

# Stability of Dissipative Optical Solitons in the 2D Complex Swift-Hohenberg Equation

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**Abstract** This article deals with stationary localized solutions of the (2D) two-dimensional complex Swift-Hohenberg equation (CSHE). Our approach is based on the semi-analytical method of collective coordinate approach. According to the parameters of the equation and a suitable choice of ansatz, the stationary dissipative solitons of the 2D CSHE equation are mapped. This approach allows to describe the influence of the parameters of the equation on the various physical parameters of the pulse and their dynamics. Finally, the major impact of spectral filtering terms on the dynamic of the solitons is demonstrated.

**Keywords:** *dissipative soliton, spatio-temporal, collective coordinate approach, Ginzburg-Landau equation, complex Swift-Hohenberg equation, spectral filtering*

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## 1. Introduction

Since several years, the stability domain of the spatio-temporal dissipative soliton self-confined in the temporal and spatial dimensions remains a serious issue of nonlinear optics [1]. The choice of relevant nonlinearity for such self-confinement and ensure a stable solution are thorny. Furthermore, the relation between the dimensionality and nonlinear effect used is especially essential.

It should be specified that dissipative systems in nonlinear optics admit stable solitons in one, two, and three dimensions [2]. These solitons can also be purely temporal, spatial or spatio-temporal. Dissipative soliton has been widely studied in nonlinear dissipative optics, from fundamental point of view and due to the clear physical meaning in particular application. Important applications are passively mode-locked laser systems and optical transmission lines.

Indeed, the formation of this dissipative structure is much more complex than that of conservative systems, because in addition to the right balance between the dispersion and the nonlinearity, dissipative solitons exchange energy and (or) matter with an external source. They exist only when there is a continuous energy supply to the system. Whenever the energy supply is stopped, soliton "stops living." Their shape, amplitude, velocity are all fixed and defined by the parameters of the system [3] rather than by the initial condition.

The properties and conditions of their existence have been studied extensively [4,5]. The theoretical study of these soliton has recently received a boost during the past

decade leading to an impressive number of works in several fields of nonlinear science [4,6,7,8]. However most of these stable soliton solutions studies use purely numerical approaches [9,10]. Solving numerically the equation for a given set of parameters and a given initial condition is an extremely lengthy and costly procedure, which can take up to several days in a standard PC.

Alternative variational semi-analytical methods can overcome this difficulty [11]. These theoretical tools can perceive soliton solutions more efficiently and envisage their domains of existence with relative flexibility [12,13,14].

Recently, it has been demonstrated in our previous works that the collective variable approach [15] is a useful tool and reduces significantly the computation time for predicting approximately the domains of existence of stable light bullets in the parameter space of the (3D) complex cubic-quintic Ginzburg-Landau equation [13].

This present work provides evidence for the stationary solutions of the (2D) complex Swift-Hohenberg equation, which, to our knowledge has not been enough reported in the literature. This equation is useful in the description of the dynamics of dissipative solitons in laser cavity in experimental situations.

## 2. Materials and Methods

### 2.1. Theory of dissipative Soliton in Swift-Hohenberg System

One of the main properties of wave is that they tend to spread out as they evolve. The principal cause for this is that distinct frequency; that are superposed to create the wave packet, propagate with different velocities and (or)

in different directions. It knows that generally the nonlinear effects accelerate the spreading of the wave. Nevertheless, under certain conditions, nonlinearity may compensate the linear effects. The resulting balanced localized pulse or beam of light, which propagates without decay, is generally known as soliton. Therefore, optical solitons are localized waves that propagate stably in nonlinear media with dispersion, diffraction or both.

Originally, the terminology soliton was reserved for conservative systems and particular set of integrable solutions existing as a result of the delicate balance between dispersion (diffraction) and nonlinearity.

However, similar classes of stable self-sustained structures can be found for a wide range of physical systems far from equilibrium. The term dissipative system has been used by Nicolis and Prigogine [16] to describe these systems far from equilibrium. As a new paradigm of nonlinear waves, solitons in real dissipative environments are known as dissipative soliton [4]. The dissipative soliton has different characteristics than those of conservative systems. They are attracting a significant surge of research activities on their spatial (temporal) complexity during the last years, particularly for systems modelled by the cubic quintic complex Ginzburg-Landau equations [4,5]. Apart from the balance between dispersion (diffraction) and nonlinearity, the separate balance between gain and loss is essential for the pattern formation of the dissipative soliton. This second balance is very crucial, and gives the dissipative soliton a markedly different dynamic from that of conservative solitons. The shape, amplitude and widths of the dissipative soliton are fixed, depend drastically on the system parameters [17] and may evolve stationary, periodically, or even chaotically [10,18,19]. One of the generic equations describing the dynamics of dissipative solitons and that we have intensely studied is the complex Cubic-quintic Ginzburg-Landau equation model [4,20,21]. This equation could be applied to the modeling of a wide-aperture laser cavity with a saturable absorber in the short pulse regime of operation. The model includes the effects of two-dimensional transverse diffraction of the beam, longitudinal dispersion of the pulse, and its evolution along the cavity. The spectral filter of this model is restricted to the term of second order and can only describe a spectral response with a single maximum.

However, experiences indicate that the gain spectrum is usually wide and can have multiple peaks.

To be more realist we need to add others terms of higher-order spectral filtering to the complex Cubic-quintic Ginzburg-Landau equation (CGLE), this lead to the complex Swift-Hohenberg equation (CSHE).

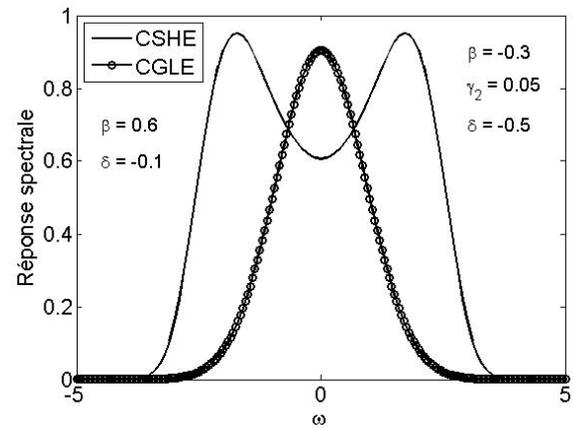
The complex Swift-Hohenberg equation is useful to describe soliton propagation in optical systems with linear and nonlinear gain and spectral filtering such as communication links with lumped fast saturable absorbers or fiber lasers with additive-pulse mode-locking or nonlinear polarization rotation. It is clear that the higher order of the spectral filter is extremely essential to analyze the generation of more complex impulse.

According to this equation, we will investigate the steady state of the 2D stationary solutions. It describes as well quantitatively as qualitatively many nonlinear effects which occur during the propagation and can be read in this normalized form [4,22]:

$$\begin{aligned} \psi_z - i\frac{D}{2}\psi_{tt} - i\frac{1}{2}\psi_{rr} - i\gamma|\psi|^2\psi - i\nu|\psi|^4\psi \\ = \delta\psi + \varepsilon|\psi|^2\psi + \beta\psi_{tt} + \mu|\psi|^4\psi + \gamma_2\psi_{ttt} \end{aligned} \quad (1)$$

where  $\mu, \delta, \beta, D, \nu, \gamma, \gamma_2$  and  $\varepsilon$  are real constants. Without the additive term  $\gamma_2\psi_{ttt}$  this equation is the same as the CGLE one. The physical meaning of each term depends on the real problem which must be examined. In optics, this equation describes the laser systems [23,24], optical regeneration for optical fibre transmission systems with soliton signals [25], nonlinear cavities with an external pump [26] and the parametric Oscillator [27]. When applied to the propagation of the pulses in a laser system, as is the case in our study  $\psi = \psi(r, t, z)$  represents the normalized optical envelope and is function of three real variables. With  $t$  the retarded time in the frame moving with the pulse,  $z$  is the propagation distance or the cavity round-trip number. And finally  $r (r = \sqrt{x^2 + y^2})$  represents the transverse coordinate, taking account of the spatial diffraction effects.

The left-hand side contains the conservative terms: namely,  $D = +1(-1)$  which is for the anomalous (normal) dispersion propagation regime and  $\nu$  which represents, if negative, the saturation coefficient of the Kerr nonlinearity. In the following, the dispersion is anomalous, and  $\nu$  is kept relatively small. The right-hand-side of equation (1) includes all dissipative terms:  $\delta, \varepsilon, \beta$  and  $\mu$  are the coefficients for linear loss (if negative), nonlinear gain (if positive), spectral filtering (if positive) and saturation of the nonlinear gain (if negative), respectively. And finally  $\gamma_2$  represents the higher-order spectral filter term, which is very important in this present study.



**Figure 1.** Evolution of the spectral response in the case of the complex Cubic-quintic Ginzburg-Landau equation (in circle) and of the complex Swift-Hohenberg equation (in solid curve). The other parameters appear inside the figure

In order to have stable pulses in the frequency domain,  $\gamma_2$  must be positive and  $\beta$  can have both sign (positive or negative) contrary to the case of the CGLE equation where  $\beta$  must be strictly positive. If  $\beta$  is greater than zero, we obtain a spectral response with a single maximum, on the other hand ( $\beta$  less than zero) we have now two maxima. To support this comments, the effect of the spectral filter is shown in Figure 1. It is described by the following transfer function:

$$T(\omega) = \exp(\delta - \beta\omega^2 - \gamma_2\omega^4). \quad (2)$$

The circle curve shows the spectral response in the case of the complex Cubic-quintic Ginzburg-Landau equation CGLE (with  $\gamma_2 = 0$ ), it is a Gaussian curve with amplitude  $\delta$  and width  $\beta$ . This curve has a single maximum. In the case of the complex Swift-Hohenberg equation (CSHE), the response of spectral filter is much affected and depends on three parameters with  $\gamma_2$  positive. This situation gives a spectral response with two distinct maximums. In this work we will carefully investigate this situation where  $\gamma_2$  is nonzero.

The parameter values are chosen according to that the width and height of the two spectral responses are not too different. However, as the spectral response is different, it goes without saying that the (2D) two-dimensional dissipative soliton in both case are different profiles for the same value of the cubic gain  $\varepsilon$ .

## 2.2. Stability Study

Our study, really, is to examine the stationary soliton of the 2D complex Swift-Hohenberg equation by semi-analytical approach, as far as we knew this has not been done previously. More specifically, our major goal is to provide an approximate mapping of the regions of existence of stable and unstable solutions in the parameter space of the equation (1), as we have done previously with the 3D complex Cubic-quintic Ginzburg-Landau equation one [13].

To achieve this goal, we use the collective variable approach, which helps to simplify the characterization of the pulse by use of a low dimensional equivalent mechanical system based on a finite number of degrees of freedom. Each degree of freedom can then be described by means of a coordinate called the *collective variable*. Indeed, the propagation of the pulse describes not only the pulse as a collective entity (localized in time and space) but also all other localized or non-localized excitations, such as noise or radiation, which are always more or less present in the real system. For a better understanding of these dynamic processes, it is important to develop analytical approaches to help to bring the dynamics of the pulse to that of a simple mechanical system with only a small number of degree of freedom.

The mean idea in the collective variable approach is to associate collective variables with the pulse's parameters of interest for which equations of motion may be derived. One may introduce  $N$  collective variables,  $z$  dependent; say  $X_i$  with  $i = 1, 2, \dots, N$ , in a way such that each of them can correctly describe a fundamental parameter of the pulse (amplitude, width, chirp ...) [15,28]. To this end, one can decompose the field  $\psi(x, y, t, z)$  in the following way:

$$\psi(r, t, z) = f(x_1, x_2, \dots, x_n, t) + q(z, t) \quad (3)$$

where  $f$  the ansatz function is a function of the collective variables and is chosen to draw, at best, the configuration of the pulse. And  $q(z, t)$  is a residual field that represents all other excitations in the system (noise, radiation, dressing field, etc.) [15]. The choice of the trial function that introduces the collective variables in the theory is important for the success of the technique. After choosing the ansatz function one can pursue the process of characterization of the pulse by neglecting the residual field. This approximation is called *the bare approximation* [15]. In this way one can consider the fact that the pulse propagation can be completely characterized that the ansatz function.

By neglecting the residual field ( $q = 0$ ) *the bare approximation*, as is the case in most practical studies [10], we chose a Gaussian function as ansatz function that is given by the following:

$$f = A \exp \left( -\frac{t^2}{w_t^2} - \frac{r^2}{w_r^2} + \frac{i}{2} c_t t^2 + \frac{i}{2} c_r r^2 + ip \right). \quad (4)$$

So, here we are assuming that all the pulses are purely Gaussian with spatial and temporal chirp and do not consider other forms of pulses.

In such case, the field is necessarily  $\psi(r, t, z) = f$ , with  $t, r$  the spatial and temporal variables along  $t$  and  $r$  axis respectively.  $A, w_t, w_r, c_t, c_r$ , and  $p$  represents the collective variables.  $A$  stands for soliton amplitude,  $\sqrt{2 \ln 2} w_t$  and  $\sqrt{2 \ln 2} w_r$  represent the temporal and spatial widths respectively.  $c_t/(2\pi)$  is the parameter of the chirp along  $t$  axis,  $c_r/(2\pi)$  the parameter of the spatial chirp and  $p$  is the global phase that evolves along with propagation. When a stationary regime is reached, the phase becomes a linear function of the propagation distance  $z$ .

After this choice of the ansatz function, variational analysis could be carried out by neglecting the residual field (*the bare approximation*). Applying the bare approximation to the 2D CSHE, consists in substituting the field  $\psi$  by the given trial function  $f$  ( $\psi = f$ ) and projecting the resulting equations in the following direction:

$$\frac{\partial f^*}{\partial X} (X = A, w_t, w_r, c_t, c_r, p).$$

A jet of six differential equations which govern the evolution of the optical pulse parameters propagating in space and time is obtained:

$$\begin{aligned} \dot{A} &= A\delta + \frac{3}{4} A^3 \varepsilon - \frac{2}{w_t^2} A\beta + \frac{5}{9} A^5 \mu - A c_t D \\ &\quad - 2A c_r + 3 \left( 2c_t^2 - w_t^2 c_t^4 + \frac{3}{w_t^4} \right) A \gamma_2, \\ \dot{w}_t &= 2w_t c_t D - \frac{1}{4} w_t A^2 \varepsilon - \frac{2}{9} A^4 w_t \mu \\ &\quad + \left( 1 - w_t^4 c_t^2 \right) \frac{2\beta}{w_t} + \left( w_t^8 c_t^4 - 1 \right) \frac{12}{w_t^3} \gamma_2, \\ \dot{w}_r &= 4w_r c_r - \frac{1}{4} w_r A^2 \varepsilon - \frac{2}{9} A^4 w_r \mu, \\ \dot{c}_t &= 2 \left( \frac{1}{w_t^4} - c_t^2 \right) D - \frac{8}{w_t^2} c_t \beta - \frac{1}{2w_t^2} A^2 \gamma \\ &\quad - \frac{4}{9w_t^2} A^4 \nu + 48c_t \left( \frac{1}{w_t^4} + c_t^2 \right) \gamma_2, \\ \dot{c}_r &= -4c_r^2 - \frac{1}{2w_r^2} A^2 \gamma + \frac{4}{w_r^4} - \frac{4}{9w_r^2} A^4 \nu, \\ \dot{p} &= 2\beta c_t + \frac{3}{4} A^2 \gamma - \frac{D}{w_t^2} - \frac{2}{w_r^2} + \frac{5}{9} A^4 \nu \\ &\quad - 12c_t \left( \frac{1}{w_t^2} - 2 - w_t^2 c_t^2 \right) \gamma_2. \end{aligned} \quad (5)$$

It is important to point out that these equations give no explicit information with regard to the different solutions of the equation CSHE (1) and their stability. Thereby, they give us the first idea on the dynamic of the light pulse. They simply reveal in detail the influence of each equation CSHE (1) parameters on the various physical parameters of the soliton.

Thus, one's can clearly see that spectral filter coefficients ( $\beta$  and  $\gamma_2$ ) affect the amplitude of the pulse, its temporal width, temporal chirp and the global phase. However, these parameters have no formal effect on the spatial variables. The temporal ( $w_t$ ) and spatial widths ( $w_r$ ) also depend on the nonlinear gain ( $\epsilon$ ) and its saturation ( $\mu$ ). As expected, the terms of spectral filtering coefficients ( $\beta$  and  $\gamma_2$ ) and dispersion term ( $D$ ) affect the temporal width and have no action on the radial component.

Similarly, the spatial  $c_r$  and temporal  $c_t$  chirp parameters are influenced in the same way by the Kerr term saturation of the optical nonlinearity ( $\nu$ ), but the temporal term is also affected by the terms of spectral filtering coefficients ( $\beta$  and  $\gamma_2$ ) and dispersion term ( $D$ ). Finally, not any parameters of the soliton are influenced by ( $p$ ), the global phase, but are governed by the second order spectral filter term ( $\gamma_2$ ).

In this way, the equation of propagation of the optical wave is transformed into a system of differential equations, describing the evolution of the physical parameters of the pulse (amplitude, width...) during the propagation.

This approach provides the basic parameters of the fixed points, and a mapping of different types of solutions, thereby reducing by several orders of magnitude the volume of calculation required usually.

The fixed points (FPs) of the system are found by imposing the left-hand side of equation (5) to be zero ( $\dot{X} = 0$  with  $X = A, w_t, w_r, c_t, c_r, p$ ). The threshold of existence of FPs can be estimated by the relation  $\epsilon_s \approx 2\sqrt{\delta\mu}$ . If  $\epsilon > \epsilon_s$ , we have in general both stable and unstable fixed points.

The stability of FPs is determined by the analysis of the eigenvalues  $\lambda_j$  ( $j = A, w_t, w_r, c_t, c_r, p$ ) of the matrix  $M_{ij} = \partial \dot{x}_i / \partial x_j$ .

The stability criterion is as follows: if the real part of at least one of the eigenvalues is positive, the corresponding FP is unstable. Hence, to have stable FP, the real parts of all the eigenvalues of the matrix  $M_{ij}$  must be negative.

The stable fixed points correspond to stationary solutions of the 2D complex Swift-Hohenberg equation (1).

In addition, the fundamental parameter which helps to control the state of the solution and study its stability is the total energy  $Q$  given by the following equation:

$$Q(z) = \int_{-\infty}^{+\infty} \int_0^{+\infty} 2\pi r |\psi(r, t, z)|^2 dr dt. \quad (6)$$

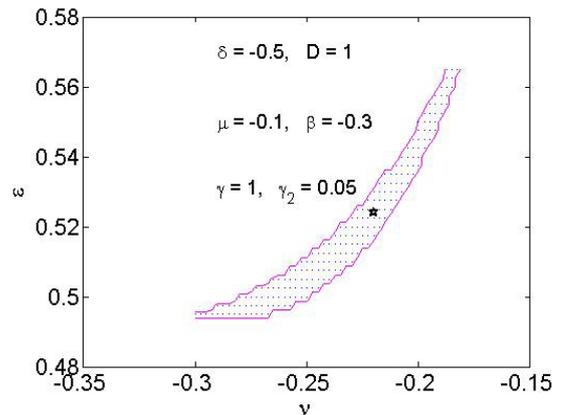
For the dissipative system, the total energy gives us the main information about the soliton dynamics. It's not conserved but evolves in accordance with the so-called balance equation. When a stationary solution is reached, the total energy converges to a constant value. However, the soliton, is a pulsating one, the total energy is an oscillating function of  $z$ . And finally, when we have unstable solutions, energy tends to infinity.

## 2.3. (2D) CSHE Stationary Dissipative Soliton

The stationary solutions of the 2D complex Swift-Hohenberg equation exist in finite regions in the parameter space. Nevertheless, it would be extremely difficult to map in extensor these regions in the all dimensional parameter in which we operate. For this reason, we restrict ourselves to fix all the parameters except for two which vary. By setting key parameters and by individually varying the nonlinear gain ( $\epsilon$ ) and the Kerr term saturation of the optical nonlinearity ( $\nu$ ), we have revealed in the ( $\nu, \epsilon$ ) plane, the stationary solution of the 2D complex Swift-Hohenberg equation. By investigating the parameter regions situated in the neighbourhood of the parameters  $D = \gamma = 1, \beta = -0.3, \delta = -0.5, \mu = -0.1$  and  $\gamma_2 = 0.05$ , with an initial pulse

$$\psi(r, t, 0) = 2.86 \exp\left(-\frac{t^2}{0.7} - \frac{r^2}{1.36}\right) \quad (7)$$

and by individually varying  $\nu$  from -0.3 to -0.16 and  $\epsilon$  from 0.49 to 0.57, so for each value pair ( $\nu, \epsilon$ ) the Newton-Raphson allows to look for the fixed point and we study its stability. Thus one's can easily realize the cartography of the solution of the equation (1). Figure 1 shows the mapping of the solutions for the range of selected values. The dotted lines correspond to the fixed points which represent the stationary solution of the 2D complex Swift-Hohenberg equation. It can be noticed that this stability area is very narrow with low value of the nonlinear gain ( $\epsilon$ ), therefore highly sensitive. That implies that a small change  $\epsilon$  of can have a real impact on the system. Thus, all the solitons parameters (amplitude, width, chirp...) stay stationary throughout propagation. The total energy of the system also has the same dynamic which does not change at all, during propagation for these values considered. Besides the stationary domain, we have instable fixed points which can be divided in two categories: the limit-cycle attractor and the instable solutions. Here, our main interest is to study the dynamic of the pulse in the dotted line domain.



**Figure 2.** cartography of the solutions of the 2D complex Swift-Hohenberg equation in the ( $\nu, \epsilon$ ) plane. The stable fixed points regions in dotted lines represents the domain of stationary solitons of the equation. Other CSHE parameters appear inside the figure

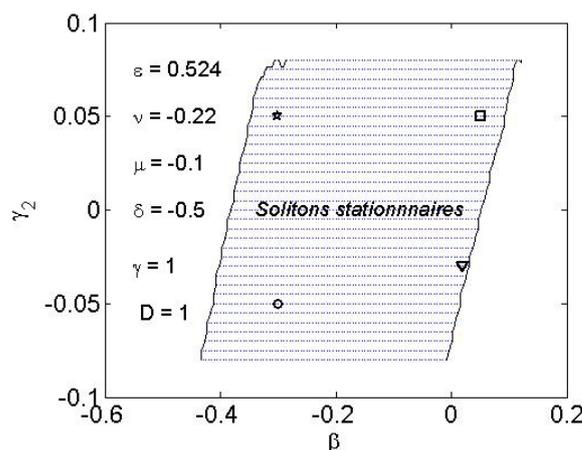
This first analysis shows that the 2D complex Swift-Hohenberg equation stationary solutions exist in the space of selected parameters. In this way, it is more revealing to map the area of stability in space of the spectral filters

namely in the  $(\beta, \gamma_2)$  plane. For this, the star point (see Figure 2) in the stability domain of the Figure 2 corresponding to  $\nu = -0.22$  and  $\varepsilon = 0.524$  is considered. From this point, the stable solutions in  $(\beta, \gamma_2)$  plane for the selected values  $\nu, \delta, \gamma, D, \mu$  and  $\varepsilon$  are mapped.

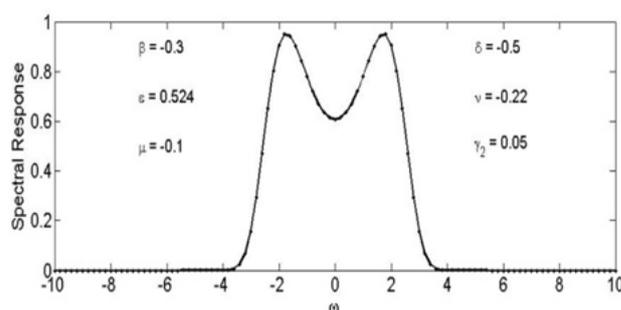
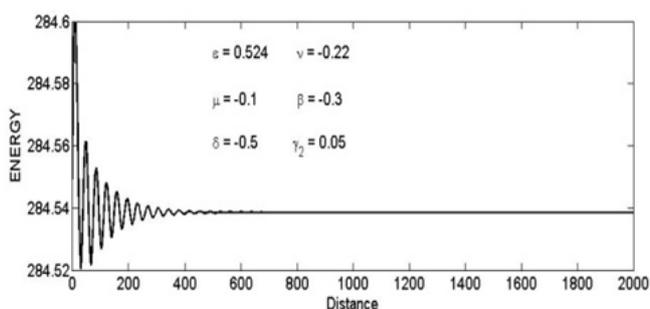
The Figure 3 summarizes this result. The dotted lines represent the stationary solutions. As will be seen from this cartography, the stationary solutions of the 2D complex Swift-Hohenberg equation exist for both positive and negative values of  $\gamma_2$ ; the same holds true for  $\beta$ . This stability domain is wide with sensitive value of  $\gamma_2$ .

This first interesting results illustrate that the stationary dissipative solutions of the 2D complex Swift-Hohenberg equation can be found for any signs of the parameters  $\beta$  and  $\gamma_2$ . Based on those results, the dynamics of the dissipative soliton depending on the signs of spectral filter and its high-order term (same signs or opposite signs) are carefully examined.

Accordingly, four points marked with a star, a circle, a square and a triangle are chosen (Figure 3).



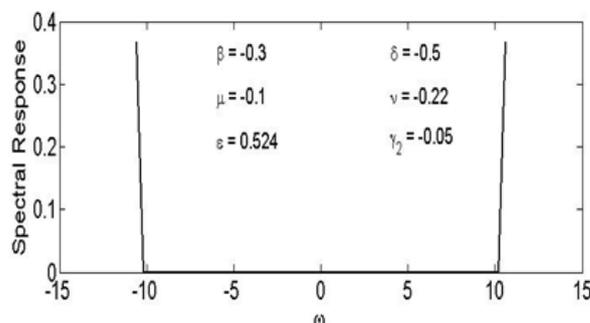
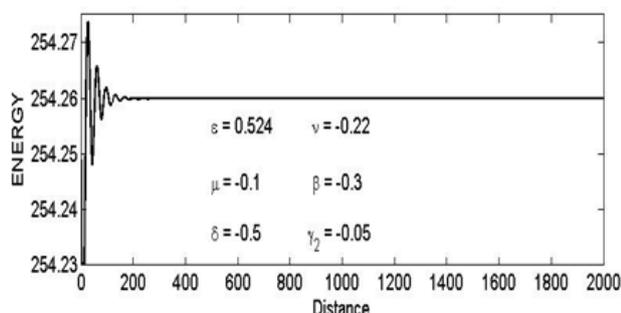
**Figure 3.** cartography of the solutions of the 2D complex Swift-Hohenberg equation in the  $(\beta, \gamma_2)$  plane. The stable fixed points regions in dotted lines represents the domain of stationary solitons of the equation. Other CSHE parameters appear inside the figure



**Figure 4.** (up) evolution of the total pulse energy of the stationary dissipative soliton, and (below) the spectral filter response in case  $\beta < 0$  and  $\gamma_2 > 0$

The star in the figure corresponds to the case where these parameters are of opposite signs [ $\beta = -0.3$  (negative) and  $\gamma_2 = 0.05$  (positive)]. The characteristics of such a pulse are represented by the Figure 4. The transfer function of the spectral filtering of that stationary soliton with  $\beta$  positive and  $\gamma_2$  negative has two maxima. The total energy of that soliton after a short oscillation remains constant over long distances. This dynamic characterizes a stationary solution.

When the spectral filtering and its high-order term are of the same negative signs [ $\beta = -0.3$  (negative) and  $\gamma_2 = -0.05$  (positive)] represented by a circle in Figure 3, the transfer function also has two maxima at its ends but with no central pulse. So these solutions do not have the same profile and features as in the situation described previously. The solution is still stationary as shown by the evolution of the total energy, but has less energy than the previous one due to the value of  $\gamma_2$ . This situation is well summarized in Figure 5.

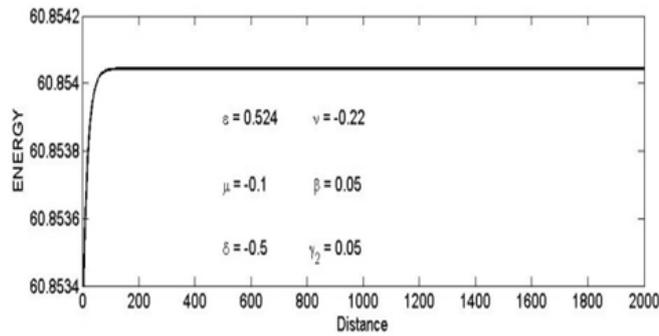


**Figure 5.** (up) evolution of the total pulse energy of the stationary dissipative soliton, and (below) the spectral filter response in case  $\beta < 0$  and  $\gamma_2 < 0$

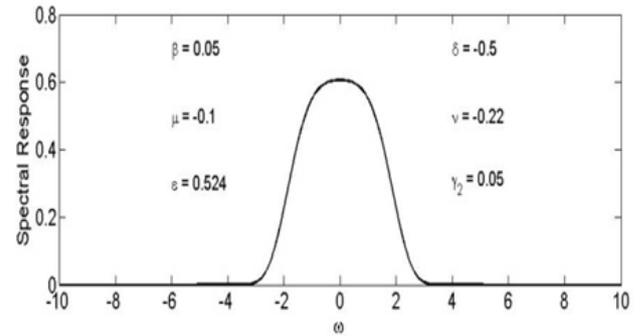
In case where the spectral filtering and its high-order term are of the same positive signs [ $\beta = 0.05$  (positive) and  $\gamma_2 = 0.05$  (positive)] represented by a square in Figure 3 the transfer function has only a single maximum. The soliton dynamic is not changed; it is still stationary but has energy much lower than that of the first two cases treated. The Figure 6 draws these behaviours.

The last case studied corresponds to the scenario where the spectral filtering and its high-order term are of opposite signs [ $\beta = 0.02$  (positive) and  $\gamma_2 = -0.03$  (negative)] represented by a triangle in Figure 3. Here, the transfer function of the spectral filtering of the pulse shown in Figure 7 has the same behaviour as that of the

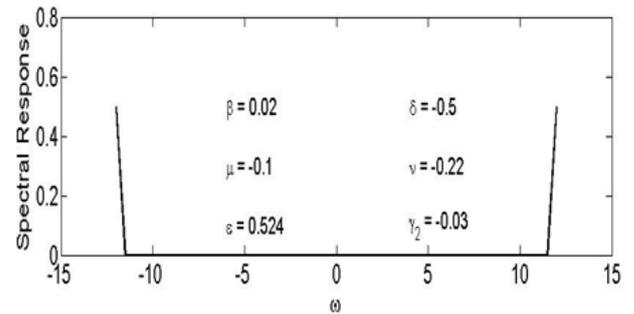
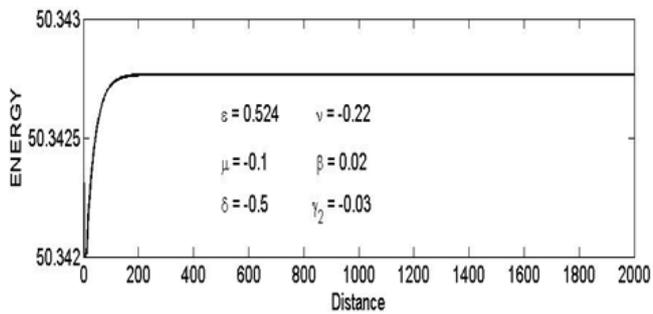
circle. The optical soliton has the same dynamic as the



previous case, but with even lower energy.



**Figure 6.** (up) evolution of the total pulse energy of the stationary dissipative soliton, and (below) the spectral filter response in case  $\beta > 0$  and  $\gamma_2 > 0$



**Figure 7.** (up) evolution of the total pulse energy of the stationary dissipative soliton, and (below) the spectral filter response in case  $\beta > 0$  and  $\gamma_2 < 0$

It appears from the different scenarios studied that the spectral filtering and its high-order term parameters ( $\beta$  and  $\gamma_2$ ) have a great influence on the dynamics of the spectral response. It is certainly true that if stationary solutions regardless the signs of  $\beta$  and  $\gamma_2$  are obtained, they have a real impact on the spectral response of the pulse.

So when the high-order term is greater than zero ( $\gamma_2 > 0$ ) and no matter the sign of the spectral filtering, the spectral response has one or two distinct maxima.

However, in the scenario where the high-order term is less than zero ( $\gamma_2 < 0$ ) the spectral response has the same behavior with zero central pulse for any signs of  $\beta$ .

These results clearly show that in the cases of experimental; the choices of the spectral filtering and its high-order term ( $\beta$  and  $\gamma_2$ ) are very crucial according to the shape of the spectrum.

### 3. Conclusion

At the end of our study, we have demonstrated, for the first time to our knowledge, the stationary dissipative solutions of the 2D complex Swift-Hohenberg equation with our collective variable approach. Particularly, the regions of existence of stationary dissipative soliton in the  $(\nu, \varepsilon)$  and  $(\beta, \gamma_2)$  planes are shown. It has been also shown that the validity of these studies is based on a careful selection of the ansatz function.

Our results reveal the essential character of the spectral filtering and its high-order term parameters ( $\beta$  and  $\gamma_2$ ). Our study shows that it is possible to observe the stationary dissipative solutions of the 2D complex Swift-Hohenberg equation, whatever the signs of  $\beta$  and  $\gamma_2$ . It has been also shown that these parameters have a real impact on the spectral response.

The collective variable approach is very efficient to obtain stable stationary solutions when a suitable trial function is chosen. This technique is incomparably quicker than direct numerical computations.

This work can be extensive and we are confident that these applications will be numerous in the fields of physics, chemistry and biology as described by the complex Swift-Hohenberg equation.

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