Calculation of Error Probability in WDM RZ Systems in Presence of Bit-Pattern-Dependent Nonlinearity and of Noise

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Abstract: We introduce a fully deterministic, computationally efficient method for calculating the error probability in wavelength-division multiplexed return-to-zero systems that accurately accounts for bit-pattern-dependent nonlinear distortions and noise.

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1 Introduction

A fundamental problem in the modeling and design of optical fiber communications systems is to accurately and efficiently compute the bit error rate (BER). Much of the recent work on this problem has focused on computing the probability of errors that are induced by noise from optical amplifiers [1], [2]. However, to accurately compute the BER for wavelength-division multiplexed (WDM) systems, it is also necessary to account for bit-pattern-dependent nonlinear signal distortions. This problem is not straightforward since, for example, in the simulation of a 9-channel system with 32 bits per channel there are $2^{288(=9\times 32)}$ possible bit patterns that must be accounted for. Although data-dependent nonlinear penalties have been characterized by several authors [3]–[8], there has not been until now an accurate calculation of the error probability due to such penalties. To compute this probability, it is necessary to calculate the probability density function (PDF) of the received current, where the sample space is the set of all possible bit patterns in all the channels. In [8], we accurately computed this distribution in the absence of noise for PDF values down to 10^{-10} by numerically solving the nonlinear Schrödinger equation (NLS) and using the multicanonical Monte Carlo (MMC) algorithm [9] to sample the bit string sequences. Although this technique is computationally much more efficient than the standard Monte Carlo method, it still requires on the order of 10^5 data realizations, which is unrealistic for extensive parameter studies. Therefore, it is essential to develop an alternative deterministic approach.

In this paper we focus on long-haul WDM return-to-zero (RZ) systems, in which collision-induced timing jitter due to interchannel cross-phase modulation (XPM) is the major nonlinear impairment. Since the distribution of the timing shift deviates significantly from its Gaussian fit in the tails [8], a knowledge of the standard deviation of XPM-induced timing shifts [4], [6] is not sufficient to accurately compute the PDF. In this paper, we develop a deterministic, computationally efficient method for calculating the PDF of collision-induced timing shifts and apply it to compute the nonlinear penalty and BER for a WDM RZ system. In particular, using the timing shift PDF and the pulse shape in the receiver, we can efficiently compute the PDF of the received current. Our results are in excellent agreement with those we obtained using MMC [8]. Moreover, by combining the timing shift PDF with the PDF due to noise for the received current in a single channel, we can obtain the PDFs of the current in the marks and spaces due to both noise and nonlinear interchannel pattern-dependent distortions of the signal.

2 Theory

To calculate the PDF of the timing shift due to interchannel XPM, we use the reduced model for the collision-induced timing shifts in WDM RZ systems introduced in [4]. For this reduced model, we choose a target pulse in the center channel and compute its timing shifts due to collisions with pulses from other channels. To do so, we numerically solve the NLS propagation equation for a single pulse taking into account pulse evolution due to nonlinearity, dispersion, fiber loss, and amplification. Using this pulse shape as a function of distance, we compute the timing shift G_{kl} of the target pulse due to its collision with a single pulse in the l-th bit of the k-th channel employing the method described in [4]. Thus we solve the propagation equation only once, which is computationally very efficient. This model assumes that the resulting timing shift ΔT of the target pulse is the sum of the timing shifts due to pairwise pulse collisions: $\Delta T = \sum \alpha_{kl} G_{kl}$, where α_{kl} is the binary data (0 or 1) and G_{kl} is the timing shift at the receiver of the target pulse due to its collision with a single pulse in the l-th bit of the k-th channel.

Our goal is to derive the PDF of the collision-induced timing shift ΔT of the target pulse. We assume that we have random data in our system so that the α_{kl} are independent random variables, each having equal probability $1/2$ of being

0 or 1. We also assume for this paper that the bit slots of all the channels are aligned at the input. However, preliminary results show that if the bit slots of different channels are not aligned, the resulting PDF will be 5% wider for PDF values of 10^{-15} and the standard deviation of the timing shift will be 1% larger. We can obtain the PDF of ΔT by computing its characteristic function $w(\xi)$ which is given by $w(\xi) = \langle \exp(i\xi\Delta T)\rangle = \langle \exp(i\xi \sum \alpha_{kl} G_{kl})\rangle$, where $\langle\cdot\rangle$ denotes the statistical average. Since the random variables α_{kl} are independent, $w(\xi) = \prod \left\{\frac{1}{2}\left[1+\exp(i\xi G_{kl})\right]\right\}$. Then the PDF of the timing shift is simply the Fourier transform of the characteristic function, $p_{\Delta T}(\Delta T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} w(\xi)\, \mathrm{e}^{-i\xi\Delta T} d\xi$. To evaluate the nonlinear penalty, we derive the distributions of the current in the marks and spaces due to the timing shifts. For a given pulse shape $i(t)$ of the electrical current in the receiver, we convert a timing shift ΔT to a value I of the sampled current using the formula $I = i(T_0 - \Delta T)$, where T_0 is the central time of the pulse. We then obtain the PDF of the current from the timing shift PDF using the properties of cumulative distribution functions.

To validate this approach, we compare the PDFs calculated this way to those computed using a full system model based on the NLS equation, in which we keep all the nonlinear interactions [10]. In order to calculate the low-probability tails of the distributions of the timing shift and received current, we use the multicanonical Monte Carlo (MMC) technique [9], [11], which increases the occurrence of rare events of interest by biasing their probability. Using a single target pulse in the center channel, we ran two MMC simulations, in which we varied the bit strings in the other channels. For the first simulation, we used the MMC algorithm to obtain the histogram of the timing shift of the target pulse, while for the second we obtained the histogram of the received current of the target pulse at the sampling time [8]. We did not include ASE noise in either MMC simulation.

To compute the PDF $p(I,t)$ of the received current I at the sampling time t due to both bit-pattern-dependent nonlinear interactions and noise, we convolve the PDF $p_{\Delta T}$ of the timing shift with the PDF due to noise, $p_{\text{noise}}(I,t)$ of the received current obtained by propagating a single-channel signal through the system: $p(I,t) = \int p_{\text{noise}}(I, t-\tau)\, p_{\Delta T}(\tau) d\tau$. To compute $p_{\text{noise}}(I,t)$, we assume that the optical noise is additive white Gaussian noise (AWGN) at the entry to the receiver, and we take into account the actual pulse shape and optical and electrical filter shapes using the model described in [1], [2]. In principle, the AWGN assumption is not restrictive since if the noise was colored and correlated due to its nonlinear interactions with the signal we could still efficiently compute p_{noise} using the noise covariance matrix method [2].

3 Results and discussion

Our results are for a typical WDM RZ system with a propagation distance of 5000 km and an amplifier spacing of 50 km, which is similar to the experimental system reported in [12]. We used a dispersion map consisting of 34 km of D_+ fiber and 16 km of D_- fiber, followed by an amplifier [13]. The values of dispersion, dispersion slope, effective core area, nonlinear index, and loss were 20.17 ps/nm-km, 0.062 ps/nm^2-km, 106.7 μm^2, 1.7×10^{-20} m^2/W, and 0.19 dB/km for the D_+ fiber and -40.8 ps/nm-km, -0.124 ps/nm^2-km, 31.1 μm^2, 2.2×10^{-20} m^2/W, and 0.25 dB/km for the D_- fiber respectively. The path average dispersion was -0.5 ps/nm-km, and the amount of dispersion pre- and post-compensation is 1000 and 1800 ps/nm respectively, which we found to be optimal for this system. We use raised-cosine pulses with a duration of 35 ps and peak power of 5 mW, and we launched nine co-polarized channels spaced 50 GHz apart, each with 32 bits. We verified with the full and reduced models that a further increase in the number of channels and number of bits per channel had a negligible effect on the system performance. It is not surprising in this context that short bit strings suffice since the bit strings in the neighboring channels are completely uncorrelated. The receiver included a 30 GHz super-Gaussian optical demultiplexer and a photodetector. For the comparison of our reduced deterministic approach with the full statistical model we used an infinite-bandwidth electrical filter and did not consider noise. For the calculation of p_{noise}, and hence $p(I,t)$ including the effect of noise, we used an 8-GHz electrical fifth-order Bessel filter and we set the OSNR to 15 dB, calculated over a 25 GHz bandwidth.

In Fig. 1(a), we compare the PDF of the timing shift obtained from our reduced deterministic model with the corresponding histogram obtained from the first MMC simulation. The agreement between the two methods is excellent.

In Fig. 1(b), we compare the PDF of the marks obtained from the deterministic timing shift PDF in Fig. 1(a), to the histogram of the current in the marks obtained from the second MMC simulation. There is a noticeable difference around the peaks of these distributions since in the reduced model the only nonlinear effects that we take into account are SPM and XPM-induced timing jitter, and so the current in the marks in the target channel cannot be larger than that of an isolated mark in a single-channel with the neighboring channels turned off. Therefore, the distribution of marks due to the pulse collisions has a sharp cut-off at a relative current of 1.0. By contrast, in the full model, since there

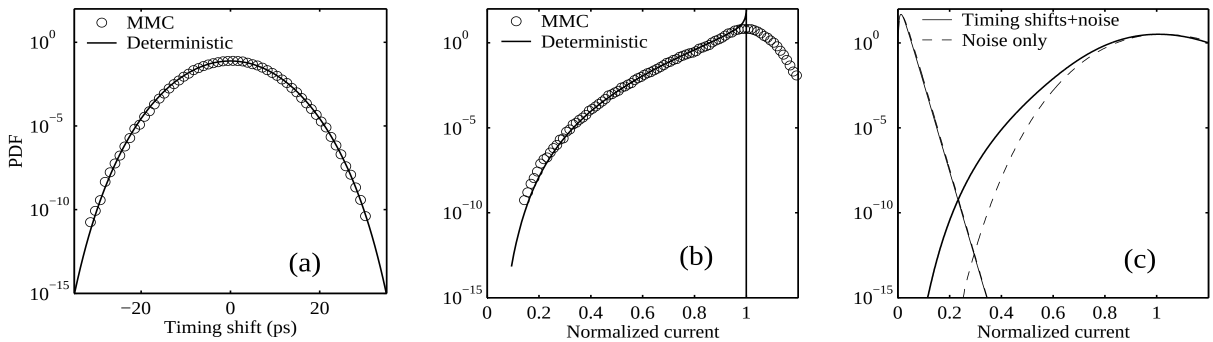

Figure 1. (a),(b) Comparison of our reduced deterministic approach with the full statistical model. (c) Comparison of the PDF $p(I,t)$ of the received current due to timing shifts and noise with the PDF $p_{\text{noise}}(I,t)$ due to noise only.

is some nonlinearly-induced amplitude jitter, the distribution is smooth about 1.0. Nevertheless, the two distributions agree well for current values smaller than 0.9, which is the region of most practical interest. Finally, in Fig. 1(c), we plot the distributions of the currents in the marks and spaces due to both noise and bit-pattern-dependent nonlinear distortions of the signal. The BER computed from these PDFs is 5.8×10^{-12}. For comparison, in Fig. 1(c), we also show the PDFs of the currents with noise but without considering the timing shifts. The BER in this case is 3.3×10^{-15}. We conclude that an accurate calculation of the BER for a WDM system must incorporate a probabilistic treatment of nonlinear signal distortions.

4 Conclusion

We developed a reduced deterministic method for calculating the PDF of the collision-induced timing shift and received current in a WDM RZ system. Although we neglected four-wave mixing and intra-channel XPM, the PDFs that we calculated agree well with the results of full numerical simulations without noise. We also demonstrated that our method can be used to calculate the PDFs of the received current for systems with noise using a realistic receiver model. Consequently, this relatively simple deterministic method allows one to reliably and efficiently calculate the BER taking into account both noise and interchannel pattern-dependent nonlinear penalties.

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