Predicting the Path of Technological Innovation: SAW Versus Moore, Bass, Gompertz, and Kryder

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Abstract

Competition is intense among rival technologies and success depends on predicting their future trajectory of performance. To resolve this challenge, managers often follow popular heuristics, generalizations, or "laws" like the Moore's Law. We propose a model, Step And Wait (SAW), for predicting the path of technological innovation and compare its performance against eight models for 25 technologies and 804 technologies-years across six markets. The estimates of the model provide four important results. First, Moore's Law and Kryder's law do not generalize across markets; none holds for all technologies even in a single market. Second, SAW produces superior predictions over traditional methods, such as the Bass model or Gompertz law, and can form predictions for a completely new technology, by incorporating information from other categories on time varying covariates. Third, analysis of the model parameters suggests that: i) recent technologies improve at a faster rate than old technologies; ii) as the number of competitors increases, performance improves in smaller steps and longer waits; iii) later entrants and technologies that have a number of prior steps tend to have smaller steps and shorter waits; but iv) technologies with long average wait time continue to have large steps. Fourth, technologies cluster in their performance by market.

Keywords: technology evolution, innovation; SAW model, Moore's Law, Kryder's Law, Bass Model, technological prediction

Introduction

Competition is intense among rival technologies in many industries. For example, which is the technology for auto-batteries of the future: lead-acid, nickel cadmium, fuel-cell, or lithium ion? Similarly, which is the technology for display monitors of the future: LCD (liquid crystal diode), LED (light emitting diode), Plasma, or OLED (organic light emitting diode)? How should firms choose among competing technologies? This is probably the pre-eminent challenge facing managers of firms in technology driven markets (Hauser, Tellis and Griffin 2007; Tellis 2008).

To resolve this challenge and predict technology change, managers often follow popular heuristics, generalizations, or “laws”. Examples of such generalizations are Moore’s Law, Kryder’s Law, and the logistic model. Some of these laws gain wide acceptance and begin to serve as self-fulfilling prophecies. For example, Moore (2003) suggests that Moore’s Law drove semiconductor firms to focus enormous energy and make large investments in a race to achieve performance predicted by the law ahead of their competitors.

However, most generalizations and long range predictions fail, offering little help in managerial decision making for at least four reasons (Armstrong 2005; Balachandra 1980; Makridakis et al 1982; Tashman 2000). First, heuristics or laws may be based on cursory observations of short term patterns instead of on a scientific study of long-term data (e.g. by Moore 1965). Such heuristics or laws may not survive careful testing. Second, the law itself may be vague in specification with many contradictory versions. For example, at least two versions of Moore’s law are popular (performance doubling every year and doubling every 18 months). The implications of this uncertainty can be substantial. For example, a technology that doubles its performance every 18 months improves to 100 times its initial performance over 10 years whereas a technology that doubles every year improves to more than 1000 times its initial performance in the same period. Third, the popularity of a law may encourage indiscriminate extension to many fields, technologies, and industries. For example, Moore's law has been claimed to apply to several metrics of technology performance including the size, cost, density and speed of components in the semiconductor industry and many other technologies besides semiconductors like biotechnology, nanotechnology, and genomics (Edwards 2008; Wolff 2004). In fact, Moore (2003, p. 1) suggests that the law has come to refer to "*almost anything related to the semiconductor industry that when plotted on a semi-log paper approximates a straight line*". Note that without the exact specification of the slope of the straight line, the law is intrinsically flexible, and susceptible to hindsight bias. Fourth, prior research is inconclusive on whether the path of technology evolution is smooth or irregular, suggesting that a data driven approach is better for prediction than dependence on generalized heuristics. All four reasons suggest the need for a better model for predicting the path of technology evolution. The current research addresses these limitations in the literature on technology evolution and addresses these research questions:

- How valid are the traditional laws and models for describing technology evolution?
- Which model can best predict the path of technological innovation?
- What are the key drivers of technology evolution?

To address these questions, we propose a new model, called Step and Wait (SAW) and test it against extant models on 25 technologies and 804 technologies-years across six markets over several decades. We make four contributions to current literature. First, we propose a model to predict the evolution of technological performance that provides better predictions than traditional models. Such prediction allows both marketing and technology managers to identify dimensions on which to focus their new product design efforts. Second, the proposed model allows for predicting the path of an entirely new technology based on the similarity of its characteristics to those of prior technologies. Third, the exercise enables us to test the validity and generalizability of some popular "laws" about technology evolution. Fourth, we identify key drivers of technology evolution.

The next five sections present the theory, hypotheses, models, method, and results. The last section discusses the findings, implications, and limitations of the research.

Theory of Technology Evolution

Technology evolution is the improvement in the performance of a technology over time. We are interested in a better understanding of the path of such improvement. Prior literature has debated the shape of the path (whether smooth or discontinuous) and the drivers of the path (explanatory variables that influence its course). We cover both of these topics next.

Shape of Path

Prior literature suggests both smooth change through incremental improvements occurring frequently (Basalla 1988; Dosi 1982) and non-smooth change through relatively stable periods of smooth change punctuated with discontinuous steps of big changes (D'Aveni 1994; Eldredge and Gould 1972; Tushman and Anderson 1986).

Proponents of smooth and incremental technological change (Bassalla 1988) argue that technology evolution is a process of continual improvement in performance of a technology through novel recombination and synthesis of existing technologies (Henderson and Clark 1990). These researchers suggest that changes in technology performance are a result of changes in a number of domains including beliefs, values, culture, technology, operating routines, organizational structure, resources, and core competencies (Gersick 1991; Tushman and Romanelli 1985; Wollin 1999). Invention is a social process that rests on the accumulation of many minor improvements not the heroic efforts of a few geniuses (Basalla 1988; Dosi 1982).

Proponents of irregular change suggest that technologies improve through eras of smooth change punctuated by discontinuous shifts (Adner 2002; Eldredge and Gould 1972; Tushman and Anderson 1986). Products that draw upon fundamentally new technologies enter an industry, and create ferment till the emergence of dominant designs (Nelson and Winter 1977; Utterback and Abernathy 1975). After a dominant design is established, firms focus more on process innovations than on product innovations (Henderson and Clark 1990). Jumps in product performance could occur both from product and process innovation related to the focal technology. Tushman and Anderson (1986) explain the discontinuous nature of technological change through two types of change – competence enhancing and competence destroying. Levinthal (1998) extends the concept of natural speciation (Eldredge and Gould 1972) to technology speciation. Substantial improvements in performance occur because a shift of a technology from one domain to another alters the relative preference for attributes, demands different price/performance ratio for older attributes, and often releases substantially higher resources for R&D (Levinthal 1998). This shift may be due to 1) changes in problem-solving heuristics, 2) fusion with other domains, and 3) other technological, social, or economic aspects. Such shifts provide access to new customers, resources, and performance metrics (Adner 2002). As a result, the technology exhibits sharp steps in performance.

In summary, even though debate in prior research is inconclusive on whether technology evolution is smooth or irregular, the question remains important to managers. Thus, good forecasting capabilities may spell the difference between success and failure in the market.

Drivers of Path of Technological Change

Our review of the theory in this area suggests four covariates that could drive the path of technological change. We discuss the role of each of these covariates next.[1]

Year of Introduction

The covariate "Year of Introduction" reflects the newness of the technology. We hypothesize that new technologies improve in larger and more frequent steps than old technologies due to the improvement in the supporting environment for innovation in recent years. In particular, improvements in supporting environment are characterized by 1) higher total R&D expenditures, 2) more researchers devoted to technology research, 3) use of better tools, 4) better laboratories, 5) better communication of research, 6) more countries focused on research.

In addition, the pace of improvement in new technologies may occur more frequently and in larger steps than old technologies for three reasons: First, after a period of rapid improvement in performance, old technologies may reach a period of maturity (Foster 1986; Brown 1992; Chandy and Tellis 2000; Sood and Tellis 2011). Foster suggests that maturation may be an innate feature of each technology. Sahal (1981) proposes that the maturity occurs because of limits of scale or system complexity. Fleming (2001) suggests that old technologies reach 'recombinant exhaustion' and improvements become smaller. Golder and Tellis 2004 suggest that maturation can result from abandonment following a cascade. Second, newer technologies attract the interest of firms. Market power acquired from successful innovation in the old technologies spur greater inventive activity in new technologies. They at one and the same time appear mysterious yet promise huge benefits. As such, they attract. Third, new technologies also introduce new performance dimensions unrelated to those offered by old technologies. For example, prior to the

[1] Other factors (e.g. market size, technological sophistication) may also affect the evolution, but have not been included in the analysis due to the lack of reliable data on these variables. We thank the anonymous reviewer for suggesting these.

advent of LCD monitors, firms making CRT monitors competed mainly on higher screen resolution. LCD monitors promised compactness as a new performance dimension. Old technologies strive to compete as customer's demand for these dimensions increases. This slows performance improvements on the existing dimension. Thus, we hypothesize:

H_1: Performance of more recent technologies increases in 1) larger steps and 2) more frequent steps (shorter wait times).

Order of Entry

After controlling for the basic effect of calendar time, the order of entry of a technology in a particular market could affect its improvement. We need to emphasize that the time effect probably holds for large time spans such as decades. The order of entry works for small time spans such as a few years within a market, within which one technology follows another pretty rapidly. We identify two rival theories: preferential attraction versus pre-commitment.

The preferential attraction theory holds that the earlier technology gets the most (or better and initially all) of the limited set of resources (dollars, locations, and researchers) than those that follow. Risk aversion of investors and researchers prevents them from investing in new technologies. Prior literature also suggests that pioneers outperform later entrants (Lambkin 1988; Urban et al 1986). If this line of reasoning is valid, the earlier technology will have larger and more frequent improvements in performance than later technologies within the same market. The above argument leads to the following hypothesis:

H_{2a}: Technology entering earlier improve with 1) larger steps and 2) more frequent steps (shorter wait times) than later technologies within the same market.

The pre-commitment argument suggests that the earlier technology enters in an environment with less information about potential markets, dimensions of performance, and available resources, than the technology that enters later. Thus, the earlier technology pre-commits to an evolutionary path that may not be the most efficient or effective. The later technology enters in an environment with greater information about markets, technologies, and resources, and chooses a more efficient and productive evolutionary path (Golder and Tellis 1993). The glamour of the "new" may also result in suppliers switching resources from the old to the new. Thus, technologies entering later to a market will have more resources and more researchers working on it than the old technology. This will result in more frequent but smaller steps in performance . The above argument leads to the following rival hypothesis:

H_{2b}: Technology entering later improve with 1) smaller steps and 2) more frequent steps (shorter wait times) than earlier technologies to a market.

Number of Competing Technologies

Controlling for the effects above, how does improvement relate to the number of competing technologies? We propose two rival theories: competition for limited resources or competition spurring breakthroughs.

The limitation of resources theory is that in any market the amount of dollars, researchers, and labs is relatively fixed in the immediate short term. Thus as the number of competing technologies increases, each gets less. This division of resources results in less frequent breakthroughs and therefore less frequent increases in performance. More competition leads firms to become more risk averse and focus on cost management instead of risky and costly product improvement. Firms generally achieve these objectives by prioritizing process innovation over product innovation (Scherer and Ross 1990). Thus, as the number of competitors increases, improvements in performance are slower.

The rival theory is of competition spurring breakthroughs. This phenomenon could occur for several reasons. First, each technology is supported by a unique set of researchers with their own egos, training, reputation, and emotional attachment. As the number of competing technologies increases, their supporters work harder to promote their own technologies and create improvements in performance. It is also possible that more firms enter a market because a) there is demand or b) because they think it is relatively easy to improve existing products (technologies). In other words, if b) is true there are more entrants because technological progress is likely to be fast[2]. As a result, the number of improvements in performance increases with the number of competition technologies in a market. Second, Rosenberg (1969) refers to a phenomenon of 'compulsive sequence' where a breakthrough in one area typically generates new technical problems creating imbalances that require further innovative effort to realize fully the benefits of the initial breakthrough. For example, the development of high speed steel improved cutting tools, and stimulated the development of sturdier and more adaptable machines to drive them (Rosenberg 1969). Third, new technologies may set up additional opportunities in new niches even for old technologies. Fourth, prior research suggests that a firm's returns from innovation at the margin are larger in an oligopolistic versus a monopolistic environment (Fellner 1961; Arrow 1962; Scherer 1967). Thus, more competition generates more funds to support innovation and faster product improvements. All these reasons suggest that an increase in the number of competitors will increase the number of improvements in technology performance. Thus, we can propose the following rival hypotheses:

H_{3a}: As the number of competitors increases, performance of technologies increases in 1) smaller steps and 2) longer wait times.

H_{3b}: As the number of competitors increases, performance of technologies increases in 1) larger steps and 2) shorter wait times.

Technology Characteristics

We include two covariates to capture technology characteristics – number of prior steps and average prior wait time. Together the two covariates capture unique patterns of technological improvement for a technology within its unique technological paradigm (Nelson and Winter 1982; Dosi 1982). A technological paradigm is the common platform on which scientists and technologists agree to do research and explain the speed and pattern of technological

[2] We thank the anonymous reviewer for suggesting this possibility.

advancement. For example, for the past 30 years, firms in the magnetic storage industry pursued higher areal density as a goal to solve design problems and achieve higher productivity. This common understanding, led firms to race to introduce improvements in areal density ahead of other firms. In this urgency, firms may not delay investments in R&D and frequently introduce products with improvements.

In technologies where such a paradigm emerges, a technology evolves with a large number of steps. However, these steps are small and frequent. Firms take advantage of inter-dependencies with components and advancements in other fields. For example, improvements in areal density of magnetic storage have partly been driven by advancements in other related disciplines like semiconductor, fiber-optic, and micro-electronics.

In the absence of a dominant technological paradigm, firms' efforts scatter in diverse directions. R&D efforts may be targeted towards improvements on diverse performance metrics leading to little synergy across firms' efforts and fewer steps. Also, competing firms within an industry may wait to introduce products to optimize commercialization costs. As a result, there are few steps with long wait times. Longer average wait times also provide firms more time to develop better products. This results in technological progress with large step sizes and long wait times. Thus, the technological paradigm theory suggests the following two hypotheses:

H_4: Technologies with a large number of *prior* steps have 1) small current step and 2) shorter current wait time.

H_5: Technologies with long average *prior* wait times have 1) large current step and 2) long current wait time.

Models

This section describes eight models in the literature that have been or could be used to predict technological change and one model (SAW) that we propose specifically for this purpose (see Table 1). One of the models is an exponential function used to fit both Moore's Law and

Kryder's Law (see Figure A1a in Appendix A). Three more models are the most popular methods used in prior literature to test an S-shaped curve: the Bass, Logistic, and Gompertz models (see Figure A1b in Appendix A). All four models are smooth and do not allow the use of explanatory variables in their popular formulation. We propose modified versions of these four models which do include explanatory variables to allow fair comparison with SAW (see Appendix B). The next two models are discontinuous and allow the use of explanatory variables: the Gupta model for buyer interpurchase behavior and the Tobit-II model used to model technology evolution (see Figure A1c in Appendix A). Appendix B provides details on the models and explains how these models predict for holdout periods and technologies. We also include two simple models for comparison – the Naïve method that does not use covariates and the DiffReg approach that implements a linear regression with covariates.

Moore's Law (Exponential Model)

First proposed by Intel co-founder Gordon E. Moore, the law suggests that the density of integrated circuits doubles in performance every year (Moore 1965). Thus, Moore's law specifies an exponential relationship between technology performance and time (see Figure A1a in Appendix A and (14) in Appendix B). Later Moore revised the law to a doubling in performance every two years (Moore 1975). Subsequently, Moore claimed that the performance of "*almost anything related to the semiconductor industry*" (Moore 1997 SPIE speech) improves at exponential rates across a number of measures like size, cost (or experience), density and speed of components. Over the last few decades, many technologies like microprocessors and DRAMs seem to have followed a revised Moore's law that suggests doubling every 18 months (Mollick 2006; Schaller 1997). Researchers suggest that the law also describes technology evolution for many other technologies besides semiconductors like biotechnology, nanotechnology, and genomics (Edwards 2008; Wolff 2004). If so, the designation of a "law" would be valid.

Kryder's law (Exponential Model)

First proposed by Seagate's Chief Technology Officer Mark Kryder, the law suggests that the density of information on hard drives, also known as areal density, "*increased by a factor of 1,000 every 10.5 years since introduction of these technologies*" (Walter 2005, pp 32). This rate is equivalent to a doubling of performance every 13 months (Shacklett 2008). Grochowski (1998) suggests that the areal density has increased at a compound annual growth rate of 60%. In effect, both Moore's Law and Kryder's Law specify the same exponential form with differing parameters on time (Figure A1a in Appendix A and (14) in Appendix B).

Logistic Model

One theory of the evolution of technology is the theory of S-curves (Foster 1986). This theory suggests that a plot of maximum performance of a technology over time follows an S-shaped curve (see Figure A1b in Appendix A and (16) in Appendix B). The S-curve results from changes in performance on one dimension over the life of the technology. In the early years after introduction, the performance improves slowly because of technical problems with mastering the new technology. Once initial bottlenecks have been resolved, the performance improves rapidly as the technology draws researchers and resources. Eventually the rate of improvement declines either because the technology reaches limits of scale or size (Sahal 1981) or firms start investing in alternate technologies (Abernathy and Utterback 1978).

Bass Model

Some researchers examining the diffusion of new products suggest a demand side explanation of the phenomenon of technology evolution (Adner 2002; Bass 1969; Rogers 1962; Young and Ord 1989; Young 1993). These researchers suggest that consumers adopt a new product based on spontaneous innovation driven by word-of-mouth diffusion. This process carves a typical S-shape of sales of a new product (Sood, James and Tellis 2009) (see Figure A1b in Appendix A and (18) in Appendix B). The demand for the new product drives the evolution of a new technology, on which the new product is based, and also follows an S-curve.

Gompertz' Model

Gompertz' Law was first proposed by British actuary Benjamin Gompertz for use in demographic studies and suggests that the rate of human mortality increases exponentially with age (Gompertz 1825). In the current context, Gompertz' Law states that maturity and exit of old technologies pave the way for the new technologies and drive technology evolution (Young and Ord 1989). The rate of change in the performance of a technology increases at an exponential rate tracing a sigmoid double exponential S-shaped path over the life of the technology from introduction till maturity (see Figure A1b in Appendix A and (21) in Appendix B). Gompertz' Law has been used extensively in prior literature to describe technology evolution because it also produces S-shaped curves that describe different phases of the evolution –acceleration, inflexion, and deceleration of growth over time (Martino 2003; Meade and Islam 1995; 1998; 2006; Young and Ord 1989). The different S-shaped curves have different implications in symmetry around the relative location of the inflection point. These differences may influence the power of these laws to predict technology evolution.

Gupta Model

The model of Gupta (1988) is a well-known and popular approach for modeling consumer purchase decisions. This model consists of three separate stages: brand choice (for modeling the probability of purchasing a particular brand), interpurchase time (for modeling time until purchase) and purchase quantity (for modeling the amount of goods purchased). We use two stages of this model, interpurchase time and quantity to model wait time and size of step, respectively. This model provides a natural approach for predicting the discontinuous nature of technology evolution (see Figure A1c in Appendix A and (23) and (24) in Appendix B).

Tobit II Model

The Tobit models the evolution of technologies as a series of step-functions with random improvements over irregular periods of time (see Figure A1c in Appendix A and (25) and (26) in Appendix B). The model includes a latent variable that represents the probability of a step as a function of explanatory variables.

Simple Models – Naïve and Diff Reg

We also include two simple alternatives. The first method, ***Naïve***, models technology curves as constant in the holdout period. In other words, we assume that the curve for each technology is horizontal i.e. if our last observation in the estimation sample is θ, we predict θ for the entire holdout period. The second method, ***Diff Reg***, performs a single linear regression on all technologies simultaneously using a technology specific indicator variable and the covariates from the previous section as the independent variables. The indicator variable is modeled as a random effect. The change in (log) technology performance between two successive periods is used as the dependent variable. So for example, if a technology remained constant between two periods, we set $Y = 0$ for the response. After fitting the linear regression model, we use the covariates of a technology to predict its change in each time period and hence, the entire trajectory.

The SAW Model

We propose a new approach which models technologies as exhibiting periods of constant performance followed by discontinuous steps (see Figure A1c in Appendix A). We call this model “Step And Wait (SAW)” because it predicts steps in performance followed by a flat “waiting” period before the next step. Hence, it is in line with the theory that technologies evolve according to irregular change. Our motivation in proposing SAW is to test whether such a discontinuous model could better predict evolution of a technology. SAW works by modeling the improvement in performance using the *Step sub-model*, and the time between changes in performance, using the *Wait sub-model*. We describe the specification and prediction of SAW here, and the fitting in Appendix C.

Specification

Let J_{ij} and t_{ij} respectively represent the size of and the duration until, the j^{th} step, for technology i. Let T_{ij} represent the time between the $j-1^{th}$ and j^{th} steps for technology i, so $t_{ij} = t_{i(j-1)} + T_{ij}$. SAW uses two sub-models – the Step sub-model and the Wait sub-model.

The Step sub-model uses a hierarchical approach to estimate the size of the j^{th} step, for the i^{th} technology, J_{ij}, as a function of three quantities M, μ_i, and γ_{ij} as follows:

(1) $$J_{ij} \sim Gamma(M, \mu_i \gamma_{ij})$$

(2) $$\mu_i^{-1} \sim Gamma(\rho, \eta)$$

(3) $$\gamma_{ij} = \exp(rT_{ij} + \alpha_1 Y_{ij1} + \cdots + \alpha_q Y_{ijq}),$$

where Y_{ijk} represents the value of the k^{th} covariate, for technology i, at time t_{ij}, that is used to predict the size of the step J_{ij}. In this formulation, ρ, η, M, r and $\alpha_1, \ldots, \alpha_q$ are parameters to be estimated from the data. The parameter M is a global value that contributes to the average step size for all technologies. The value of r controls the level and type of correlation between the step at time j, J_{ij}, and the wait until this step, T_{ij}. For $r > 0$ increased wait times imply larger steps. The term γ_{ij}, is a function of the various covariates, such as the last wait time.

The random effect term, μ_i, is unique to each technology and reflects its typical step size. SAW builds strength across all the data by estimating μ_i using both the previously observed step sizes for the i^{th} technology and the typical step sizes of the other technologies. Modeling μ_i as a random effect allows us to borrow strength across multiple technologies by assuming the μ_i for each technology is drawn from a common distribution.

In theory, one could model J_{ij} or μ_i as coming from a variety of distributions. However, the Gamma distribution has the advantage that: a) it is extremely flexible (it can model the memoryless exponential and the chi-square distributions, and provides good approximations to Normal and *t*-distributions). b) Using a Gamma allows us to calculate an exact likelihood function for the Step and Wait sub-models which, in turn, provide a relatively simple way of fitting the models by computing the maximum likelihood estimates. For a given μ_i the expected step size is a function of the covariates, Y_{ijk}, the wait time, T_{ij}, and μ_i.

$$E\left(J_{ij}\middle|\mu_i\right) = M\mu_i\gamma_{ij} = M\mu_i\exp(rT_{ij} + \alpha_1 Y_{ij1} + \cdots + \alpha_q Y_{ijq})$$

Hence, a technology with a small μ_i will tend to have small step sizes, and vice-versa, but this effect can be moderated by the observed covariates (e.g. a large investment in research and development at time t_{ij}) through the parameter γ_{ij}. Since $E(\mu_i^{-1}) = \rho\eta$, ρ and η provide information about the typical step size over all technologies. However, the individual covariates for each technology will also affect the step size. The coefficients, $\alpha_1, \dots, \alpha_q$ dictate the relationship between the covariates and the step size so, for example, a positive value for α_k indicates that increases in the k^{th} covariate are associated with larger step sizes while $\alpha_k = 0$ would suggest no relationship.

The Wait sub-model works in a similar fashion, estimating the wait until the $j+1^{th}$ step for technology i, $T_{i,(j+1)}$, as a function of three quantities, λ_i, ω_{ij} and K as follows:

(4) $$T_{ij} \sim Gamma(K, \lambda_i\omega_{i(j-1)})$$

(5) $$\lambda_i^{-1} \sim Gamma(\kappa, \theta)$$

(6) $$\omega_{ij} = \exp\left(sJ_{ij} + \beta_1 X_{ij1} + \cdots + \beta_p X_{ijp}\right),$$

where X_{ijk} represents the value of the k^{th} covariate used to predict T_{ij} for technology i at time t_{ij} and $K, \kappa, \omega, \theta, s$ and $\beta_1, \dots, \beta_p$ are parameters. The parameter K is a global value that contributes to the average wait time for all technologies, while s controls the correlation between the j^{th} step, J_{ij}, and the wait time until the $j+1^{th}$ step. A positive value of s implies longer wait times after larger steps. The term ω_{ij}, is a function of the various covariates for technology i (including the step size, J_{ij}) at time t_{ij}.

The random effect, λ_i, is unique to each technology and reflects its typical wait between steps. Again, SAW builds strength across all the data by estimating λ_i using both the previously observed wait times for the i^{th} technology and the typical wait times of the other technologies. For a given λ_i, the expected wait until the next step is a function of the covariates X_{ijk}, K and λ_i ,

$$E\left(T_{i(j+1)}\middle|\lambda_i\right) = K\lambda_i\omega_{(ij)} = K\lambda_i \exp\left(sJ_{ij} + \beta_1 X_{ij1} + \cdots + \beta_p X_{ijp}\right).$$

Hence, a technology with a small λ_i will tend to have short time periods between steps, and vice-versa, but this effect can be moderated by the observed covariates at time t_{ij}, through the parameter ω_{ij}. For example, a technology may have a large λ_i, and hence typically experience long waits between steps, but at a given time, this might be moderated by a change in the number of competing technologies, resulting in a small ω_{ij} and, hence, a smaller wait time. The expected value of λ_i^{-1} is $\kappa\theta$. So κ and θ provide information about the typical wait time over all technologies. However, the individual covariates for each technology also affect the wait time. The coefficients, $\beta_1, \dots, \beta_p$, dictate the relationship between the covariates and the wait time. For example, a positive value for β_k indicates that increases in the k^{th} covariate are associated with a longer wait, while $\beta_k = 0$ suggests no relationship between the k^{th} covariate and the wait time. Since the covariates can change over time, the typical T_{ij} may increase or decrease.

Predictions

Suppose for a given technology i we observe n_i steps, $J_{i.} = (J_{i1}, \dots, J_{in_i})$ with wait times $T_{i.} = (T_{i1}, \dots, T_{in_i})$. Note that t_{i0} represents the time of introduction. So T_{i1} corresponds to the duration from introduction of the technology until the first step, and J_{i1} is the size of the first step. Then natural estimates for the size of the next step, $J_{i(n_1+1)}$, and the wait until the next step, $T_{i(n_i+1)}$, are $E\left(J_{i(n_1+1)}\middle|J_{i.}\right)$ and $E\left(T_{i(n_i+1)}\middle|T_{i.}\right)$. Using the Step sub-model given by Equations (1) through (3), by the law of iterated expectations and the fact that $J|\mu$ has a gamma distribution,

$$E\left(J_{i(n_i+1)}\middle|J_i.\right) = E\left(E\left(J_{i(n_i+1)}\middle|\mu_i\right)\middle|J_i.\right) = E\left(M\gamma_{i(n_i+1)}\mu_i|J_i.\right) = M\gamma_{i(n_i+1)}E(\mu_i|J_i.)$$

In order to compute the final expectation we need to derive the expected value of $\mu|J$. The distribution of μ^{-1} conditional on $J_.$ is given by

$$\begin{aligned} f(\mu^{-1}|J.) &\propto f(J.|\mu^{-1})f(\mu^{-1}) \\ &= \frac{\prod_{j=1}^{n_i}\left(\mu_i\gamma_{ij}\right)^{-M} J_{ij}^{M-1}\, exp\left(-\mu_i^{-1}\sum_{j=1}^{n_i} J_{ij}\gamma_{ij}^{-1}\right)\eta^{-\rho}\mu_i^{-(\rho-1)}exp\left(-\mu_i^{-1}/\eta\right)}{\Gamma(\mathrm{M})^{\mathrm{n_i}}\Gamma(\rho)} \\ &\propto \mu_i^{-(Mn_i+\rho-1)}exp\left(-\mu_i^{-1}\left(\frac{1}{\eta}+\sum_{j=1}^{n_i} J_{ij}\gamma_{ij}^{-1}\right)\right) \end{aligned}$$

Hence $\mu_i^{-1}|J. \sim \text{Gamma}\left(Mn_i+\rho, \frac{1}{\eta^{-1}+\sum_{j=1}^{n_i} J_{ij}\gamma_{ij}^{-1}}\right)$ but the expected value of the inverse of a Gamma (α, β) random variable is equal to $\frac{1}{\beta(\alpha-1)}$. Therefore $E(\mu_i|J_i.) = \frac{\eta^{-1}+\sum_{j=1}^{n_i} J_{ij}\gamma_{ij}^{-1}}{Mn_i+\rho-1}$ and the expected size of the next step conditional on previous steps is

(7) $$E\left(J_{i(n_i+1)}\middle|J_{i.}\right) = M\gamma_{i(n_i+1)}\frac{\eta^{-1}+\sum_{j=1}^{n_i} J_{ij}\gamma_{ij}^{-1}}{Mn_i+\rho-1}\,.$$

Similarly, using the Wait sub-model given by Equations (4) through (6) the expected wait until the next step conditional on previous steps is (derivation is identical to that for (7)),

(8) $$E\left(T_{i(n_i+1)}\middle|T_{i.}\right) = K\omega_{in_i}E(\lambda_i|T_{i.}) = K\omega_{in_i}\frac{\theta^{-1}+\sum_{j=1}^{n_i}\omega_{i(j-1)}^{-1}\,T_{ij}}{Kn_i+\kappa-1}\,.$$

From equation (8) we can predict that the next step in technology *i* will occur at time

$t_{i(n_i+1)} = t_{in_i} + E\left(T_{i(n_i+1)} \middle| T_{i.}\right) = t_{in_i} + K\omega_{in_i} \frac{\theta^{-1}+\sum_{j=1}^{n_i} \omega_{i(j-1)}^{-1} T_{ij}}{Kn_i + \kappa - 1}$ and the following step at time

$t_{i(n_i+2)} = t_{i(n_i+1)} + K\omega_{i(n_i+1)} \frac{\theta^{-1}+\sum_{j=1}^{n_i} \omega_{i(j-1)}^{-1} T_{ij}}{Kn_i + \kappa - 1}$ and so on.

Together Equations (7) and (8) can be used to predict the entire remaining trajectory. Note that this approach will work even for a curve for which we have no data. SAW can be used to estimate the size of the first step and the duration until the first step after the introduction of a new technology. In this case $n_i = 0$ so Equations (7) and (8) simplify as:

(9) $$E(T_{i1}) = K\omega_{i0} \frac{\theta^{-1}}{\kappa - 1}$$

(10) $$E(J_{i1}) = M\gamma_{i1} \frac{\eta^{-1}}{\rho - 1}$$

Thus, given estimates for $\mu_i, \gamma_{ij}, \lambda_i, K, M$ and ω_{ij} one can predict the evolution of a technology as far into the future as desired by combining the predicted wait time ($K\lambda_i\omega_{ij}$) with the predicted step size ($M\mu_i\gamma_{ij}$).

Connections to Renewal Reward Process

Our SAW model has similarities to a Renewal Reward Process (Cox 1970). In particular for fixed values of λ_i and μ_i, SAW fits a separate non-homogeneous Renewal Reward Process (RRP) to each technology. The non-homogeneous component is introduced by virtue of the time varying covariates. However, while conditional on λ_i and μ_i each technology is independent, these parameters are unobserved in practice. So SAW models the processes (technologies) as unconditionally related via the Gamma distributions given by (2) and (5). In this sense SAW can be considered to be a generalization of a standard Reward Renewal Process because it is building strength across the technologies by jointly modeling a series of related processes.

Extensions of the Exponential, Logistic, Bass and Gompertz' Models

In their standard forms, the Exponential, Logistic, Bass, and Gompertz models are all fit individually to a single technology, and do not incorporate covariates in their specification. This specification places them at a potential disadvantage relative to SAW, which both utilizes the covariate information and builds strength across technologies by fitting all curves simultaneously. In order to ensure a fair comparison we fit modified versions of these methods. In particular we implemented two new versions of each approach.

In the first implementation, we used a non-linear mixed effects model (Pinheiro and Bates, 2000), which fitted the standard functional forms of each method but modeled the various parameters as random effects coming from a Gaussian distribution. The parameters for the Gaussian distribution were estimated using all technologies simultaneously. Hence it built strength across technologies in a similar fashion to SAW. Our second implementation also modeled the parameters using a random effects formulation; but, in addition, incorporated the covariates as a multiplicative adjustment to the original prediction. In this implementation we modeled each technology using,

$$P_{ij} = f_i(t_{ij})\ \exp\left(\beta_0 + \textstyle\sum_{k=1}^{q} \beta_k X_{ijk}\right) e^{\epsilon_{ij}} \tag{11}$$

where P_{ij} is the performance of technology i at time t_{ij}, $f_i(t)$ is the general formulation of the Exponential, Logistic, Bass, or Gompertz model, exclusive of covariates, and X_{ijk} is the kth covariate for technology i at time t_{ij}. For example, the Exponential model, (11) becomes,

$$P_{ij} = \tau_{i1} e^{\tau_{i2} t_{ij}} \times \exp\left(\beta_0 + \sum_{k=1}^{q} \beta_k X_{ijk}\right) e^{\epsilon_{ij}}, \quad \tau_{i1} \sim N(\mu_1, \sigma_1^2), \ \ \tau_{i2} \sim N(\mu_2, \sigma_2^2)$$

with τ_{i1} and τ_{i2} modeled as coming from a Gaussian distribution. Equivalently, using a log transformation,

$$\log(P_{ij}) = \log(\tau_{i1}) + t_{ij}\tau_{i2} + \beta_0 + \sum_{k=1}^{q} \beta_k X_{ijk} + \epsilon_{ij}.$$

When $f_i(t_{ij})$ is set to the Bass Model (11) has a similar form to the Generalized Bass Model (Bass et al. 1994) though the latter method does not use a mixed effects fitting procedure.

We used a multiplicative covariate adjustment to f_i because this ensured the basic shape for each model was maintained while still allowing the covariates to influence the fit. This second implementation had the twin advantages of building strength by simultaneously fitting all curves and incorporating the covariates. Hence, these models can be seen as a direct competitor to SAW. To our knowledge, neither the first nor second mixed effects formulations have been previously implemented in such a setting, though in the Bass . So our specification can be considered as a contribution in its own right. For more details of our fitting procedure see Appendix B.

Method

This section describes the data collection and the method of prediction.

Data

We collected data on 26 technologies drawn from six markets - external lighting, desktop printers, display monitors, desktop memory, data transfer, and automotive battery technologies (see Table 2). We chose these six markets to ensure sufficiently long periods of study, wide variety of technologies, and diversity of markets. We collected the data using the historical method (Sood and Tellis 2005). The primary sources of our data are technical journals, white papers, press releases, timelines of major firms, records in museums on the development of industries, and annual reports of industry associations.

For each technology, we collected the performance of the technology on the most important attribute to consumers – the primary basis of competition among technologies within a market (see Table 2). We identified these important attributes based on articles collected through the historical method. We recorded the maximum performance for any commercialized product based on the technology at each time period. Our sample includes technologies introduced more than a hundred years ago and those introduced only in the last decade. It also includes markets from basic utilities, medical therapeutics, and the digital industry. Figure 1 shows the performance of all technologies in three of the six markets.

We define a step as an improvement in performance *however small,* of *any product in the market* based on a technology. We make the following assumptions: 1) The performance of a technology in the market is based on the best performance of any commercialized product based on that technology. Because of constraints in production, competitive agreements, or regulation, the performance of products in the market often does not change at all in some years. Hence the performance curve is flat in these years. 2) We have identified all products in the market based on all technologies. 3) The performance of these products is correctly reported by manufacturers.

We used the following rules to ensure reliable and consistent data. First, we measure the performance of a technology based only on commercialized products of that technology. Second, if two sources provide conflicting performance for a technology in a period, we choose the one whose values are more consistent with the rest of the series. Third, if no record is available for a certain year, but a later record confirms that performance has not changed since the last available record, we assume that the performance has not changed in the intervening years. Fourth, if no record is available for a certain year, but a later record confirms that performance has changed since the last available record, we treat the intervening years as missing data. Using these rules, we were able to collect data on only 804 technology years as compared to the total of 901 technology years in our original sample (89%).

Method of Prediction

A direct comparison of the statistical models across markets on all these technologies is not possible unless the performance plots are modified to convert absolute performance to some sort of relative performance. Since we are interested in analyzing how a technology improves over time, we calculate the ratio of current performance to its performance in the year of first introduction. We fit all methods after transforming the data onto a log scale. This transformation reduces skewness in the data and generally gave lower prediction errors for all methods. We explain the specific procedure for carrying out the prediction in two parts: partitioning of sample and evaluation of predictive accuracy.

Partitioning of Sample

To test the accuracy of predictions for future technology innovation using SAW and the six alternate models, we divided the technologies into training (in sample) and testing (out of sample) time periods. We could use data on only 25 technologies because the ESL technology had only one observation by 2009. For each technology, we aimed to predict the performance for the most recent *5* years. The training period consisted of the remaining data (see Figure A2a in Appendix A). For the SAW approach we fitted the model using the training observations for all technologies except the one for which we wished to make predictions. We then used the training observations from the curve for which we were forming predictions to make predictions using equations (7) and (8). This approach guaranteed a fair comparison with the other models by ensuring that the out of sample data for a particular curve was never used, directly or indirectly, to form estimates for a given technology.

Evaluation of Predictive Accuracy

We compare the predictions on the test time period with the actual evolution of the technology using two measures. The first is the average absolute deviation (AAD),

(12) $$AAD_i = \frac{1}{Z}\sum_{t=1}^{Z} |P_{it} - \widehat{P_{it}}|$$

where Z is the length of the testing period, P_{it} is the performance level at time t of the testing period, for technology i, and $\widehat{P_{it}}$ is the corresponding estimate using a given model.

The second approach standardizes the curves according to the absolute values of the technology (Percentage AAD). This method scales the error relative to the level of performance in the technology. Specifically we compute,

(13) $$Percentage\ AAD_i = \frac{1}{Z}\sum_{t=1}^{Z} \frac{|P_{it} - \widehat{P_{it}}|}{P_{it}}\ .$$

We report the median values of AAD and Percentage AAD averaged over all the technologies.

Results

We first present the results on the drivers of technological change. Next, we compare the performance of SAW with alternative models in predicting technology evolution. We then present the findings on the step size, wait time, and growth rate for all technologies. Finally, we present plots of the patterns of technology evolution for all markets combined.

Drivers of Technological Change

Table 3 presents the parameter estimates for the Step and Wait sub-models. The year of introduction covariate has a positive sign for the Step sub-model but a negative sign for the Wait sub-model. The results support H_1 that products introduced in later years tend to have shorter waits and larger steps.

The order of entry covariate has negative signs for both the Step and Wait sub-models. The results indicate that, after controlling for year of entry, later entrants to a market tend to have a shorter wait, but smaller steps. The negative coefficient for the step size is highly statistically significant and is consistent with the preferential attraction theory (H_{2b}).

The number of competing technologies covariate has a negative sign for the Step sub-model and a positive sign for the Wait sub-model. The results suggest that after controlling for the effects above, our results support H_{3a} and reject H_{3b}.

The number of prior steps covariate has a negative sign for both the Step and Wait sub-models. The results support H_4 and suggest that technologies that have a number of prior steps continue to have small steps that happen at frequent intervals.

The average prior wait time covariate has a positive sign for the Step sub-model but a slightly negative sign for the Wait sub-model. The results partially support H_5; suggesting that, after conditioning on the other covariates, technologies with long average prior wait time also have larger step sizes but may not continue to have long wait times.

Finally, the last step size and the last wait time covariates are statistically significant in the Step sub-model; providing evidence that there is a correlation between step sizes and wait times, even after adjusting for the other covariates.

Comparison with Alternative Models

Table 4 presents the median errors, over all technologies, comparing SAW with the alternative models. We use the final five years for each technology as the testing period, i.e. Z=5. We found that the alternative models all generally gave superior results using the log transformed data so we report only these results. We also adjusted the competing methods so that their predicted curve passed through the final training data point (see Figures A2b and A2c in Appendix A). This generally gave superior results and made the models comparable to SAW, which forms its predictions in the holdout period starting from the final training data point.

Table 4 contains two sets of results for each method. The first is the random effects fit with no covariates while the second is the fit that incorporates the covariates from Table 3 using (11). Among the models without covariates, SAW is significantly superior on both metrics. When incorporating covariates, SAW improves further on the AAD metric, both in absolute terms and relative to the competing methods. SAW is the best in AAD and equal to the best in percentage AAD. Figure 2 plots the median AAD by year for models with covariates and demonstrates that SAW outperforms most models in every year during the testing period. The only exceptions are in 2005 and 2006, where the SAW and Gupta models both have zero AAD. In all other years, and for all other methods, SAW is superior. We also compare the per technology performance of SAW relative to the competing methods (see table 5). The SAW model is first equal in performance on 40% of technologies, and has the lowest Median AAD across all technologies for the 5 hold-out (most recent) years.

We also implemented the Exponential, Logistic, Bass, and Gompertz models using fixed effects for the parameters, i.e. fitting the models separately to each curve. The results (not shown here) were generally inferior to those reported in Table 4, suggesting that building strength by fitting all curves simultaneously using random effects improves prediction accuracy. However, since the alternate models were still inferior to SAW, we can conclude that SAW is performing well partly because of its ability to build strength across technologies but also because of its functional form which more accurately matches the observed data.

Step Size, Wait Time, and Growth Rate

Equations (7) and (8) provide predicted step size and wait times which can be used to predict the future evolution of a technology. Table 6 presents the average predicted step size, on a log scale, and wait time, in years, for each technology (Columns 4 and 5). By taking the ratio of predicted step size and wait time, we can also assess the average long run growth rate for each technology (Column 6). The final column of Table 6 contains the estimates for τ_2, the exponent when using a fixed effects model to fit an exponential curve to each technology, along with the associated standard error, σ_{τ_2}. Kryder's law predicts that $\tau_2 = \frac{12}{13}\log 2 = 0.64$ while Moore's law implies $\tau_2 = \frac{12}{18}\log 2 = 0.46$. Almost all technologies exhibited rates of growth considerably slower than these values. The lone exceptions were the Fiber optics and Wireless technologies which had estimated coefficients of $\tau_2 = 0.44$ and $\tau_2 = 0.60$ respectively. Thus, contrary to claims in the literature, Kryder's Law and Moore's Law appear to be neither applicable to the magnetic storage technology nor generalizable across markets.

Figure 3 provides a plot of the predicted step sizes and wait times for each of the 25 technologies on a two-dimensional graph. Several aspects stand out: First, there is clear clustering, with technologies from the same markets generally showing similar predicted step sizes and wait times. We might expect this form of clustering since technologies within the same market will tend to have similar properties. Second, the unconditional correlation between step size and wait time is negative (-0.32).

Figures A3a and A3b, in the Appendix A, plot the step size and wait times for each technology as a function of calendar year respectively. The positive slope of the trend line in Figure A3a suggests that the step size is increasing over time and the negative slope in Figure A3b suggests that the wait time is decreasing over time. Figure A3c plots the growth rate (on a log scale) over calendar time and shows a very clear trend of exponentially increasing growth rates over calendar time, with a correlation of over 0.6 with a p-value below 1%. These results suggest that technology evolution is occurring at a faster pace with calendar time.

Discussion

This section summarizes the findings and discusses the implications and limitations.

Summary of Findings

The current research leads to four major findings:

1. The traditional laws of technology evolution like Moore's Law and Kryder's law do not generalize across markets; none holds for all technologies even in a single market.
2. SAW produces superior predictions over traditional methods, such as the Bass model or Gompertz law, and can form predictions for a completely new technology, by incorporating information from other categories on time varying covariates.
3. The signs of the significant drivers of technology evolution suggest that:
   i. recent technologies improve at a faster rate than old technologies;
   ii. as the number of competitors increases, performance of technologies increases in smaller steps and longer waits;
   iii. later entrants to a market and technologies that have a number of prior steps tend to have smaller steps and shorter waits
   iv. technologies with long average prior wait time continue to have large step sizes
4. Technologies cluster in their performance by market.

Implications

This study has several implications for managers. First, our results suggest that popular laws and models like Moore’s Law, Kryder’s Law, Gompertz Law, and the logistic model are naive generalizations of what seems to be a complex phenomenon. Such theories make simplistic assumptions about the path of technology evolution (e.g., exponential or S-shaped), and so are inadequate in predicting technology change well. Surprisingly, over the period covered in our analysis, it took 28 months for magnetic storage technology to double in performance, which is much longer than the commonly espoused versions of Moore’s law claiming doubling every 18 months (recent) or 12 months (original). Hence, while such laws may serve as long term guideposts for industry evolution, using them to predict the performance of a technology is quite risky and potentially misleading. On the other hand, SAW explicitly models the discontinuous nature of the technology evolution curves observed empirically.

Second, SAW can help managers to reduce the nature and extent of uncertainty regarding the future path of technology evolution. SAW can be easily fit by a simple maximum likelihood approach and incorporates time-varying covariates for each technology. Thus, managers can use it to assess the nature of the threat posed by a competing technology by classifying it as one that is a long-wait-small-step technology or vice versa. As an example, consider the competition between LCD and CRT monitors (see Figure 1b). Sony kept investing in CRT even after LCD first crossed CRT in performance in 1996. Instead of considering LCD, Sony introduced the FD Trinitron/WEGA series, a flat version of the CRT. CRT crossed LCD for a few years, but ultimately lost decisively to LCD in 2001. In contrast, by backing LCD, Samsung grew to be the world's largest manufacturer of LCD, while the former leader Sony had to seek a joint venture with Samsung in 2006 to manufacture LCD. Prediction of the next step size and wait time using SAW could have helped Sony’s managers make a timely investment in LCD technology.

Third, SAW overcomes limitations of prior models of depending on only environmental scanning (e.g., survey or the Delphi method) or extrapolation (e.g., trend analysis). SAW incorporates both environmental scanning by incorporating data from multiple technologies and extrapolation by incorporating past data from the target technology in making predictions. Further, SAW is flexible enough to allow for large periods of no change punctuated by big steps or small periods of small changes approximating a smooth curve. As such, it partially resolves the controversy in the literature between technology evolution via a smooth curve (Basalla 1988; Dosi, 1982) or via stable periods punctuated with big steps (Eldredge and Gould 1972; Tushman and Anderson 1986). For example, inkjet printers became the dominant technology in the market even though they had the lowest performance at introduction through a series of small but frequent steps.

Fourth, our results suggest that the competitive landscape is becoming more intense. An increasing number of new technologies are entering the market. The rate of technology evolution is increasing at a faster pace. Thus, managers need a method and model to predict technology evolution to guide their multi-million dollar investments. SAW serves such a purpose. SAW can easily make predictions for a new technology with no prior data. This discussion brings us back to the key question that managers face. Which technology to back? In GM's case, it turned out to be a billion dollar question. GM spent over a billion dollars on the hydrogen fuel cell. Yet the technology that leapt ahead in the 2000s was Lithium-ion. Tesla based its battery on the Lithium-ion and had a car on the market in 2006. GM saw the need for Lithium-ion only after the Tesla was launched and launched a car using a Lithium-ion battery only in December 2010. Many firms were taken by surprise by the sudden dominance of Lithium-ion. Managers could possibly have presaged the improvements in Lithium-ion technology before 2006 by using our model.

Limitations

This study has four limitations. First we had to limit our analysis to only six markets due to the time and difficulty of data collection. Second, our analysis does not include the impact of investments in R&D on technology evolution. This is a limitation of the data, rather than of SAW, since it could certainly include R&D budgets as a covariate, which should increase its predictive accuracy even more. Third, our analysis does not include the cost of the technology to buyers. Fourth, it is not possible to exactly estimate the step size and wait times for the years with missing data. However, given the small percentage of such data this is unlikely to have a significant effect on the results. Fifth, we assume firms announce all improvements in performance and there are no minor improvements between steps. A possible extension may relax this assumption and allow for a low level of growth during the wait period. All of these limitations are potential opportunities for future research.

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Table 1: Unifying Framework for Models for Predicting Technology Evolution

	Smooth (Continuous)	**Discontinuous (Irregular)**
Symmetric	Logistic, Bass, Gompertz (S-shaped)	NA
Asymmetric	Moore, Kryder (Exponential-shaped)	SAW, Tobit, Gupta, Diff Reg (irregular step sizes with irregular wait times)

Table 2: Technologies Sampled and Primary Dimensions of Competition*

Market	**Primary Basis of Competition**	**Metric**
External Lighting	Lighting Efficacy	Lumens per Watt
Desktop Memory	Storage capacity	Bytes per square inch
Display Monitors	Screen resolution	Dots per square inch
Desktop Printers	Print resolution	Pixels per square inch
Data Transfer	Transfer Speed	Megabits per second
Automotive Battery	Energy Density	Watt-hour/kg

Note: * Adapted from Sood and Tellis (2005)

Table 3: Drivers of Step Size and Wait Time

Covariate	**Step Size**		**Wait Time**	
	Est.	**t-val**	**Est.**	**t-val**
Year of Introduction (H_1)	.19	39.7	-.12	-247.3
Order of Entry (H_2)	-.31	-8.0	-.05	-1.3
Number of Competing Technologies (H_3)	-.11	-3.0	.42	12.4
No of Prior Steps (H_4)	-.01	-1.3	-.06	-6.4
Average Prior Wait Time (H_5)	.08	3.4	-.003	-.1
Last Step Size (*r* – Equation *3*)	.02	2.9	.002	.3
Last Wait Time (*s* – Equation *6*)	-.04	-2.8	-.01	-.8

Table 4: Comparison with Alternative Models: Median of Test Errors (Z=5 years)

Model	Path of Tech. Change	No Covariates		Hypothesized covariates	
		AAD	% AAD	AAD	% AAD
Moore/ Kryder	Exponential	.45	.12	.30	.07
Logistic	S-shaped	.27	.08	.31	.07
Bass	S-shaped	.28	.08	.56	.21
Gompertz	S-shaped	.31	.09	.32	.07
Gupta	Irregular	.26	.07	.31	.08
Tobit II	Irregular	.41	.14	.34	.16
SAW	Irregular	.20	.05	.13	.07
Naïve	No Change	.24	.06	NA	
Diff Reg		NA		.55	.13

Note: Refer Table 3 for hypothesized covariates.

Table 5: Average AAD in testing period for all models and technologies

Technology	Exp	Logistic	Bass	Gompertz	Gupta	Tobit II	SAW	DiffReg
Incandescent	.10	.22	.72	.28	.08	.06	.05*	1.16
ArcD	.12	.20	1.47	.24	.00	.02	.00*	.05
GasDischarge	.09	.11	.85	.11	.12	.04	.11	.03
LED	1.32	1.37	1.47	1.47	1.47	1.26	1.39	1.13
MED	.05	.03	.00	.00	.00	.24	.07	.01
Magnetic	.32	.32	.62	.62	.25	.23	.26	.74
Optical	.22	.25	.02	.00	.46	.72	.00*	.21
Magneto.Optical	1.30	1.30	1.71	1.61	1.09	.88	1.61	1.30
Holographic	.28	.12	.39	.31	.30	.29	.37	.15
Semiconductor	.78	.65	.86	.75	.76	.72	.86	.74
CRT	.35	.01	.03	.01	.42	.32	.13	.72
LCD	.24	.10	.23	.18	.25	.45	.10*	.37
OLED	.16	.05	.58	.07	.31	.22	.08	.55
PDP	.30	.31	.37	.37	.37	.30	.26*	.32
ELD	.09	.32	.05	.33	.06	.14	.04*	.04
Dot.Matrix	.47	.48	.52	.48	.29	.34	.48	.50
Ink.Jet	.32	.55	.69	.47	.31	.46	.58	.68
Laser	.96	1.11	1.42	1.35	1.13	.83	1.39	1.08
Thermal	.63	.71	1.16	1.07	1.01	.82	.87	.60
Cu.Al	.93	.63	.00	.62	1.27	1.06	.00*	2.07
Fiber.optics	.77	.76	.51	.66	.45	1.84	1.21	1.00
Wireless	.50	.50	1.38	.32	.32	.47	.05*	.85
Galvanic.cell	.13	.05	.56	.07	.00	.13	.00*	.39
Fuel.Cell	.08	.16	.07	.17	.25	.61	.30	.25
Flow.Cell	.03	.01	.03	.00	.00	.12	.00*	.08
# times best	**1**	**5**	**2**	**2**	**4**	**2**	**10**	**3**
Median	**0.30**	**0.31**	**0.56**	**0.32**	**0.31**	**0.34**	**0.13**	**0.55**

Note: ***** Lowest AAD across all models

Table 6: Step Size, Wait Times, and Growth Rates

Category	Technology	Year of Introduction	Mean Step Size	Mean Wait Time	Growth Rate From SAW (Equations 7 and 8)	Growth Rate from Exponential Model (SE)
External Lighting	Incandescent	1879	0.11	19.73	0.01	0.02 (0.001)
	Arc Discharge	1908	0.10	10.11	0.01	0.03 (0.001)
	Gas Discharge	1932	0.27	14.02	0.02	0.02 (0.001)
	LED	1965	0.34	3.63	0.09	0.13 (0.005)
	MED	1989	0.34	6.98	0.05	0.01 (0.002)
Desktop memory	Magnetic	1937	0.35	1.25	0.28	0.31 (0.007)
	Optical	1982	1.28	10.83	0.12	0.12 (0.011)
	MO	1986	0.88	4.47	0.20	0.24 (0.011)
	Holographic	2002	0.79	4.06	0.19	0.12 (0.017)
	Semiconductor	2002	0.91	5.00	0.18	0.26 (0.066)
Display Monitors	CRT	1929	0.38	3.59	0.10	0.19 (0.016)
	LCD	1967	0.50	3.29	0.15	0.21 (0.011)
	OLED	1971	0.52	5.13	0.10	0.11 (0.011)
	PDP	1984	0.60	4.13	0.15	0.20 (0.018)
	ELD	2004	0.47	4.52	0.10	0.02 (0.007)
Data Transfer	Dot Matrix	1953	0.56	5.02	0.11	0.07 (0.005)
	Ink Jet	1975	0.91	2.19	0.41	0.32 (0.013)
	Laser	1976	0.83	5.21	0.16	0.19 (0.014)
	Thermal	1979	0.69	3.37	0.20	0.29 (0.018)
Desktop Printers	Cu Al	1962	2.47	5.17	0.48	0.38 (0.021)
	Fiber Optics	1977	2.19	1.88	1.16	0.44 (0.016)
	Wireless	1982	1.83	2.69	0.68	0.60 (0.051)
Automotive Batteries	Galvanic Cell	1780	0.34	5.74	0.06	0.06 (0.008)
	Fuel Cell	1838	0.49	2.62	0.19	0.10 (0.008)
	Flow Cell	1980	0.30	7.60	0.04	0.02 (0.002)

Figure 1: Empirical Path of Technology Evolution In 3 Markets

Figure 1a: Desktop Memory

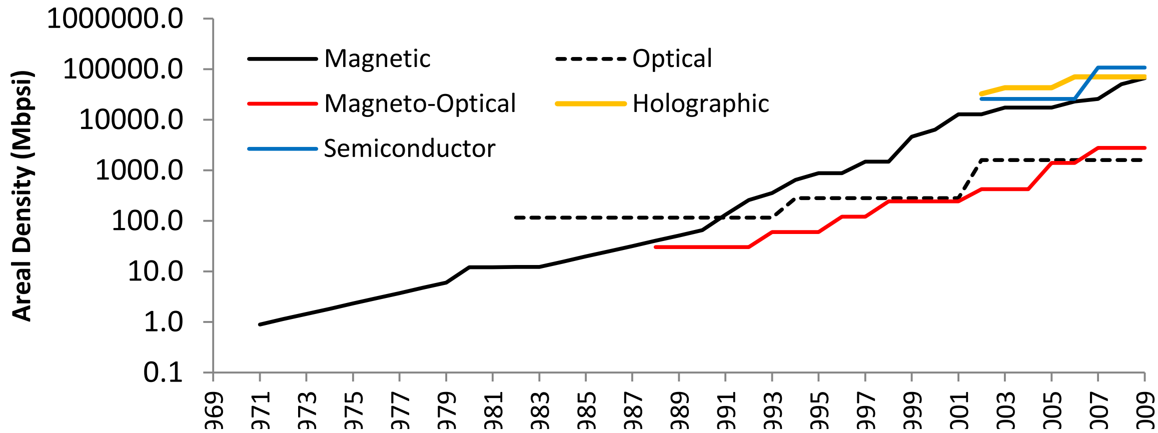


Figure 1b: Display Monitors

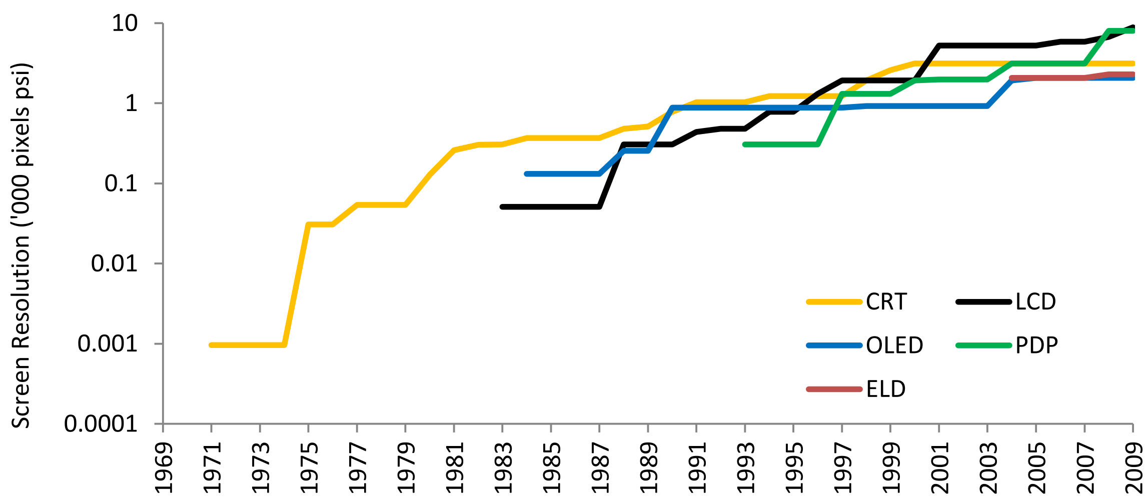


Figure 1c: Automotive Batteries

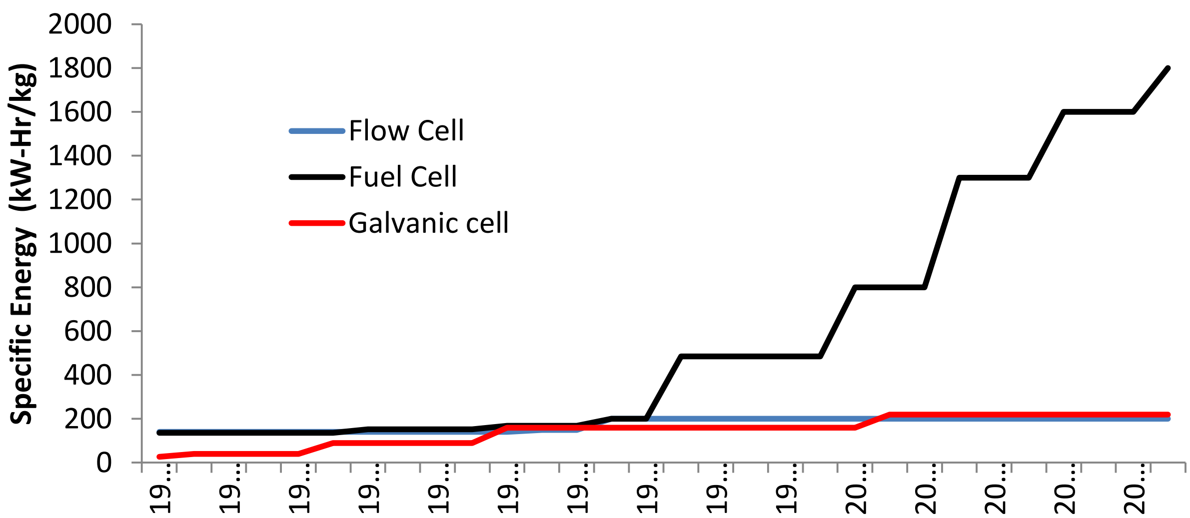


Figure 2: Median AAD for Models with Covariates for 5 Year Prediction

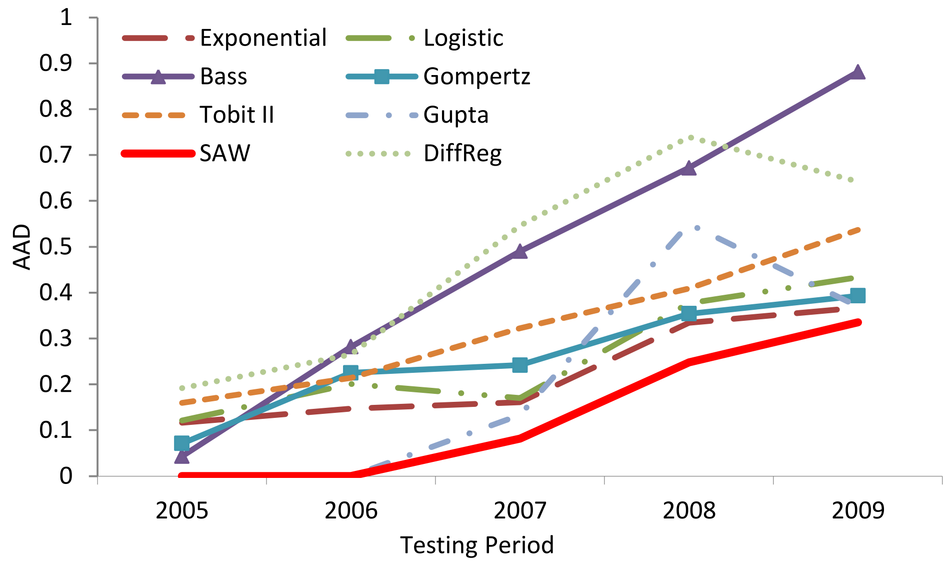
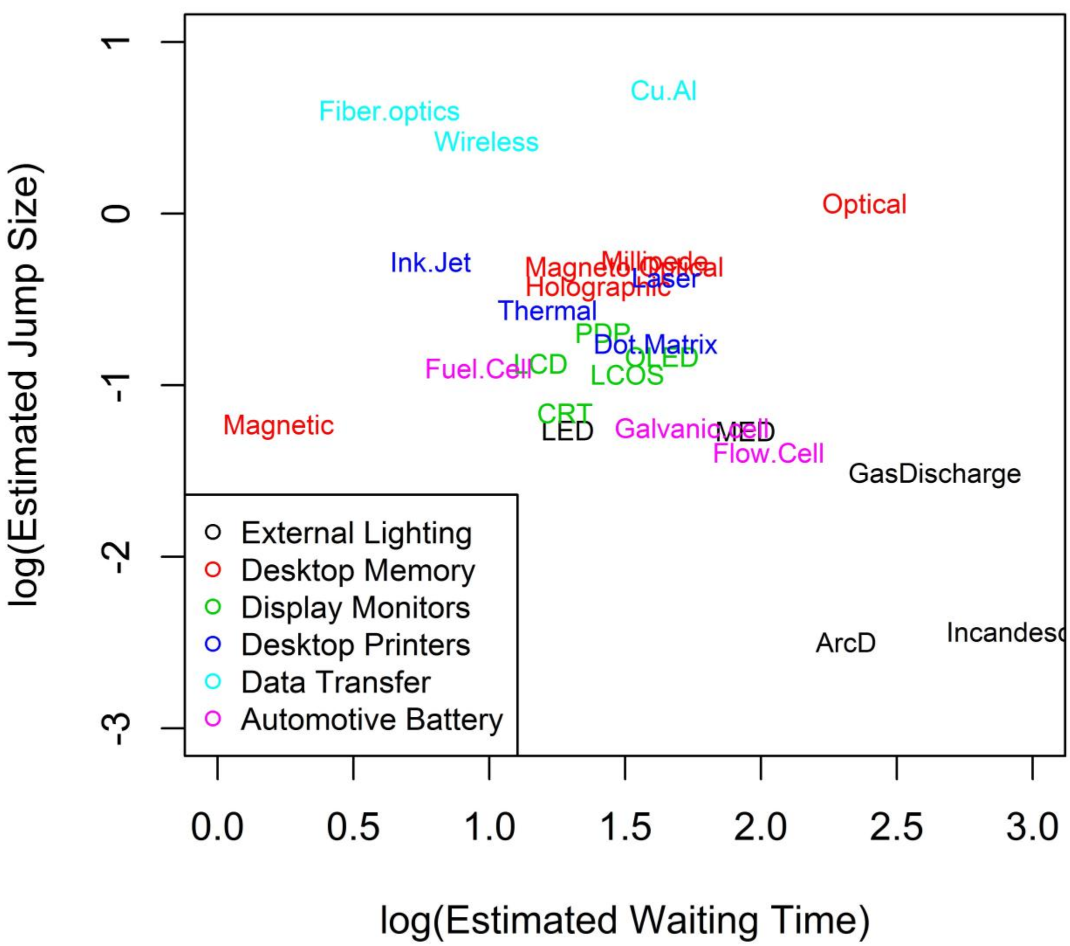


Figure 3: Predicted Step Vs. Wait Patterns



Appendix A: Supplementary Figures

Figure A1: Hypothesized Paths of Technological Evolution

Figure A1a: Exponential curve representing the Moore's and Kryder's Law

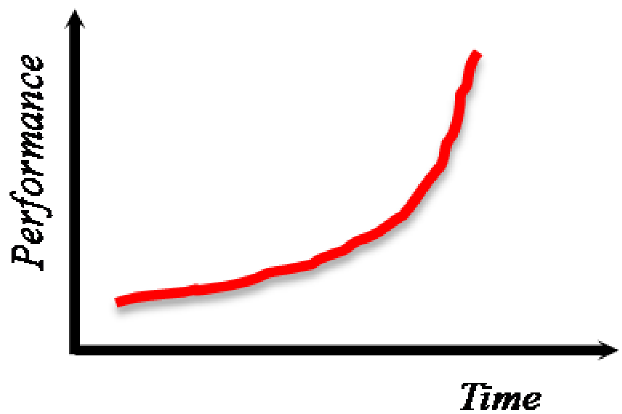


Figure A1b: Sigmoid curve representing the Logistic, Bass and Gompertz Law

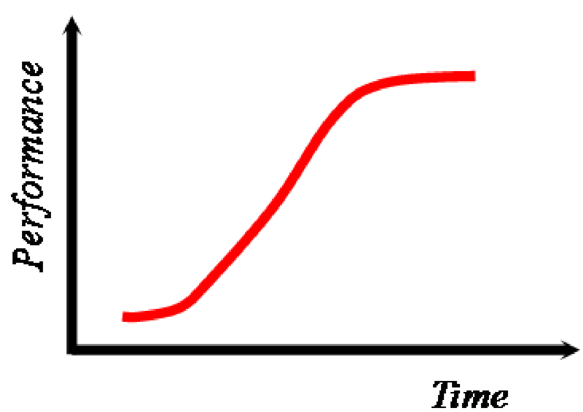


Figure A1c: Step functions representing the Gupta, Tobit II, and SAW models

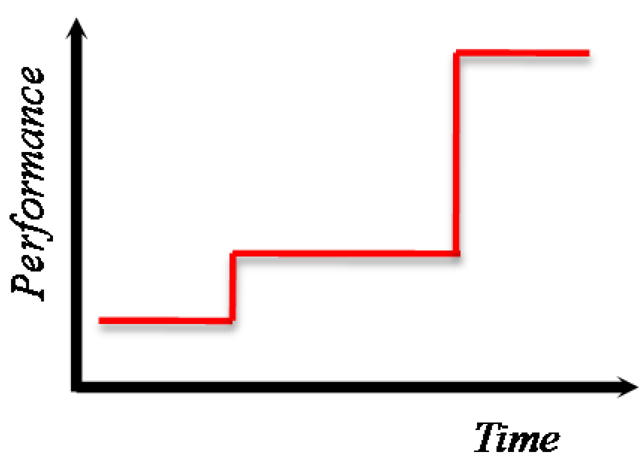


Figure A2: Experimental Setup

Figure A2a: Sampling of Technologies and Time Periods For Prediction

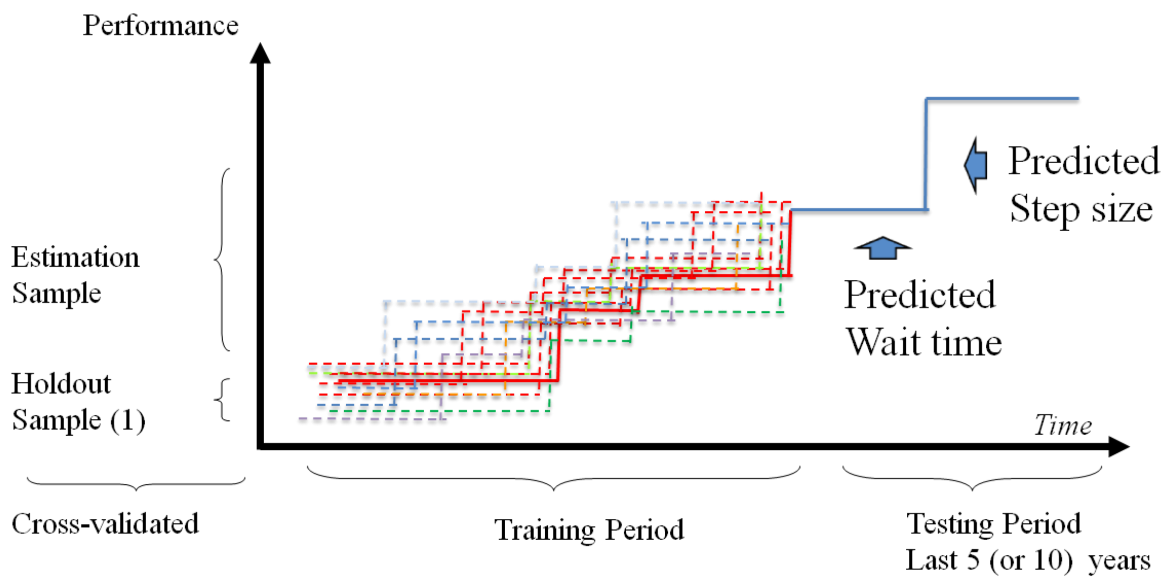


Figure A2b: Unconstrained Fits

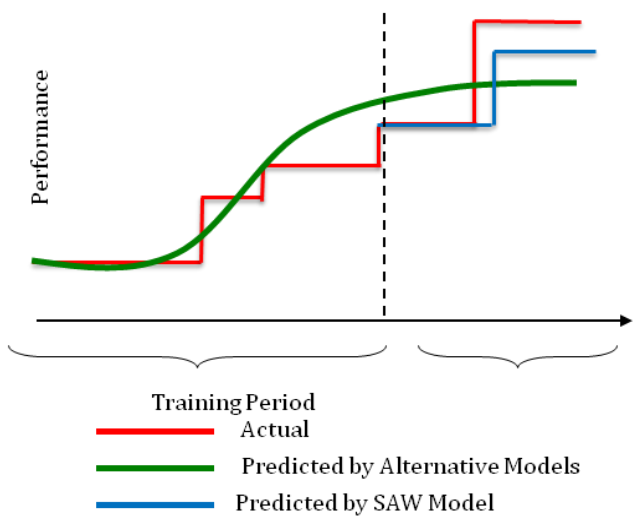


Figure A2c: Constrained Fits

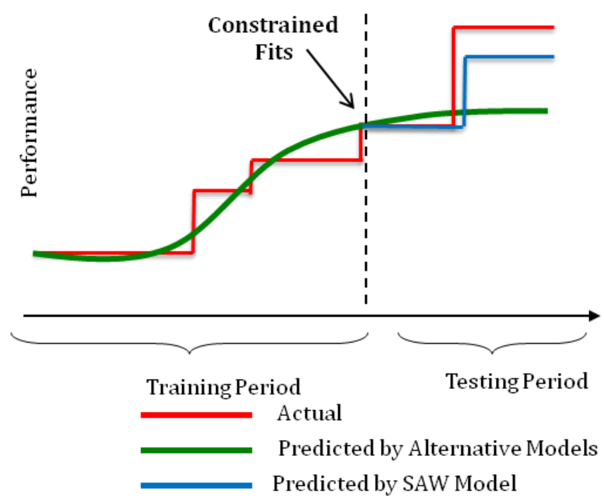


Figure A3: Estimated Parameters (Step Size and Wait Time)

Figure A3a: Size of Steps

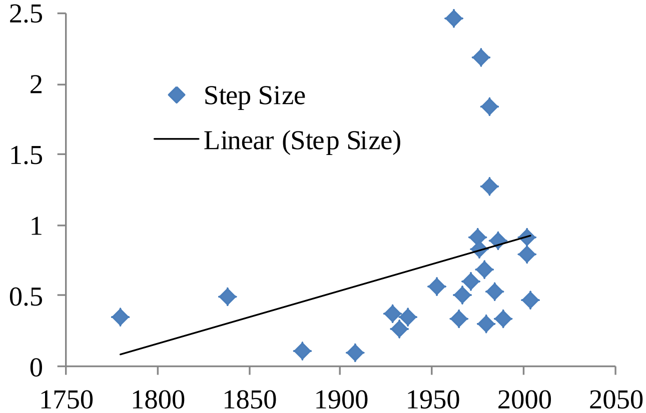


Figure A3b: Wait Times

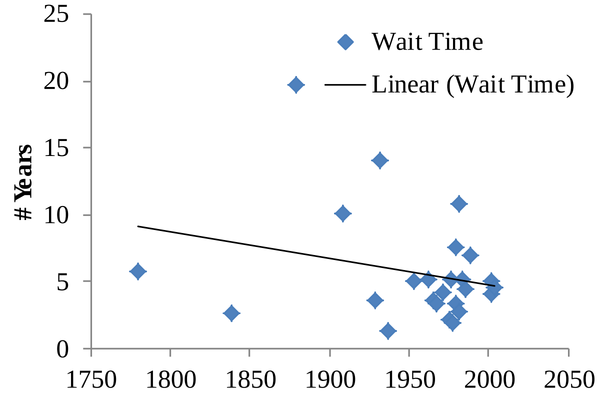
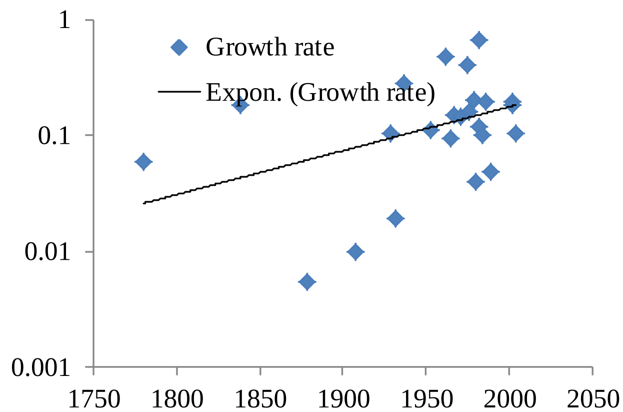


Figure A3c: Growth Rate: Log (Ratio of Steps and Waits)



Appendix B:
Prediction and Fitting of Comparison Models

In their standard forms, Moore's law, Kryder's law and the logistic, Bass, and Gompterz model do not directly incorporate covariates into their predictions. In order to provide a comparison with SAW, which does allow for the inclusion of covariates, we fit two modified versions of these methods. In the first implementation we used a non-linear mixed effects model which fit the standard functional forms of each method but modeled the various parameters as random effects coming from a Gaussian distribution. The parameters for the Gaussian distribution were estimated using all technologies simultaneously and hence built strength in a similar fashion to SAW. Our second implementation of these methods also modeled the parameters using a random effects formulation but in addition incorporated the covariates as a multiplicative adjustment to the original prediction using (11). Next, we discuss both the *Mixed Effects Model* and the *Mixed Effects Model with Covariate Effects* for each method. By comparison the Gupta and Tobit II models involve covariates so must be fit to all curves simultaneously to estimate the population level covariate coefficients.

Extensions of Moore's Law and Kryder's Law (Exponential Model)

Mixed Effects Model

Moore's Law and Kryder's Law each state that the rate of change in the performance of a technology is exponential with given constants. Thus, if we model performance of the technology as an exponential function of time, the coefficient of time represents the constant rate at which the technology improves. To test the applicability of Moore's Law and Kryder's Law, we model the relationship between time and technology performance using the following exponential function:

$$P_{ij} = \tau_{i1} e^{\tau_{i2} t_{ij}} e^{\epsilon_{ij}} \tag{14}$$

where, P_{ij} is performance for technology i at time t_{ij}, and τ_{i1} and τ_{i2} are modeled as random variables coming from Gaussian distributions.

We estimate the coefficients for the exponential model associated with Moore's Law and Kryder's Law using linear mixed effects software applied to the log-transformed data. Kryder's Law assumes $\tau_2 = \frac{12}{13}\log 2$ while Moore's Law assumes $\tau_2 = \frac{12}{18}\log 2$. After fitting the mixed effects model to each technology we use the fitted parameters to form predictions and compare the estimate for $\tau_{i2}$with that predicted by Kryder's and Moore's Laws.

Mixed Effects Model with Covariate Effects

Let $X_{ij1}, X_{ij2}, \dots, X_{ijq}$ represent q different covariates measured at time t_{ij}. We incorporate these covariates into the exponential model in a multiplicative fashion using the following formulation,

(15) $$P_{ij} = \tau_{i1} e^{\tau_{i2} t_{ij}} \times \exp\left(\beta_0 + \sum_{k=1}^{q} \beta_k X_{ijk}\right) e^{\epsilon_{ij}}, \quad \tau_{i1} \sim N(\mu_1, \sigma_1^2), \;\; \tau_{i2} \sim N(\mu_2, \sigma_2^2)$$

Note that the parameter τ_{i1} are modeled as random effects because they are specific to a particular technology but the β coefficeints are treated as fixed effects because they are common to all technologies.

Extensions of Logistic Model

Mixed Effects Model

The generalized form of the logistic curve is a relatively flexible model that can also capture an S-shape of the path of technological evolution. This model has been widely used in prior literature (e.g. Young 1993; Meade and Islam 1998).

(16) $$P_{ij} = \frac{\tau_{i3}}{1 + \tau_{i4} \times \exp(-\tau_{i5} t)} e^{\epsilon_{ij}}$$

where, P_{ij} is the performance of technology i at time t_{ij}, and τ_3, τ_4 and τ_5 are modeled as random variables coming from Gaussian distributions.

We fit equation (16) using non-linear mixed effects software applied to the log-transformed data.

Mixed Effects Model with Covariate Effects

We incorporate the covariates into the logistic model using a multiplicative formulation, thus

$$P_{ij} = \frac{\tau_{i3}}{1+\tau_{i4}\times\exp(-\tau_{i5}t_{ij})} \times \exp\left(\beta_0 + \sum_{k=1}^{q} \beta_k X_{ijk}\right) e^{\epsilon_{ij}} \quad (17)$$

Note that the parameters $\tau_{31}, \tau_{i4}, \tau_{i5}$ are modeled as random effects because they are specific to a particular technology but the β coefficients are treated as fixed effects because they are common to all technologies.

Extensions of Bass Model

Mixed Effects Model

The Bass model (Bass 1969) is a special case of the Gamma/shifted-Gompertz distribution that can capture an S-shape plus a variety of other shapes that approximate the S-curve depending on the values of the parameters. We use the operational form of the Bass model used previously for modeling technology evolution (Young and Ord 1989; Young 1993):

$$p_{ij} = \left(\tau_{i6} + \tau_{i7}P_{ij} + \tau_{i8}P_{ij}^2\right)e^{\epsilon_{ij}} \quad (18)$$

where, P_{ij} is the performance of technology i at time t_{ij},

p_{ij} is marginal performance improvement at time t_{ij},

τ_{i6}, τ_{i7} and τ_{i8} are modeled as random variables coming from a Gaussian distribution. We fit equation (18) using non-linear mixed effects software applied to the log-transformed data.

Mixed Effects Model with Covariate Effects

We incorporate the covariates into the Bass model using a multiplicative formulation, thus:

$$p_{ij} = (\tau_{i6} + \tau_{i7}P_{ij} + \tau_{i8}P_{ij}^2) \times \exp\left(\beta_0 + \sum_{k=1}^{q} \beta_k X_{ijk}\right) e^{\epsilon_{ij}} \tag{19}$$

Note that the parameters $\tau_{i6}, \tau_{i7}, \tau_{i8}$ are modeled as random effects because they are specific to a particular technology but the β coefficients are treated as fixed effects because they are common to all technologies.

Extensions of Gompertz' Model

Mixed Effects Model

The Gompertz' Model used to estimate Gompertz' Law takes the functional form:

$$P'_{ij} = rP_{ij} \log \frac{K}{P_{ij}} \tag{20}$$

Where, P_{ij} is the performance of technology i at time t_{ij},

P'_{ij} is the corresponding derivative of the performance,

r is the intrinsic growth rate,

K is the final technology level.

Notice that this equation gives slow growth when P_{ij} is either low or close to K, and rapid growth in between. The solution to this differential equation is the following double exponential function:

$$P_{ij} = \tau_{i9} \exp(-\tau_{i10} e^{-\tau_{i11} t_{ij}})\, e^{\epsilon_t} \tag{21}$$

where τ_9, τ_{10} and τ_{11} are modeled as random variables coming from a Gaussian distribution. We fit equation (21) using non-linear mixed effects software applied to the log-transformed data.

Mixed Effects Model with Covariate Effects

We incorporate the covariates into the Gompertz model using a multiplicative formulation, thus:

(22) $$P_{ij} = \tau_{i9}\exp(-\tau_{i10}e^{-\tau_{i11}t_{ij}}) \times \exp\left(\beta_0 + \sum_{k=1}^{q}\beta_k X_{ijk}\right)e^{\epsilon_t}$$

Note that the parameters $\tau_{i9}, \tau_{i10}, \tau_{i11}$ are modeled as random effects because they are specific to a particular technology but the β coefficients are treated as fixed effects because they are common to all technologies.

Constrained Parameters

Several of the parameters in the above mentioned models are constrained to by positive. To operationalize this constraint we parameterized the corresponding coefficients as $\tau = \exp(\tau^*)$ where τ^* was modeled as coming from a Gaussian distribution. This formulation ensured that τ would always be positive.

Extensions of Gupta Model

The interpurchase time model uses an Erlang-2 distribution to model the time until a purchase, or in our case wait time until a jump in technology, with the Erlang parameter modeled as a function of a set of explanatory variables. Specifically,

(23) $$f_{ij}(t) = \alpha_{ij}^2 t \exp(-\alpha_{ij}t)$$

(24) $$\alpha_{ij} = \exp(-\delta_0 - \delta_1 X_{ij1} - \cdots - \delta_p X_{ijp})$$

where $f_{ij}(t)$ is the probability density, at time t_{ij}, of the wait until the next jump for technology i, α_{ij} is the Erlang-2 scale parameter and X_{ijk} is the kth covariate at t_{ij}. The time until purchase, or technology jump, is predicted using the Erlang-2 mean, $\frac{2}{\alpha_{ij}}$.

Gupta (1988) models the purchase quantity using a logistic distribution because his sales data is categorical in terms of size e.g. 16 oz, 32 oz, etc. However, our jump size data is continuous so we follow Gupta (1988)'s recommendation to model such data using a standard linear regression, with explanatory variables $Y_{ij1}, Y_{ij2}, \ldots, Y_{ijq}$. Consistent with our other comparison models, we use the log transformed jump size as the dependent variable.

The interpurchase time and purchase quantity models are fit separately. The interpurchase time model is fit using an iterative reweighted least squares algorithm that maximizes the likelihood function associated with the Erlang-2 distribution. The purchase quantity model is fit using a standard linear regression least squares procedure with the log transformed jumps as the dependent variable and the values of the covariates at the associated time points as the independent variables. Combining the interpurchase time model, which predicts time until the next jump, with the purchase quantity model, which predicts jump size, we can use the explanatory variables to estimate the remaining evolution for any given technology.

Tobit II Model

For the i^{th} technology, at time t_{ij}, Tobit II models the probability of a step (P_1^*) as a function of explanatory variables $X_{ij1}, X_{ij2}, \ldots, X_{ijp}$, and the size of the step (P_2^*) as a function of explanatory variables $Y_{ij1}, Y_{ij2}, \ldots, Y_{ijq}$ thus:

(25) $$P_{1ij}^* = \frac{e^{\delta_0 + \delta_1 X_{ij1} + \cdots + \delta_p X_{ijp}}}{1 + e^{\delta_0 + \delta_1 X_{ij1} + \cdots + \delta_p X_{ijp}}} + U_{ij}$$

(26) $$P_{2ij}^* = \chi_0 + \chi_1 Y_{ij1} + \cdots + \chi_q Y_{ijq} + V_{ij}$$

where $$(U, V) \sim N\left(0, \begin{bmatrix} 1 & \rho\sigma \\ \rho\sigma & \sigma^2 \end{bmatrix}\right)$$

We observe a step conditional on the probability of a jump exceeding a cutoff value, for example $P_{1ij} = 1(P^{*}_{1ij} > 0.1)$ always, but observe the size of step P_{2ij} only when the step occurs i.e., $P_{1ij} = 1$. We use standard software to estimate the joint outcome – probability and size of step – in the Tobit II model (Tellis 1988).

Appendix C: Fitting of SAW

We provide details on fitting of SAW:

Fitting of SAW

Fitting SAW requires estimating a number of parameters. We use a maximum likelihood approach. Suppose we have observed n_i steps of technology i at times $t_{i1},\ldots,t_{in_i}$. Let $J_{i.} = (J_{i1}, \ldots, J_{in_i})$ represent the series of observed steps and $T_{i.} = (T_{i1}, \ldots, T_{in_i})$ be the times between these steps. In addition we assume covariates X_{ijk} and Y_{ijk} have been observed at times t_{ij}.

Conditional on λ_i and ω the distribution of $T_{i\cdot}$ is

$$f(T_{i.}|\lambda_i, \omega) = \frac{1}{\Gamma(\mathrm{K})^{n_i}}\left(\prod_{j=1}^{n_i} \omega_{i(j-1)}^{-K} T_{ij}^{K-1}\right)\lambda_i^{-Kn_i} \exp\left(-\lambda_i^{-1}\sum_{j=1}^{n_i} \omega_{i(j-1)}^{-1} T_{ij}\right)$$

Similarly, the distribution of λ_i^{-1} conditional on κ and ω is

$$f(\lambda_i^{-1}|\kappa, \theta) = \frac{\lambda_i^{-(\kappa-1)}}{\Gamma(\kappa)\theta^\kappa} \exp\left(\frac{-\lambda_i^{-1}}{\theta}\right)$$

Hence, the distribution of $T_{i\cdot}$ conditional on ω, κ, K and θ is

(27)

$f(T_{i.}|\kappa, \theta, \omega, K) =$

$$\int \frac{1}{\Gamma(\mathrm{K})^{n_i}}\left(\prod_{j=1}^{n_i} \omega_{i(j-1)}^{-K} T_{ij}^{K-1}\right)\lambda_i^{-Kn_i} \exp\left(-\lambda_i^{-1}\sum_{j=1}^{n_i} \omega_{i(j-1)}^{-1} T_{ij}\right)\frac{\lambda_i^{-(\kappa-1)}}{\Gamma(\kappa)\theta^\kappa} \exp\left(\frac{-\lambda_i^{-1}}{\theta}\right) d\lambda_i^{-1}$$

$$= \frac{\frac{\Gamma(Kn_i + \kappa)}{\Gamma(\mathrm{K})^{\mathrm{n_i}}\Gamma(\kappa)}\left(\prod_{j=1}^{n_i} \theta^K T_{ij}^{K-1} \omega_{i(j-1)}^{-1}\right)}{\left(1 + \sum_{j=1}^{n_i} \theta \omega_{i(j-1)}^{-1} T_{ij}\right)^{\kappa + Kn_i}}$$

We can use Equation (27) to write down the log likelihood function,

(28) $l_T(\kappa,\theta,\beta,K) = \sum_{i=1}^{N} \log f(T_{i.}|\kappa,\theta,\omega,K) = \sum_{i=1}^{N} \log\left(\frac{\Gamma(Kn_i+\kappa)}{\Gamma(K)^{n_i}\Gamma(\kappa)}\right) -$

$\sum_{i=1}^{N}\sum_{j=1}^{n_i}\left((1-K)\log(T_{ij}) + K(sJ_{i(j-1)} + \beta_0 + \sum_{k=1}^{p}\beta_k X_{i(j-1)k})\right) - \sum_{i=1}^{N}(\kappa + Kn_i)log\Big(1 +$

$\sum_{j=1}^{n_i} T_{ij}\exp\big(-(sJ_{i(j-1)} + \beta_0 + \sum_{k=1}^{p}\beta_k X_{i(j-1)k})\big)\Big)$

where $\theta = \exp(-\beta_0)$. Equation (28) is a convex function provided the n_i's are large enough. Hence standard optimization techniques can maximize Equation (28) in terms of κ and the β's.

An analogous argument shows that the log likelihood function for the Step sub-model is

(29) $l_J(\rho,\eta,\alpha,M) = \sum_{i=1}^{N} \log f(J_{i.}|\rho,\eta,\alpha,M) = \sum_{i=1}^{N} \log\left(\frac{\Gamma(Mn_i+\rho)}{\Gamma(M)^{n_i}\Gamma(\rho)}\right) -$

$\sum_{i=1}^{N}\sum_{j=1}^{n_i}\left((1-M)\log(J_{ij}) + M(rT_{ij} + \alpha_0 + \sum_{k=1}^{q}\alpha_k Y_{ijk})\right) - \sum_{i=1}^{N}(\rho + Mn_i)log\Big(1 +$

$\sum_{j=1}^{n_i} J_{ij}\exp\big(-(rT_{ij} + \alpha_0 + \sum_{k=1}^{q}\alpha_k Y_{ijk})\big)\Big)$

where $\eta = \exp(-\alpha_0)$, which can similarly be optimized by standard techniques. We calculate maximum likelihood estimates for all the parameters and produce future predictions using Equations (8) and (9). Note that the joint log likelihood of both T and J is equal to,

$$l_{T,J}(\kappa,\theta,\beta,K,\rho,\eta,\alpha,M) = \sum_{i=1}^{N} \log f(T_{i.},J_{i.}|\kappa,\theta,\omega,K,\rho,\eta,\alpha,M)$$

$$= l_T(\kappa,\theta,\beta,K) + l_J(\rho,\eta,\alpha,M).$$

Hence, the joint likelihood is separable into the sum of the Wait and Step likelihoods.

Comparison of one step and two step approaches to estimation

As a direct consequence the two step approach of maximizing the Wait and Step likelihoods individually is mathematically identical to the one step approach of maximizing the joint SAW likelihood. This is an advantage of the SAW model because it significantly reduces the fitting procedure's complexity.