Introduction to Multi-Criteria Decision Making and the Evidential Reasoning Approach

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Contents

ABSTRACT

In this paper, the main features of multiple criteria decision making (MCDM) problems are first summarised followed by a list of typical techniques used in MCDM analysis. To tackle the main features of MCDM problems, the Evidential Reasoning (ER) approach, a recent development and a breakthrough in handling hybrid MCDM problems with uncertainties, is introduced. Then the Intelligent Decision System, a software developed to implement the ER approach, is briefly demonstrated with an example followed by a comparison between the ER approach and the AHP method, one of the most widely used MCDM methods. Finally some terminologies related to the ER approach and the IDS software are defined and relevant references are listed.

1 INTRODUCTION

Multiple criteria decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting, criteria. MCDM problems are common in everyday life. In personal context, a house or a car one buys may be characterised in terms of price, size, style, safety, comfort, etc. In business context, MCDM problems are more complicated and usually of large scale. For example, many companies in Europe are conducting organisational self-assessment using hundreds of criteria and sub-criteria set in the EFQM (European Foundation for Quality Management) business excellence model. Purchasing departments of large companies often need to evaluate their suppliers using a range of criteria in different area, such as after sale service, quality management, financial stability, etc..

Although MCDM problems are widespread all the time, MCDM as a discipline only has a relatively short history of about 30 years. The development of the MCDM discipline is closely related to the advancement of computer technology. In one hand, the rapid development of computer technology in recent years has made it possible to conduct systematic analysis of complex MCDM problems. On the other hand, the widespread use of computers and information technology has generated a huge amount of information, which makes MCDM increasingly important and useful in supporting business decision making.

There are many methods available for solving MCDM problems as reviewed by Hwang and Yoon [1981], though some of the methods were criticised as ad hoc and to certain degree unjustified on theoretical and/or empirical grounds [Stewart; 1992]. There were calls in early 1990s to develop new methods that could produce consistent and rational results, capable of dealing with uncertainties and providing transparency to the analysis processes [Stewart; 1992 and Dyer et al, 1992].

As part of the effort to deal with MCDM problems with uncertainties and subjectivity, the Evidential Reasoning (ER) has been devised, developed, and finally implemented into a window based software called Intelligent Decision Systems by Yang and his collaborators in a time span of more than 10 years [Zhang, Yang and Xu, 1989; Yang and Singh, 1994; Yang and Xu 2000a; Yang & Xu, 2000b; Yang 2001]. The ER approach and the software are now widely used in many areas.

In the following section, the main characteristics of MCDM problems are summarised first, followed by a list of typical techniques used in MCDM analysis. In **Section 3**, the Evidential Reasoning (ER) approach is described, followed by an overview of the IDS software in **Section 4**. In **Section 5**, a comparison is conducted between the ER approach and the AHP method, one of the most widely used MCDM methods. In **Section 6**, a list of terminologies related to the ER approach and the IDS software are defined and finally a list of relevant references is given in **Section 7**.

2 MCDM METHODS – AN OVERVIEW

2.1 Main Features of MCDM

In general, there exist two distinctive types of MCDM problems due to the different problems settings: one type having a finite number of alternative solutions and the other an infinite number of solutions. Normally in problems associated with selection and assessment, the number of alternative solutions is limited. In problems related to design, an attribute may take any value in a range. Therefore the potential alternative solutions could be infinite. If this is the case, the problem is referred to as multiple objective optimisation problems instead of multiple attribute decision problems. Our focus will be on the problems with a finite number of alternatives.

A MCDM problem may be described using a decision matrix. Suppose there are m alternatives to be assessed based on n attributes, a decision matrix is a $m \times n$ matrix with each element Y_{ij} being the j-th attribute value of the i-th alternative.

Although MCDM problems could be very different in context, they share the following common features.

- **Multiple attributes/criteria often form a hierarchy**.

  Almost any alternatives, such as an organisation, an action plan, or a product of any kind, can be evaluated on the basis of attributes. An attribute is a property, quality or feature of alternatives in question. Some attributes may break down further into lower levels of attributes, called sub-attributes. To evaluate an alternative, a criterion is set up for each attribute. Because of the one to one correspondence between attribute and criterion, sometimes attributes are also referred to as **criteria** and used interchangeably in the MCDM context. MCDM itself can also be referred to as Multiple Attribute Decision Analysis (MADA) if there are a finite number of alternatives.

- **Conflict among criteria**.

  Multiple criteria usually conflict with one another. For example, in designing a car, the criteria of higher fuel economy might mean a reduced comfort rating due to the smaller passenger space.

- **Hybrid nature**

  1) **Incommensurable units**.

     An attribute may have a different unit of measurement. In the car selection problem, fuel economy is measured by miles per gallon, and price is expressed by pound sterling etc. In many decision problems, attributes may even be non-quantitative, such as the safety feature of a car may be indicated in a non-numerical way.

2) **Mixture of qualitative and quantitative attributes**.

   It is possible that some attributes can be measured numerically and other attributes can only be described subjectively. For instance, the price of a car is numerical and the comfort rating is qualitative.

3) **Mixture of deterministic and probabilistic attributes**.

   For example, in the car selection problem, car price is deterministic and fuel economy could be random. Fuel economy changes depending on road conditions, traffic conditions and weather.

- **Uncertainty**

  1) **Uncertainty in subjective judgments**

     It is common that people may not be 100% sure when making subjective judgments.

  2) **Uncertainty due to lack of data or incomplete information**

     Sometimes information of some attributes may not be fully available or even not available at all.

- **Large Scale**

  A real life MCDM problem may consist of hundreds of attributes. For example, in the European Foundation for Quality Management (EFQM) business excellence model, there are 3 levels of criteria, 9 criteria in level 1, 32 in level 2, and 174 in level 3. In a supplier assessment model for a large international company, there are 10 level 1 criteria and more than 900 sub-criteria.

- **Assessment may not be conclusive**

  Due to lack of information, the conflict among criteria, the uncertainties in subjective judgment and different preferences among different decision makers, the final assessment results may not be conclusive. There could be many solutions to a MCDM problem as listed below.

2.2 MCDM Solutions

MCDM problems may not always have a conclusive or unique solution. Therefore different names are given to different solutions depending on the nature of the solutions [Hwang and Yoon, 1981].

2.2.1 Ideal solution

All criteria in a MCDM problem can be classified into two categories. Criteria that are to be maximised are in the profit criteria category, although they may not necessarily be profit criteria. Similarly criteria that are to be minimised are in the cost criteria category. An **ideal solution** to a MCDM problem would maximise all profit criteria and minimise all cost criteria. Normally this solution is not obtainable. The question is what would be a best solution for the decision maker and how to obtain such a solution?

2.2.2 Non dominated solutions

If an ideal solution is not obtainable, the decision maker may look for non-dominated solutions. An alternative (solution) is **dominated** if there are other alternatives that are better than the solution on at least one attribute and as good as it on other attributes. An alternative is called **non-dominated** if it is not dominated by any other alternatives.

2.2.3 Satisfying solutions

Satisfying solutions are a reduced subset of the feasible solutions with each alternative exceeding all the expected criteria. A satisfying solution may not be a non-dominated solution. Whether a solution is satisfying depends on the level of the decision maker’s expectation.

2.2.4 Preferred solutions

A **preferred solution** is a non-dominated solution that best satisfies the decision maker’s expectations.

2.3 MCDM Methods

There are two types of MCDM methods. One is compensatory and the other is non-compensatory [Hwang and Yoon, 1981].

2.3.1 Non-compensatory Methods

Non-compensatory methods do not permit tradeoffs between attributes. An unfavourable value in one attribute cannot be offset by a favourable value in other attributes. Each attribute must stand on its own. Hence comparisons are made on an attribute-by-attribute basis. The MCDM methods in this category are credited for their simplicity. Examples of these methods include:

Dominance method: Eliminate all dominated alternatives. There could be more than one solutions generated by this method.

Maxmin method: Find the weakest attribute value (min) of each alternative and then choose the alternative with the best (max) weakest attribute value. The logic is that a chain is as strong as its weakest link. This method is applicable only when attribute values are comparable with one another, either measured in the same unit or transformed to a common scale.

Maxmax Method: In contrast to the Maxmin method, the Maxmax method selects an alternative by its best attribute value. It is also applicable only when attributes are comparable.

Conjunctive constraint method: By setting up a minimum standard for each attribute, the alternative selection or evaluation process is simplified to compare each attribute against its standard. If the standard reflects the decision maker’s expectations, the obtained solutions are satisfying solutions.

Disjunctive constraint method: This method evaluates an alternative on its best attribute regardless of all other attributes.

These techniques may have their application domains in which they are reasonable, but they may not be very useful for general decision making.

2.3.2 Compensatory Methods

Compensatory methods permit tradeoffs between attributes. A slight decline in one attribute is acceptable if it is compensated by some enhancement in one or more other attributes. Compensatory methods can be classified into the following 4 subgroups.

2.3.2.1 Scoring Methods

The scoring method selects or evaluates an alternative according to its score (or utility). **Utility** or score is used to express the decision maker's preference. It transforms attribute values into a common preference scale such as [0,1] so that comparisons between different attributes becomes possible. A very popular method in this category is the Simple Additive Weighting method. This method calculates the overall score of an alternative as the weighted sum of the attribute scores or utilities. The Analytical Hierarchy Process (AHP) is another popular method in this category. This method calculates the scores for each alternative based on pairwise comparisons [Saaty, 1988.].

2.3.2.2 Compromising Methods

The compromising method selects an alternative that is closest to the ideal solution. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method belongs to this category. This method first normalises the decision matrix of a MCDM problem. Then based on the normalised decision matrix, it calculates the weighted distances of each alternative from an ideal solution and a nadir solution. A solution relatively close to the ideal solution and far from the nadir solution is evaluated to be the best [Hwang and Yoon, 1981].

2.3.2.3 Concordance Methods

The concordance method generates a preference ranking which best satisfies a given concordance measure. The Linear Assignment Method is one of the examples in this family. In this method it is believed that an alternative having many highly ranked attributes should be ranked high [Hwang and Yoon, 1981].

2.3.2.4 Evidential Reasoning Approach

The evidential Reasoning (ER) approach is the latest development in the MCDM area [Yang and Xu 2000, Yang 2001; Yang and Singh, 1994]. It is different from the above 3 conventional methods. Instead of describing a MCDM problem with a decision matrix, the ER approach uses a extended decision matrix, in which each attribute of an alternative is described by a distributed assessment using a belief structure. For example, the distributed assessment result of the quality of a car engine could be {(Excellent, 60%), (Good, 40%), (Average, 0%), (Poor, 0%), (Worst, 0%)}, which means the quality of the car engine is assessed to be Excellent with 60% of belief degree and Good with 40% of belief degree.

The advantages of using a distributed assessment include that it can model precise data and meanwhile capture various types of uncertainties such as probabilities and vagueness in subjective judgements. For example, in a consumer survey, if 20% of the customers evaluate the after sale service of a computer shop to be excellent, 30% good and 50% average, one is not required to aggregate the information before using it. The distributed assessment accepts the raw information as it is.

The ER approach is the only method so far capable of handling MCDM problems with uncertainties and hybrid nature as characterised in **Section 2.1**. It describes and handles uncertainties by using the concept of the degrees of belief.

- **Absence of data.**

  Absence of data is used to describe a situation where there is no data available to assess an attribute. If this is the case, the total sum of **belief degrees** in the **distributed assessment** for that attribute will be 0.

- **Incomplete description of an attribute**

  This is a situation where data for describing an attribute are partially available. If this is the case, the total sum of **belief degrees** in the **distributed assessment** for that attribute will be between 0% and 100%.

- **Random nature of an attribute**

  Some attributes are of random nature. For example, fuel consumption of a car in mile per gallon is not a deterministic number. Depending on road conditions, traffic conditions and seasons of a year, the figure will vary. The nature of the fuel consumption will be described by a probability distribution. If this is the case, the probability distribution will be transformed into the **degrees** of belief in the **distributed assessment** for the attribute.

The distributed assessment concept was proposed by Zhang, Yang and Xu in 1989. To syntheses the distributed assessment information, the ER approach was developed and enhanced over the 1990s [Yang & Singh, 1994, Yang & Sen, 1994, Yang and Sen 1997, Yang and Xu 2000, Yang 2001]. ER approach employs the Dampster-Shafer Evidence Combination rule for criteria aggregation. The logic behind the approach is that if an object has a good (or bad) sub-attribute, then the object must be good (or bad) to certain extent. The extent is measured by both the degree to which that sub-attribute is important to the object and the degree to which the sub-attribute belongs to the good (or bad) category.

In the late 1990s, the ER approach is fully integrated into a window based software referred to as the Intelligent Decision System (IDS). Because the ER approach and the IDS software are used as the main tool for a number of research projects in the Manchester School of Management, UMIST, the principle of ER and an overview of IDS will be briefly discussed below.

3 ER APPROACH

The ER approach is different from most conventional MCDM modelling methods in that it employs a belief structure to represent an assessment as a distribution. For example, when you choose a car, you may have the following four evaluation grades:

$$
\begin{aligned}
H &= \{H_1,\ H_2,\ H_3,\ H_4\} \\
&= \{\textit{Slightly preferred},\ \ \textit{Modertaely preferred}, \\
&\quad \textit{Preferred},\ \textit{Greatly preferred}\}
\end{aligned}
$$

Suppose there are K alternatives, O_j (j=1, …, K), to choose from and M attributes, A_i (i=1, …, M), to consider. Using the four evaluation grades, the assessment of an attribute A_1 on an alternative O_1, denoted by $S(A_1(O_1))$, can be represented using the following belief structure:

$$S(A_1(O_1)) = \{(\beta_{1,1}, H_1), (\beta_{2,1}, H_2), (\beta_{3,1}, H_3), (\beta_{4,1}, H_4)\} \tag{1}$$

where $1 \geq \beta_{n,1} \geq 0$ (n=1, …, 4) denotes the degree of belief that the attribute A_1 is assessed to the evaluation grade H_n. $S(A_1(O_1))$ reads that the attribute A_1 is assessed to the grade H_n to a degree of $\beta_{n,1} \times 100\%$ (n=1, …, 4) for the alternative O_1.

There must not be $\sum_{n=1}^{4} \beta_{n,1} > 1$. $S(A_1(O_1))$ can be considered to be a complete distributed assessment if $\sum_{n=1}^{4} \beta_{n,1} = 1$ and an incomplete assessment if $\sum_{n=1}^{4} \beta_{n,1} < 1$. In the ER framework, both complete and incomplete assessments can be accommodated (Yang, 2001).

In the ER framework, a MCDM problem with M attributes A_i (i=1, …, M), K alternatives O_j (j=1, …, K) and N evaluation grades H_n (n=1, …, N) for each attribute is represented using an extended decision matrix with $S(A_i(O_j))$ as its element at the i-th row and j-th column where $S(A_i(O_j))$ is given as follows:

$$S(A_i(O_j)) = \{(H_n, \beta_{n,i}(O_j)),\ n = 1, \cdots, N\} \quad i\text{=1, …, }M, \quad j\text{=1, …, }K \tag{2}$$

It should be noted that an attribute could have its own set of evaluation grades that may be different from those of other attributes [Yang, 2001].

Instead of aggregating average scores, the ER approach employs an evidential reasoning algorithm developed on the basis of decision theory and the evidence combination rule of the Dempster-Shafer theory to aggregate belief degrees [Yang & Singh, 1994)], [Yang & Sen, 1994] [Yang, 2001]. Thus, scaling grades is not necessary for aggregating attributes in the ER approach and in this way it is different from traditional MCDM approaches, most of which aggregate average scores.

Suppose ω_i is the relative weight of the attribute A_i and is normalised so that $1 \geq \omega_i \geq 0$ and $\sum_{i=1}^{L} \omega_i = 1$ where L is the total number of attributes in the same group sharing the same upper level attribute in the attribute hierarchy. To simplify the discussion, only the combination of complete assessments is examined. The description of the recursive ER algorithm capable of aggregating both complete and incomplete assessments is detailed in [Yang & Sen, 1994][Yang, 2001]. Without loss of generality and for illustration purpose, the ER algorithm is presented below for combining two attribute assessments only.

Suppose the first assessment is given in equation (1) and the second $S(A_2(O_1))$ is given by

$$S(A_2(O_1)) = \{(H_1,\beta_{1,2}),(H_2,\beta_{2,2}),(H_3,\beta_{3,2}),(H_4,\beta_{4,2})\} \tag{3}$$

The problem is to aggregate the two assessments $S(A_1(O_1))$ and $S(A_2(O_1))$ to generate a combined assessment $S(A_1(O_1)) \oplus S(A_2(O_1))$. Suppose $S(A_1(O_1))$ and $S(A_2(O_1))$ are both complete. Let

$$m_{n,1} = \omega_1\beta_{n,1} \text{ (}n\text{=1, …, 4) and } m_{H,1} = 1-\omega_1\sum_{n=1}^{4}\beta_{n,1} = 1-\omega_1$$

$$m_{n,2} = \omega_2\beta_{n,2} \text{ (}n\text{=1, …, 4) and } m_{H,2} = 1-\omega_2\sum_{n=1}^{4}\beta_{n,2} = 1-\omega_2$$

where each $m_{n,j}$ (j = 1, 2) is referred to as basic probability mass and each $m_{H,j}$ (j = 1,2) is the remaining belief for attribute j unassigned to any of the H_n (n = 1, 2, 3, 4).

The ER algorithm is used to aggregate the basic probability masses to generate combined probability masses, denoted by m_n (n=1, …, 4) and m_H using the following equations:

$$m_n = k(m_{n,1}m_{n,2} + m_{H,1}m_{n,2} + m_{n,1}m_{H,2}), \qquad (n\text{=1, …, 4})$$

$$m_H = k(m_{H,1}m_{H,2})$$

where

$$k = \left(1-\sum_{t=1}^{4}\sum_{\substack{n=1\\n\neq t}}^{4} m_{t,1}m_{n,2}\right)^{-1}$$

The combined probability masses can then be aggregated with the third assessment in the same fashion. The process is repeated until all assessments are aggregated. The final combined probability masses are independent of the order in which individual assessments are aggregated.

If there are only two assessments, the combined degrees of belief β_n (n=1, …, 4) are generated by:

$$\beta_n = \frac{m_n}{1-m_H} \qquad (n\text{=1, …, 4})$$

The combined assessment for the alternative O_1 can then be represented as follows:

$$S(O_1) = \{(H_1,\beta_1),(H_2,\beta_2),(H_3,\beta_3),(H_4,\beta_4)\}$$

An average score for O_1, denoted by $u(O_1)$, can also be provided as the weighted average of the scores (utilities) of the evaluation grades with the belief degrees as weights, or

$$u(O_1) = \sum_{i=1}^{4} u(H_i)\beta_i$$

where $u(H_i)$ is the utility of the *i*-th evaluation grade H_i. If evaluation grades are assumed to be equidistantly distributed in the utility space, for example, the utilities of the evaluation grades can be given as follows:

$u(H_1) = u$ (*Slightly preferred*) = 0.25.

$u(H_2) = u$ (*Moderately preferred*) = 0.50.

$u(H_1) = u$ (P*referred*) = 0.75.

$u(H_1) = u$ (*Greatly preferred*) = 1.00.

An intelligent decision system (IDS[1]) has been developed on the basis of the ER approach [Yang & Xu, 2000]. The IDS software is designed to transform the lengthy and tedious model building and result analysis process into an easy window-based click and design activity. The following section is devoted to demonstrating the modelling and solution process of a MCDM problem using the IDS software.

4 INTELLIGENT DECISION SYSTEM (IDS) OVERVIEW

Taking a motorcycle selection problem as an example, the main window of IDS for solving a MCDM problem is a model display window shown in **Figure 1**, which has a menu bar, a toolbar and a model display window. The hierarchy of the assessment criteria can be readily constructed using the modelling menu or the related short cuts in the toolbar. IDS also provides an assistant model builder for building large-scale models that may have hundreds of criteria and options.

In the model display window, each attribute object is coloured in blue and has three boxes for showing and editing the attribute name at the top and its weight at the bottom left and displaying average score at the bottom right. Each alternative object is coloured in yellow and also has three boxes for showing and editing the alternative name at the top and displaying its ranking at the bottom left and overall average score at the bottom right. Apart from an average score, IDS is capable of generating a distributed assessment for each alternative on any attribute. **Figure 2** shows the overall distributed assessment of Yamaha in the motorcycle selection sample that is generated by the IDS software. In **Figure 2**, the degrees of belief to the evaluation grades clearly show the merits and drawbacks of the motorcycle.

In IDS, many dialog windows are designed to support model building, data input, result analysis, reporting and sensitivity analysis. For example, **Figure 2** is generated using an IDS dialog window for reporting results graphically. **Figure 3** shows an IDS

[1] A free demo version of IDS can be obtained from Dr J B Yang via email: jian-bo.yang@umist.ac.uk

dialog window for data input for BMW on **Price** attribute. All data can be entered using similar dialog windows, whether they are precise numbers, random numbers with probabilities, or subjective assessments. **Figure 4** shows the visual cross comparison of the four alternatives of the motorcycle selection problem on **Price**, **Engine** and **Operation** generated using the IDS visual comparison dialog window.

In IDS, AHP and other methods are used for generating relative weights of criteria and the evidential reasoning approach is used to aggregate criteria from the bottom level of criteria to the top level criterion. The overall assessment for each alternative can be characterised as shown for Yamaha in **Figure 2**. In IDS, dialog windows are designed to support visually scaling the evaluation grades or estimating the utilities of the grades. For example, **Figure 5** shows a utility curve for the five evaluation grades for motorcycles. The curve can be changed onscreen by drag and drop the marked point on the curve to suit the decision maker's preferences. For the given weights and utility curve, the average scores for the four motorcycles are generated as shown in **Table 1**.

Table 1 Overall Assessment of Motorcycles

	Kawasaki	Yamaha	Honda	BMW
Overall Performance	0.62	0.67	0.71	0.58

Based on the overall scores of **Table 1**, the ranking of the four options are given as follows:

$$\text{Honda} \succ \text{Yamaha} \succ \text{Kawasaki} \succ \text{BMW}$$

5 COMPARISON BETWEEN IDS AND THE AHP METHOD

Because AHP is one of the most popular MCDM methods, it will help to better understand the ER approach by comparing the two methods. Because both ER and AHP use hierarchical structure to model a MADA problem, it is also natural to compare the two methods. Although both methods use a hierarchical structure, ER differs from AHP in the following ways.

1) In the ER framework, alternatives are not in the hierarchical structure, while in AHP all alternatives consist of the bottom level of the hierarchy. This is where the principle of AHP has been criticised the most.
2) ER approach uses a generalised decision matrix. Each element of the matrix is a distributed assessment of an attribute using the degrees of belief concept. A conventional decision matrix is a special case of the ER framework, where each element of the matrix is a single number. In AHP, however, the decision matrix becomes a comparison matrix with each element, also a number, describing the relative importance of one attribute over another.
3) ER uses the Evidential Reasoning approach (**Section 3**) to aggregate the distributed assessment (degrees of belief) of the lower level attributes to higher

level attributes progressively. AHP aggregates average scores generated from pairwise comparison matrices.

As a consequence of the above differences in modelling a MCDM problem, the ER approach can handle not only what AHP can, but also what AHP cannot.

1) IDS can handle very large scale MADM problems. There is no limit on the number of attributes in the IDS hierarchy. There is no limit on the number of alternatives to be assessed. The only limit is the computer memory. For example, IDS has been used for conducting organisational self-assessment and supplier assessment. These assessment problems have about 1000 attributes. IDS can handle them easily with a 128 MB RAM PC. To handle such problems will be a daunting task for AHP.

2) IDS can assess newly added alternatives independently. Because IDS calculates the absolute ranking score for each alternative independently, when new alternatives are added to IDS, IDS does not need to re-evaluate the previously assessed alternatives. AHP, on the other hand, is based on pairwise comparisons. When new alternatives are added to AHP, the assessments done on the old alternatives have to be discarded. A new assessment procedure has to start from the beginning, taking into account all the alternatives.

3) IDS produces consistent ranking after new alternatives are added into the assessment procedure. It has been shown that the use of AHP may lead to problems like rank reversal [Belton, 1986; Islei & Lockett, 1988; Stewart, 1992; Barzilai, 1997] when new alternatives are added for assessment. That is the newly added alternatives may cause the unexpected and irrational change of the ranking of the previously assessed alternatives.

4) In addition to the ranking score of all alternatives, IDS produces a distributed assessment, which provides the decision maker with a panoramic view about the diversity of the performance of an alternative, thereby helping the decision maker to identify areas for improvement and to design and test action plans to make improvement.

5) IDS can be used to assess an alternative against a set of standards, while AHP can only compare the relative importance between attributes.

6) IDS can handle mixed information with uncertainties in the same ER framework. The mixed information includes random and deterministic, qualitative and quantitative and incomplete or absence of data for some attributes.

7) IDS uses AHP as one of its attribute weighting assignment methods.

8) The evidential reasoning approach and the information conversion techniques used in IDS have sound theoretical foundations [Yang, 2000 & 2001; Yang and Singh, 1994].

6 GLOSSARY AND ABBREVIATIONS RELATED TO ER APPROACH

Alternatives

Alternatives are objects or options to be assessed or evaluated. An alternative may be assessed or evaluated based on its **attributes**.

Attributes

An attribute is a property, quality or feature of an alternative. To evaluate an alternative, a criterion is set up for each of its attributes and the attribute is examined against the criterion. Because of the one to one correspondence between an attribute and a criterion, sometimes attributes are also referred to as **criteria**. In the context of MCDM, the word attributes and criteria are used interchangeably.

There are two types of attributes. One is **quantitative** and the other is **qualitative**.

Attributes may break down further into one or more levels of sub-attributes to form a **hierarchy** structure.

Attribute Hierarchy

A hierarchy or an attribute hierarchy is composed of multi-level **attributes**. The top-level attribute of the hierarchy is normally an overall qualitative attribute. This high level attribute can be decomposed into more specific sub-attributes. The sub-attributes can be further decomposed if necessary until the bottom level attributes can be evaluated directly.

It should be noted that in contrast with AHP method, alternatives are not in the hierarchy of the **ER framework**.

Criteria

See **Attribute**.

Evidential Reasoning (ER) Framework

ER framework including the concept of

- the **attribute hierarchy**,
- the **distributed assessment** structure using degree of belief and
- the **evidential reasoning approach** used in aggregating degrees of belief from lower level attributes to higher level attributes.

The most important feature of the ER framework is that it employs a degree of belief structure to represent an assessment as a distribution.

Evidential Reasoning (ER) Approach

ER Approach is used for aggregating distributed assessment results from lower level attributes to higher level attributes. The ER approach employs an evidential reasoning algorithm developed on the basis of decision theory and the evidence combination rule of the Dempster-Shafer theory to aggregate belief degrees.

Grades

Grades are used for assessing a qualitative attribute of an alternative. A commonly used set of grades for assessing the quality of a car engine, for example, could be {Excellent, Good, Average, Poor, Worst} and for assessing the price of the car, could be {Very Low, Low, Average, High, Very High}.

It should be noted that there are no restrictions on how many grades and what grade names can be used for each attribute. Different number of grades can be used for different attributes.

Degrees of Belief or **Belief Degrees**

Degrees of belief or belief degrees are subjective probabilities associated with assessment **grades.** It describes the confidence level of an attribute being evaluated to a grade. For example, a car engine quality could be assessed to be Excellent with 60% of belief degree and Good with 40% of belief degree. The belief degrees could be generated from a survey, group decision-making or by mapping evidence related to the standards of each grade.

It should be noted that the total sum of degree of beliefs could be less than or equal to 100% but should never be more than 100%.

Assessment and **Distributed Assessment**

For bottom level **attributes**, assessment is the process or the result of assigning **grades** and the associated **degrees of belief** to an attribute based on guidelines and evidence.

For other levels of attributes, it is the process or the results of aggregating **belief degrees** of lower level attributes to higher level attributes using the **evidential reasoning approach**.

Because the result of an **assessment** from the **ER approach** is of a **distributed** structure, the assessment is sometimes referred to as **distributed assessment**. For example, the distributed assessment result of the quality of a car engine could be {(Excellent, 60%), (Good, 40%), (Average, 0%), (Poor, 0%), (Worst, 0%)}. It means that the quality of the car engine is assessed to be Excellent with 60% of belief degree and Good with 40% of belief degree.

Complete and **Incomplete Distributed Assessment**

If the total sum of belief degrees for all the grades of a **distributed assessment** is 100%, then the distributed assessment is said to be complete.

If the sum is less than 100%, then the distributed assessment is said to be incomplete. The incompleteness is a reflection of some **uncertainties** in the assessment. In the extreme circumstances where there is no information available for an attribute (**absence of data**), the sum will be 0.

Uncertainties

The **ER approach** can handle the following types of uncertainties in a multiple attribute decision analysis problem.

- **Absence of data.**

  Absence of data is used to describe the situation where there is no data available to assess an attribute. If this is the case, the total sum of **belief degrees** in the **distributed assessment** for that attribute will be 0.

- **Incomplete description of an attribute**

  This is the situation where data for describing an attribute are partially available. If this is the case, the total sum of **belief degrees** in the **distributed assessment** for that attribute will be between 0% and 100%.

- **Random nature of an attribute**

  Some attributes are of random nature. For example, fuel consumption of a car in mile per gallon is not a deterministic number. Depending on road conditions, traffic conditions and seasons of a year, the figure will vary. The nature of the fuel consumption can be described using a probability distribution. If this is the case, the probability distribution will be transformed into the degrees of belief in the **distributed assessment** for the attribute.

- **Fuzziness in grade definitions**

  It is natural that the grades “excellent” and “very good” may have some overlap in meanings. If it is the case, the overlap can be described using fuzzy sets in the grade definitions. The extent of the overlap can be described by the membership functions of a set of fuzzy grades.

Utility or **Utility Function**

Utility is a measure of the preference of the decision maker. It is a number within a predefined range assigned by the decision maker to

- an assessment **grade** if the attribute is **qualitative** or
- an attribute value if the attribute is **quantitative**.

Normally the range can be one of the following: [0, 1], [0, 10] or [0, 100]. The highest number is assigned to the most preferred grade or value while the lowest number is assigned to the least preferred grade or value.

Utility function is used to map all the grades or values of an attribute to the predefined range of utilities. The function could be linear or non linear for a quantitative attribute and equidistant or non-equidistant for a qualitative attribute. The utility function can be displayed as a continuous curve on a two-dimension chart, with the X-axis as the grades or values and Y-axis as the utility values. A utility function has to be monotonously increasing or monotonously decreasing. Either type can be linear or nonlinear, and if nonlinear, concave or convex.

Sensitivity Analysis

Sensitivity analysis is used to investigate how sensitive the ranking of alternatives is to changes in weights and belief degrees for certain attributes.

Decision Matrix

Suppose there are M alternatives and each alternative has N attributes values. A decision matrix is an $M \times N$ matrix whose element x_{ij} indicates a value or an assessment of the i-th alternative on the j-th attribute.

Pairwise Comparison Matrix

Suppose there are M alternatives in a MCDM problem. A pairwise comparison matrix is an $M \times M$ matrix whose element x_{ij} indicates the relative preference of alternative i over alternative j for a given attribute.

Extended Decision Matrix

Extended decision matrix is specific to the **ER approach**. Suppose there are M alternatives in a MCDM problem and each alternative has N attributes. An extended decision matrix is an $M \times N$ matrix whose element x_{ij} is the **distributed assessment** of the i-th alternative on the j-th attribute. It is similar to a decision matrix except that each element is an array of assessment grades, not a single number or grade.

MCDM, MCDA, MADM and MADA

They are the acronyms for Multiple Criteria Decision Making, Multiple Criteria Decision Analysis, Multiple Attribute Decision Making and Multiple Attribute Decision Analysis. They are different terms for the same thing and can be used interchangeably. The last two terms are normally used for assessment problems with a finite number of alternatives.

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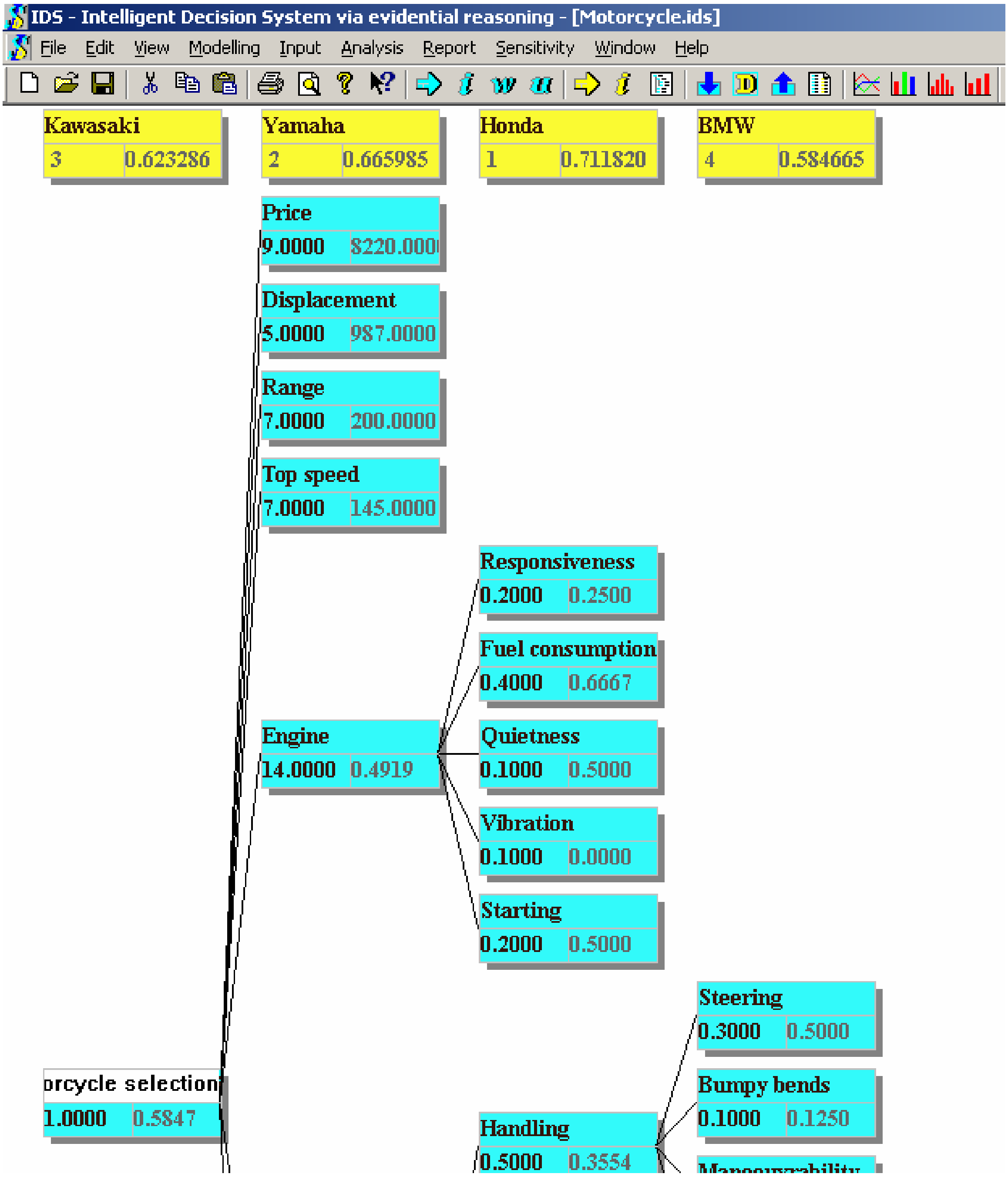

Figure 1 IDS Model Display Window

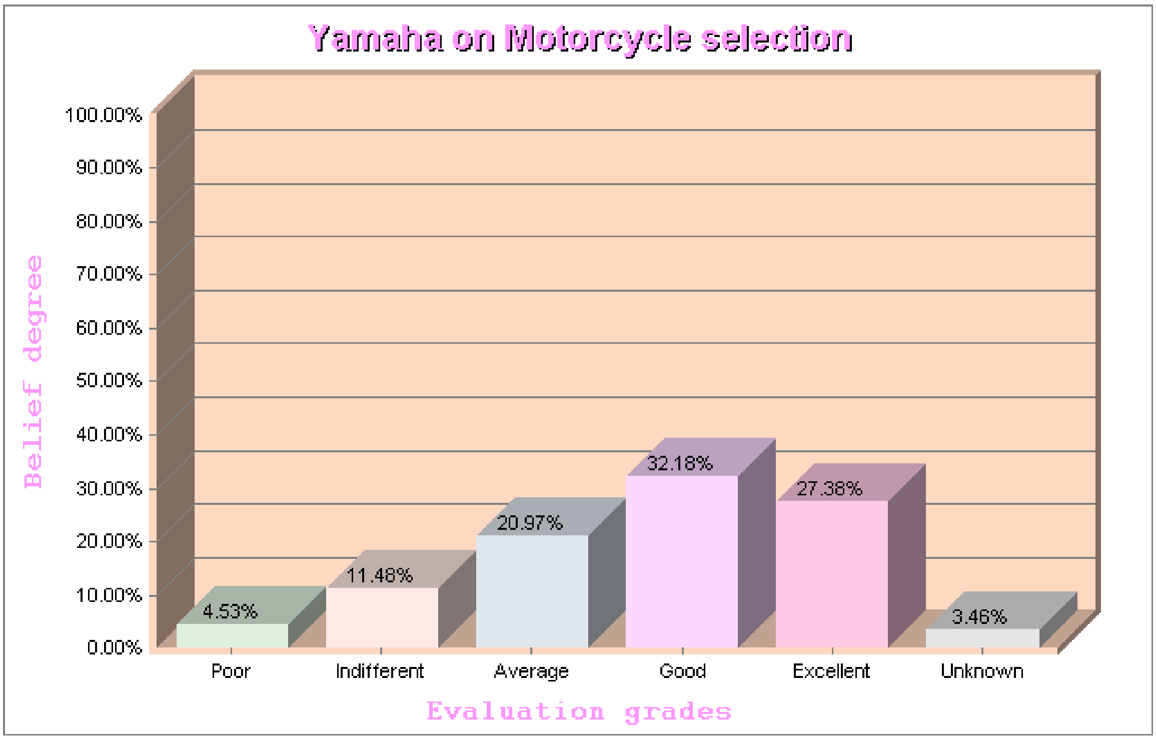

Figure 2 Overall Distributed Assessment of Yamaha Generated by IDS

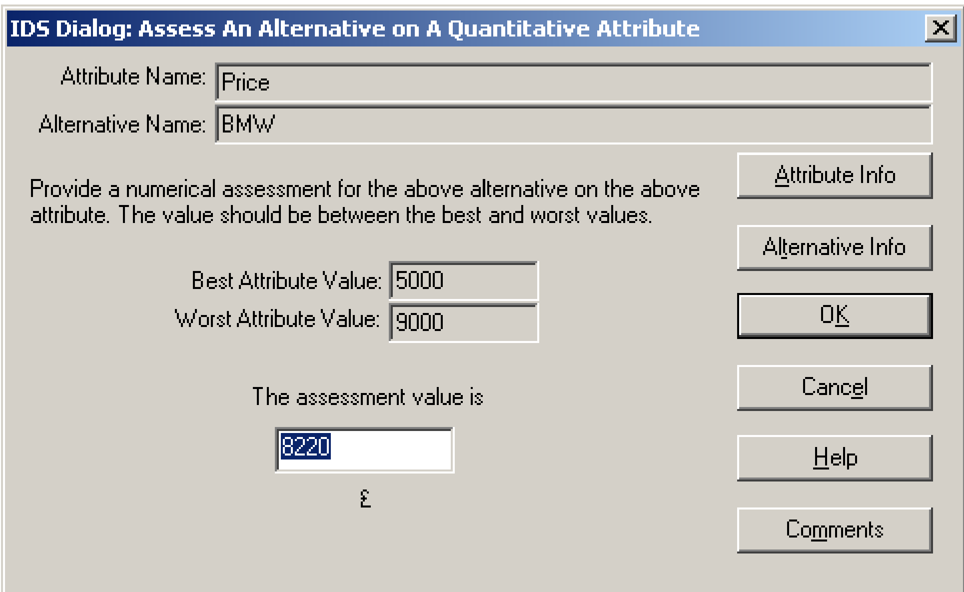

Figure 3 IDS Data input dialog window

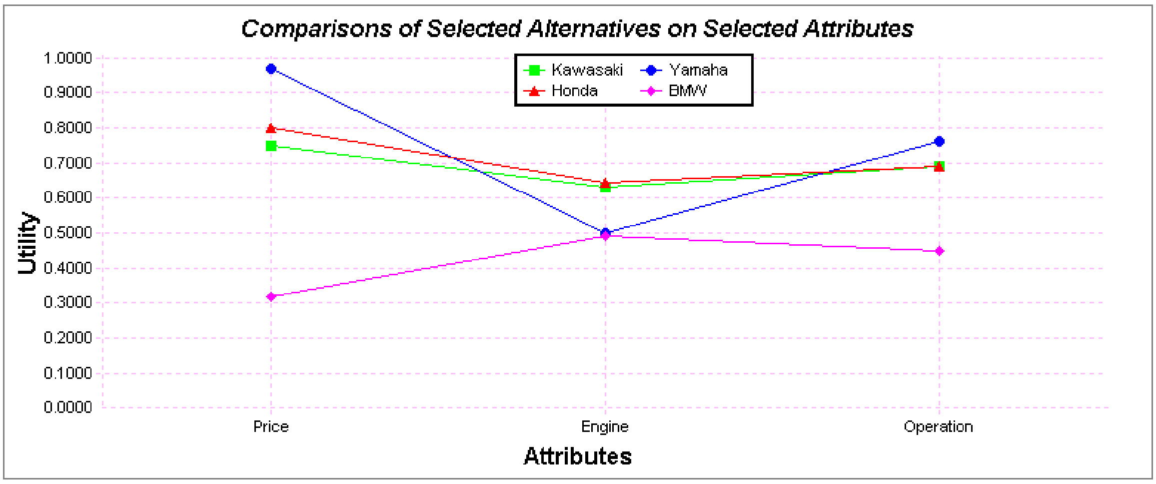

Figure 4. Comparison of 4 Motorcycles on Price, Engine and Operation

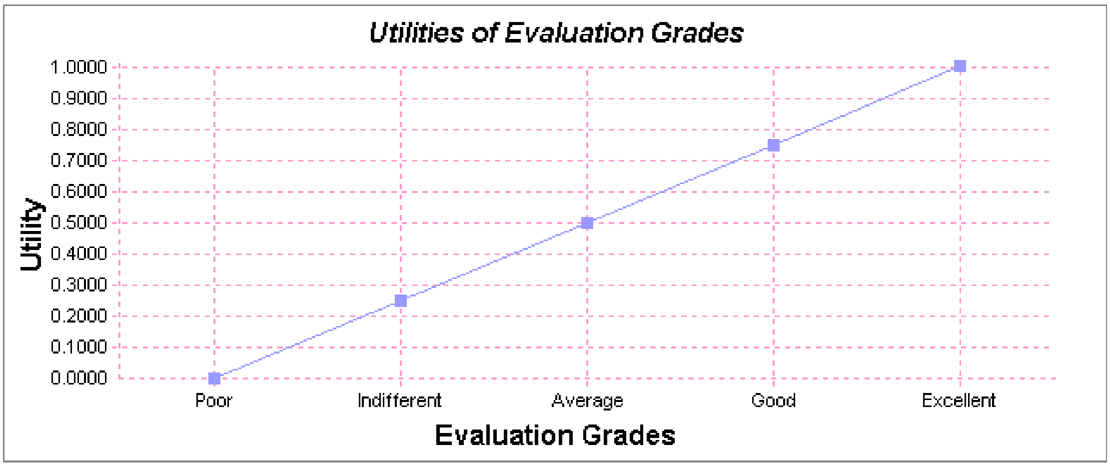

Figure 5 Utility Curve of Evaluation Grades Generated by IDS