3} \tag{61}$$

This may be contrasted with the estimate by Amelino-Camelia (see section 2) in which no light clock was explicitly used:

$$\delta l \ge const \cdot l_p \cdot (l/s)^{1/2} \tag{62}$$

and in which the size of the clock-lightgun system still explicitly appears. The form of the Ng-van-Dam estimate seems to convey a deep (functional) relation between length measurement and Planck scale physics, in particular as it contains the symbol \gtrsim instead of, say, \gg . We first should investigate if this connection does really exist or, on the other hand, if it is only apparent.

We have seen that, first of all, in the approach of Ng et al. the precision of length measurement is fundamentally limited by the period of the light clock or, by the same token, by b. The smallest conceivable clock of the kind Ng et al. are envisioning is, in our opinion, a clock consisting of one atom or atomic nucleous. With our rough approximation, a_0 , we thus get

$$\delta l \gtrsim \delta l_{techn.} \approx b \gtrsim a_0 = 10^{-15} [m]$$
 (63)

We previously calculated l_s for such an atom and got

$$l_s(M_0) \approx 10^{-53} [m] \tag{64}$$

That is, for such an atomic clock its natural size exceeds its Schwarzschild radius by many orders of magnitude, put differently, for such a clock it would be inappropriate to use the symbol \gtrsim in the respective estimates.

On the other hand, we derived in the preceding sections a relation between the size of an object, made from ordinary matter, and its mass. In three dimensions it reads (by assuming $a = a_0$):

$$s \gtrsim (m/M_0)^{1/3} \cdot a_0$$
 (65)

That is, if we make the clock heavier, the Schwarzschild radius would also increase but, by the same token, the size of the clock would increase too. As l_s grows linearly with m while s is proportional to $m^{1/3}$ in three dimensions, there exists a unique value where l_s and s become identical. In the Ngvan Dam approach the size of the clock is however rigidly related to the parameter b which is a lower bound to the uncertainty δl_{techn} which is smaller than δl . So we arrive at a certain dilemma as the uncertainty in time measurement would also increase with mass and size of the light clock.

Conclusion 4.4 In our view it is reasonable to use clocks with the parameters s and b being decoupled, that is, one should use clocks with the diameter of the mirrored cavity as small as possible in order to make $\delta l_{techn.}$ as small as possible but making their mass and, by the same token, their size large. Or, what seems to be even better, to use clocks with $l_{techn.}$ not limited by som geometric parameter b.

Remark: As we are no expert in time metrology we do not know if there perhaps exist ingenious methods to make the period of the time clock shorter than the value we assumed, i.e. $\Delta t \approx 10^{-15} [m]/c$. This seems to be, at least in our view, difficult for the type of light clock Ng et al. are employing but may be possible for other types of clocks. One can learn from e.g. the analysis in [21] that the sensitivity of the modern interferometers can be increased by various ingenious methods, but this seems to apply rather to the observation of the (qualitative) change in interference patterns, not so much to the exact measurement of distances.

If we loosen the connection between the size of the mirrored cavity, b, and the size of the whole clock system, s, we have more possibilities. We can try to make b as small as we can, or even better, use a different sort of clock, and, on the other hand, make s as large as possible in order to avoid the Schwarzschild constraint while we make l_c as small as possible. We then fall back on the relations derived in section 2, i.e.

$$\delta l \gtrsim l_c^{1/2} l^{1/2} \quad , \quad m \le const \left(c^2 s/G \right) = m_s \tag{66}$$

as long as we insist on independent freely falling clock and mirror systems. One should however remark that so far these devices are treated as essentially structureless objects. The kind of internal fluctuations being always present in these objects if their quantum nature is taken into account has been treated in section 3 and will further be treated in the following.

Inserting now (as e.g. Amelino-Camelia did) $m \approx m_s$ in $l_c = \hbar/mc$, we get

$$\delta l \gtrsim const \cdot l_p \, (l/s)^{1/2} \tag{67}$$

From our previous calculations with $a = a_0$ we learned that both m_s and s are of considerable size. On the other hand, we think, this is not totally unrealistic as in our framework these devices can be considered to be more or less rigidly fixed onto for example the earth itself. This is certainly the case, as we already emphasized above, for the large interferometers the authors themselves invoked in their arguments. That is, there is in our view no real need to resort to small clocks and mirrors. Even if the length to be measured is small, the clock-mirror system can ultimately be taken to have the size of the earth.

Conclusion 4.5 With clock and mirror being parts of some large devices which, on their side, being rigidly attached to e.g. the earth itself, both the Schwarzschild-constraint and the wandering-away effect can be essentially avoided so that, in the end, we get at least thought-experimentally an estimate of the kind

$$\delta l \gtrsim const \cdot l_p \tag{68}$$

Even if in practice the term const may not really be of order one but some small power of ten, the result is certainly independent of the length l itself.

4.3 The Statistical Mechanics of Relative Position Fluctuations of the Components of Large Measuring Devices

What we and also the other authors have so far only superficially discussed are the relative position fluctuations of parts of a larger device relative to each other or relative to the larger device they are embedded in. If, for example, the mirror is part of a larger device or is used as a component in a clock system, the uncertainty of distance mesurement is of course enhanced by the unavoidable statistical movements of these parts relative to each other. To formulate this problem in a more general way we will analyse in the following the relative statistical movement of parts of a larger many-body system, more precisely, of the respective centers of mass or of the movement of the center of mass of a subsystem with respect to the total system.

One should note that there exist in the literature various quite heuristic statements concerning this point which are however not satisfying in our context as they usually only apply to freely moving objects. In our context the subsystem is in contact with a larger system and, furthermore, there exist delicate and even long-range correlations among the constituents of the object. Under such conditions the problem is no longer totally trivial.

In section 3 we got already estimates on the individual fluctuations of the atoms of a crystallic body. At zero temperature and ordinary densities and velocities of sound we had roughly

$$\Delta u_i \approx 10^{-11} [m] \tag{69}$$

while for the extreme values, $c_s = c$ $a = a_0$, we got

$$\Delta u_i \approx 10^{-16} [m] \tag{70}$$

In [16] we already introduced the idea to attach e.g. clock and/or mirror to larger parts of the under-structure in order to further reduce the degree of position fluctuations, as in general the center of mass of a subsystem, containing itself a substantial number of atoms, is expected to display a smaller degree of fluctuations than its individual constituent atoms. The quantitative analysis will however depend on the general context.

Let us start with our standard example, the harmonic crystal. We will see that in this case the problem turns out to be quite intricate. We take a subcluster, S, of, say, N atoms in the crystal with $N \gg 1$. The corresponding center of mass coordinate is

$$\mathbf{R} = \sum_{i=1}^{N} M_0 \, \mathbf{x}_i / N M_0 \tag{71}$$

The expected fluctuation of this coordinate can then be written as

$$\langle (\mathbf{R} - \mathbf{R}_0)^2 \rangle = N^{-2} \left\langle \left(\sum_{i=1}^N (\mathbf{x}_i - \mathbf{x}_{i,0}) \right)^2 \right\rangle$$
(72)

with $\mathbf{x}_{i,0}$ the equilibrium positions of the atoms and \mathbf{R}_0 the corresponding

position of the center of mass. This yields

$$\langle (\mathbf{R} - \mathbf{R}_0)^2 \rangle = N^{-1} \langle N^{-1} (\sum_{i=1}^N \mathbf{u}_i)^2 \rangle = N^{-1} (N^{-1} \sum_{i=1}^N \mathbf{u}_i^2 + 2N^{-1} \sum_{i\neq j=1}^N \mathbf{u}_i \cdot \mathbf{u}_j) \quad (73)$$

As

$$N^{-1}\left(\sum_{i=1}^{N} \mathbf{u}_{i}\right)^{2} = \Delta u_{i}^{2} \approx const\left(\hbar a/c_{s} M_{0}\right)$$
(74)

(see section 3), the first term would essentially yield the result which also follows from general handwaving arguments, i.e.

$$\Delta \mathbf{R} \approx N^{-1/2} \,\Delta \mathbf{u}_i \tag{75}$$

Problematical is however the second sum, as we know that the \mathbf{u}_i happen to be long-range correlated in a crystal with the correlations in 3-dim. only decaying in leading order proportional to $|\mathbf{x}_i - \mathbf{x}_j|^{-1}$. On the other hand, we know that there is a tendency of an oscillating behavior of correlations, that is, to some extent the individual terms may compensate each other. But it is very difficult to estimate this in a rigorous way.

The whole section 4 of [16] was devoted to this point in connection with the question of (anti)correlations in the geometric fluctuations of spacetime on the Planck scale. We furthermore mentioned in that section some older literature where such questions have been systematically treated in the framework of statistical mechanics and quantum field theory. The general problem consists in estimating the behavior of autocorrelations of certain space-integrals (or sum) over some physically relevant (operator)density, q(x), i.e.

$$Q_V := \int_V q(x) d^3x \quad \text{or} \quad \sum_V q(x_i) \tag{76}$$

In many cases one is interested in the behavior of $\langle Q_V \cdot Q_V \rangle$ when V becomes large or approaches the whole space, \mathbb{R}^3 . Obviously the correlation function

$$\langle q(x) q(y) \rangle$$
 or rather $\langle (q(x) - \langle q \rangle) (q(y) - \langle q \rangle) \rangle$ (77)

enters in this expression with $\langle q \rangle$, the expectation of q(x), being subtracted.

If the individual fluctuations are uncorrelated or only weakly correlated (an integrable decay of correlations is sufficient) we get a behavior

$$\langle Q_V \cdot Q_V \rangle \sim R^3$$
 (78)

in 3-dim. with R the diameter of the integration volume. The situation frequently becomes better if certain covariance properties are present (for example, q(x) being the zero-component of a conserved current); see section 4 of [16]. On the other hand, if the correlations are of long-range character, the situation becomes more complicated. Such a problem was for example analysed in [28] section 3 or [29] section 5 in the field of the statistical mechanics of phase transitions.

In that case we got roughly a result that

$$\langle Q_V/V \cdot Q_V/V \rangle \lesssim R^{-1}$$
 (79)

for correlations decaying weakly like

$$\langle q(x) q(y) \rangle \sim |x - y|^{-1} \tag{80}$$

in 3-dim. and with q(x) normalized to $\langle q(x) \rangle = 0$. In the above fluctuation result possible anticorrelation effects (i.e. oscillations) are not included, only the decay property has been used. That is, it is possible that the situation is actually better but we do not know for sure.

Replacing now q(x) by our \mathbf{u}_i and the integral by the corresponding sum, this result can be taken over for the calculation of the fluctuation of our center of mass variable, i.e. we have

Conclusion 4.6 As the atomic position fluctuations in our crystal are longrange correlated, we get the rigorous bound

$$\langle (\mathbf{R} - \mathbf{R}_0)^2 \rangle^{1/2} \lesssim N^{-1/6} \cdot \Delta u_i$$
 (81)

with **R** denoting the center of mass of some macroscopic part of the whole crystal and $R \sim N^{1/3}$. It is however possible that the estimate is better if (anti)correlations do effectively cooperate.

We learned however from our analysis in section 4 of [16] that one can reduce the noise of the position fluctuations in solids, that is, by the same token, in our measuring devices, if one uses substances with short-range position fluctuations. Macroscopically these short-range correlations work as damping mechanisms. So our idea is to embed e.g. clocks and mirrors in components of the total measuring system which display short-range correlations. These may be for example viscous fluids or some dissordered systems. For such systems we can use the results in section 4 of [16]. **Conclusion 4.7** For systems with e.g. integrable correlations we get the much better result for the center of mass motion

$$\langle (\mathbf{R} - \mathbf{R}_0)^2 \rangle^{1/2} \sim N^{-1/2} \cdot \Delta u_i$$
(82)

with Δu_i being of atomic order. This implies that for sufficiently large N the random movement of mirrors and clocks become very small compared to typical atomic values and may come near the Planck scale under ideal conditions.

5 Commentary

To sum up what we have finally attained; we have shown that by using compound systems as measuring devices, in which critical parts like clocks and mirrors are embedded in components which effectively damp the unavoidable random fluctuations of atomic positions via short-range correlations, we can reach, at least in principle, a level of precision regarding distance measurements, which may come near the Planck level. In any case, we think, we have convincingly shown that experimentally there is no indication that the precision of distance measurements displays a functional dependence on the distance to be measured, as has been claimed by e.g. Amelino-Camelia and Ng-van Dam.

These authors attributed this dependence to some Brownian-motion like behavior of the geometric fluctuations in the micro-structure of space-time. This is certainly a very interesting topic, but we will show in a forthcoming paper (and have already argued in this direction in previous work, cited above) that due to strong anticorrelations these microscopic fluctuations have rather the tendency to compensate each other, so that in the end we get a result which corroborates our above analysis.

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