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Comparison of a Case Study of Uncertainty Propagation using Possibility-Probability Transformation

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ABSTRACT

Uncertainty parameters in risk assessment can be modeled by different ways viz. probability distribution, possibility distribution, belief measure, depending upon the nature and availability of the data. Different transformations exist for converting expression of one form of uncertainty to another form. They differ from one another substantially, ranging from simple ratio scaling to more sophisticated transformation based upon various principles. These transformations should satisfy certain consistency principles. Several researchers viz., Zadeh, Klir, Dubois & Prade have given such type of consistency laws. The weakest consistency rule that any probability-possibility transformation should satisfy is pro(A)$\leq$ pos(A) i.e., probability of any event is less than or equal to possibility of that event. The strongest among such transformation law Pro(A) > 0$\Longrightarrow$Pos(A) =1. Though possibility and probability capture different types of uncertainty, still transformations are used because it is essential in solving many practical problems. In this paper, we reviewed the consistency principles as given by the above authors. Then we have made a comparative case study of uncertainty propagation by three different methods using probability- possibility transformation satisfying consistency conditions.

Keywords

Uncertainty, Risk Assessment, Hybrid method, Probability-possibility transformation

1. INTRODUCTION

Risk assessment methods have become more and more popular support tools in decision making process. The goal of risk assessment is to estimate the severity and likelihood of harm to humans' health from exposure to a substance or activity that under plausible circumstances can cause harm to human health. Uncertainty in risk assessment may arise from many different sources such as scarce or incomplete information or data, measurement error or data obtain from expert judgment or subjective interpretation of available data or information. Here we will consider four different types of uncertainties: firstly, random variable observed with total precision which can be represented by a classical probability measure. Secondly, deterministic parameters whose value is imprecisely known, which can be modeled in a natural way by possibility distribution. Thirdly, imprecisely known observed random variables which can be represented by a p-box. Fourthly, there may be a case in which we do not know the representation of the parameters. i.e., it is random variable or deterministic but we know only the range of the values of the parameter and the most likely value. That kind of uncertainty can be either modeled by a possibility distribution or a fuzzy number. As in the last type, where we do not have the proper idea about the representation of the parameter, so we can perform probability/possibility transformation.

Human being is always exposed to radiation either from natural or anthropogenic sources in the environment. While there have been natural nuclides since the beginning the earth's existence, manmade nuclides have been released from nuclear installations and fallouts from the nuclear test and nuclear accident. Also produced water is the most significant source of waste generated in the production phase of oil and gas operations. Once discharged into the ocean, a number of heavy metals and poly aromatic hydrocarbon in produced water may introduce toxicity and bioaccumulation in marine organisms. These compounds are harmful to fish and therefore human can be affected through intake of such fishes. Consequently, we can say that human health can also be indirectly (or directly) affected through different pathways such as inhalation, ingestion, submersion and dermal contact. For this purpose, risk assessment is performed to quantify the potential detriment to human and evaluate the effectiveness of proposed remediation measures.

To demonstrate and make use of the transformations a hypothetical case study for non-cancer human health risk assessment is presented here by considering three scenarios and each scenario contains three cases. In the first case, the representation of the some parameters are taken to be possibilistic (fuzzy number) while some are taken to be probabilistic and some are considered as constants. In the second case, we transform the possibilistic distribution (fuzzy number) to triangular probability distribution. In the third case, we will consider the triangular fuzzy numbers as uniform probability distribution with the same support. All the calculations have been performed using Risk calc 4 [7].

2. PROBABILITY THEORY

Probability theory frequently used in uncertainty analysis. If parameters used in prescribed models are random in nature and followed well define distribution, then probabilistic methods are most suitable and well accepted approach for risk assessment.

A random variable is a variable in a study in which subjects are randomly selected. Let X be a discrete random variable.

A probability mass function is a function such that

(i) $f(x_i) \geq 0$, (ii) $\sum_{i=1}^{n} f(x_i) = 1$, (iii) $f(x_i) = p(x = x_i)$

The cumulative distribution function of a discrete random variable X, denoted as F(x) is

$$F(x) = P(X \le x) = \sum_{X \le x_i} f(x_i)$$

Let X be a continuous random variable. A probability density function of X is a non-negative function *f*, which satisfies

$$P(X \in B) = \int_B f(x)dx$$

for every subset B of the real line.

As X must assume some value, f must satisfy

$$P(X \in (-\infty, \infty)) = \int_{-\infty}^{\infty} f(x)dx = 1$$

This means the entire area under the graph of the PDF must be equal to unit.

In particular, the probability that the value of X falls within an interval [a, b] is

$$P(a \le X \le b) = \int_a^b f(x)dx$$

The CDF of a continuous random variable X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$$

$\forall A, B$ we have, $P(A \cup B) = P(A) + P(B) - P(P \cap B)$

3. POSSIBILITY THEORY

Possibility theory normally associated with some fuzziness, either in the background knowledge on which possibility is based or in the set for which possibility is asserted. This constitute a method of formalizing non-probabilistic uncertainties on events i.e., a mean of assessing to what extent the occurrence of an event is possible and to what extent we are certain of its occurrence, without knowing the evaluation of the possibility of its occurrence.

A possibility distribution [1], denoted by π, here is a mapping from the real line to the unit interval, unimodal and upper semicontinuous. A possibility distribution describe the more or less plausible values of some uncertain variable *X*. Possibility theory provides two evaluations of the likelihood of an event, for instance that the value of a real variable *X* should lie within a certain interval: possibility $\prod$ and the necessity *N*. Possibility measure $\prod$ and necessity measure *N* are defin

$$\Pi(A) = \sup \pi_{x \in A}(x)$$

$\mathrm{N}(A) = 1 - \pi(A^c)$, A^c is the complement of A.

$\prod$ satisfies the following conditions

$$\Pi(A \cup B) = \max(\Pi(A), \Pi(B)), \forall A, B \subseteq R$$

$$\Pi(A \cap B) = \min(\Pi(A), \Pi(B)), \forall A, B \subseteq R$$

For triangular (trapezoidal) fuzzy numbers, possibility and necessity measures are straight lines. For example, if a continuous possibility distribution is a triangular fuzzy number say, [a, b, c] then possibility measure is given by $\frac{x-a}{b-a}, a \le x \le b$ and necessity measure is given by $\frac{x-b}{c-b}, b \le x \le c$.

In particular, consider a fuzzy number A = [10, 20, 30]. Then the possibility measure of the fuzzy number A is $\frac{x-10}{10}, 10 \le x \le 20$ and necessity measure of the fuzzy number A is $\frac{x-20}{10}, 20 \le x \le 30$. Which are depicted below:

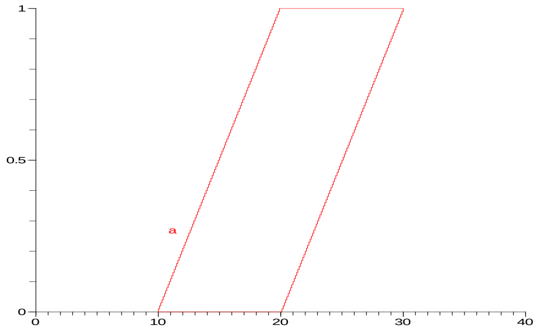

Figure 1: possibility and necessity measure of the fuzzy number A

4. POSSIBILITY- PROBABILITY TRANSFORMATION

Transforming possibility measure [3] into probability measure or conversely can be useful in any problem where heterogeneous uncertain and imprecise data must be dealt with (e.g. subjective, linguistic like evaluation and statistical data).The possibilistic representation is weaker because it explicitly handles imprecision (i.e., incomplete knowledge) and because possibility measure are based on ordering structure rather than additive one. Therefore, it can be concluded that transforming a probabilistic representation to possibilistic representation, some information is lost because we go from point value probabilities to interval values ones. The converse transformation from possibility adds information to some possibilistic incomplete Knowledge.

When information regarding some phenomenon is given in both probabilistic and possibilistic terms, the two descriptions should be in some sense consistent. That is, given probabilistic representation p_i and possibilistic representation π_i on X, the two representations should satisfy some consistency condition. Although various consistency conditions may be required, the weakest one acceptable on intuitive groups can be expressed as follows:

An event that is probable to some degree must be possible at least to the same degree. That is, the weakest consistency condition is expressed formally by the inequality

$$p_i \le \pi_i$$

On the other hand, the strongest consistency condition would require that any event with nonzero probability must be fully possible.

$$p_i > 0 \Rightarrow \pi_i = 1.$$

Transforming probabilistic [6] data to possibilistic data is useful when weak source of information make probabilistic data unrealistic. Also, it is useful in order to explore the advantages of possibilistic theory at combination steps, or perhaps to reduce the complexity of the solution when computing with possibility values rather than with probability values.

Transforming from possibility [6] to probability may be meaningful in the case of decision making where a precise outcome is often preferred, such that, the decision maker is interested to know "what is likely to happen in future", instead of "what is possible in future".

4.1. Transformation consistency principles

In this section different consistency principles [2], [4], [5], [8] are reviewed

4.1.1. Zadeh consistency principle

Zadeh defined the probability-possibility consistency principle such as "a high degree of possibility does not imply a high degree of probability, nor does a low degree of probability imply a low degree of possibility" (Zadeh 1978).

Let *U* be a finite set. *X* is a variable taking a value in *U*. π_i and p_i are possibility and probability that $X = u_i \in U$ respectively. Then, Zadeh's consistency principle can be expressed by

$$\pi_i = 0 \Rightarrow p_i = 0 \text{ and } \pi_i > \pi_j \Rightarrow p_i > p_j .$$

He defined the degree of consistency between a probability $p = (p_1, p_2, p_3 \ldots\ldots p_n)$ and a possibility distribution $\pi = (\pi_1, \pi_2, \pi_3 \ldots. \pi_n)$ as:

$$r = \sum_{i=1}^{n} \pi_i p_i \text{........(1)}$$

From (2), it can be check that

(i) if $\pi_i = 0, \forall i$ then $r = 0$, no consistency available. An impossible event cannot be probable.

(ii) If $\pi_i = 1, \forall i$ then $r = 1$, the maximum consistency value is reached. Any probability measure is still consistent with this probability measure.

Maximizing the degree of consistency, however, poses us a very restrictive condition that; $\pi_i < 1 \Rightarrow p_i = 0.$

4.2.2: Klir consistency principle

Let $X = \{w_1, w_2, \ldots. w_n\}$ be a finite universe, let $p_i = p_i(w_i)$ and $\pi_i = \pi_i(w_i)$. Assume that the elements of X are ordered in such a way that: $\forall i = 1, 2, \ldots n : p_i > 0, p_i \geq p_{i+1}$ and $\pi_i > 0, \pi_i \geq \pi_{i+1}$ with $p_{n+1} = 0$ and $\pi_{n+1} = 0$.

According to Klir the transformation from p_i to π_i must preserve some appropriate scale and the amount of information contained in each distribution (Klir 1993).The information contained in p or π can be expressed by the equality of their uncertainties. Klir has considered the principle of uncertainty preservation under two scales.

The ratio scale: This is a normalization of the probability distribution. The transformation $p \rightarrow \pi$ and $\pi \rightarrow p$ are named the normalized transformations and they are defined by

$$\pi_i = \frac{p_i}{p_1}, \; p_i = \frac{\pi_i}{n\sum_{i=1}^{n} \pi_i} \text{........(2)}$$

The log-interval scales: the corresponding transformation $p \rightarrow \pi$ and $\pi \rightarrow p$ are define by:

$$\pi_i = (\frac{p_i}{p_1})^{\alpha}, \; p_i = \frac{\pi_i^{\frac{1}{\alpha}}}{\sum_{i=1}^{n} \pi_i^{\frac{1}{\alpha}}} \text{........(3)}$$

These transformations are known as Klir transformation satisfying the uncertainty preservation principle defined by Klir (1993). α is a parameter that belongs to the open interval (0, 1). According to Klir any transformation should be based on the following three assumptions:

- A scaling assumption that forces each value $(\pi_i)_i$ to be a function of p_i/p_1 (where $p_1 \geq p_2 \geq \ldots\ldots \geq p_n$) that can be ratio scale, interval scale, log-interval scale transformation etc.
- An uncertainty invariance assumption according to which the entropy *H(p)* should be numerical equal to the measure of information $E(\pi_i)$ contained in the transformation π_i to *p*.
- The transformation should satisfy the consistency condition $\forall\, \pi(u) \geq p(u), \forall\, i$, starting that what is probable must be possible.

Dubois and Prade gave an example to show that the scaling assumption of Klir may some time lead to violation of the consistency principle that requires $\Pi \geq P$ for all events. The second assumption is also debatable because it assumes possibilistic and probabilistic information measures are commensurate.

4.2.3: Dubois and Prade consistency Principle:

The transformation $p \rightarrow \pi$ is guided by the principle of maximum specificity, which aims at finding the most informative possibility distribution. While the transformation $\pi \rightarrow p$ is guided by the principle of insufficient reason which aims at finding the possibility distribution that contains as much as uncertainty as possible but that retains the features of possibility distribution (Dubois 1993). This leads to write the consistency principle of Dubois and Prade such as:

$$\forall A \subset X : \pi(A) \geq p(A) \text{.......(4)}$$

The transformation $p \rightarrow \pi$ and $\pi \rightarrow p$ are define by

$$\pi_i = \sum_{j=i}^{n} p_j ; p_i = \sum_{j=i}^{n} \frac{(\pi_j - \pi_{j+1})}{j} ....(5)$$

The two transformations define by (5) are not converse of each other because they are not based on same informational principle. Therefore, the transformation defined by (5) can be named as asymmetric. Dubois and Prade suggested a symmetric $p \to \pi$ transformation which is define by:

$$u_i = \sum_{j=i}^{n} \min(p_i, p_j) ....(6)$$

Dubois and Prade proved that the symmetric transformation $p \to \pi$, define by (6), is the most specific transformation which satisfies the condition of consistency of Dubois and Prade define by (4).

6. THE COMPARATIVE STUDY

In this section we discuss a hypothetical case study to demonstrate and make use of the transformations for non-cancer human health risk assessment.

A lot of organic and inorganic pollutants exist in produced water. However, in this paper we consider only the heavy metal arsenic (As) because of its toxicity and high concentration in produced water.

The general form of a comprehensive food chain risk assessment model as provided by EPA, 2001 [9] is follows:

$$CDI = \frac{C_f \times FIR \times FR \times EF \times ED \times CF}{BW \times AT} ....(7)$$

Where CID = Chronic daily intake (mg/kg-day), FIR = fish ingestion rate (g/day), FR = fraction of fish from contaminated source, EF = exposure frequency (day/year), ED = exposure duration (years), CF = conversion factor (= 10^{-9}), BW = body weight (kg), AT = averaging time (days) and C_f = chemical concentration of fish tissue (mg/kg). The chemical concentration in fish tissue (C_f) can be computed as

$$C_f = PEC \times BCF ....(8)$$

Where PEC = predicted environmental concentration (mg/l) and BCF is the chemical bioaccumulation factor in fish (l/kg).

The non-cancer risk model for fish ingestion is expressed as:

$$Risk_{non-cancer} = \frac{CDI}{Rfd} ....(9)$$

Where, *Rfd* is the reference dose.

Scenario1:

In this scenario, non-cancer human health risk assessment is performed by considering three cases.

Here, the parameters fish ingestion rate (FIR) and reference dose (Rfd) are considered in normal probabilistic mode with mean 170 and standard deviation 50 (because ingestion rate varies person to person) and triangular probabilistic mode respectively. Also representation of the parameters predicted environmental concentration (PEC) and chemical bioaccumulation factor (BCF) are considered to be fuzzy number. Other parameters are taken to be constant.

Case 1:

Values of the parameters for the calculation of non-cancer risk are given in the table 1.

Table 1: Parameters used in the risk assessment

parameter	Units	Type of variable	Value/distribution
Average Time (AT)	days	constant	25550
Body Weight (BW)	Kg	constant	70
Exposure Duration (ED)	Years	constant	30
Exposure frequency (EF)	Days/year	constant	350
Fraction of contaminated Fish (FR)	-	constant	0.5
Fish Ingestion Rate (FIR)	g/day	Random	Normal(170,50)
Conversion Factor (CF)	-	constant	1E-09
PEC for As	ug/l	Fuzzy	[1.5,4.7,9.0]
BCF for As	l/kg	Fuzzy	[30, 44, 60]
Oral Rfd for As	mg/(kg.day)	Random	[2.0E-04,3.0E-04,4.0E-04]

Results of the calculation is

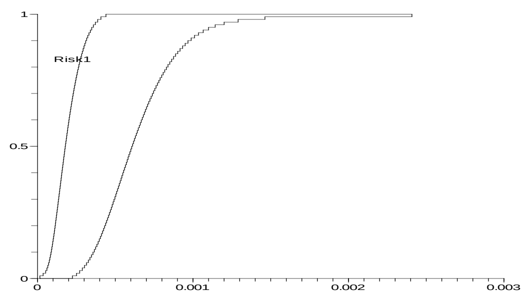

Figure 2: Risk estimation with hybrid parameters

Case 2:

In this case, we have transformed the triangular possibility distribution to triangular probabilistic distribution. i.e., parameters *PEC* and *BCF* are transformed to triangular probability distribution. Other parameters are kept same.

A fuzzy number can be transform to triangular probability distribution as follows. Consider a triangular fuzzy number A= [a, b, c] whose membership function is given as:

$$\mu_A = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \end{cases} .... (10)$$

Integrating the Fuzzy set μ_A with respect to x on [a, c] we have,

$$=\frac{1}{b-a}\int_a^b (x-a)dx+\frac{1}{c-b}\int_b^c (c-x)dx$$

$$=\frac{1}{b-a}\left[\frac{x^2}{2}-ax\right]_a^b+\frac{1}{c-b}\left[cx-\frac{x^2}{2}\right]_b^c$$

$$=\frac{1}{b-a}\left[\frac{b^2}{2}-ab-\frac{a^2}{2}+a^2\right]+\frac{1}{c-b}\left[c^2-\frac{c^2}{2}-bc+\frac{b^2}{2}\right]$$

$$=\frac{1}{b-a}\left[\frac{1}{2}(b^2-a^2)-a(b-a)\right]+\frac{1}{c-b}\left[\frac{1}{2}(b^2-c^2)-c(b-c)\right]$$

$$=\frac{1}{b-a}(b-a)\left[\frac{1}{2}(b+a)-a\right]-\frac{1}{c-b}(c-b)\left[\frac{1}{2}(b+c)-c\right]$$

$$=\frac{1}{2}(b-a)-\frac{1}{2}(b-c)$$

$$=\frac{1}{2}(c-a)$$

Dividing the Fuzzy set (8) by $\frac{1}{2}(c-a)$, we get

$$f(x\,|\,a,b,c)=\begin{cases}\dfrac{2(x-a)}{(b-a)(c-a)}, & a\le x\le b\\[2ex] \dfrac{2(b-x)}{(c-b)(c-a)}, & b\le x\le c\end{cases}$$

Which is a probability distribution function (pdf) of a triangular probability distribution with lower limit 'a', mode 'b' and upper limit 'c'. This transformation follows consistency principle of Dubois and Prade as well as that of Zadeh.

Result of the calculation for non-cancer human health is depicted below:

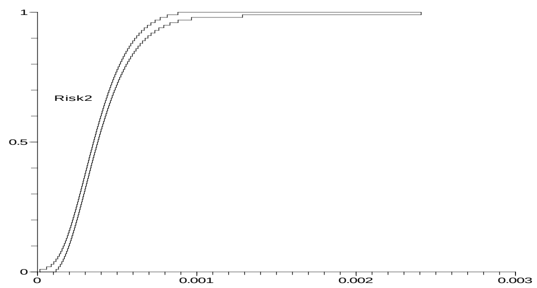

Figure 3: Risk estimation after possibility- probability transformation form triangular fuzzy number to triangular probability distribution.

Case 3:

It has been already proved that [3] the triangular probability distribution is a legitimate transformation of the uniform probability distribution with the same support, and that is upper bounds of all possibility transformations associated with all bounded symmetric unimodal probability distribution with the same support. Also a possibility measure encodes a family of probability measures. Therefore, we can consider a triangular possibility distribution as uniform probability distribution with the same support. Result of the calculation is depicted below:

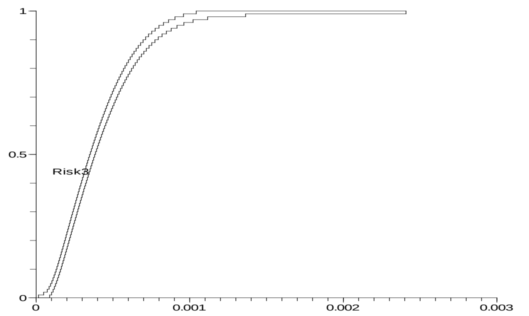

Figure 4: Risk estimation after possibility-probability transformation from fuzzy number to uniform probability distribution.

On Superimposition of the three graphs we obtain the following diagram:

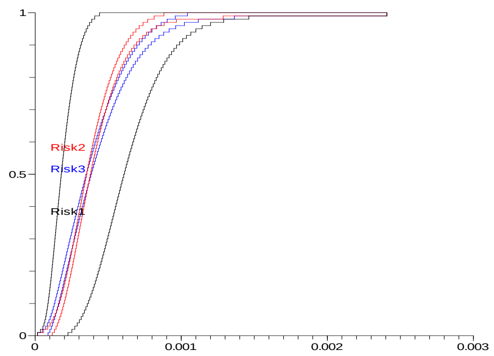

Figure 5 Superimposition of the three cases.

Scenario2:

In this, scenario the parameter fish ingestion rate (FIR) and chemical bioaccumulation factor (BCF) are considered in normal probabilistic mode with mean 170 and standard deviation 50 and triangular probabilistic mode respectively. Also the parameters reference dose (Rfd) and predicted environmental concentration (PEC) are considered to be fuzzy number. Other parameters are taken to be constant.

Case 1:

Values of the parameters for the calculation of non-cancer risk are given in the table 2.

Table 2: Parameters used in the risk assessment

parameter	Units	Type of variable	Value/distribution
Average Time (AT)	days	constant	25550
Body Weight (BW)	Kg	constant	70
Exposure Duration (ED)	Years	constant	30
Exposure frequency (EF)	Days/year	constant	350

Fraction of contaminated Fish (FR)	-	constant	0.5
Fish Ingestion Rate (FIR)	g/day	Random	Normal(170,50)
Conversion Factor (CF)	-	constant	1E-09
PEC for As	ug/l	Fuzzy	[1.5,4.7,9.0]
BCF for As	l/kg	Random	[30, 44, 60]
Oral Rfd for As	mg/(kg.day)	Fuzzy	[2.0E-04,3.0E-04,4.0E-04]

Result of the calculation is depicted below

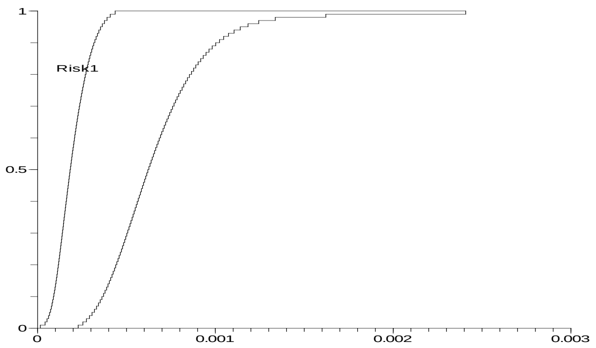

Figure 6: Risk estimation with hybrid parameters

Case 2:

In this case, we have transformed the triangular possibility distribution to triangular probabilistic distribution. i.e., parameters *PEC* and *Rfd* are transformed to triangular probability distribution. Other parameters are kept same. Result of the non-cancer human health risk assessment is given below.

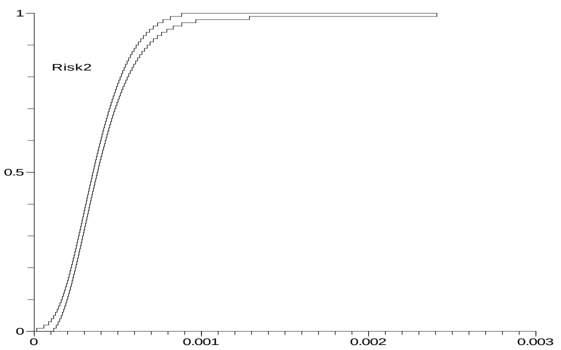

Figure 7: Risk estimation after possibility- probability transformation form triangular fuzzy number to triangular probability distribution.

Case 3:

In this case, we have considered a triangular possibility distribution as uniform probability distribution with the same support. Result of the calculation is depicted below:

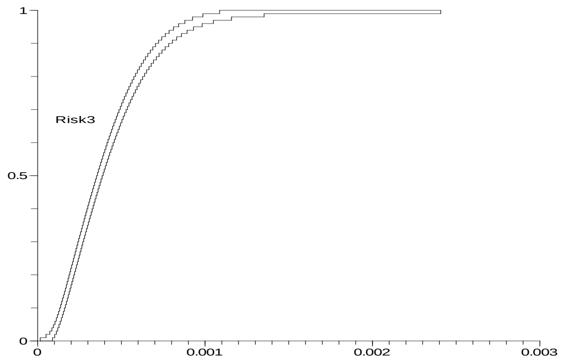

Figure 8: Risk estimation after possibility-probability transformation from fuzzy number to uniform probability distribution.

On superimposition of the results of the non-cancer human health risk assessments are depicted below:

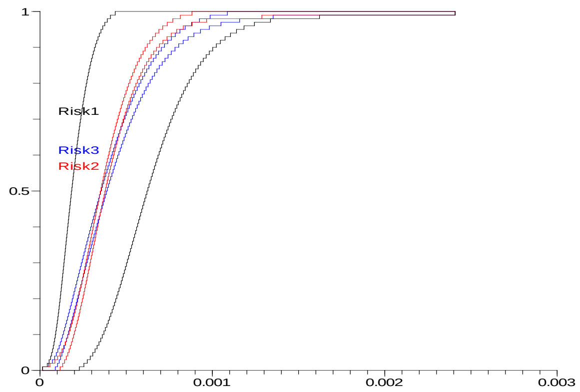

Figure 9: Superimposition of the three cases.

Scenario3:

Here, the parameters fish ingestion rate (FIR), chemical bioaccumulation factor (BCF) and predicted environmental concentration (PEC) are considered in probabilistic mode. Only the parameter reference dose (Rfd) is considered to be fuzzy number. Other parameters are taken to be constant.

Case 1:

Values of the parameters for the calculation of non-cancer risk are given in the table 3.

Table 3: Parameters used in the risk assessment

parameter	Units	Type of variable	Value/distribution
Average Time (AT)	days	constant	25550
Body Weight (BW)	Kg	constant	70
Exposure Duration (ED)	Years	constant	30
Exposure frequency (EF)	Days/year	constant	350
Fraction of	-	constant	0.5

contaminated Fish (FR)			
Fish Ingestion Rate (FIR)	g/day	Random	Normal(170,50)
Conversion Factor (CF)	-	constant	1E-09
PEC for As	ug/l	Random	[1.5,4.7,9.0]
BCF for As	l/kg	Random	[30, 44, 60]
Oral Rfd for As	mg/(kg.day)	Fuzzy	[2.0E-04,3.0E-04,4.0E-04]

Result of the calculation is depicted below

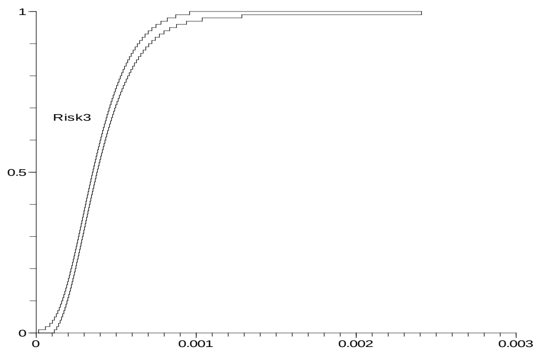

Figure 10: Risk estimation with hybrid parameters

Case 2:

In this case, we have transformed the triangular possibility distribution to triangular probabilistic distribution. i.e., parameter *Rfd* is transformed to triangular probability distribution. Other parameters are kept same. Result of the non-cancer human health risk assessment is given below.

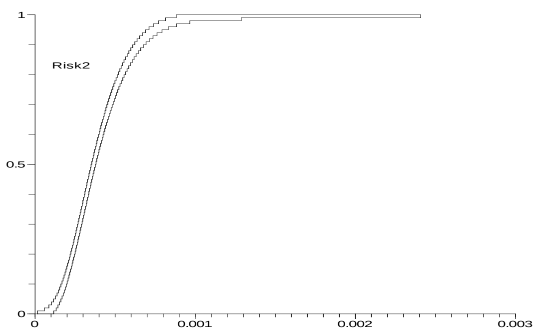

Figure 11: Risk estimation after possibility- probability transformation form triangular fuzzy number to triangular probability distribution.

Case 3:

In this case, we have considered a triangular possibility distribution as uniform probability distribution with the same support. Result of the calculation is depicted below:

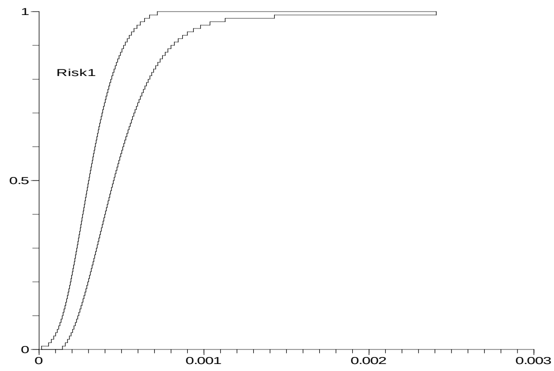

Figure 12: Risk estimation after possibility-probability transformation from fuzzy number to uniform probability distribution

On superimposition of the results of the non-cancer human health risk assessments we have:

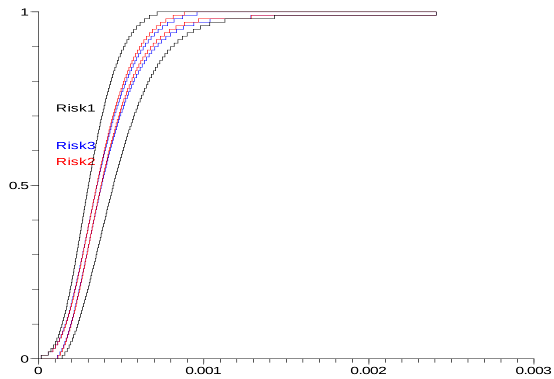

Figure 13: Superimposition of the three cases.

Where

Black: Hybrid approach (when representation of parameters are probabilistic as well as possibilistic)

Blue: Taking fuzzy numbers as uniform distribution

Red: Transforming fuzzy numbers to triangular probability distribution

7. CONCLUSION

The motivation for study of probability-possibility transformations arises not only from a desire to comprehend the relationship between the two theories of uncertainty, but also for some practical problems. For **e**xample: to construct a membership grade function of a fuzzy set from statistical data, to construct a probability measure from a given possibility measure in the context of decision making or system modeling, to combine probabilistic and possibilistic information in expert systems, or to transform probabilities to possibilities to reduce computational complexity. To deal with these problems, various probability-possibility transformations satisfying different consistency principles have been suggested in the literature. In this paper we have used two such transformations to study uncertainty propagation of a single case. First we transformed triangular fuzzy number to triangular probability distribution which satisfies the consistency condition $\pi_i \geq p_i$. In another case, we have transformed triangular fuzzy number to uniform probability distribution. This uniform probability distribution is a legitimate representative of the fuzzy number from which it is obtained. The superimposition of the results of three cases in each scenario show that bound obtained from the hybrid method encodes that obtained when transformation is used. In other words, using probability-possibility transformation we obtain more precise result.

8. ACKNOWLEDGEMENT:

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