

## Cuckoo Search Algorithm Using Different Distributions for Short-Term Hydrothermal Scheduling with Reservoir Volume Constraint

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**Abstract:** This paper proposes a cuckoo search algorithm (CSA) using different distributions for solving the short-term hydrothermal scheduling (ST-HTS) problem with reservoir storage constraint on hydropower plants. The CSA method is a new meta-heuristic algorithm inspired from the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds of other species for solving optimization problems. The advantages of the CSA method are few control parameters and effective for optimization problems with complicated constraints. In the proposed CSA, three distributions have been used including Lévy distribution, Gaussian distribution and Cauchy distribution. The proposed method has been tested on two test systems and the obtained results have been compared to that from other methods available in the literature. The result comparisons have indicated that the proposed method is a very favorable method for solving the short term hydrothermal scheduling problems with reservoir constraint.

**Keywords:** Cuckoo search algorithm, short-term hydrothermal scheduling, reservoir volume constraint.

### 1. Introduction

The short term hydro-thermal scheduling (ST-HTS) problem is to determine the power generation among the available thermal and hydro power plants so that the total fuel cost of thermal units is minimized over a schedule time of a single day or a week satisfying power balance equations, total water discharge constraint as the equality constraints and reservoir storage limits and the operation limits of the hydro as well as thermal generators as the inequality constraints [1].

Several methods have been implemented for solving the hydrothermal scheduling problem such as evolutionary programming technique (EP) [1-5], genetic algorithm (GA) [6-7], gradient search techniques (GS) [8], simulated annealing approach (SA) [9], and clonal selection algorithm (CSA) [10]. In [1, 2], ST-HTS problem has been solved by using EP with Gauss mutation. The method can reach a reasonable solution (suboptimal near globally optimal) with reasonable computation time. However, this method is only useful for solving simple problems, which do not contain many constraints [4]. Furthermore, it does not always guarantee the globally optimal solution. The GS method [8] has been applied to the problem as conventionally hydro generation models were represented as piecewise linear or polynomial approximation with a monotonically increasing nature. However, such an approximation may be too rough and seems impractical [2]. SA technique seems to be better than GS via

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comparison of total fuel cost reported in [9]. However, appropriate setting of the relevant control parameters of the SA based algorithm is a difficult task and often the speed of the algorithm is slow when applied to a practical sized power system. It is reported in [11] that EP outperforms GA.

The cuckoo search algorithm (CSA) developed by Yang and Deb in 2009 [12] is a new meta-heuristic algorithm inspired from the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds of other species for solving optimization problems. The advantages of the CSA method are few control parameters and effective for optimization problems with complicated constraints. Recent years, CSA has been applied for solving non-convex economic dispatch (ED) problems [13-14] and micro grid power dispatch problem [14], short-term hydrothermal scheduling problem [15]. In [15], CSA has been applied for solving the short-term hydrothermal scheduling problem where a set of cascaded reservoirs is considered, hydro generation is a function of water discharge and reservoir volume, and the continuity water constraints consider the delay time that water from the upper reservoirs flow into the lower reservoirs. The result comparisons reported in the papers have shown that CSA is an efficient method for solving optimization problems.

In this paper, a cuckoo search algorithm (CSA) with different distributions including Lévy distribution, Gaussian distribution and Cauchy distribution is proposed for solving short-term hydrothermal scheduling problem considering reservoir volume constraint on hydropower plants. On the contrary to study in [[15], water discharge is a function of hydro generation and the delay time is neglected in the continuity water constraint due to the reservoirs located on different rivers in the paper. Therefore, in the implementation of CSA for the considered problem, each egg corresponding to a solution in [15] is represented by thermal plant generations and water discharge meanwhile it is represented by thermal plant generations and reservoir volume in the paper. The effectiveness of the proposed CSA method has been tested on two systems and the obtained results have been compared to those from methods reported in the paper.

## 2. Problem Formulation

In this section, the mathematical formulation of the short-term HTS problem consisting of  $N_1$  thermal units and  $N_2$  hydro units scheduled in  $M$  time sub-intervals with  $t_m$  hours for each is formulated. The objective of the problem is to minimize total cost of thermal units subject to the system and unit constraints.

The mathematical model of the problem is formulated as follows:

$$F = \sum_{m=1}^M \sum_{i=1}^{N_1} t_m \left[ a_{si} + b_{si} P_{si,m} + c_{si} P_{si,m}^2 + \left| d_{si} \times \sin \left( e_{si} \times (P_{si,\min} - P_{si,m}) \right) \right| \right] \quad (1)$$

where  $a_{si}$ ,  $b_{si}$ ,  $c_{si}$ ,  $d_{si}$  and  $e_{si}$  are fuel cost coefficients of thermal plant  $i$ ;  $P_{si,\min}$  is the minimum generation of thermal unit  $i$ .

subject to:

- Load Demand Equality Constraint

The total power generation from thermal and hydro units must satisfy the load demand neglecting power losses in transmission lines:

$$\sum_{i=1}^{N_1} P_{si,m} + \sum_{j=1}^{N_2} P_{hj,m} - P_{D,m} = 0 \quad (2)$$

where  $P_{D,m}$  is total system load demand at subinterval  $m$ .

- The Total Discharge Constraints for Each Duration of  $t_m$

$$Q_{j,m} = t_m q_{j,m} \quad (3)$$

where  $q_{j,m}$  is water discharge rate obtained by

$$q_{j,m} = a_{hj} + b_{hj} P_{hj,m} + c_{hj} P_{hj,m}^2 \quad (4)$$

- The Reservoir Volume Constraints

$$V_{j,m-1} - V_{j,m} + I_{j,m} - Q_{j,m} - S_{j,m} = 0 \quad (5)$$

where  $V_{j,m}$ ,  $I_{j,m}$  and  $S_{j,m}$  are reservoir volume, water inflow and spillage discharge rate of  $j^{th}$  hydropower plant in  $m^{th}$  interval.

- Initial and Final Reservoir Storage

$$V_{j,0} = V_{j,initial}; \quad V_{j,M} = V_{j,End} \quad (6)$$

- Reservoir Storage Limits

$$V_{j,min} \leq V_{j,m} \leq V_{j,max}; \quad j=1,2,...,N_2; m=1,2,...,M \quad (7)$$

where  $V_{j,max}$  and  $V_{j,min}$  are the maximum and minimum reservoir storage of the hydro plant  $j$ , respectively.

- Water discharge rate

$$q_{j,min} \leq q_{j,m} \leq q_{j,max}; \quad j=1,2,...,N_2; m=1,2,...,M \quad (8)$$

where  $q_{j,max}$  and  $q_{j,min}$  are the maximum and minimum water discharge of the hydro plant  $j$ .

- Generator operating limits

$$P_{si,min} \leq P_{si,m} \leq P_{si,max}; \quad i=1,2,...,N_1; m=1,2,...,M \quad (9)$$

$$P_{hj,min} \leq P_{hj,m} \leq P_{hj,max}; \quad j=1,2,...,N_2; m=1,2,...,M \quad (10)$$

where  $P_{si,max}$ ,  $P_{si,min}$  and  $P_{hj,max}$ ,  $P_{hj,min}$  are maximum, minimum power output of thermal plant  $i$  and hydro plant  $j$ , respectively.

### 3. Cuckoo Search Algorithm for ST-HTS Problems

#### A. Cuckoo Search Algorithm

CSA was developed by Yang and Deb in 2009 [12]. During the search process, there are mainly three principle rules as follows.

- Each cuckoo lays one egg (a design solution) at a time and dumps its egg in a randomly chosen nest among the fixed number of available host nests.

- The best nests with high a quality of egg (better solution) will be carried over to the next generation.
- The number of available host nests is fixed, and a host can discover an alien egg with a probability  $p_a \in [0, 1]$ . In this case, the host bird can either throw the egg away or abandon the nest so as to build a completely new nest in a new location.

In the paper, a nest or an egg is an optimal solution consisting of generation of thermal plants and reservoir volume of hydro plants at subinterval  $m$ , namely each egg is called  $X_d$  and  $X_d = [P_{si,m,d} \ V_{j,m,d}]$ . In nature, a host bird builds only one nest and lays its eggs in the nest. The number of eggs that each host bird lays is normally much higher than two ones dependent on species of bird. In Cuckoo behaviour, each Cuckoo lays eggs and dumps one egg into one nest of another bird species. The phenomenon is included in Cuckoo search algorithm where each Cuckoo egg is represented as an optimal solution and each nest, which contains only one cuckoo egg, is also an optimal solution. In fact, each nest at each iteration holds two old eggs and two new eggs, where one is obtained from the Lévy flights and one is from the Alien Egg Discovery and Randomization. Only one egg corresponding to one solution at the time is retained by comparing the fitness function. Consequently, cuckoo egg and cuckoo nest are also an optimal solution.

#### B. Units Calculation of Power Output for Slack Thermal Unit

Suppose that the power output of  $(N_1-1)$  thermal plants and  $N_2$  hydro plants are known. To exactly meet the power balance constraints (2), a slack thermal unit is arbitrarily selected and therefore its power output will be dependent on the power output of remaining  $(N_2+N_1-1)$  hydro and thermal units in the system. The power output of the slack thermal unit 1 is calculated by:

$$P_{s1,m} = P_{D,m} - \sum_{i=2}^{N_1} P_{si,m} - \sum_{j=1}^{N_2} P_{hj,m} \quad (11)$$

#### C. Implementation of Cuckoo Search Algorithm

Based on the three rules in section 3.A, the standard cuckoo search algorithm for solving short-term hydrothermal scheduling problems is as follows:

##### - Initialization

A population of  $N_p$  host nests is represented by  $X = [X_1, X_2, \dots, X_{N_p}]^T$ , in which each  $X_d$  ( $d = 1, \dots, N_p$ ) represents a solution vector of variables given by  $X_d = [P_{si,m,d} \ V_{j,m,d}]$ .

In the CSA methods, each egg can be regarded as a solution which is randomly generated in the initialization. Therefore, each element in nest  $d$  of the population is randomly initialized as follows:

$$P_{si,m,d} = P_{si,\min} + rand_1 * (P_{si,\max} - P_{si,\min}); \quad i = 2, \dots, N_1; \quad m = 1, \dots, M \quad (12)$$

$$V_{j,m,d} = V_{j,\min} + rand_2 * (V_{j,\max} - V_{j,\min}); \quad j = 1, \dots, N_2; \quad m = 1, \dots, M-1 \quad (13)$$

where  $rand_1$  and  $rand_2$  are uniformly distributed random numbers in  $[0,1]$ .

The total water discharge over the  $t_m$  hours is then calculated using (5) above as follows.

$$Q_{j,m} = V_{j,m-1} - V_{j,m} + I_{j,m} - S_{j,m}; \quad m = 1, \dots, M \quad (14)$$

The water discharge  $q_{j,m}$  is calculated using (3) and then hydro generation  $P_{hj,m}$  can be obtained using (4) as follow.

$$P_{hj,m} = \frac{-b_{hj} \pm \sqrt{b_{hj}^2 - 4c_{hj}(a_{hj} - q_{j,m})}}{2c_{hj}}; j = 1, 2, \dots, N_2 \quad (15)$$

where  $b_{hj}^2 - 4 \times c_{hj} \times (a_{hj} - q_{j,m}) \geq 0$

The slack thermal unit is obtained using section 3.B.

Based on the initial population of nests, the fitness function to be minimized corresponding to each nest for the considered problem is calculated.

$$FT_d = \left( \sum_{m=1}^M \sum_{i=1}^{N_1} F_i(P_{si,m,d}) + K_s \sum_{m=1}^M (P_{s1,m,d} - P_{s1}^{\lim})^2 + K_q \sum_{j=1}^{N_2} \sum_{m=1}^M (q_{j,m,d} - q_j^{\lim})^2 \right) \quad (16)$$

where  $K_s$  and  $K_q$  are penalty factors for the slack thermal unit 1 and water discharge, respectively;  $P_{s1,m,d}$  is power output of the slack thermal unit calculated from Section 3.B corresponding to nest  $d$  in the population.

The limits for the slack thermal unit and water discharges in (16) are determined as follows:

$$P_{s1}^{\lim} = \begin{cases} P_{s1,\max} & \text{if } P_{s1,m,d} > P_{s1,\max} \\ P_{s1,\min} & \text{if } P_{s1,m,d} < P_{s1,\min} \\ P_{s1,m,d} & \text{otherwise} \end{cases}; m = 1, \dots, M \quad (17)$$

$$q_j^{\lim} = \begin{cases} q_{j,\max} & \text{if } q_{j,m,d} > q_{j,\max} \\ q_{j,\min} & \text{if } q_{j,m,d} < q_{j,\min} \\ q_{j,m,d} & \text{otherwise} \end{cases}; j = 1, \dots, N_2; m = 1, \dots, M \quad (18)$$

where  $P_{s1,\max}$  and  $P_{s1,\min}$  are the maximum and minimum power outputs of the slack thermal unit, respectively.

The initial population of the host nests is set to the best value of each nest  $X_{bestd}$  ( $d = 1, \dots, N_d$ ) and the nest corresponding to the best fitness function in (16) is set to the best nest  $G_{best}$  among all nests in the population.

- Generation of New Solution Via Lévy Distribution, Cauchy Distribution and Gaussian Distribution

The new solution is calculated based on the previous best nests via Lévy flights. In the proposed method, the optimal path for the Lévy flights is calculated by Mantegna's algorithm. The new solution by each nest is calculated as follows:

$$X_d^{new} = Xbest_d + \alpha \times rand_3 \times \Delta X_d^{new} \quad (19)$$

where  $\alpha > 0$  is the updated step size;  $rand_3$  is a normally distributed stochastic number; and the increased value  $\Delta X_d^{new}$  is determined by using Lévy distribution, Cauchy distribution and Gauss distribution as follows:

- Lévy distribution

$$\Delta X_d^{new} = \nu \times \frac{\sigma_x(\beta)}{\sigma_y(\beta)} \times (Xbest_d - Gbest) \quad (20)$$

$$\nu = \frac{rand_x}{|rand_y|^{1/\beta}} \quad (21)$$

where  $rand_x$  and  $rand_y$  are two normally distributed stochastic variables with standard deviation  $\sigma_x(\beta)$  and  $\sigma_y(\beta)$  given by:

$$\sigma_x(\beta) = \left[ \frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}} \right]^{1/\beta} \quad (22)$$

$$\sigma_y(\beta) = 1 \quad (23)$$

where  $\beta$  is the distribution factor ( $0.3 \leq \beta \leq 1.99$ ) and  $\Gamma(\cdot)$  is the gamma distribution function. Cauchy distribution [16]

$$\Delta X_d^{new} = \sum_{j=1}^{NoStep} (\mu + s * (\tan(\pi * (rand_4 - 0.5)))) \quad (24)$$

where median  $\mu=0$  and scale  $s=1$ ;  $rand_4$  is a normally distributed stochastic number. Gaussian distribution

$$\Delta X_d^{new} = \sum_{j=1}^{NoStep} [2 * \sqrt{-\log(rand_5)} * \sin(\pi * rand_6)] \quad (25)$$

where  $rand_5$  and  $rand_6$  are uniformly distributed random numbers in  $[0,1]$

For the case of using Lévy distribution, the method is called CSA-Lévy. Similarly, the two remaining methods are called CSA-Cauchy and CSA-Gauss corresponding to Cauchy distribution and Gauss distribution.

For the newly obtained solution, its lower and upper limits should be satisfied according to the unit's limits:

$$V_{j,m,d} = \begin{cases} V_{j,\max} & \text{if } V_{j,m,d} > V_{j,\max} ; j = 1, \dots, N_2, \\ V_{j,\min} & \text{if } V_{j,m,d} < V_{j,\min} \quad m = 1, \dots, M - 1 \\ V_{j,m,d} & \text{otherwise} \end{cases} \quad (26)$$

$$P_{si,m,d} = \begin{cases} P_{si,\max} & \text{if } P_{si,m,d} > P_{si,\max} ; i = 2, \dots, N_1 \\ P_{si,\min} & \text{if } P_{si,m,d} < P_{si,\min} \quad m = 1, \dots, M \\ P_{si,m,d} & \text{otherwise} \end{cases} \quad (27)$$

The power output of  $N_2$  hydro units and the slack thermal unit are then obtained as in Sections 3.C and 3.B, respectively. The fitness value is calculated using equations (16). The nest corresponding to the best fitness function is then set to the best nest  $Gbest$ .

#### - Alien Egg Discovery and Randomization

The action of discovery of an alien egg in a nest of a host bird with the probability of  $p_a$  also creates a new solution for the problem similar to the Lévy flights. The new solution due to this action can be found out in the following way:

$$X_d^{dis} = Xbest_d + K \times \Delta X_d^{dis} \quad (28)$$

where  $K$  is the updated coefficient determined based on the probability of a host bird to discover an alien egg in its nest:

$$K = \begin{cases} 1 & \text{if } rand_7 < p_a \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

and the increased value  $\Delta X_d^{dis}$  is determined by:

$$\Delta X_d^{dis} = rand_8 \times [randp_1(Xbest_d) - randp_2(Xbest_d)] \quad (30)$$

where  $rand_7$  and  $rand_8$  are the distributed random numbers in  $[0, 1]$  and  $randp_1(Xbest_d)$  and  $randp_2(Xbest_d)$  are the random perturbation for positions of the nests in  $Xbest_d$ . For the newly obtained solution, its lower and upper limits should be also satisfied constraints (26) and (27). The value of the fitness function is calculated using (16) and the nest corresponding to the best fitness function is set to the best nest  $Gbest$ .

#### - Stopping Criteria

The above algorithm is stopped when the maximum number of iterations is reached.

#### D. Overall Procedure

The overall procedure of the proposed CSA for solving the short-term HTS problem is described as follows.

- Step 1: Select parameters for the CSA including number of host nests  $N_p$ , probability of a host bird to discover an alien egg in its nest  $p_a$ , and maximum number of iterations  $N_{max}$ .
- Step 2: Initialize a population of  $N_p$  host nests as in Section 3.C and calculate the power output for the slack unit 1 as in Section 3.B.
- Step 3: Evaluate the fitness function using (16) and store the best value for each nest  $Xbest_d$  and the best value of all nests  $Gbest$  in the population. Set the initial iteration counter  $n = 1$ .

- Step 4: Generate a new solution via Lévy flights and calculate the power output for the slack unit as in Section 3.B
- Step 5: Evaluate the fitness function using (16) for the newly obtained solution and determine the new  $Xbest_d$  and  $Gbest$  via comparing the values of the fitness function.
- Step 6: Calculate a new solution based on the probability of  $p_a$  and calculate the power output for the slack unit 1 as in Section 3.B.
- Step 7: Evaluate the fitness function using (16) and determine the newly best  $Xbest_d$  and  $Gbest$  for the new obtained solution.
- Step 8: If  $n < N_{max}$ ,  $n = n + 1$  and return to Step 4. Otherwise, stop.

#### 4. Numerical Results

The proposed cuckoo search algorithm has been applied for solving two systems where system 1 comprises one hydro plant and one thermal plant with quadratic fuel cost function, and system 2 consists of four hydro plants and four thermal plants with nonconvex fuel cost function. The both systems are scheduled in three days with six intervals and 12 hours for each. Transmission losses are neglected for system 1 but considered for system 2. The data of system 1 is taken from [1] meanwhile the data of system 2 given in Appendix is obtained by modifying system 1. The proposed CSA is coded in Matlab platform and run on a 1.8 GHz PC with 4 GB of RAM.

##### A. Selection of Parameters

In the proposed CSA method, three main parameters which have to be predetermined are the number of nests  $N_p$ , maximum number of iterations  $N_{max}$ , and the probability of an alien egg to be discovered  $p_a$ .

Among the three parameters, the number of nests significantly effects on the obtained solution quality. Normally, the larger number of  $N_p$  is chosen the higher probability for a better optimal solution is obtained. However, the simulation time for obtaining the solution in case of the large numbers is long. Thus, the selection of  $N_p$  is an important task. By experience, the number of nests in this paper is set to 30 for system 1 and 50 for system 2. Similar to  $N_p$ , the maximum number of iterations  $N_{max}$  also has an impact on the obtained solution quality and computation time. It is chosen based on the complexity and scale of the considered problems. For the test systems above, the maximum number of  $N_{max}$  is set to 400 for system 1 and 3500 for system 2. The value of the probability for an alien egg to be discovered can be chosen in the range [0, 1]. However, different values of  $p_a$  may lead to different optimal solutions for a problem. For the complicated or large-scale problems, the selection of value for the probability has an obvious effect on the optimal solution. In contrast, the effect is inconsiderable for the simple problems, that is different values of the probability can also lead the same optimal solution. In this paper, the value of the probability is selected in range from 0.1 to 0.9 with a step of 0.1 whereas the number of nests and the maximum number of iterations are predetermined in advance.

##### B. Obtained Results

- Case 1: System 1 with one thermal plant and one hydropower plant

Table 1. Summary of the obtained result from CSA-Lévy with different values of  $P_a$

For the system, each version of the proposed CSA method including CSA- Lévy, CSA-Cauchy and CSA-Gauss is run ten independent trials with each of nine values of  $P_a$  in range from 0.1 to 0.9, and the number of nests and maximum number of iterations are set to fixed values of 30 and 400, respectively. The results including minimal total cost, average total cost, maximal total cost, standard deviation, and average computational time obtained by CSA-Lévy, CSA-Cauchy and CSA-Gauss are respectively given in Tables 1, 2 and 3. As indicated in the tables, CSA-Lévy gets optimal solutions at  $P_a = 0.1-0.9$ , CSA-Cauchy gets optimal



solutions at  $P_a=0.8-0.9$  and CSA-Gauss obtains an optimal solution at  $P_a=0.9$  only. Furthermore, CSA-Lévy can obtain less average total cost, less maximum total cost and less standard deviation than CSA-Cauchy and CSA-Gauss. Consequently, it can be concluded that CSA-Lévy is more favorable than CSA-Cauchy and CSA-Gauss. The optimal solutions obtained by the three versions of the CSA method are shown in Tables 4, 5 and 6. The convergence characteristic of the CSA methods shows in figure 1.

$p_a$	Min cost (\$)	Avg. cost (\$)	Max cost (\$)	Std. dev. (\$)	Avg. CPU (s)
0.1	709862.0489	709862.0490	709862.0498	0.0003	0.32
0.2	709862.0489	709862.0492	709862.0511	0.0006	0.27
0.3	709862.0489	709862.0490	709862.0493	0.0001	0.26
0.4	709862.0489	709862.0490	709862.0493	0.0001	0.28
0.5	709862.0489	709862.0490	709862.0494	0.0002	0.34
0.6	709862.0489	709862.0492	709862.0505	0.0005	0.33
0.7	709862.0489	709862.0491	709862.0504	0.0005	0.36
0.8	709862.0489	709862.0491	709862.0494	0.0003	0.29
0.9	709862.0489	709862.0492	709862.0496	0.0002	0.28

Table 2. Summary of the obtained result from CSA-Cauchy with different values of  $P_a$

$p_a$	Min cost (\$)	Avg. cost (\$)	Max cost (\$)	Std. dev. (\$)	Avg. CPU (s)
0.1	709926.4514	710092.1485	710407.738	140.6969	0.23
0.2	709884.4939	709933.3626	710015.2688	41.24073	0.26
0.3	709866.524	709874.7255	709886.5772	5.779156	0.32
0.4	709863.0979	709865.4688	709873.088	3.258535	0.31
0.5	709862.1021	709862.6537	709864.2928	0.666199	0.32
0.6	709862.0504	709862.1187	709862.1989	0.049269	0.27
0.7	709862.0506	709862.0686	709862.1442	0.026691	0.27
0.8	709862.0489	709862.0522	709862.0596	0.003328	0.28
0.9	709862.0489	709862.0499	709862.0514	0.000893	0.3

Table 3. Summary of the obtained result from CSA-Gauss with different values of  $P_a$

$p_a$	Min cost (\$)	Avg. cost (\$)	Max cost (\$)	Std. dev. (\$)	Avg. CPU (s)
0.1	709912.3772	710103.8473	710443.2205	178.7315581	0.25
0.2	709896.2653	709947.9924	710053.2134	50.92232719	0.26
0.3	709864.998	709881.1832	709901.0048	13.67540428	0.32
0.4	709863.1651	709866.5598	709881.5191	6.0827853	0.28
0.5	709862.1356	709862.5409	709863.1357	0.305317756	0.31
0.6	709862.0547	709862.1584	709862.2613	0.070826696	0.27
0.7	709862.0495	709862.0746	709862.1779	0.036520219	0.28
0.8	709862.0496	709862.0547	709862.0774	0.007925554	0.3
0.9	709862.0489	709862.0506	709862.0593	0.002980419	0.3

Table 4. The optimal solutions obtained by CSA-Lévy

$m$	$P_{Dm}$ (MW)	$V_m$ (acre-ft)	$q_m$ (arce-ft/hr)	$P_{sm}$ (MW)	$P_{hm}$ (MW)
1	1200	101928.0846	1839.326281	896.31262	303.68738
2	1500	85963.8659	3330.35156	896.30753	603.69247
3	1100	93855.9115	1342.329532	896.31197	203.68803
4	1800	60000	4821.32596	896.31268	903.68732
5	950	70437.1382	1130.238479	788.98622	161.01378
6	1300	60000	2869.761521	788.98159	511.01841

Table 5. The optimal solutions obtained by CSA-Cauchy

$m$	$P_{Dm}$ (MW)	$V_m$ (acre-ft)	$q_m$ (arce-ft/hr)	$P_{sm}$ (MW)	$P_{hm}$ (MW)
1	1200	101928.5456	1839.28786	896.32035	303.67965
2	1500	85964.12674	3330.36824	896.30418	603.69582
3	1100	93855.56403	1342.38023	896.30177	203.69823
4	1800	60000	4821.297	896.31851	903.68149
5	950	70436.71519	1130.27373	788.97913	161.02087
6	1300	60000	2869.72627	788.98868	511.01132

Table 6. The optimal solutions obtained by CSA-Gauss

$m$	$P_{Dm}$ (MW)	$V_m$ (acre-ft)	$q_m$ (arce-ft/hr)	$P_{sm}$ (MW)	$P_{hm}$ (MW)
1	1200	101927.9993	1839.3334	896.31119	303.68881
2	1500	85963.54261	3330.37139	896.30354	603.69646
3	1100	93856.28171	1342.27174	896.32359	203.67641
4	1800	60000	4821.35681	896.30648	903.69352
5	950	70437.16814	1130.23599	788.98672	161.01328
6	1300	60000	2869.76401	788.98108	511.01892

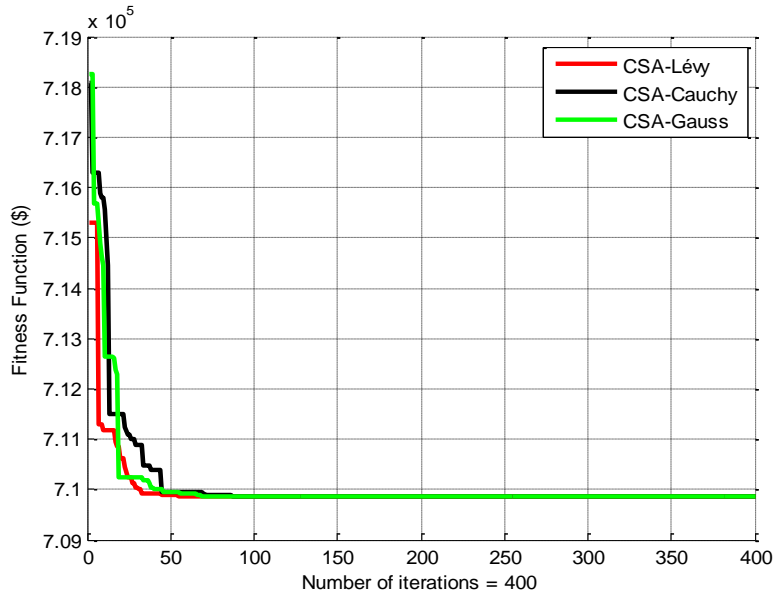


Figure 1. Convergence characteristic of the three proposed CSA methods.

Table 7. Comparison of the results obtained by the proposed CSA methods with others.

Method	Fuel cost (\$)	CPU time (s)	Computer
EP [1]	709863.29	264	PC 486
EP [2]	709862.06	8	PC-486
CEP [3]	709862.05	159.2	Pentium-II,128MB Ram
FEP [3]	709862.05	101.4	Pentium-II,128MB Ram
IFEP [3]	709862.05	59.7	Pentium-II,128MB Ram
RIFEP [4]	709862.05	-	1.83 GHz, 1GB Ram
GS [8]	709877.38	-	-
SA [9]	709874.36	901	PC-486
CSA [10]	709862.05	4.54	Pentium IV, 256 MB Ram
CSA-Lévy	709862.0489	0.26	1.8 GHz, 4 GB RAM
CSA-Cauchy	709862.0489	0.28	
CSA-Gauss	709862.0489	0.3	

The best minimum total cost and average computational time from the versions of CSA method are compared to those from other methods including EP [1], EP [2], CEP [3], FEP [3], IFEP [3], RIFEP [4], GS [8], SA [9], and CSA [10] as shown in Table 7. Obviously, the total cost obtained by CSA- Lévy, CSA-Cauchy and CSA-Gauss is equal to that gotten by CEP [3], FEP [3], IFEP [3], RIFEP [4] and CSA [10], and less than that obtained by EP [1], EP [2], GS [8] and SA [9]. Besides, the three versions of the proposed method are faster than all methods.

Therefore, the proposed method shown in the paper is very effective for solving short-term hydrothermal scheduling with reservoir volume constraints.

Normally, to evaluate the performance of an optimization algorithm two main factors obtained from the search process consisting of quality of solution and execution time are employed to compare with those from other algorithms. Thus, optimal fuel cost and execution time from the three versions of CSA are compared to those from other methods and Table 7 has shown the comparison. The comparison has indicated that the CSA methods have approximate or less cost than other methods. Although the minimum cost from the proposed methods has no significant improvement over other methods, the solution quality from the proposed ones is very high, especially CSA-Lévy where standard deviation cost shown in Table 1 is nearly equal to zero for most cases of  $Pa$ . Furthermore, the average computational time from the proposed methods is also shorter than that from others. However, it may not directly compare the computational times among the applied methods for solving the problem due to different programming language and computer processors used. Therefore, a fair comparison of the execution time among the methods using different computer processors may be performed converting the provided CPU times from methods into a common base. The adjusted CPU time in pu is determined as follows [17]:

$$\text{adjusted CPU time} = \frac{\text{Given CPU speed (GHz)}}{1.8(\text{GHz})} \times \frac{\text{Given CPU time (second)}}{\text{CPU time from CSA-Lévy (second)}} \quad (31)$$

It is noted that the value of 1.8 (GHZ) is the processor of the CPU chip used to run three versions of CSA and the CPU time obtained by CSA-Lévy is used to be a common base time. Therefore, the adjusted CPU time determined for other methods is a time number of the CPU time of the proposed CSA-Lévy as shown in Table 8. It is obvious that the adjusted CPU time that CSA-Lévy spends for searching optimal solution is faster than that from other methods; especially it is from 29.7 to 97 times faster than other methods except EP [2], which is slightly faster than CSA-Lévy.

Table 8. Adjusted computational time comparison for the test system 1

Method	Processor used (GHz)	CPU speed (pu)	Given CPU time (sec)	Given CPU time (pu)	Adjusted CPU time (pu)
EP [1]	0.05	0.03	2640	1015.38	28.4
EP [2]	0.05	0.03	8.00	30.77	0.9
CEP [3]	NA	NA	159.2	612.3	NA
FEP [3]	NA	NA	101.40	390.00	NA
IFEP [3]	NA	NA	59.70	229.62	NA
RIFEP [4]	1.83	1.02	NA	NA	NA
GS [8]	NA	NA	NA	NA	NA
SA [9]	0.05	0.03	901.00	3465.38	97.0
CSA [10]	3.06	1.70	4.54	17.46	29.7
CSA-Lévy	1.80	1.00	0.26	1.00	1.0
CSA-Cauchy	1.80	1.00	0.28	1.08	1.0
CSA-Gauss	1.80	1.00	0.30	1.15	1.1

NA: not available

- Case 2: system 2 with four thermal plants and four hydropower plants

In the case, a large system with four hydro plants and four thermal plants with nonconvex fuel cost function is employed to test the performance of CSA methods. To run the CSA methods fifty independent trials for each value of  $P_a$ , the number of nests and the maximum number of iterations are respectively set to 50 and 3500. The best minimum cost and the average cost, maximum cost, and standard deviation cost corresponding to the best minimum cost for the CSA methods are shown in Table 9 below. The results have shown that the three versions of CSA can deal with the large system with nonconvex fuel cost function of thermal units. In addition, it can be sated that CSA-Lévy is the best one since it can obtain the lowest minimum cost and the second best standard deviation. The optimal solution obtained by CSA-Lévy is given in Tables 10, 11 and 12. Figure 2 has shown the fitness convergence characteristic obtained by the CSA methods.

Table 9. The obtained result by CSA methods for system 2 with nonconvex fuel cost function of thermal plants

Method	$P_a$	Min. cost (\$)	Avg. cost (\$)	Max. cost (\$)	Std. dev. (\$)	Avg. time (s)
CSA-Lévy	0.3	387725.553	396692.5	468641.1	11167.31	47.6
CSA-Cauchy	0.4	388887.678	394450.2	409167.751	4415.6579	49.9
CSA-Gauss	0.4	389213.469	400034.09	495150.953	22239.3865	48.3

Table 10. The optimal volume obtained by CSA-Lévy for system 2 with nonconvex fuel cost function of thermal plants

Sub-interval	$V_{1m}$ (acre-ft)	$V_{2m}$ (acre-ft)	$V_{3m}$ (acre-ft)	$V_{4m}$ (acre-ft)
1	120000	80860	94400	78620
2	963410	776690	905280	759020
3	109960	82480	107110	84570
4	600000	662860	684640	700220
5	619770	720320	600000	621450
6	60000	60000	60000	60000

Table 11. The optimal water discharge obtained by CSA-Lévy for system 2 with nonconvex fuel cost function of thermal plants

Sub-interval	$q_{1m}$ (arce-ft/hr)	$q_{2m}$ (arce-ft/hr)	$q_{3m}$ (arce-ft/hr)	$q_{4m}$ (arce-ft/hr)
1	333.3	3594.8	3466.9	3781.4
2	4471.6	5266.2	3322.5	3226.7
3	865.2	4599.5	1618	1277.6
4	5163.2	5349.1	5220.6	5212.5
5	2835.2	4521.2	2705.3	1656.4
6	5164.8	5002.6	2000	5178.8

Table 12. The optimal generation obtained by CSA-Lévy for system 2 with nonconvex fuel cost function of thermal plants

Sub-interval	$P_{h1}$ (MW)	$P_{h2}$ (MW)	$P_{h3}$ (MW)	$P_{h4}$ (MW)	$P_{s1}$ (MW)	$P_{s2}$ (MW)	$P_{s3}$ (MW)	$P_{s4}$ (MW)
1	0.6707	648.4367	623.3445	685.0148	255.061	608.9945	548.8908	500
2	819.7913	974.0993	594.9889	576.1568	340.6456	599.7214	550	499.3011
3	107.4594	844.6925	257.8245	189.9284	412.2978	675	550	500
4	954.1573	990.1575	965.2757	963.7033	491.9889	674.8039	549.4815	500
5	499.059	829.4481	473.4236	265.4652	98.8813	667.258	550	496.4483
6	954.461	923.0273	333.7745	957.1699	94.8431	363.3373	289.953	319.3386

In summary, cuckoo search algorithm has two new solution generations including the first generation via Lévy flights and the second generation via the replacement of alien eggs. In fact, there are three distributions employed in the paper consisting of Lévy distribution, Cauchy distribution and Gaussian distribution. The performance of the three distributions is tested on two systems above. The obtained results shown in Tables 1, 2 and 3 for system 1 have indicated that the three distributions can lead to the same minimum cost. However, the standard deviation costs for each value of  $Pa$  have revealed that the Lévy distribution is superior to the Cauchy and Gaussian distributions since the standard deviation from CSA-Lévy is nearly equal to zero whereas that from CSA-Cauchy and CSA-Gauss is much higher. In addition, when applied to the large-scale system 2, CSA-Lévy has obtained much less minimum cost than the two other distributions. Obviously, the Lévy distribution has high performance when applied to the cuckoo search algorithm.

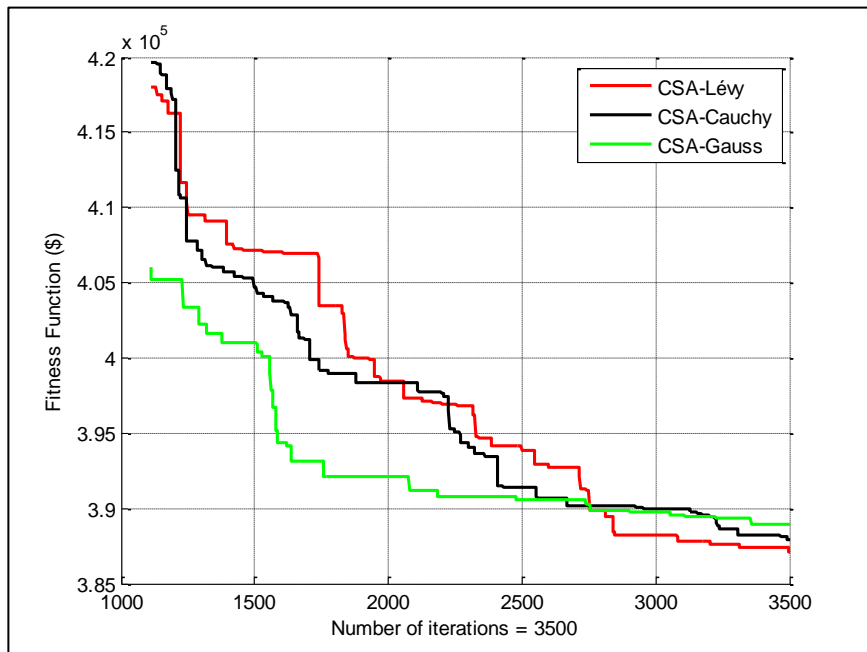


Figure 2. Fitness convergence characteristic for the system 2

## 5. Conclusions

In this paper, the three versions of Cuckoo Search Algorithm including CSA-Lévy, CSA-Cauchy, and CSA-Gauss have been applied for solving short-term hydrothermal scheduling problem with reservoir capacity constraint. The proposed algorithms have been tested on two test systems where the first one consists of one hydropower plant and one thermal plant with quadratic fuel cost function and the second one comprises four hydropower plant and four thermal plants considering valve point loading effect. The comparison of the results obtained by the proposed CSA methods with that from other methods has indicated that the proposed CSA methods can obtain better total cost with faster computational time than the other methods. Among the three versions of CSA proposed in the paper, CSA-Lévy is the best one with the lowest minimum for the test systems. Therefore, the proposed CSA methods, especially the CSA with Lévy distribution, are very favourable and powerful methods for solving short-term hydrothermal scheduling problem with reservoir volume constraint.

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## APPENDIX

Table A1. Data of thermal units for test system 2

Thermal plant	$a_{si}$ (\$/h)	$b_{si}$ (\$/MWh)	$c_{si}$ (\$/MW <sup>2</sup> h)	$d_{si}$ (\$/h)	$e_{si}$ (rad/MW)	$P_{si,min}$ (MW)	$P_{si,max}$ (MW)
1	10	3.25	0.0083	12	0.0450	20	125
2	10	2.00	0.0037	18	0.0370	30	175
3	20	1.75	0.0175	16	0.0380	40	250
4	20	1.00	0.0625	14	0.0400	50	300

Table A2. The data of hydropower plants for test system 2

Hydro plant	$a_{hj}$ (acre-ft/h)	$b_{hj}$ (acre-ft/MWh)	$c_{hj}$ (acre-ft/MW <sup>2</sup> h)	$P_{hj,min}$ (MW)	$P_{hj,max}$ (MW)	$V_{hj0}$ acre-ft	$V_{hjEnd}$ acre-ft	$V_{hjmin}$ acre-ft	$V_{hjmax}$ acre-ft
1	330	4.97	0.0001	0	1000	100000	60000	60000	120000
2	330	4.97	0.0001	0	1000	100000	60000	60000	120000
3	330	4.97	0.0001	0	1000	100000	60000	60000	120000
4	330	4.97	0.0001	0	1000	100000	60000	60000	120000

Table A3. Load demand and the reservoir inflows for test system 2

Subinterval	Duration (h)	Load demand (MW)	$I_{1m}$ (acre-ft/h)	$I_{2m}$ (acre-ft/h)	$I_{3m}$ (acre-ft/h)	$I_{4m}$ (acre-ft/h)
1	12	3600	2000	2000	3000	2000
2	12	4500	2500	5000	3000	3000
3	12	3300	2000	5000	3000	2000
4	12	5400	1000	4000	2000	4000
5	12	3600	3000	5000	2000	1000
6	12	3900	5000	4000	2000	5000

Transmission loss coefficients for system 2:

$$B = \begin{bmatrix} 0.000049 & 0.000014 & 0.000015 & 0.000015 & 0.000020 & 0.000017 \\ 0.000014 & 0.000045 & 0.000016 & 0.000020 & 0.000018 & 0.000015 \\ 0.000015 & 0.000016 & 0.000039 & 0.000010 & 0.000012 & 0.000012 \\ 0.000015 & 0.000020 & 0.000010 & 0.000040 & 0.000014 & 0.000010 \\ 0.000020 & 0.000018 & 0.000012 & 0.000014 & 0.000035 & 0.000011 \\ 0.000017 & 0.000015 & 0.000012 & 0.000010 & 0.000011 & 0.000036 \\ 0.000020 & 0.000018 & 0.000012 & 0.000014 & 0.000035 & 0.000011 \\ 0.000017 & 0.000015 & 0.000012 & 0.000010 & 0.000011 & 0.000011 \end{bmatrix}$$





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