

M175-WR, M275-WR, M375-WR CONFERENCE COURSE WRITING COMPONENT

M 301 COLLEGE ALGEBRA

M302 INTRODUCTION TO MATHEMATICS

M303D APPLICABLE MATHEMATICS

M305G ELEMENTARY FUNCTIONS AND COORDINATE GEOMETRY

M 316K FOUNDATIONS OF ARITHMETIC

M 316L FOUNDATIONS OF GEOMETRY, STATISTICS, AND PROBABILITY

408C DIFFERENTIAL AND INTEGRAL CALCULUS

408D DIFFERENTIAL AND INTEGRAL CALCULUS

408K DIFFERENTIAL CALCULUS

408L INTEGRAL CALCULUS

408M MULTIVARIABLE CALCULUS

M427K ADVANCED CALCULUS FOR APPLICATIONS I

M427L ADVANCED CALCULUS FOR APPLICATIONS II

M325K DISCRETE MATHEMATICS

M328K INTRODUCTION TO NUMBER THEORY

M340L MATRICES AND MATRIX CALCULATIONS

M 341 LINEAR ALGEBRA AND MATRIX THEORY

M343K INTRODUCTION TO ALGEBRAIC STRUCTURES

M361 THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

M361K INTRODUCTION TO REAL ANALYSIS

MATH 362K PROBABILITY I

M365C REAL ANALYSIS I

M365D REAL ANALYSIS II

M367K TOPOLOGY I

Math 373K ALGEBRAIC STRUCTURES I

M373L ALGEBRAIC STRUCTURES II

M175-WR, M275-WR, M375-WR Syllabus

Conference Course Writing Component

Prerequisites and degree relevance:

Vary with the topic, and are given in the course schedule.

Course description:

It is the responsibility of the student to select a professor and make individual arrangements with the professor regarding the meeting, time, and course content. One, two, three, or four meetings a week for one semester. Signed permit forms must be submitted. The forms may be obtained in the mathematics office, RLM 8.100.

NUMBER AND DESCRIPTION OF WRITING ASSIGNMENTS: There will be four writing assignments totaling at least sixteen pages. The first is to be an autobiographical paper relating the student to mathematics, the second will be an exposition of a mathematical topic, the third will be a paper of a historical nature, and the fourth will be a technical mathematical paper. (To get this course approved, I submitted this list of assignments. Other instructors would be able to define their own assignments, as long as the writing requirements for courses in the College of Natural Sciences is met; see below.)

ADDITIONAL COMMENTS: The course may be offered as a supplement to selected organized lower-division undergraduate mathematics classes, or may be taken independently.

COLLEGE WRITING REQUIREMENTS: Each course must include at least three writing activities per semester, exclusive of in-class quizzes and examinations. These three or more writing activities must total at least 16 typewritten, double-spaced pages (about 4000 words). A major rewriting of a paper (requiring additional original writing, not merely editing) may be considered a separate writing activity.

During the course, each student must receive a timely critique concerning the quality of the student's written expression and ways in which the paper may be improved. The quality of the student's written expression must be a component in determining the course grade.

Don Edmondson; modified 9/17/00 kathy davis

M 301 Syllabus

COLLEGE ALGEBRA

Prerequisite and degree relevance:

May not be included in the major requirement for the Bachelor of Arts or Sciences degree with a major in mathematics. In some colleges M301 cannot be counted toward the Area C requirement or toward the total hours required for a degree. Credit for M301 may NOT be earned after a student has received credit for any calculus course with a grade of at least C. Mathematics Level I test is not required.

Course description:

Topics include a brief review of elementary algebra; linear, quadratic, exponential, and logarithmic functions; polynomials; systems of linear equations; applications. Usually offered only in the summer session.

Text: Durbin, College Algebra, preliminary third edition, McGraw-Hill College Custom Series, 1993.

M 301 is the lowest-level "precalculus" course we offer. It should be an honest college algebra course, that is, not an intermediate algebra course (which is offered by community colleges and some four-year colleges and which is often equivalent to second year high school algebra.)

This syllabus is written for use in summer school (the only time we offer M 301). It assumes 26 lectures.

Chapter 1	Five Fundamental Themes	5 sections 4 lectures
Chapter 2	Algebraic Expressions	5 sections 4 lectures
Chapter 3	Equations and Inequalities	5 sections 5 lectures
Chapter 4	Graphs and Functions	4 sections 4 lectures
Chapter 5	Polynomial and Rational Functions	4 sections 4 lectures
Chapter 6	Exponential , Logarithmic Functions	4 sections 3 lectures
Chapter 7	Systems of Equations, Inequalities	3 sections 2 lectures

June 1993

M302 Syllabus

INTRODUCTION TO MATHEMATICS

Prerequisite and degree relevance:

Three units of high school mathematics at the level of Algebra I or higher. The Mathematics Level I test is not required. It may be used to satisfy Area C requirements for the Bachelor of Arts degree under Plan I or the mathematics requirement for the Bachelor of Arts degree under Plan II.

M302 is intended primarily for general liberal arts students. It may not be included in the major requirement for the Bachelor of Arts or the Bachelor of Science degree with a major in mathematics. In some colleges M302 cannot be counted toward the Area C requirement nor toward the total hours required for a degree. Only one of the following may be counted: M302, 303D, or 303F. A student may not earn credit for Mathematics 302 after having received credit for any calculus course.

Course Description:

Introduction to Mathematics is a terminal course satisfying the University's general-education requirement in mathematics. Topics may include: number theory (divisibility, prime numbers, the Fundamental Theorem of Arithmetic, gcd, Euclidean Algorithm, modular arithmetic, special divisibility tests), probability (definition, laws, permutations and combinations), network theory (Euler circuits, traveling salesman problem, bin packing), game theory. Some material is of the instructor's choosing.

Texts: For All Practical Purposes or, The Heart of Mathematics

There is a broad spectrum of students who take M302. Some are quite good at math and may even have had some calculus in high school. These, however are greatly outnumbered by the students who have weak math skills and poor backgrounds. It is not at all uncommon for the students to exhibit a fear of and dislike for math and most have very low self-confidence about their ability to succeed in a math class. In answer to this, the goal of the course should be to demonstrate that math is not about memorizing formulas, but is rather a process of thinking which is relevant to them on a daily basis. The two recommended books, , both are geared toward this type of course. For All Practical Purposes emphasizes applications of math in today's world such as scheduling problems and consumer finance models, for example. The Heart of Mathematics, while dealing with more theoretical topics such as number theory and topology, emphasizes that the problem-solving strategies used to solve mathematical problems are universal and can be applied to solving day-to-day problems. Both texts have proven to be successful at engaging this population of students and giving them a new appreciation of math as well as boosting their self-confidence.

The topics to be covered will depend on the choice of text. Both texts cover probability and statistics and at least 3 weeks of the course should be devoted to this topic. The coverage in For All Practical Purposes is more thorough, especially in the area of statistics. If this is the chosen text, then the syllabus should include chapters 5 and 7. Chapter 6 can be covered lightly, if at all, and chapter 8 should be considered optional. If The Heart of Mathematics is the chosen text, then all of chapter 7 should be covered.

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Sample syllabus for For All Practical Purposes:

Chapter 1 Street Networks All sections (3 days)
Chapter 2 Visiting Vertices(omit Minimum cost spanning trees) (4 days)
Chapter 3 Planning and Scheduling(omit Bin Packing) (4 days)
Chapter 5 Producing Data All sections (4 days)
Chapter 6 Exploring Data Cover lightly (2 days)
Chapter 7 Probability All sections (5 days)
Chapter 10 Transmitting Information (supplement the modular arithmetic and cover cryptography only) (4 days)
Chapter 15 Game Theory All sections (5 days)
Chapter 20 Consumer Finance Models All section (time permitting) (6 days)

Notes: For All Practical Purposes

Chapter 1 is an introduction to graph theory and is a good chapter for establishing the course as one which is not "formula-based." Chapters 2 and 3 then follow up with some applications of graph theory.

As mentioned above, Chapters 5 and 7 should be covered thoroughly and Chapter 6 lightly.

Chapters 9 and 10 introduce the concept of modular arithmetic with applications to error detecting codes and cryptography. Students tend to find the arithmetic challenging, but in general they enjoy the ideas in these chapters.

Chapter 13 on Fair Division is fun to do, however it is difficult to get the ideas across. Students tend to get lost in the logic and may end up simply memorizing procedures.

Chapter 15, Game Theory, also gives the students a work-out in the area of following a logical argument and again they tend to memorize algorithms for finding good strategies. This chapter does give a chance to revisit expected value and they also appreciate the real-world applications of the "Prisoner's Dilemma" problems.

Chapter 20 deals with compound interest and annuities. The relevance of this material to their lives makes it one of the most widely-appreciated chapters on the part of the students.

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Sample syllabus for The Heart of Mathematics

Chapter 1: Fun and Games All sections (3 days)
Chapter 2: Number Contemplation All sections (11 days)
Chapter 3: Infinity Sections 3.1-3.3 (4 days)
Chapter 4: Geometric Gems 4.1, 4.3, 4.5, 4.7 (6 days)
Chapter 5: Contortions of Space 5.1, 5.3 (3 days)
Chapter 7: Risky Business All sections (10 days)

Notes: The Heart of Mathematics:

Chapter 1 is excellent for setting the tone of the class and illustrating some problem-solving strategies. The puzzles also tie in with the material from later chapters.

Chapter 2 covers some topics from number theory and gives an appreciation of number theory as an ancient area of mathematics as well as one which is essential today in the areas of cryptography and error detecting codes.

Chapter 3, on infinity, is guaranteed to provoke lively discussions as well as controversy.

Chapter 4 contains some nice sections on geometry. The section on the Pythagorean theorem give the students several examples of geometric proofs. In the section on the Platonic solids, the students are encouraged to build the solids and explore the concept of duality. The section on the fourth dimension gives them the opportunity to experience an abstract idea through the process of generalization.

Chapter 5 deals with some ideas from topology. The section on rubber sheet geometry has some fun and surprising results, but the students will probably need a model to convince them that the results are indeed true. The section on the Euler characteristic ties in with chapter 4's section on Platonic Solids.

As mentioned above, Chapter 7 should be covered in its entirety.

Altha Rodin July, 2000

M303D Syllabus

APPLICABLE MATHEMATICS

Prerequisite and degree relevance:

The prerequisite is three units of high school mathematics at the level of Algebra I or higher or a Mathematics Level I or IC score of at least 430/400, or Mathematics 301 with a grade of at least C.

May not be included in the major requirement for the Bachelor of Arts or Bachelor of Science degree with a major in mathematics. Only one of the following may be counted: Mathematics 302, 303D, and 303F. A student may NOT earn credit for Mathematics 303D after having received credit for Mathematics 305G nor any calculus course.

Course description:

The course treats some of the techniques which allow mathematics to be applied to a variety of problems. It is designed for the non-technical student who needs an entry level course developing such mathematics skills. Topics include: linear and quadratic equations, systems of linear equations, matrices, probability, statistics, exponential and logarithmic functions, and mathematics of finance.

Text Barnett and Ziegler, Finite Mathematics, eighth edition

This course is an entry-level course for non-technical students. It deals with some of the techniques that allow mathematics to be applied to a variety of problems and seeks to develop these skills for this type of student. It typically will contain students from fashion design, interior design, psychology, government, communications and assorted other disciplines. It is also taken by elementary education students before M316K.

A pocket calculator will be required to work problems at some stages of the course; a "scientific" one with x^y , and $\ln x$ functions is the type needed.

A suggested allocation of time is given for the chapters of the book treated. A few sections may be omitted at the instructor's discretion. The allocations given require 33 days.

(davis, schultz 10/22/00)

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Chapters 1 and 2: Libraries of Elementary Functions (7 days)
§§1.1 through 1.4; 2.1 through 2.3

Chapter 3 Mathematics of Finance (4 days)
§§3.1 through 3.4

Chapter 4 Systems of Linear Equations (7 days)
§§4.1 through 4.7

Chapter 5 Linear Inequalities and Linear Programming (2 days)
§§5.1 and 5.2 (covering the simplex method is unrealistic)

Chapter 6 Probability (8 days)
§§6.2 through 6.7

Chapter 7 Data Description and Probability Distributions (5 days)
§§7.1 through 7.5

M 305G SYLLABUS

The purpose of this course is to prepare students for calculus courses (M408C and M408K). Some students are taking this course for review; many because they did not score high enough on the Math Level I test to enter calculus directly.

While some students in the course may not be headed to calculus, the emphasis should be on techniques that will be needed in calculus and among the problems assigned should be those that look forward to calculus. The instructor should not skip difficult material, assuming “they won’t need this.”

It may be assumed that the students have had at least three and a half years of high school mathematics.

Required text: Sullivan, Precalculus. Prentice-Hall, 6th edition, 2001

Chapter 1 Graphs

Sections 1.1, 1.2, 1.3

Chapter 2 Functions and Their Graphs

Sections 2.1, 2.2, 2.3, 2.4, 2.5

Chapter 3 Polynomials and Rational Functions

Sections 3.1, 3.2, 3.3, 3.4, (skip 3.5), 3.6

Chapter 4 Exponential and Logarithmic Functions

Sections 4.1, 4.2, 4.3, 4.4(excluding curve fitting pps. 263-265), 4.5, 4.6, 4.7

Chapter 5 Trigonometric Functions

Sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.6(excluding curve fitting)

Chapter 6 Graphs of Trigonometric Functions

Sections 6.1, 6.2, 6.3, 6.4, 6.5, 6.6

If there is time, you could include the material on partial fractions and completing the square. Other topics worth considering are conic sections (Chapter 9) and the law of cosines (Section 7.3) it is better to cover the listed sections above carefully than to cover more sections superficially.

(kathy davis 8/17/01)

M 316K SYLLABUS

FOUNDATIONS OF ARITHMETIC

Prerequisite and degree relevance:

The prerequisite is Mathematics 303D, 305G, or 316 with a grade of at least C. M316K is intended for prospective elementary teachers and other students whose degree programs require it; it treats basic concepts of mathematical thought. May not be included in the major requirement for the Bachelor of Arts or Bachelor of Science degrees with a major in mathematics. Credit for Mathematics 316K may not be earned after the student has received credit for any calculus course with a grade of C or better, unless the student is registered in the College of Education.

Course description:

An analysis, from an advanced perspective, of the concepts and algorithms of arithmetic, including sets; numbers; numeration systems; definitions, properties, and algorithms of arithmetic operations; and percents, ratios, and proportions. Problem solving is stressed.

Textbook:

There is no really good textbook for the course. The default textbook is Bassarear , Mathematics for Elementary School Teachers. However, you are not required to use this or any other textbook. Some instructors (especially in M316L) have used Musser & Burger, Mathematics for Elementary Teachers: a Contemporary Approach. If you use one of these textbooks, be sure to obtain the accompanying activities book as well. Some instructors have used just the activities book without the main textbook.

Many instructors do not use a textbook for this course. Indeed, there are many things that can be done without a textbook that are very well suited to the course. Whether or not you decide to use a textbook, please contact Martha Smith (RLM 10.136, 471-6142, mks@math.utexas.edu) for a list of other resources for these classes. In particular, three previous instructors (Tracy Rusch, Mary Hannigan, and Jayne Ann Harder) have created a set of classroom-ready activities that can be used instead of a textbook for M 316K.

Math content courses for elementary teachers are the subject of nation-wide interest, so new textbooks or electronic alternatives may appear that might be good choices.

Intended Audience:

Students planning to be elementary school teachers.

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Purpose:

To enrich these students' understanding of elementary school mathematics. This includes providing mathematical background to help prepare them for:

- Their math teaching methods courses.
- Teaching with understanding.
- Flexibility in adapting to new math curricula
- The math portion of the Excet exam required for certification.

Challenges involved in teaching this course:

- Many students enrolled in the courses have poor mathematics backgrounds and a history of negative experiences with mathematics. Part of the instructor's job is to make progress in turning this around. Thus, teaching these courses requires patience and tact. Connecting the course to the students' future career can help tremendously here. Suggestions on doing this are given in Chapter 8 of Basic Reference 1 below.
- Often the things that are most important for these students to learn are things that the instructor may take as obvious. For example, setting up equations to fit a word problem may be second nature to you; how can you teach someone to do it? (Chapter 8 of Basic Reference 1 below has some suggestions.) Awareness of the problem, patience, and willingness to listen to your students and to learn from others' experience can help you meet this challenge.
- Student backgrounds vary widely. Although many students will be as described above, there will also be students who have had calculus and plan to become math specialists. Connecting the course to the students' future career can help here also, since often the mathematically better-prepared students may have the same difficulties as the instructor in assuming that things are "obvious."
- Many students will expect this to be a course in how to teach. It definitely should be relevant to teaching, but is not a methods course.

General suggestions:

- Don't feel you have to "cover the entire syllabus." Treating a few topics with the appropriate depth will be better in the long run than a superficial treatment of many topics.
- Problem solving, conjecturing, justifying, seeking counterexamples, and reasoning should be routine parts of class discussion.
- Keep lecture to a minimum; use hands-on activities whenever possible. But remember that activities usually require a whole-class summing-up and discussion to be sure students have learned what they are supposed to learn from them.
- Listen to your students and be willing to adapt your plans as their needs require. (For example, if you are studying a data set in M 316L and discover that many of your students have trouble comparing decimals, take some time out -- a few minutes or a couple of class periods if needed -- to work on this.)
- Tie the course content as much as possible to elementary teaching. (Chapter 8 of Basic Reference 1 below can provide some elementary classroom scenes that can help you do this.)
- Be aware that many students may be weak in proportional reasoning (in particular, in seeing when a situation involves a multiplicative comparison rather than an additive one). Take advantage of opportunities in both courses (e.g., fractions in M 316K, similarity and probability in M 316L) to build their understanding in this area.

Basic References:

These syllabi have been heavily influenced by the following two documents. You are encouraged to consult them yourself.

1. Chapters 3 and 8 (and to a lesser extent, 4 and 9) of the draft report of the Conference Board in the Mathematical Sciences Mathematical Education of Teachers Project, available at <http://www.maa.org/cbms/metdraft/>.

2. The draft Mathematics Teacher Certification Guidelines of the State Board for Teacher Certification, downloadable in pdf format from http://www.sbec.state.tx.us/certstand/stand_math.pdf

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Resources:

1. Faculty coordinator for the course is Martha Smith, RLM 10.136, mks@math.utexas.edu, 471-6142. She can give you a list of resources for M 316K and L and tries to be available for discussion, encouragement, suggestions, hand-holding, etc.
2. Previous instructors can be valuable resources.
3. Susan Empson (empson@mail.utexas.edu) in the math ed group specializes in elementary education and is glad to serve as a resource, especially in how M 316K and L can help prepare students for their math methods courses.

Note:

Assistant Instructors teaching one of these courses for the first time may want to explore the possibility of signing up for an independent study course under either Dr. Smith or Dr. Empson to read some of the relevant literature on these courses.

Reminder:

It is better to cover only part of the syllabus well than to “cover” everything superficially.

I. Number and Operations

Understanding operations on whole numbers, including:

- interpretations of addition, subtraction, multiplication and division in various contexts.
- understanding relationships among operations, particularly how subtraction and division are derived from addition and multiplication

Developing a solid understanding of place value in the base-ten number system, including:

- understanding how place value permits efficient representation of large numbers
- recognizing each place value as ten times as large as the next place to the right
- seeing how the operations of addition, multiplication, and exponentiation are used in representing numbers

Understanding multi-digit calculations, including standard algorithms, "mental math," and non-standard methods commonly devised by students, including:

- understanding how the standard algorithms depend on the place-value system
- developing acquaintance with alternate algorithms and the ability to detect whether a student using a nonstandard method is using a valid procedure or misunderstanding a standard algorithm
- recognizing how decimal notation allows for approximation of numbers by "round numbers" (numbers with fewer non-zero digits) and that this can facilitate mental arithmetic and approximate solutions
- developing skill at rounding numbers up or down to facilitate computation and then compensating for having done so

Extending the notion of number to integers and rationals, including:

- understanding what integers, fractions, and decimals are and developing a sense of their relative size
- understanding what the operations on integers and fractions mean and understanding rules for calculation
- understanding how place value applies to numbers less than 1
- recognizing that the arithmetic of decimals is essentially the same as that of whole numbers

II. Foundations of algebra and functions

Generalizing arithmetic and quantitative reasoning, including:

- learning to use a variety of representations, including conventional algebraic notation, to articulate and justify generalizations
- seeing computation algorithms as applications of commutativity, associativity, and distributivity

Using inductive reasoning to identify, extend, and create patterns using concrete models, figures, numbers, and algebraic expressions

Translating, modeling, and solving problems using concrete, numerical, tabular, graphic, and algebraic methods, and moving between different problem representations.

Making, testing, validating, and using conjectures about patterns and relationships in data presented in tables, sequences, or graphs

Understanding the concepts of variable and function

M 316L SYLLABUS

FOUNDATIONS OF GEOMETRY, STATISTICS, AND PROBABILITY

Prerequisite and degree relevance:

The prerequisite is M316K. May not be included in the major requirement for the Bachelor of Arts or Bachelor of Science degrees with a major in mathematics. Credit for Mathematics 316L may not be earned after a student has received credit for any calculus course with a grade of C or better, unless the student is registered in the College of Education.

Course description:

An analysis, from an advanced perspective, of the basic concepts and methods of geometry, statistics, and probability, including representation and analysis of data; discrete probability, random events, and conditional probability; measurement; and geometry as approached through similarity and congruence, through coordinates, and through transformations. Problem solving is stressed.

Textbook:

There is no really good textbook for the course. The default textbook is Bassarear , Mathematics for Elementary School Teachers. However, you are not required to use this or any other textbook. Some instructors (especially in M316L) have used Musser & Burger, Mathematics for Elementary Teachers: a Contemporary Approach. If you use one of these textbooks, be sure to obtain the accompanying activities book as well. Some instructors have used just the activities book without the main textbook.

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To enrich these students' understanding of elementary school mathematics. This includes providing mathematical background to help prepare them for:

- Their math teaching methods courses.
- Teaching with understanding.
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- The math portion of the Excet exam required for certification.

Challenges involved in teaching this course:

- Many students enrolled in the courses have poor mathematics backgrounds and a history of negative experiences with mathematics. Part of the instructor's job is to make progress in turning this around. Thus, teaching these courses requires patience and tact. Connecting the course to the students' future career can help tremendously here. Suggestions on doing this are given in Chapter 8 of Basic Reference 1 below.
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- Many students will expect this to be a course in how to teach. It definitely should be relevant to teaching, but is not a methods course.

General suggestions:

- Don't feel you have to "cover the entire syllabus." Treating a few topics with the appropriate depth will be better in the long run than a superficial treatment of many topics.
- Problem solving, conjecturing, justifying, seeking counterexamples, and reasoning should be routine parts of class discussion.
- Keep lecture to a minimum; use hands-on activities whenever possible. But remember that activities usually require a whole-class summing-up and discussion to be sure students have learned what they are supposed to learn from them.
- Listen to your students and be willing to adapt your plans as their needs require. (For example, if you are studying a data set in M 316L and discover that many of your students have trouble comparing decimals, take some time out -- a few minutes or a couple of class periods if needed -- to work on this.)
- Tie the course content as much as possible to elementary teaching. (Chapter 8 of Basic Reference 1 below can provide some elementary classroom scenes that can help you do this.)
- Be aware that many students may be weak in proportional reasoning (in particular, in seeing when a situation involves a multiplicative comparison rather than an additive one). Take advantage of opportunities in both courses (e.g., fractions in M 316K, similarity and probability in M 316L) to build their understanding in this area.

Basic References:

These syllabi have been heavily influenced by the following two documents. You are encouraged to consult them yourself.

1. Chapters 3 and 8 (and to a lesser extent, 4 and 9) of the draft report of the Conference Board in the Mathematical Sciences Mathematical Education of Teachers Project, available at <http://www.maa.org/cbms/metdraft/>.

2. The draft Mathematics Teacher Certification Guidelines of the State Board for Teacher Certification, downloadable in pdf format from http://www.sbec.state.tx.us/certstand/stand_math.pdf

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Resources:

1. Faculty coordinator for the course is Martha Smith, RLM 10.136, mks@math.utexas.edu, 471-6142. She can give you a list of resources for M 316K and L and tries to be available for discussion, encouragement, suggestions, hand-holding, etc.
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Note:

Assistant Instructors teaching one of these courses for the first time may want to explore the possibility of signing up for an independent study course under either Dr. Smith or Dr. Empson to read some of the relevant literature on these courses.

Reminder: It is better to cover only part of the syllabus well than to “cover” everything superficially.

I Geometry and Measurement

Developing spatial sense , including

- decomposing two- and three-dimensional objects into component parts and recombining them
- recognizing shapes as the same if they have been rotated, translated, or reflected.
- recognizing rotational, line, and plane symmetry of two- and three- dimensional shapes

Developing familiarity with basic shapes and their properties , including

- knowing names and basic properties of common two- and three- dimensional shapes
- developing an understanding of angles and how they are measured
- understanding congruence and similarity

Recognizing basic geometric shapes and concepts in a variety of settings (everyday objects, arts and crafts, architecture, nature)

Communicating geometric ideas , including

- learning technical vocabulary and understanding its value
- understanding the role of definition

Understanding concepts of area and volume, including

- understanding the idea of a unit of area or volume and the relationship between linear dimensions and area or volume of two-and three-dimensional rectangular arrays
- extending these ideas to find the area and volume of rectangular figures or solids with non-integral dimensions
- devising formulas for the area of triangles, parallelograms, and trapezoids
- understanding intuitively the formulas for the area and circumference of a circle and why pi occurs in both
- understanding formulas for volumes and surface areas of prisms, cylinders, and other three-dimensional objects
- exploring the complex relationships among perimeter and area; surface area and volume

Developing, justifying, and using conversions within and between different measurement systems.

Estimating quantities using both standard and ad hoc units of measurement

II. Data Analysis, Statistics, and Probability

Designing simple data investigations , including

- understanding the kinds of questions that can be addressed by data
- differentiating between categorical and numerical data
- Deciding what to measure and how to measure it
- moving back and forth between the question (the purpose of the study) and its design

Describing data , including

- describing spread: range, outliers, clusters, gaps, skewness, and what these indicate about the question to be addressed by the data
- using summary statistics: mode, median, and mean and what these indicate about the question to be addressed by the data
- interpreting and using various forms of data representation appropriately (bar charts and pie charts for categorical data; boxplots, line plots, stemplots, scatter plots, and histograms for quantitative data)
- recognizing that different forms of representation communicate different features of the data
- comparing two sets of data (not always of the same size)

Drawing Conclusions, including

- developing beginning intuitive ideas of sampling and inference
- choosing among representations and summary statistics to communicate conclusions

Developing initial concepts of probability , including

- making judgments with uncertainty
- recognizing probability as occurring on a continuum (first, ranking events as “impossible, not very likely, likely, certain; then expressing probabilities in numerical terms, including situations where outcomes are not equally likely)
- **becoming familiar with the idea of randomness**

Syllabus: M408C

DIFFERENTIAL AND INTEGRAL CALCULUS

Text: Stewart, Calculus, Fifth Edition

Responsible Parties: Kathy Davis, John Gilbert, Gary Hamrick June 19 2003

Prerequisite and degree relevance:

Either a 560 on the Mathematics IC Test or 560 on Mathematics Level IIC or a grade of at least C in M304E or M305G. Note: Students who score less than 600 on the Mathematics Level IC Test should be aware that studies show taking M305G first is likely to improve their grade in M408C.

Only one of the following may be counted: M 403K, 408C, 408K, 308K.

M408C and M408D (or the equivalent sequence M408K, M408L, M408M) are required for mathematics majors, and mathematics majors are required to make grades of C or better in these courses.

Course description:

M408C is our standard first-semester calculus course. It is directed at students in the natural and social sciences and at engineering students. The emphasis in this course is on problem solving, not on the presentation of theoretical considerations. While the course necessarily includes some discussion of theoretical notions, its primary objective is not the production of theorem-provers.

The syllabus for M408C includes most of the elementary topics in the theory of real-valued functions of a real variable: limits, continuity, derivatives, maxima and minima, integration, area under a curve, volumes of revolution, trigonometric, logarithmic and exponential functions and techniques of integration.

Overview and Course Goals

The following pages comprise the syllabus for M408C, and advice on teaching it. Calculus is a service course, and the material in it was chosen after interdepartmental discussions. Please do not make drastic changes (for example, skipping techniques of integration). You will do your students a disservice and leave them ill equipped for subsequent courses.

This is not a course in the theory of calculus; the majority of the proofs in the text should not be covered in class. At the other extreme, some of our brightest math majors found their first passion in calculus; one ought not to bore them. Remember that 408C/D is the 'fast' sequence for students with good algebra skills; students who cannot maintain the pace are encouraged to take the M408KLM sequence.

Resources for Students

Some of our students have weak study skills. The Learning Skills Center in Jester has a wide variety of material (drills, video-taped lectures, computer programs, counseling, math anxiety workshops, algebra and trig review, calculus review) as well as tutoring options, all designed to

help students through calculus. On request, (471-3614) they'll come to your classroom and explain their services.

You can help your students by informing them of LSC services.

Timing and Optional Sections

A 'typical' semester has 43 MWF days; a day or so will be lost to course-instructor evaluations, etc. The syllabus contains material for 39 days; you cannot afford to lose class periods. If you plan to give exams in lecture rather than in TA section, you will surely have to cut time somewhere. We have added some flexibility to the syllabus by designating all the application sections as optional. It is expected that you will choose three or four of these and cover them well; it is also expected that you will not have time to do all of them.

Those teaching on TTh should adjust the syllabus; a MWF lecture lasts 50 min; a TTh therefore 75.

Forty Class Days As:

Appendixes *(for reference and reading by student)*

- A Numbers, Inequalities, and Absolute Values
- B Coordinate Geometry and Lines
- C Graphs of Second-Degree Equations
- D Trigonometry
- E Sigma Notation

1 Functions and Models *(for reference and reading by student)*

- 1.1 Four Ways to Represent a Function
 - 1.2 Mathematical Models: A Catalog of Essential Functions
 - 1.3 New Functions from Old Functions
- Principles of Problem Solving

2 Limits and Rates of Change *(Five Days)*

- 2.1 The Tangent and Velocity Problems
- 2.2 The Limit of a Function
- 2.3 Calculating Limits Using the Limit Laws
- 2.4 The Precise Definition of a Limit (*briefly*)
- 2.5 Continuity
- 2.6 Tangents, Velocities, and Other Rates of Change

3 Derivatives (*Seven Days*)

- 3.1 Derivatives
- 3.2 The Derivative as a Function
- 3.2 Differentiation Formulas
- 3.4 Rates of Change in the Natural and Social Sciences
- 3.5 Derivatives of Trigonometric Functions
- 3.6 The Chain Rule
- 3.7 Implicit Differentiation
- 3.8 Higher Derivatives
- 3.9 Related Rates

4 Applications of Differentiation (*Seven Days*)

- 4.1 Maximum and Minimum Values
- 4.2 The Mean Value Theorem
- 4.3 How Derivatives Affect the Shape of a Graph
- 4.4 Limits at Infinity; Horizontal Asymptotes
- 4.5 Summary of Curve Sketching
- 4.7 Optimization Problems
- 4.10 Antiderivatives

5 Integrals (*Six Days*)

- 5.1 Areas and Distances
- 5.2 The Definite Integral
- 5.3 The Fundamental Theorem of Calculus
- 5.4 Indefinite Integrals and the Net Change Theorem
- 5.5 The Substitution Rule

6 Applications of Integration (*Two Days*)

- 6.1 Areas between Curves
- 6.2 Volumes

7 Inverse Functions: Exp Log and Inverse Trig (*Six Days*)

- 7.1 Inverse Functions

Sections 7.2-7.4 or Sections 7.2-7.4* at discretion of instructor*

- 7.2 Exponential Functions Their Derivatives
- 7.3 Logarithmic Functions

7.4 Derivatives of Logarithmic Functions

7.5 Inverse Trigonometric Functions

8 Techniques of Integration (*Seven Days*)

8.1 Integration by Parts

8.2 Trigonometric Integrals

8.3 Trigonometric Substitution

8.4 Integration of Rational Functions by Partial Fractions

8.5 Strategy for Integration

408D SYLLABUS

DIFFERENTIAL AND INTEGRAL CALCULUS

Prerequisite and degree relevance:

A grade of C or better in M408C or the equivalent. (Note: The pace of M408C and M408D is brisk. For this reason, transfer students with one semester of calculus at another institution are requested to consult with the Undergraduate Adviser for Mathematics to determine whether M408D or an alternative, M308L, is the appropriate second course.) 408D may not be counted by students with credit for Mathematics 403L, 308M. M408C and M408D (or the equivalent sequence M308K, M308L, M308M) are required for mathematics majors, and mathematics majors are required to make grades of C or better in these courses.

Certain sections of this course are reserved as advanced placement or honors sections; they are restricted to students who have scored well on the advanced placement AP/BC exam, or are honors students, or who have the approval of the Mathematics Advisor. Such sections and their restrictions are listed in the Course Schedule for each semester.

Course description:

M408C, M408D is our standard first-year calculus sequence. It is directed at students in the natural and social sciences and at engineering students. The emphasis in this course is on problem solving, not on the presentation of theoretical considerations. While the course necessarily includes some discussion of theoretical notions, its primary objective is not the production of theorem-provers. M408D contains a treatment of infinite series, and an introduction to vectors and vector calculus in 2-space and 3-space, including parametric equations, partial derivatives, gradients and multiple integrals.

Text: Salas and Hille: Calculus, eighth edition

Overview and Course Goals

The following pages comprise the syllabus for M408D, and advice on teaching it. Calculus is a service course, and the material in it was chosen after interdepartmental discussions. Please do not make drastic changes (for example, skipping multiple integrals). You will do your students a disservice and leave them ill equipped for subsequent courses. This is not a course in the theory of calculus; the majority of the proofs in the text should not be covered in class. The purpose of this course is to develop computational skills and reasoning abilities in calculus. Geometrical understanding is nice, too.

Student Background

In general 408D students are somewhat better than 408C students; on the other hand 408D is a more difficult course. The Learning Skills Center in Jester has a wide variety of material (drills, video-taped lectures, computer programs, counseling, math anxiety workshops, algebra and trig review, calculus review) as well as tutoring options, all designed to help students through calculus. On request, (471-3614) they'll come to your classroom and explain their services. You can help your students by informing them of LSC services.

A very basic thing you can do is encourage students to work together in solving problems. The Emerging Scholars Program accomplishes this by providing 'Worksheets' of problems, and devoting part of the TA section to group work on those problems. These worksheets are available on the Math Department Web Site, at www.ma.utexas.edu/users/erichsu/index.html We've also put a variety of other calculus texts in the library; students may be encouraged to consult these.

Timing and Optional Sections

A 'typical' semester has 43 MWF days; a day or so will be lost to course-instructor evaluations, etc. The syllabus contains material for 39 days; you cannot afford to lose class periods. If you plan to give exams in lecture rather than in TA section, you will surely have to cut time somewhere. We have added some flexibility to the syllabus by designating all the application sections as optional. It is expected that you will choose three or four of these and cover them well; it is also expected that you will not have time to do all of them. Those teaching on TTh should adjust the syllabus; a MWF lecture lasts 50 min; a TTh therefore 75 min.

(continued next page) (9/17/00 kathy davis)

CH 10 SEQUENCES; INDETERMINATE FORMS (5 days)

10.2 SEQUENCES OF REAL NUMBERS

- 10.3 Limit of a Sequence (remember: this isn't an epsilon-delta course)
- 10.4 Some Important Limits
- 10.5 The Indeterminate Form (0/0)
- 10.6 More Indeterminate Forms
- 10.7 Improper Integrals (emphasize unbounded intervals, preparing for §11.3)

CHAPTER 11 INFINITE SERIES (10 days)

- 11.1 Infinite Series (students get sequences & series, and convergence of each, confused/)
- 11.2 Integral Test; Comparison Theorems (de-emphasize comparison theorems)
- 11.3 The Root Test; the Ratio Test
- 11.4 Absolute & Conditional Convergence; Alternating Series
- 11.5 Taylor Polynomials, Series in x
- 11.6 Taylor Polynomials, Series in $(x-a)$ (emphasize numerical applications)
- 11.7 Power Series
- 11.8 Differentiation and Integration of Power Series

CH 9 POLAR & PARAMETRIC EQUATIONS (3 days) (note change in sequence!!)

- 9.3 Polar Coordinates (some students may not have seen this before)
- 9.4 Graphing in Polar Coordinates
- 9.5 Area in Polar Coordinates
- 9.6 Curves Given Parametrically
- 9.7 Tangents to Curves Given Parametrically

CHAPTER 12 VECTORS (5 days)

- 12.1 Cartesian Space Coordinates (assign)
- 12.2 Displacements; Forces and Velocities
- 12.3 Vectors
- 12.4 The Dot Product
- 12.5 The Cross Product
- 12.6 Lines
- 12.7 Planes

CHAPTER 13 VECTOR CALCULUS (2 days)

- 13.1 Vector Functions
- 13.2 Differentiation Formulas
- 13.3 Curves
- 13.4 Arc Length

CH 14 FUNCTIONS OF SEVERAL VARIABLES (3 days)

- 14.1 Elementary Examples
- 14.2 A Brief Catalog of the Quadric Surfaces; Projections (assign or cover quickly)
- 14.3 Graphs; Level Curves and Level Surfaces
- 14.4 Partial Derivatives
- 14.6 Limits and Continuity; Equality of Mixed Partial Derivatives (emphasis on mixed partials)

CH 15 GRADIENTS; EXTREME VALUES (7 days) 15.1 Differentiability and Gradient

- 15.2 Gradients and Directional Derivatives
- 15.3 Mean-Value Theorem; Chain Rules
- 15.4 The Gradient as a Normal; Tangent Lines and Planes
- 15.5 Maximum and Minimum Values
- 15.6 Maxima and Minima with Side Conditions

CH 16 DOUBLE AND TRIPLE INTEGRALS (4 days)

- 16.1 Multiple-Sigma Notation (assign)

16.2 DOUBLE INTEGRALS

- 16.3 Evaluation of Double Integrals by Repeated Integrals

Syllabus: M408K
DIFFERENTIAL CALCULUS

Text: Stewart, Calculus, Fifth Edition

Responsible Parties: Kathy Davis, John Gilbert, Gary Hamrick June 19 2003

Prerequisite and degree relevance:

Either four years of high school mathematics and a Mathematics Level I or IC Test score of at least 520, or M 305G with a grade of at least C. Note: Students who score less than 600 on the Mathematics Level I or IC Test are advised to take the M408KLM sequence rather than M408CD.

Only one of the following may be counted: M 403K, 408C, 408K.

Calculus is offered in two equivalent sequences: a two-semester sequence, M 408C/408D, which is recommended only for students who score at least 600 on the mathematics Level IC Test, and a three-semester sequence, M 408K/408L/408M.

For some degrees, the two-semester sequence M 408K/408L satisfies the calculus requirement . This sequence is also a valid prerequisite for some upper-division mathematics courses, including M325K, 427K, 340L, and 362K.

M408C and M408D (or the equivalent sequence M408K, M408L, M408M) are required for mathematics majors, and mathematics majors are required to make grades of C or better in these courses.

Course description:

M408K is one of two first-semester calculus courses. It is directed at students in the natural and social sciences and at engineering students. In comparison with M408C, it covers fewer chapters of the text. However, some material is covered in greater depth, and extra time is devoted the development of skills in algebra and problem solving. This is not a course in the theory of calculus.

The syllabus for M408K covers differential calculus: limits, continuity, derivatives, maxima and minima, trigonometric, logarithmic and exponential functions.

Timing and Optional Sections

A 'typical' semester has 43 MWF days; a day or so will be lost to course-instructor evaluations, etc. The syllabus contains material for 40 days; you cannot afford to lose class periods. Those teaching on TTh should adjust the syllabus; a MWF lecture lasts 50 min; a TTh therefore 75 min.

Forty Class Days As:

Appendixes (*assigned as reading and reference*)

- A Numbers, Inequalities, and Absolute Values
- B Coordinate Geometry and Lines
- C Graphs of Second-Degree Equations
- D Trigonometry

1 Functions and Models (*three days*)

- 1.1 Four Ways to Represent a Function
- 1.2 Mathematical Models: A Catalog of Essential Functions
- 1.3 New Functions from Old Functions

2 Limits and Rates of Change (*seven days*)

- 2.1 The Tangent and Velocity Problems
- 2.2 The Limit of a Function
- 2.3 Calculating Limits Using the Limit Laws
- 2.4 The Precise Definition of a Limit (*briefly*)
- 2.5 Continuity
- 2.6 Tangents, Velocities, and Other Rates of Change

3 Derivatives (*eleven days*)

- 3.1 Derivatives
- 3.2 The Derivative as a Function
- 3.2 Differentiation Formulas
- 3.4 Rates of Change in the Natural and Social Sciences
- 3.5 Derivatives of Trigonometric Functions
- 3.6 The Chain Rule
- 3.7 Implicit Differentiation
- 3.8 Higher Derivatives
- 3.9 Related Rates
- 3.10 Linear Approximations and Differentials

4 Applications of Differentiation (*eleven days*)

- 4.1 Maximum and Minimum Values
- 4.2 The Mean Value Theorem
- 4.3 How Derivatives Affect the Shape of a Graph
- 4.4 Limits at Infinity; Horizontal Asymptotes
- 4.5 Summary of Curve Sketching
- 4.7 Optimization Problems
- 4.8 Applications to Business and Economics

- 4.9 Newton's Method
- 4.10 Antiderivatives (*light*)

7 Exponential Logarithmic and Inverse Trigonometric Functions (*eight days*)

- 7.1 Inverse Functions
- 7.2 Exponential Functions Their Derivatives (*skip material with integration*)
- 7.3 Logarithmic Functions
- 7.4 Derivatives of Logarithmic Functions (*skip material with integration*)
- 7.5 Inverse Trigonometric Functions (*skip material with integration*)
- 7.7 Indeterminate Forms and L'Hospital's Rule

Syllabus: M408L
INTEGRAL CALCULUS

Text: Salas and Hille, Calculus, Eighth Edition

Responsible Parties: Kathy Davis, John Gilbert, Gary Hamrick June 19 2003

Prerequisite and degree relevance:

A grade of C or better in either M408C or in M408K.

Only one of the following may be counted: M 403L, 408D, 408L.

Calculus is offered in two equivalent sequences: a two-semester sequence, M 408C/408D, which is recommended only for students who score at least 600 on the mathematics Level I or IC Test, and a three-semester sequence, M 408K/408L/408M.

For some degrees, the two-semester sequence M 408K/408L satisfies the calculus requirement . This sequence is also a valid prerequisite for some upper-division mathematics courses, including M325K, 427K, 340L, and 362K.

M408C and M408D (or the equivalent sequence M408K, M408L, M408M) are required for mathematics majors, and mathematics majors are required to make grades of C or better in these courses.

Course description:

M408L is one of two first-year calculus courses. It is directed at students in the natural and social sciences and at engineering students. In comparison with M408D, it covers fewer chapters of the text. However, some material is covered in greater depth, and extra time is devoted the development of skills in algebra and problem solving. This is not a course in the theory of calculus.

Introduction to the theory and applications of integral calculus of functions of one variable; topics include integration, the fundamental theorem of calculus, transcendental functions, sequences, and infinite series.

Timing and Optional Sections

A 'typical' semester has 43 MWF days; a day or so will be lost to course-instructor evaluations, etc. The syllabus contains material for 40 days; you cannot afford to lose class periods. Those teaching on TTh should adjust the syllabus; a MWF lecture lasts 50 min; a TTh therefore 75 min.

Thirty Nine Class Days As:

CHAPTER 5 INTEGRATION (7 days)

5.1 The Definite Integral of a Continuous Function

5.2 The Function $F(x) = \text{Int}[f(t) \{t, a, x\}]$

- 5.3 The Fundamental Theorem of Integral Calculus
- 5.4 Some Area Problems
- 5.5 Indefinite Integrals
- 5.6 The u-Substitution; Change of Variables
- 5.7 Additional Properties of the Definite Integral

CH 6 APPLICATIONS OF THE INTEGRAL (2 days)

- 6.1 More on Area
- 6.2 Volume by Parallel Cross Sections

CH 7 TRANSCENDENTAL FUNCTIONS (3 days)

(cover the integration material in these sections)

- 7.3 The Logarithm Function, Part II
- 7.4 The Exponential Function
- 7.7 The Inverse Trigonometric Functions

CH 8 TECHNIQUES OF INTEGRATION (6 days)

- ~~8.1 Integral Tables and Review~~
- 8.2 Integration by Parts
- 8.3 Powers and Products of Trigonometric Functions
- 8.4 Trigonometric Substitutions
- 8.5 Partial Fractions *(may omit repeated quadratic factors)*

CH 14 FUNCTIONS OF SEVERAL VARIABLES (2 days)

- 14.1 Elementary Examples
- 14.2 A Brief Catalog of the Quadric Surfaces; Projections
- 14.4 Partial Derivatives

CH 15 GRADIENTS; EXTREME VALUES (1 day)

- 15.5 Maximum and Minimum Values

CH 16 DOUBLE AND TRIPLE INTEGRALS (2 days)

- 16.3 Evaluation of Double Integrals by Repeated Integrals

CH 10 SEQUENCES; INDETERMINATE FORMS (5 days)

- ~~10.1 The Least Upper Bound Axiom~~
- 10.2 Sequences of Real Numbers
- 10.3 Limit of a Sequence
- 10.4 Some Important Limits

10.5 The Indeterminate Form (0/0)

10.6 More Indeterminate Forms

10.7 Improper Integrals

CHAPTER 11 INFINITE SERIES (*11 days*)

11.1 Infinite Series

11.2 Integral Test; Comparison Theorems

11.3 The Root Test; the Ratio Test

11.4 Absolute & Conditional Convergence; Alternating Series

11.5 Taylor Polynomials, Series in x

11.6 Taylor Polynomials, Series in $(x - a)$

11.7 Power Series

11.8 Differentiation and Integration of Power Series

Syllabus: M408M

Text: Salas and Hille, Calculus, Eighth Edition

Responsible Parties: Kathy Davis, John Gilbert, Gary Hamrick June 19 2003

Prerequisite and degree relevance:

Mathematics 408L or the equivalent with a grade of at least C.

Only one of the following may be counted: Mathematics 403L, 408D, 408M (or 308M).

Calculus is offered in two equivalent sequences: a two-semester sequence, M 408C/408D, which is recommended only for students who score at least 600 on the mathematics Level I or IC Test, and a three-semester sequence, M 408K/408L/408M.

For some degrees, the two-semester sequence M 408K/408L satisfies the calculus requirement . This sequence is also a valid prerequisite for some upper-division mathematics courses, including M325K, 427K, 340L, and 362K.

M408C and M408D (or the equivalent sequence M408K, M408L, M408M) are required for mathematics majors, and mathematics majors are required to make grades of C or better in these courses.

Course description:

M408M is one of two first-year calculus courses. It is directed at students in the natural and social sciences and at engineering students. In comparison with M408D, it covers fewer chapters of the text. However, some material is covered in greater depth, and extra time is devoted the development of skills in algebra and problem solving. This is not a course in the theory of calculus.

Introduction to the theory and applications of differential and integral calculus of functions of several variables. Includes parametric equations, polar coordinates, vectors, vector calculus, functions of several variables, partial derivatives, gradients, and multiple integrals.

Thirty Nine Class Days As:

CH 9 POLAR & PARAMETRIC EQUATIONS

(4 days) (**note change in sequence!!**)

~~9.1 Translations; The Parabola~~

~~9.2 The Ellipse and the Hyperbola~~

9.3 Polar Coordinates (*some students may not have seen this before*)

9.4 Graphing in Polar Coordinates

9.5 Area in Polar Coordinates

9.6 Curves Given Parametrically

9.7 Tangents to Curves Given Parametrically

9.8 Arc Length and Speed

~~9.9 The Area of a Surface of Revolution~~

CHAPTER 12 VECTORS (5 days)

12.1 Cartesian Space Coordinates (*assign*)

12.2 Displacements; Forces and Velocities

12.3 Vectors

- 12.4 The Dot Product
- 12.5 The Cross Product
- 12.6 Lines
- 12.7 Planes

CHAPTER 13 VECTOR CALCULUS (3 days)

- 13.1 Vector Functions
- 13.2 Differentiation Formulas
- 13.3 Curves
- 13.4 Arc Length
- 13.5 Curvilinear Motion; Mechanics

CH 14 FUNCTIONS OF SEVERAL VARIABLES (4 days)

- 14.1 Elementary Examples
- 14.2 A Brief Catalog of the Quadric Surfaces; Projections
- 14.3 Graphs; Level Curves and Level Surfaces
- 14.4 Partial Derivatives
- ~~14.5 Open Sets and Closed Sets~~
- 14.6 Limits and Continuity; Equality of Mixed Partial

CH 15 GRADIENTS; EXTREME VALUES (9 days)

- 15.1 Differentiability and Gradient
- 15.2 Gradients and Directional Derivatives
- 15.3 Mean-Value Theorem; Chain Rules
- 15.4 The Gradient as a Normal; Tangent Lines and Planes
- 15.5 Maximum and Minimum Values
- 15.6 Maxima and Minima with Side Conditions

CH 16 DOUBLE AND TRIPLE INTEGRALS (7 days)

- 16.1 Multiple-Sigma Notation (assign)
- 16.2 Double Integrals
- 16.3 Evaluation of Double Integrals by Repeated Integrals
- 16.6 Triple Integrals
- 16.7 Reduction to Repeated Integrals
- 16.10 Jacobians; Changing Variables in Multiple Integration

CH 18 SOME ELEMENTARY DIFFERENTIAL EQUATIONS (7 days)

- 18.1 Introduction
- 18.2 First Order and Numerical Methods
- 18.3 The equation $y'' + ay + by = 0$
- 18.4 The equation $y'' + ay + by = f(x)$
- 18.5 Mechanical Vibrations

M427K Syllabus

ADVANCED CALCULUS FOR APPLICATIONS I

Prerequisite and degree relevance:

The prerequisite is 408D or the equivalent with a grade of at least C.

Course description:

M427K is a basic course in ordinary and partial differential equations, with Fourier series. It should be taken before most other upper division, “applied” mathematics courses. The course meets three times a week for lecture and twice more for problem sessions. Geared to the audience primarily consisting of engineering and science students, the course aims to teach the basic techniques for solving differential equations which arise in applications. The approach is problem-oriented and not particularly theoretical. Most of the time is devoted to first and second order ordinary differential equations with an introduction to Fourier series and partial differential equations at the end. Depending on the instructor, some time may be spent on applications, Laplace transformations, or numerical methods. Five sessions a week for one semester.

Text: Boyce and DiPrima Elementary Differential Equations and Boundary Value Problems, Seventh Edition
This text is required for most sections; honors, computer supplement sections may use other texts.

Required Topics

It will be impossible to cover everything here adequately. The core material which must be covered is selected sections from Chapters 1, 2, 3, 5, 10. Chapter 7 is so important that it ought to be covered, but be aware that most students have not already had matrix methods, and you will likely find yourself covering the 2 by 2 case. You might then do stability, etc. Numerical methods are becoming increasingly important, and covering this topic here is a good lead in to the department’s new computational science degree. Engineers like their students to have seen some Laplace transforms. This will leave time for other topics, and you may choose to emphasize some over others: stability, higher order equations, applications. Whichever approach you take, you will have to carefully plan your sections and time to be spent on them.

Resources

If you are new to this course, you might talk to the senior faculty who teach this course regularly: Beckner, de la Llave, Dollard, Friedman, Gamba, Koch, Showalter, Uhlenbeck and others.

(continued next page)

Chapter I Introduction (2 - 3 weeks for Chapters 1 and 2)

- 1.1 Some Basic Mathematical Models; Direction Fields
- 1.2 Solutions of Some Differential Equations 9
- 1.3 Classification of Differential Equations 17
- 1.4 Historical Remarks 23

Chapter 2 First Order Differential Equations (2 - 3 weeks for Chapters 1 and 2)

- 2.1 Linear Equations with Variable Coefficients 29
- 2.2 Separable Equations 40
- 2.3 Modeling with First Order Equations 47 (optional)
- 2.4 Differences Between Linear and Nonlinear Equations
- 2.5 Autonomous Equations and Population Dynamics 74 (optional)
- 2.6 Exact Equations and Integrating Factors 89
- 2.7 Numerical Approximations: Euler's Method 96 (optional unless you do Ch 8)
- 2.8 The Existence and Uniqueness Theorem 105
- 2.9 First Order Difference Equations 115 (optional unless you do stability)

Chapter 3 Second Order Linear Equations (2 - 3 weeks)

- 3.1 Homogeneous Equations with Constant Coefficients 129
- 3.2 Fundamental Solutions of Linear Homogeneous Equations 137
- 3.3 Linear Independence and the Wronskian 147
- 3.4 Complex Roots of the Characteristic Equation 153
- 3.5 Repeated Roots; Reduction of Order 160
- 3.6 Nonhomogeneous Equations; Method of Undetermined Coefficients 169
- 3.7 Variation of Parameters 179 (optional)
- 3.8 Mechanical and Electrical Vibrations 186 (optional)
- 3.9 Forced Vibrations 200 (optional)

Chapter 4 Higher Order Linear Equations (cover quickly)

- 4.1 General Theory of n th Order Linear Equations 209
- 4.2 Homogeneous Equations with Constant Coefficients 214
- ~~4.3 The Method of Undetermined Coefficients 222~~
- 4.4 The Method of Variation of Parameters 226

Chapter 5 Series Solutions of Second Order Linear Equations (2 weeks)

- 5.1 Review of Power Series 231 (optional)
- 5.2 Series Solutions near an Ordinary Point, Part I 238
- 5.3 Series Solutions near an Ordinary Point, Part II 249
- 5.4 Regular Singular Points 255
- 5.5 Euler Equations 260
- 5.6 Series Solutions near a Regular Singular Point, Part I

Chapter 6 The Laplace Transform (1 week)

- 6.1 Definition of the Laplace Transform 293
- 6.2 Solution of Initial Value Problems 299
- 6.3 Step Functions 310
- 6.4 Differential Equations with Discontinuous Forcing Functions
- 6.5 Impulse Functions 324
- 6.6 The Convolution Integral 330

Chapter 7 Systems of First Order Linear Equations (1 – 2 weeks)

- 7.1 Introduction 339
- 7.2 Review of Matrices 348
- 7.3 Systems of Linear Algebraic Equations; Linear Independence, Eigenvalues, Eigenvectors 357
- 7.4 Basic Theory of Systems of First Order Linear Equations 368
- 7.5 Homogeneous Linear Systems with Constant Coefficients 373
- 7.6 Complex Eigenvalues 384
- 7.7 Fundamental Matrices 393
- 7.8 Repeated Eigenvalues 401

Chapter 8 Numerical Methods (1 week if covered) (optional)

- 8.1 The Euler or Tangent Line Method
- 8.2 Improvements on the Euler Method
- 8.3 The Runge-Kutta Method 435
- 8.4 Multistep Methods 439
- 8.5 More on Errors; Stability 445

Chapter 9 Nonlinear Differential Equations and Stability 459

- 9.1 The Phase Plane; Linear Systems 459
- 9.2 Autonomous Systems and Stability 471
- 9.3 Almost Linear Systems 479
- 9.4 Competing Species 491
- 9.5 Predator-Prey Equations 503
- 9.6 Liapunov's Second Method 511
- 9.7 Periodic Solutions and Limit Cycles 521
- 9.8 Chaos and Strange Attractors; the Lorenz Equations

Chapter 10 Partial Differential Equations and Fourier Series (3 weeks)

- 10.1 Two-Point Boundary Value Problems 541
- 10.2 Fourier Series 547
- 10.3 The Fourier Convergence Theorem 558
- 10.4 Even and Odd Functions 564
- 10.5 Separation of Variables; Heat Conduction in a Rod 573
- ~~10.6 Other Heat Conduction Problems 581~~
- 10.7 The Wave Equation; Vibrations of an Elastic String 591 (optional)
- 10.8 Laplace's Equation 604 (optional)
- Appendix A. Derivation of the Heat Conduction Equation (optional)
- Appendix B. Derivation of the Wave Equation 617 (optional)

(remarks by Beckner, de la Llave, Showalter; typesetting Davis 8/24/00)

M427L Syllabus

ADVANCED CALCULUS FOR APPLICATIONS II

Prerequisite and degree relevance:

The prerequisite is 408D or the equivalent with a grade of at least C.

Course description:

Topics include matrices, elements of vector analysis and calculus functions of several variables, including gradient, divergence, and curl of a vector field, multiple integrals and chain rules, length and area, line and surface integrals, Green's theorem in the plane and space; If time permits, topics in complex analysis may be included. This course has three lectures and two problem sessions each week. It is anticipated that most students will be engineering majors. Five sessions a week for one semester.

Text: Marsden & Tromba, Vector Calculus, Fourth Edition
(continued next page)

I THE GEOMETRY OF EUCLIDEAN SPACE (6 days)

- 1.1 Vectors in two- and three-dimensional space
- 1.2 The inner product, length, and distance
- 1.3 Matrices, determinants, and the cross product
- 1.4 Cylindrical and spherical coordinates
- 1.5 n-dimensional Euclidean space

2 DIFFERENTIATION (5-6 days)

(add discussion of linear maps, matrices)

- 2.1 The geometry of real-valued functions
- 2.2 Limits and continuity (assign to read)
- 2.3 Differentiation
- 2.4 Introduction to paths
- 2.5 Properties of the derivative
- 2.6 Gradients and directional derivatives

3 HIGHER-ORDER DERIVATIVES (3 days)

- 3.1 Iterated partial derivatives (briefly)
- 3.2 Taylor's theorem
- 3.3 Extrema of real-valued functions
- 3.4 Constrained extrema and Lagrange multipliers
- 3.5 The implicit function theorem(if time permits) 3,5

4 VECTOR-VALUED FUNCTIONS (5 days)

- 4.1 Acceleration and Newton's Second Law
- 4.2 Arc length
- 4.3 Vector fields
- 4.4 Divergence and curl

5 DOUBLE AND TRIPLE INTEGRALS (3 days)(cover first three sections in one lecture)

- 5.1 Introduction
- 5.2 The double integral over a rectangle
- 5.3 The double integral over more general regions
- 5.4 Changing the order of integration
- 5.6 The triple integral

6 THE CHANGE OF VARIABLES FORMULA (3 days)

- 6.1 The geometry of maps (not crucial)
- 6.2 The change of variables theorem (lightly)
- 6.3 Applications of double, triple integrals(if time permits)

7 INTEGRALS OVER PATHS AND SURFACES (7 days) (next chapter depends heavily on this)

- 7.1 The path integral
- 7.2 Line integrals
- 7.3 Parametrized surfaces
- 7.4 Area of a surface
- 7.5 Integrals of scalar functions over surfaces
- 7.6 Surface integrals of vector functions

8 THEOREMS OF VECTOR ANALYSIS (5-6 days) (may reorder as §§8.1, 8.4, 8.2, 8.3)

- 8.1 Green's theorem
- 8.2 Stokes' theorem
- 8.3 Conservative fields
- 8.4 Gauss' theorem

M325K Syllabus

Discrete Mathematics

Prerequisite and degree relevance:

M408D with a grade of at least C, or consent of instructor. This is a first course that emphasizes understanding and creating proofs. Therefore, it provides a transition from the problem-solving approach of calculus to the entirely rigorous approach of advanced courses such as M365C or M373K. The number of topics required for coverage has been kept modest so as to allow adequate time for students to develop theorem-proving skills.

Text: Faculty have a choice among three recommended texts. The preferred texts are Epp, Discrete Mathematics with Applications, second edition; Scheinerman Mathematics: A Discrete Introduction, first edition. A text in use before these was Grimaldi, Discrete and Combinatorial Mathematics, current edition. Of the three, Grimaldi is the most directed towards applications in computer Science and Electrical Engineering. He also tends to integrate his applications directly into the flow of the text rather than discussing them separately.

Students in M325K are a mixture: 30-40% Mathematics majors, 10% Computer Science majors, 30-40% Engineering majors, many from Electrical Engineering; another 10% may be from Elementary Education with a Mathematics concentration. Some of the math majors may have changed their majors from Business or Engineering or Computer Science – opting to try Mathematics since they have accumulated more hours in math than in any other area.

The course should serve our majors as a transition from early computational courses, to the more problem-solving, abstract and rigorous courses they will encounter in the BA and BS degree programs. It serves the EE's as a required course in discrete techniques; they are expected to deal with proof techniques in a discrete context. The course is not intended to be all rigour, but proofs and the cognitive skills requisite to read, comprehend and do proofs are a major theme. Students are expected to become familiar with the language and techniques of proof (converse, if and only if, proof by contradiction, etc); they should also see detailed, rigorous proofs presented in class. More importantly, they need to develop the ability to read and understand proofs on their own, and they must begin doing proofs; this cannot be slighted.

Discrete mathematics offers a variety of contexts in which the student can begin to understand mathematical techniques and appreciate mathematical culture. Abstraction per se is not the goal; discrete mathematics offers very concrete computational contexts, and this can be exploited to develop a feeling for what it is that proofs, and proof techniques, say and do.

Topics may include: fundamentals of logic and set theory; functions and relations; basic properties of integers, and elementary number theory; recursion and induction; counting techniques and combinatorics; introductory graph theory.

The instructor should focus on depth of understanding rather than breadth of coverage, and subsequent courses will assume that students have seen induction and set theory in this course.

responsible party: Kathy Davis and Martha Smith, 2001

M328K Syllabus

INTRODUCTION TO NUMBER THEORY

Prerequisite and degree relevance:

One of M311 or M341 is required, with a grade of at least C.

This is a first course that emphasizes understanding and creating proofs; therefore, it must provide a transition from the problem-solving approach of calculus to the entirely rigorous approach of advanced courses such as M365c or M373K. The number of topics required for coverage has been kept modest so as to allow instructors adequate time to concentrate on developing the students' theorem-proving skills.

A list of texts from which the instructor may choose is maintained in the text office.

The choice of text will determine the exact topics to be covered. The following subjects should definitely be included:

Divisibility: divisibility of integers, prime numbers and the fundamental theorem of arithmetic.

Congruences: including linear congruences, the Chinese remainder theorem, Euler's ϕ -function, and polynomial congruences, primitive roots.

The following topics may also be covered, the exact choice will depend on the text and the taste of the instructor.

Diophantine equations: (equations to be solved in integers), sums of squares, Pythagorean triples.

Number theoretic functions: the Mobius Inversion formula, estimating and partial sums $\sum_{n \leq x} f(n)$ of other number theoretic functions.

Approximation of real numbers by rationals: Dirichlet's theorem, continued fractions, Pell's equation, Liouville's theorem, algebraic and transcendental numbers, the transcendence of e and/or π .

March, 1989

M340L Syllabus

MATRICES AND MATRIX CALCULATIONS

Prerequisite and degree relevance:

The prerequisite is one semester of calculus or consent of instructor. Only one of M311, M341, M340L may be counted.

Course description:

The goal of M340L is to present the many uses of matrices and the many techniques and concepts needed in such uses. The emphasis is on understanding and using techniques and concrete concepts rather than on learning proofs and abstractions. The course is designed for applications-oriented students such as those in the natural and social sciences, engineering, and business. Topics might include matrix operations, systems of linear equations, introductory vector-space concepts (e.g., linear dependence and independence, basis, dimension), determinants, introductory concepts of eigensystems, introductory linear programming, and least square problems.

Text: David C. Lay, Linear Algebra and its Applications, 2nd edition.

Background: M341 (Linear Algebra and Matrix Theory) and M340L (Matrices and Matrix Calculations) cover similar material. However, the emphasis in M340L is much more on calculational techniques and applications, rather than abstraction and proof. (M341 is the linear algebra course required of math majors and contains a substantial introduction to proof component). Credit cannot be received for both M341 and M340L.

The syllabus is essentially the first seven chapters of Lay, namely 1) Systems of linear equations, 2) Vector and matrix equations, 3) Matrix algebra, 4) Determinants, 5) Vector spaces, 6) Eigenvalues and eigenvectors, and 7) Orthogonality and least-squares.

Each section is designed to be covered in a single 50-minute lecture. However, in practice chapters 1, 3 and 4 should be covered more quickly, allowing more time for chapters 2, 5, 6 and 7. Most incoming 340L students are already quite adept at solving systems of equations, and it is important to move quickly at the beginning of the term to material that does challenge them, reserving time to tackle the more difficult vector-space concepts of chapters 2 and 5. Many of the essential concepts, such as linear independence, are covered twice: once in chapter 2 for \mathbb{R}^n , then again in chapter 5 for a general vector space.

Read the "Note to the Instructor" at the beginning of the book. The core of 340L is indeed the "core topics" listed on pages x-x/, plus sections 4.1 and 4.2. Various faculty members disagree strongly about which of the remaining "supplementary topics" and "applications" are most important; use your own judgement. You will probably have time for about half a dozen of these supplementary topics and applications.

Computers: Linear algebra lends itself extremely well to computerization, and there are many packages that students can use. Once students have learned the theory of row-reduction and matrix multiplication (which they pick up very quickly), they should be encouraged to use Maple, Matlab, Mathematica, or a similar package. There are also many "projects" on the departmental computers that students can learn from. Many concepts in the book, especially in the later chapters (e.g. understanding the long-time behavior of a dynamical system from its eigenvalues), can be absorbed quite easily through numerical experimentation.

Sadun April 1996

M 341 Syllabus

LINEAR ALGEBRA AND MATRIX THEORY

Prerequisite and degree relevance:

M408D or the equivalent or consent of instructor. (Credit may not be received for both M341 and M340L. M341 is required for math majors; furthermore, math majors must make a grade of at least C in M341.)

The department allows instructors to choose between two texts. Whichever text you choose, it is recommended that you read both syllabi, as each represents a different view of the course. The choice of text constrains which topics you can reasonably cover, affecting in particular determinants and eigenvalues.

Primary Text - Andrilli & Hecker, Elementary Linear Algebra 2nd edition

This course has three purposes and the instructor should give proper weight to all three. The students should learn some linear algebra - for most of them, this will be the only college linear algebra course they take. This is one of the first proof courses these students will take and they need to develop some proof skills. Finally, this is, for almost all students, the introductory course in mathematical abstraction and provides a necessary prerequisite for a number of our upper division courses. To teach this course successfully, the instructor should establish modest goals on all three fronts. On one hand, a student should not be able to pass this course simply by doing calculational problems well, but on the other hand, overly ambitious proof and abstraction goals simply discourage teacher and student alike.

To teach proofs, the instructor should cover Section 1.3 thoroughly to introduce various proof techniques. Afterwards, a liberal (but not overwhelming) number of proofs should be sprinkled in the lectures, homework, and tests.

In teaching abstraction, it is critical to remember that almost no students are capable of becoming truly comfortable with it in a single semester; it is self-defeating to establish this as a goal. The study of abstract vector spaces is a unified treatment of various familiar vector spaces and students in this course should never be taken very far from the concrete. Linear algebra is the perfect subject for teaching students that abstraction can be a friend. For example, it underlines nicely how the solutions to a homogeneous system are better behaved than the solutions to a non-homogeneous system. However, amusing examples of unnatural algebraic systems that may or may not be vector spaces should be avoided.

A warning should be given concerning the calculational homework problems. The authors, intending the students to take full advantage of technology, have made no effort to make problems come out neatly.

Suggested Coverage:

Chapter 1 - The first two sections provide necessary definitions for Section 1.3. The entire chapter should be covered. Generally move quickly but cover 1.3 meticulously.

Chapter 2 - Cover all sections but again move reasonably to have enough time for Chapters 4 and 5.

Chapter 3 - For those instructors familiar with the first edition, it should be noted that this chapter has been redone and the unusual treatment of determinants has been replaced by a more conventional one.

Cover Sections 3.1 and 3.2. Section 3.3 is optional - you might also choose to cover parts of this section. Section 3.4 is new and is a fairly reasonable attempt to introduce eigenvalues before introducing linear transformations. The instructor should cover at least part of this section, all if desired.

Chapter 4 - This chapter is the meat of the course and the instructor should plan to take a good deal of time here. Sections 4.1-4.4 should be covered thoroughly. The material in Section 4.5 is also important; however, it is probably the most poorly written section in the book. An alternate presentation is recommended. Sections 4.6 and 4.7 should also be covered.

Chapter 5 - In a perfect world, the entire chapter should be taught. Realistically, at least Sections 5.1 and 5.2 should be covered.

Heitmann 8/4/990

(continued next page)

Alternate Text: Johnson, Riess and Arnold, Introduction to Linear Algebra, 4th edition.

This course has three aims and the instructor needs to properly balance all three. One, of course, is to learn linear algebra; for most of the students, this will be the only college linear algebra class they will take (although they should be encouraged to continue with M346). Second, this is an "introduction to proof" course, and the students need to develop basic proof-writing skills. This needs to be stressed; having the instructor prove theorems at the board is not enough. The instructor should look over section 1.3 of Andrilli and Hecker on methods of proof, and adapt whatever material seems useful. Finally, this is one of the first courses on mathematical abstraction. Indeed, the point where students have the most trouble is in making the transition from \mathbb{R}^n to an abstract vector space.

It is next to impossible to cover all three areas thoroughly; you have to make compromises on each score. On learning proofs, we recommend spending some time on explicit proof techniques, and then sprinkling a liberal (but not overwhelming) number of proofs in the lectures, homework and tests. Avoid the temptation of giving complete proofs in the lectures in place of demanding proofs from the students proofs in homework and on exams are more important than proofs in lecture.

On abstraction, the book does a good job of ramping up gradually. Most of the difficult concepts, like linear independence, basis, and linear transformations are covered first in \mathbb{R}^n (Chapter 2) and then repeated for abstract vector spaces (Chapter 4). This is one of the strengths of the book. Ideally, a student should finish the course understanding that matrix manipulations are mechanical and easy, that computations in \mathbb{R}^n reduce to matrix manipulations, and that a basis makes an abstract vector space look just like \mathbb{R}^n . However, these connections will not sink in immediately, and what looks like repetition to you will look new to the student. Allow enough time for both repetitions, at the expense (if necessary) of covering other material.

Finally, on learning linear algebra, the syllabus is modest. Unlike M340L, eigenvalues are not part of the core syllabus. The syllabus is essentially Chapters 1, 2 and 4, with optional sections and least-squares skipped. If there is time at the end, you can dabble in eigenvalues (Chapter 3) or return to least squares. The theory of determinants (Chapter 5) and applications of eigenvalues (Section 3.8 and Chapter 6) are simply not part of this course.

Suggested core coverage:

Chapter 1. Skip sections 1.4 and 1.8. Students are quick to learn the mechanics of GaussJordan elimination, matrix multiplication, and obtaining inverses, and you should be able to move quickly through these sections. However, they are much slower to understand what information is contained in a row-reduced matrix, especially when the matrix is not square.

Chapter 2. Only cover sections 2.1-2.7. The material in this chapter makes for simple and instructive proofs of nontrivial theorems.

Chapter 4. Skip section 4.6 for now. This chapter is the guts of the course. The challenge here is abstraction. If proofs have been sufficiently stressed in Chapter 2, you can probably back off the proofwriting here.

Additional topics if time permits

There are two recommended directions to go, and there is almost certainly not time for both. One is to cover inner products, least squares and Gram-Schmidt (Sections 2.8, 2.9, and 4.6). The second, and recommended, option is to provide a brief exposure to eigenvalues and determinants (Chapter 3). The book is written with Chapters 3 and 4 independent, so there is no problem with doing Chapter 4 first and only then returning to Chapter 3.

Orthogonality and least squares are extremely important, eigenvalues even more so. Both sets of topics are covered thoroughly in M346. Whichever you choose, remember that the goal in 341 is to provide a first exposure to these topics, not depth.

(Lorenzo Sadun 8/4/99)

M343K Syllabus

INTRODUCTION TO ALGEBRAIC STRUCTURES

Prerequisite and degree relevance:

Either consent of Mathematics Advisor, or two of M341, 328K, 325K (Philosophy 313K may be substituted for M325K), with a grade of at least C. This course is designed to provide additional exposure to abstract rigorous mathematics on an introductory level. Students who demonstrate superior performance in M311 or M341 should take M373K RATHER THAN 343K Those students whose performance in M311 or M341 is average should take M343K before taking M373K Credit for M343K can NOT be earned after a student has received credit for M373K with a grade of at least C.

Course description:

Elementary properties of the integers, groups, rings, and fields are studied.

Possible Texts:

Durbin, John R., Modern Algebra: An Introduction, 4th edition, John Wiley & Sons, 2000.

Rotman, Joseph J., A First Course in Abstract Algebra, 2nd edition, Prentice Hall, 2000.

The number of topics should be kept modest to allow adequate time to concentrate on developing the students' theorem-proving skills. Some instructors will prefer to introduce groups before rings and some will reverse the order. In any case, below are some reasonable choices of topics. One should not try to cover all of these topics. *It is very important to avoid superficial coverage of too many topics.* All potential graduate students will take M373K, where it is possible to expect more and to do more.

Topics:

Groups: Axioms, basic properties, examples, symmetry, cosets, Lagrange's Theorem, isomorphism. Optional: Homomorphisms, quotient groups, and the Fundamental Homomorphism Theorem.

Rings: Axioms, basic properties, examples, integral domains, and fields. Optional: More about polynomial rings and properties of fields.

Other options: Groups acting on sets, characterization of the familiar number systems in terms of ring and field properties, and other applications of groups.

Durbin July 2000

M361 Syllabus

THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

Prerequisite and degree relevance:

The prerequisite is M427K or M427L with a grade of at least C, or consent of the instructor.

Course description:

M361 consists of a study of the properties of complex analytic functions. Students are mainly from physics and engineering, with some mathematics majors and joint majors. Representative topics are Cauchy's integral theorem and formula, Laurent expansions, residue theory and the calculation of definite integrals, analytic continuation, and asymptotic expansions. Rigorous proofs are given for most results, with the intent to provide the student with a reliable grasp of the results and techniques.

Text: a reasonable text is Brown and Churchill, Complex variables and Applications, sixth edition.

Topics:

Complex Numbers

Analytic Functions

Elementary Functions

Integrals

Series

Residues and poles

Applications of Residues

responsible party: John Dollard 2001

M361K Syllabus

INTRODUCTION TO REAL ANALYSIS

Prerequisite and degree relevance:

Either consent of Mathematics Advisor, or two of M341, 328K, 325K (Philosophy 313K may be substituted for M325K), with a grade of at least C. May not be counted by students with credit for M365K with a grade of C or better.

Course description:

This is a rigorous treatment of the real number system, of real sequences, and of limits, continuity, derivatives, and integrals of real-valued functions of one real variable.

Text: A reasonable text is "Introduction to Real Analysis" by Bartle and Sherbert. The course might cover the bulk of chapters one through six in that book.

Topics:

The real number system: the axiomatic description of the real number system as the unique complete ordered field, with special emphasis on the completeness axiom; the elementary topology of the real line.

Real sequences: the definition and elementary properties of sequential limits; subsequences and accumulation points; monotone sequences; inferior and superior limits; the Bolzano-Weierstrass theorem.

Limits and continuity of functions: the definition and elementary properties of limits of functions, including the usual variations on the basic theme (e.g., one-sided limits, infinite limits, limits at infinity); continuity; the fundamental facts concerning continuous functions on intervals (e.g., Intermediate Value Theorem, Maximum-Minimum Theorem, continuity of inverse functions, uniform continuity on closed intervals).

Differentiation: the definition and geometric significance of the derivative; differentiation rules; the Mean Value Theorem and its consequences; Taylor's Theorem; L'Hospital's rules; convexity.

Riemann Integration: the definition and elementary properties of the Riemann integral; the integrability of continuous functions and monotone functions; the Fundamental Theorems of Calculus.

March, 1989

MATH 362K Syllabus

PROBABILITY I

Prerequisite and degree relevance:

M408D with a grade of at least C. A student may not receive credit for M316 after completing M362K with a grade of C or better.

Course description:

This is an introductory course in the mathematical theory of probability, thus it is fundamental to further work in probability and statistics. Principles of set theory and a set of axioms for probability are used to derive some probability density and/or distribution functions. Special counting techniques are developed to handle some problems. Properties associated with a “random variable” are developed for the usual elementary distributions. Both theorem proving and problem solving are required.

Suggested Textbook : A First Course in Probability, 5th ed., by Sheldon Ross (Prentice Hall, 1998).

The following course outline refers to section numbers in Ross' book, and assumes a MWF lecture format (it must be modified for TTh classes)

Some Alternate Textbooks:

1. Charles M. Grinstead and J. Laurie Snell, Introduction to Probability, 2nd revised ed., AMS, 1997. This book has an interesting style that is different from the more standard format of Ross. It introduces some important ideas in examples and exercises, so the instructor needs to know what not to omit. There is too much emphasis on computation for this course, but otherwise it is very well written, with many good examples and exercises.

2. Saeed Ghahramani, Fundamentals of Probability, Prentice Hall, 1996. Similar to Ross' text.

Background: M362K is required of all undergraduate mathematics majors, and it is a prerequisite for courses in statistics. However, many of the students are majoring in other subjects (e.g., computer science or economics), and have little preparation in abstract mathematics. Calculus skills (integration and infinite series) tend to be weak, even at this level. The course tends to be relatively easier for the first three to four weeks, so some students get the wrong impression as to its difficulty. Clarifying this early for the students can avoid unpleasant surprises later.

Course Content: Emphasize problem solving and intuition. Some advanced concepts should be presented without proof, so as to devote more attention to the examples.

Basic combinatorics: Counting principle, permutations, combinations.

Basic concepts: Sample spaces, events, basic axioms and theorems of probability, finite sample spaces with equally likely probabilities.

Conditional probability: Reduced sample space, independence, Bayes' Theorem.

Random variables: Discrete and continuous random variables, discrete probability functions and continuous probability density functions, distribution functions, expectation, variance, functions of random variables.

Special distributions: Bernoulli, Binomial, Poisson, and Geometric discrete random variables. Uniform, Normal, and Exponential continuous random variables. Approximation of Binomial by Poisson or Normal.

Jointly distributed random variables: Joint distribution functions, independence, conditional distributions, expectation, covariance

Sums of independent random variables: expectation, variance.

Inequalities and Limit theorems: Markov's and Chebyshev's inequalities, Weak and Strong Law of Large Numbers, Central Limit Theorem.

(continued next page)

§1.1-4	3 lectures	Limit this material to one week.
§2.1-5,7	4 lectures	Do not get bogged down in §2.5; limit it to about one lecture.
§3.1-4	4 lectures	Students like tree diagrams for Bayes' Theorem, and need more help and examples to learn how to extract information from word problems.
§4.1-6	4 lectures	
§4.7-8,9.1	3 lectures	Omit §4.7.2 and §4.8.1. One could delay §4.8 to §5.4.1. §4.9.2-3 are optional.
§5.1-5,7	7 lectures	Omit §5.5.1. §5.6.1 is optional.
§6.1-5	4 lectures	
§7.1-3	2 lectures	§7.4,6,7 are optional, as is correlation.
§8.1-4	3 lectures	Do not let any optional material crowd out the limit theorems. Emphasize intuitive understanding of the Central Limit Theorem by examples, and omit the proof, especially if optional §7.6 is not covered.
§9,10	0 lectures	One or two topics are optional.

There are a wealth of examples in the text, so the instructor has time to present only some of them. The outline above allows room for 34 lectures, 3 in class exam days, and 3 review days, for a total of 40 days. A typical semester has 42 MWF class days in the fall and 44 in the spring, so a few days for make-up or optional material are provided. It is likely that an instructor will find no time for any of the optional material.

T. Arbogast, J. Luecke, and M. Smith, April 1999

M365C Syllabus

REAL ANALYSIS I

Prerequisite and degree relevance:

Either consent of Mathematics Advisor, or two of M341, 328K, 325K (Philosophy 313K may be substituted for M325K), with a grade of at least C.

Students who receive a grade of C in M325K or M328K are advised to take M361K before attempting M365C.

Course description:

This course is an introduction to Analysis. Analysis together with Algebra and Topology form the central core of modern mathematics. Beginning with the notion of limit from calculus and continuing with ideas about convergence and the concept of function that arose with the description of heat flow using Fourier series, analysis is primarily concerned with infinite processes, the study of spaces and their geometry where these processes act and the application of differential and integral to problems that arise in geometry, pde, physics and probability.

Text: An appropriate text is Rudin "Principles of Mathematical Analysis" and the course should roughly cover its first seven chapters. The main difference between M361K and M365C lies in the more abstract metric space point of view in the latter. A strong student should be able to handle M365C without first taking M361K.

The real number system and Euclidean spaces: the axiomatic description of the real number system as the unique complete ordered field; the complex numbers; Euclidean space \mathbb{R}^n .

Metric spaces: elementary metric space topology, with special emphasis on Euclidean spaces; sequences in metric spaces - limits, accumulation points, subsequences, etc.; Cauchy sequences and completeness; compactness in metric spaces; compact sets in \mathbb{R}^n ; connectedness in metric spaces; countable and uncountable sets.

Continuity: limits and continuity of mappings between metric spaces, with particular attention to real-valued functions defined on subsets of \mathbb{R}^n ; preservation of compactness and connectedness under continuous mapping; uniform continuity.

Differentiation on the line: the definition and geometric significance of the derivative of a real-valued function of a real variable; the Mean Value Theorem and its consequences; Taylor's Theorem; L'Hospital's rules.

Riemann integration on the line: the definition and elementary properties of the Riemann integral; existence theorems for Riemann integrals; the Fundamental Theorems of Calculus.

Sequences and series of functions: uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation.

March, 1989

M365D Syllabus

REAL ANALYSIS II

Prerequisite and degree relevance:

Mathematics 365C, with a grade of at least C.

Course description:

A rigorous treatment of selected topics in real analysis, such as Lebesgue integration, or multivariable integration and differential forms.

Possible Texts:

Spivak, Calculus. Ross, Elementary Analysis; the Theory of Calculus.

Fulks, Advanced Calculus.

This is a continuation of 365C with emphasis on functions of several variables. The treatment should be reasonably simple (for example, it is inappropriate to use Banach space language). The teacher can select his own textbook, and should weigh his/her choice carefully in light of the above remarks and not get too ambitious.

March, 1989

M367K Syllabus

TOPOLOGY I

Prerequisite and degree relevance:

Mathematics 361K or 365C or consent of instructor.

Course description:

This will be a first course that emphasizes understanding and creating proofs; therefore, it provides a transition from the problem-solving approach of calculus to the entirely rigorous approach of advanced courses such as M365C or M373K. The number of topics required for coverage has been kept modest so as to allow instructors adequate time to concentrate on developing the students' theorem proving skills. The syllabus below is a typical syllabus. Other collections of topics in topology are equally appropriate. For example, some instructors prefer to restrict themselves to the topology of the real line or metric space topology.

Cardinality: 1-1 correspondance, countability, and uncountability.

Definitions of topological space: basis, sub-basis, metric space.

Countability properties: dense sets, countable basis, local basis.

Separation properties: Hausdorff, regular, normal.

Covering properties: compact, countably compact, Lindelof.

Continuity and homeomorphisms: properties preserved by continuous functions, Urysohn's Lemma, Tietze Extension Theorem.

Connectedness: definition, examples, invariance under continuous functions.

Notes containing definitions, theorem statements, and examples have been developed for this course and are available. The notes include some topics beyond those listed above.

March, 1989

Math 373K Syllabus

ALGEBRAIC STRUCTURES I

Prerequisite and degree relevance:

Either consent of Mathematics Advisor, or two of M341, 328K, 325K (Philosophy 313K may be substituted for M325K), with a grade of at least C.

Students who receive a grade of C in M325K or M328K are advised to take M343K before attempting M373K.

Course description:

M373K is a rigorous course in pure mathematics. The syllabus for the course includes topics in the theory of groups and rings. The study of group theory includes normal subgroups, quotient groups, homomorphisms, permutation groups, the Sylow theorems, and the structure theorem for finite abelian groups. The topics in ring theory include ideals, quotient rings, the quotient field of an integral domain, Euclidean rings, and polynomial rings.

This course is generally viewed (along with 365C) as the most difficult of the required courses for a mathematics degree. Students are expected to produce logically sound proofs and solutions to challenging problems.

Text: Herstein, Topics in Algebra

Material to be covered: Chapters 1, 2, 3 and if time permits some topics in Chapters 4 and 5.

This includes: properties of the integers, including divisibility and prime factorization; properties of groups, including subgroups, homomorphisms, permutation groups, the Sylow theorems; properties of rings, including subrings and ideals, homomorphisms, domains, especially Euclidean, principal ideal and unique factorization domains, polynomial rings.

If time permits: Fields, elementary properties of vector spaces including concept of dimension, field extensions.

We will be glad to discuss any questions or listen to any comments which you may have now or during the term on the course, the text, or the syllabus.

The Undergraduate Curriculum Committee

M373L Syllabus

ALGEBRAIC STRUCTURES II

Prerequisite and degree relevance:

The prerequisite is M373K. M373L is strongly recommended for undergraduates contemplating graduate study in mathematics.

Course description:

M373L is a continuation of M373K, covering a selection of topics in algebra chosen from field theory and linear algebra. Emphasis is on understanding theorems and proofs.

Text: Herstein, Topics in Algebra

Material to be covered: Chapters 4, 5, (sections 1-3), 6.

This includes: elementary properties of vector spaces and fields, including bases and dimension, elementary properties of linear transformations, relations to matrices, change of bases, dual spaces, characteristic roots, canonical forms, inner product spaces, normal transformations, quadratic and bilinear forms.

If time permits: topics left up to the instructor.

The Undergraduate Curriculum Committee

M175-WR, M275-WR, M375-WR CONFERENCE COURSE WRITING COMPONENT

M 301 COLLEGE ALGEBRA

M302 INTRODUCTION TO MATHEMATICS

M303D APPLICABLE MATHEMATICS

M305G ELEMENTARY FUNCTIONS AND COORDINATE GEOMETRY

M 316K FOUNDATIONS OF ARITHMETIC

M 316L FOUNDATIONS OF GEOMETRY, STATISTICS, AND PROBABILITY

M403K CALCULUS I FOR BUSINESS AND ECONOMICS

M403L CALCULUS II FOR BUSINESS AND ECONOMICS

408C DIFFERENTIAL AND INTEGRAL CALCULUS

408D DIFFERENTIAL AND INTEGRAL CALCULUS

408K DIFFERENTIAL CALCULUS

308L INTEGRAL CALCULUS

308M MULTIVARIABLE CALCULUS

M427K ADVANCED CALCULUS FOR APPLICATIONS I

M427L ADVANCED CALCULUS FOR APPLICATIONS II

M325K DISCRETE MATHEMATICS

M328K INTRODUCTION TO NUMBER THEORY

M340L MATRICES AND MATRIX CALCULATIONS

M 341 LINEAR ALGEBRA AND MATRIX THEORY

M343K INTRODUCTION TO ALGEBRAIC STRUCTURES

M361 THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

M361K INTRODUCTION TO REAL ANALYSIS

MATH 362K PROBABILITY I

M365C REAL ANALYSIS I

M365D REAL ANALYSIS II

M367K TOPOLOGY I

Math 373K ALGEBRAIC STRUCTURES I

M373L ALGEBRAIC STRUCTURES II