Liquidity in Equity and Option Markets – A Hedging Perspective

Stefan Sandberg

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Abstract

When options and other derivatives are issued, the issuer seeks risk neutral positions. These positions are obtained through an analysis of the sensitivity of the derivative's price w.r.t. the targeted parameters. Risk neutral positions acquire a time continuous price process as a good proxy to ensure more or less explicit hedging costs. This thesis describes what happens with the hedging costs if the price process is not continuous or if there is a discrete event (a jump) between time zero and maturity. We show how much the hedging cost increases and for which positions the issuers is most vulnerable, and how the profit and loss deviation increases for discontinuous processes. We document for the importance of no major jumps in the underlying time process, when hedging.

Acknowledgments

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Table of Contests

Introduction

This thesis describes what happens if the continuous time process for a standard market in equities and options suddenly disappears or reduces heavily, from a derivative issuer perspective. Institutions that issue options under discontinuous time processes cannot rely on the pricing models. Simply because the used price models rely on a time continuous processes in the underlying market, which is important from a risk neutral perspective [1,4,5]. Every time a contract is issued, the issuer gets paid from the holder. The price of the contract is a fair price between the issuer and holder, and the cash is used to cover the cost for the issuer to be risk neutral [6].The definition of risk neutral simply are that the issuers have hedged them against further payments. The issuers always follow the fundamental parameters in the pricing model for then take a position in the underlying.

So let us say an option expires deeply in-the-money (ITM) which means for the issuer to pay quite a bit of cash to the holder, and seems very costly for the issuer. But that does not need to be true. If the risk neutral positions had been made correct, the cash from the hedging will cover the cost of the deeply ITM expired option at delivery. But, let us draw this to the discontinuous time process case. The time continuous process in the underlying market suddenly drops heavily, this will create lager movements in the underlying market for each time process, i.e. larger movements for every risk neutral position, which is increasing the future costs. This is of course important to study, see how the profit and loss distribution changes when risk neutral positions are taken from time continuous processes to a discrete time process, so the issuer can put aside cash for these special events.

Models

Before going any further and describing more deeply about the problem, a closer look into the basic theory is needed. The examples are described from a plain vanilla option view. So the general model for pricing a plain vanilla option is by using *Black-Scholes* model for option pricing [4]. From the model it is possible to show the different risk neutral strategies. The value of a call option for an underlying stock or stock index is

$$C(S,t) = N(d_1)Se^{-qt} - N(d_2)Ke^{-r(T-t)} \, . \tag{1.1}$$

The price of a corresponding put option is

$$P(S,t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)Se^{-qt} \, , \tag{1.2}$$

where d_1 and d_2 are

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \, , \tag{1.3}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \, . \tag{1.4}$$

For the equations above

- *N(·)* is the cumulative distribution function of the standard normal distribution.
- *r* is the annualized risk free interest rate and assumed constant.
- *S* is the spot price of underlying asset.
- *K* is the strike price.
- σ is the implied volatility of the underlying asset and assumed time independent.
- *T-t* is the time to maturity.
- *Q* is the dividend yield.

Now when the *Black-Scholes* model is introduced a further look at the risk management properties can be done, and we can introduce “The Greeks” [7]. The Greeks are important properties in risk management when someone buying or selling options, and they measures the sensitivity of the option value when the underlying parameters changes. The Greeks that are most important for an options issuer are delta and vega, which we will focus on. These two Greeks tell the option issuer to change the hedge position when the parameters change in the underlying, to accomplish a theoretical risk neutral stage.

Delta

Delta measures the rate of change of the option value with regard to changes in underlying spot price. Delta is the first derivative with respect to the underling spot price. Delta is shown bellow

$$\Delta = \frac{\partial V}{\partial S}. \tag{2.1}$$

For a vanilla call option, one receives

$$e^{-qt}N(d_1), \tag{2.2}$$

and for a vanilla put option

$$-e^{-qt}N(-d_1). \tag{2.3}$$

If no dividends are paid, then one can exclude $-e^{-qt}$.

Let us first focus on a vanilla option. The delta in this case will be a value between 0 and 1 for a call option or between -1 and 0 for a put option. If an option has delta close or equal to 1, it equals that the option price behaves equal to the underlying asset. If an option has delta close or equal to -1, it equals that the option price behaves opposite as the underlying.

The delta relationship between call and put options with respect on same underlying, strike price and time to maturity gives a value equal to 1 if one sums up the absolute delta values

$$\Delta_c + \left|\Delta_p\right| = 1. \tag{2.4}$$

This can be shown by the put-call parity, the call minus the put fold back a forward, which must has a delta of 1 if non-arbitrage condition is satisfied.

Let us show a basic example when an option issuer obtain a risk neutral position by hedging delta. An institution has issued a vanilla call option and wants to be risk neutral against future possible payments. In the start the call option has a delta of 0.25, which tells the issuer to buy 0.25 underlying per issued call option to protect against the future payments. Under the time to maturity, the parameters change and of course the delta change. The issuer following continuous the exact delta position in the underlying to obtain an optimal delta hedge i.e. the delta neutral. If the institution issues put options the delta neutral position will be a negative number of underlings. Hence, the institution shorts the underlying to obtain the optimal delta hedge.

Delta is not always a number between 0 and 1 for calls, or -1 and 0 for puts, even if there is a vanilla option pay-off structure. There are several exotic options whose pay-offs are equal or close to a vanilla pay-off function, but with other conditions to obtain the pay-off. The change of delta for an exotic option can be far faster than for the vanilla case. Let us look on a barrier option.

A barrier option is a vanilla option that is split up in two options with same strike a maturity but with a barrier, knock-in and a knock-out barrier option. The knock-out enters always as a vanilla option and the knock-in always enter worthless. If the underlying spot price passes the barrier the knock-out become worthless and can never go back active. But the knock-in barrier option knocks-in to a vanilla option and become a vanilla option permanent.

The value of a knock-in and knock-out barrier option with same strike and time to maturity compare with the similar vanilla option is shown in equation (5.1)

$$V_V = V_{KI} + V_{KO}. \tag{2.5}$$

V_V is the value for the vanilla option, V_{KI} is the value for the knock-in barrier option and $V_{KO}$is the value for the knock-out barrier option. If one of the barriers becomes worthless the value of the active one must be equal to the vanilla or else the non-arbitrage argument is not satisfied.

The delta for let us say the knock-in barrier option can be quite stationary and relative low for a spot price beyond the barrier, however if the underling spot price suddenly moves close to the barrier the delta can suddenly shifts from values about 0.4 to values like 3 or 4. The high values of delta is created because the pay-off function is not linear, there will be a Dirac pulse when the barrier is knocked. If the delta can obtain higher values and rapid changes the hedging cost can accelerate quickly if the underlying market is not a time continuous process during the hedge.

Vega

The second presented Greek is vega. Vega measures the sensitivity of the option price, when volatility changes by one percent. Vega is the first derivative with respect on volatility. Vega is presented bellow

$$\nu = \frac{\partial V}{\partial \sigma}. \tag{3.1}$$

Vega is equal for both vanilla call and put options, one receives

$$Se^{-qt}\phi(d_1)\sqrt{t} = Ke^{-rt}\phi(d_2)\sqrt{t}\,, \tag{3.2}$$

where

$$\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}. \tag{3.3}$$

When issuers focus on vega, they get an idea of much the options value rise or falls when the volatility rise or falls 1%. The issued option has a specific vega, a vega that issuers need to hedge and is called vega hedging. The vega hedge is done by buying or selling similar options in the options market. Vega cannot be hedged from the underlying direct, only by similar options. If the vega hedge is not feasible or done correctly, higher losses are to be expect.

Let us say a call option cost 10 and has a vega of 0.5, if then the volatility rises by 1% the option price will shift to 10.5. Similarly if the vega is -0.5 the option will then lose in value to 9.5 if the volatility rise by 1%, or gain 0.5 to 10.5 if the volatility drops 1%. Vega falls when the option gets closer to maturity, and has the highest value when the underlying spot price is equal to the strike price for a constant time point.

Other Greeks

There are two other first order Greeks to introduce, theta and rho. Theta is the first derivative with respect on time, and is shown bellow

$$\Theta = -\frac{\partial V}{\partial t}. \qquad (4.1)$$

Theta shows the sensitivity of the option value to the passage of time, time decay. Issuers does not focus so much on theta because the cost of theta is quite expensive, which may not recompense at maturity.

The forth first order Greek is rho. Rho is the first derivative with respect on the risk free rate, and is shown bellow

$$\rho = \frac{\partial V}{\partial r}. \qquad (4.2)$$

Rho shows the sensitivity of the option value with respect to the risk free interest rate falls or rises by 1%. Usually issuers do not hedge rho, because the changes are too small, only if there are extreme movements in the interest rate market, hedging rho is made.

Of no major importance of hedging rho and theta, the examples and explanation of the two Greeks ends here.

There are more Greeks than the four presented fist order. Usually higher ordered Greeks are not so interesting, simply because they tell only about the sensitivity of the sensitivity. But, there is one higher ordered Greek than will be used later on in the thesis, which is gamma. Gamma is a second ordered Greek, and is the derivative on delta with respect on the spot price. Gamma for a vanilla option is presented bellow

$$\frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2} = \frac{e^{-qt}\phi(d_1)}{S\sigma\sqrt{t}}. \qquad (4.3)$$

Gamma measures the rate of change on delta with respect to the underlying spot price. Gamma can be useful when talking about the fast changes in delta for especially exotic derivatives, whose behavior is more extreme than vanillas.

Change in Time Series

The main focus is in when a normal market or a time continuous process suddenly transform into un-continuous time process, or the hedging process is not a time continuous processes. Let us suppose there is a stock on the exchange, Company A. Company A is one of the most liquid stocks on the exchange and therefore lots of vanilla options and different exotic options are available. Suppose the daily time series for Company A have a mean of 2500 shifts in the spot price per day. The figure of this example is shown in figure 1 bellow.

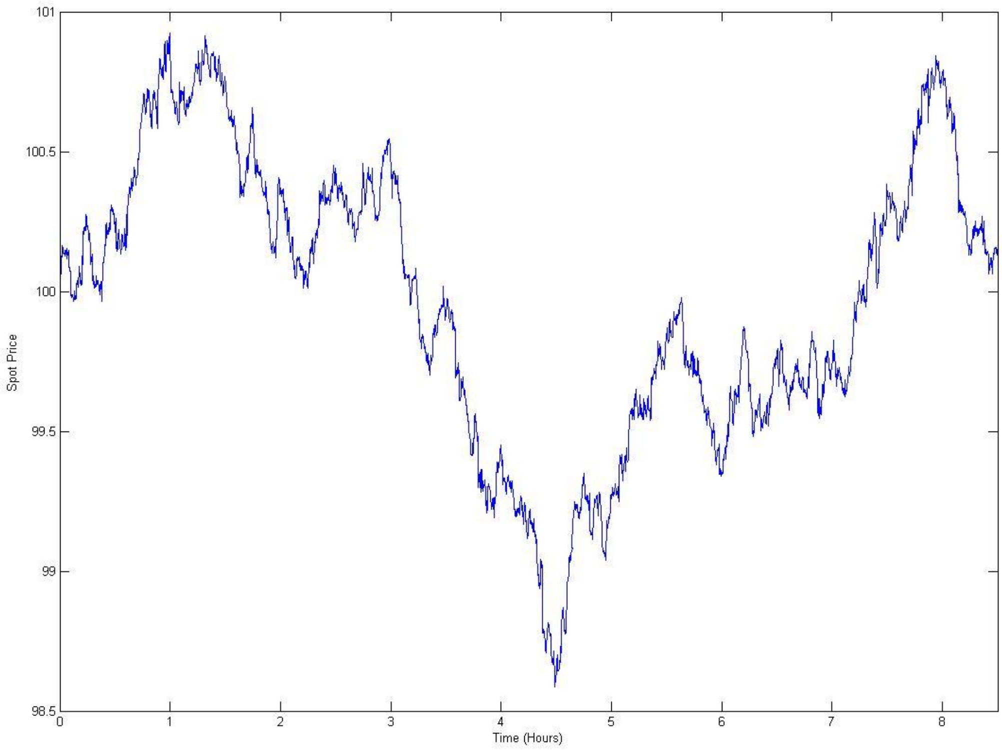

Figure 1, 2500 shifts per a day for Company A.

Company A is simulated in figure 1 with student's t distribution of 4 degrees of freedom which equals an underlying volatility of 22.5 percent. The simulation represents 2500 daily shifts, and can be considered as a time continuous process. So for the delta hedging issuer this time series will not create any hedging costs problem due to the quite continuous shifts and of course Company A's high liquidity. But let us draw this to the un-continuous time process case. The daily volume drops heavily and the daily shifts are now 99 percent lower than a normal, and there is not possible to make a time continuous delta hedge, is shown in figure 2. Figure 2 is created with similar time series as in figure 1, but instead with 99 percent lesser shifts.

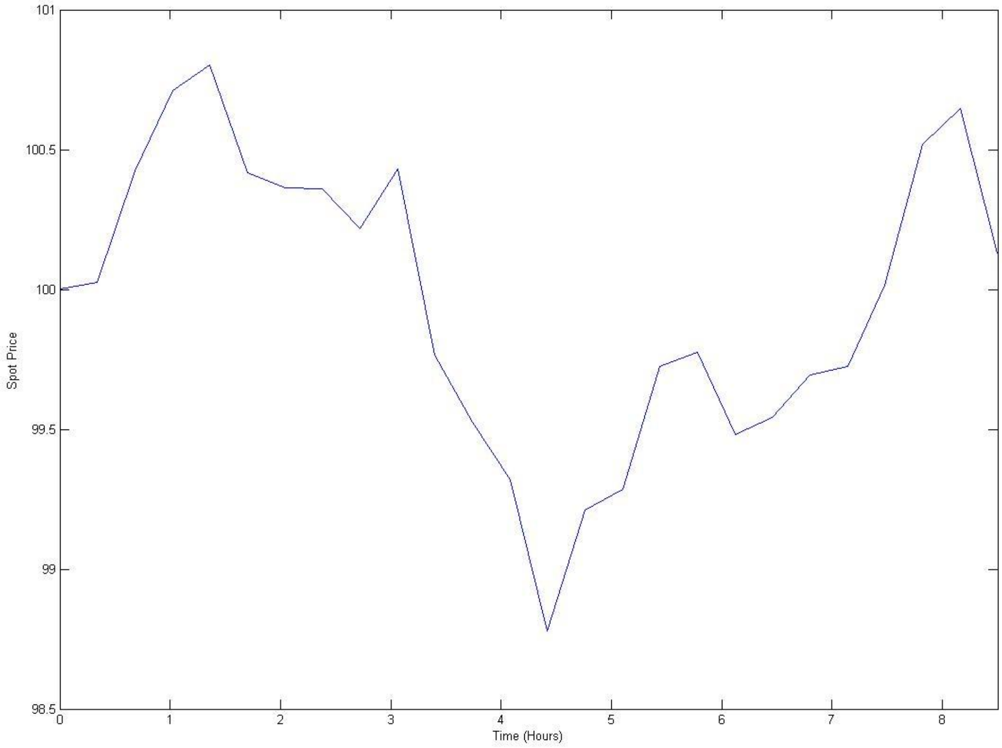

Figure 2, 25 shifts per a day for Company A.

Figure 2 shows when the issuer is not able to delta hedge the underlying asset frequently as figure 1, which of course increases the cost of the delta hedge. The hedging cost will only increases if the underlying time process is getting more discontinuous.

But, there can also be that figure 2 illustrates the delta hedge process. That Company A's time series in illiquid periods could have a time process as in figure 1, but the issuer is not able to follow every shift, because each shift represents a very low volume. If the issuer should follow each shift, the issuer may create a market impact that increases the hedging cost even more than the discontinuous delta hedge from figure 2 [2].

Delta Hedging Cost

The cost from the delta hedge is going to differ from event to event. Sometimes the issuer might gain and sometimes the issuer might lose. The importance is the shape of the profit and loss distribution, if the distribution has heavy tails it shows that the probability of a given expected profit and loss value is harder to estimate, and higher losses is more common. This is going to be hard for the issuer to expect how much extra money to put aside which claims in illiquid periods. Before going any further with the importance of the distribution shape, an illustration of delta hedging a vanilla call option is illustrated in table 1 bellow

Current Day	Day 0	Day 1	Day 2	···	Day n
Sold Call Option	+10	+10	+10	···	+10
Delta Position	0.3	0.35	0.32	···	0.012
Cash for Delta Zero	-30	-36	-31.5	···	-1
Delta Cost	0	1.3	0.5	···	-8.23
Profit and Loss	10	11.3	10.5	···	1.77

Table 1; Delta hedge from start to maturity.

At day zero a vanilla call option is issued on Company A with a strike price of 110. Company A have a current spot price of 100 and the issuer gets 10 for the option which gives a profit and loss of 10, and the option has a delta of 0.3. The issuer then buys 0.3 underlying per issued option which is done from borrowed money from the institution, or it cloud be just a borrowed underlying from a fund. Then of course the fund wants an extra fee, but let us ignoring the possible fee. At day one, the underlying has increased in value which increased the delta to 0.35. The issuer needs to borrow more underlying, exactly an extra 0.05 underlying per issued option to be delta neutral. The issuer has done a gain due to the increase in underlying spot price, and the gain has increased the issuers' cash desk for possible further payments at maturity, which illustrates in the profit and loss box. At day two the underlying spot price has fall and gives a lower delta than the day before. The issuer need to sell some underlying to compensate for the fall in delta. The probability of further payments at maturity has fall compare to the day before, which has of course decreases in the profit and loss box. And then it goes on to maturity. In this case the vanilla call option expire worthless and gives a profit of 1.77. An important mention is that profit and loss includes the maturity payments, *max(S-K, 0)* if vanilla call option or *max(K-S, 0)* if vanilla put option.

This is a basic example of delta hedging an option, but needs to be re-simulated with different time processes to acquire a profit and loss distribution. A deeper illustration of this will be study in later chapters.

Vega Hedging

The second type of hedging is the vega hedge. Vega hedging refers to the hedge against the fluctuations in volatility. When an issuer hedge vega, it can only be done by buy or selling similar options. Here is important to mention that there is not just one market that issuers use in hedging. The delta hedge for instance, issuers usually hedge the instrument with the underlying direct, because it is general more liquid and time continuous processes in the equity market. Suppose an institution issues an option on Company A, the issuer here goes to use the delta hedge in the equity market, not in the options market. The issuer can of course delta hedge by buying or selling similar options.

One important constraint when constructing the program is that the option market is frozen. The assumption is realistic, because the equity market compare to the options market is much more liquid and time continuous. Let us suppose the equity market average volume has dropped 95% or more, then of course the options market that is depending on the trade volume of equities, will also be affected in similar scale, and can therefore be consider as unavailable. The unavailable vega hedge will create an major impact on the profit and loss distribution and is shown in a later chapter.

Empirical Data

Before one can start simulate different time processes, a time series of data sets has to be created. First and a normal thought are by using empirical data in the time process. Historical data on stocks or indexes many years back is not hard to find, and even public publisher can offer free data a couple years back.

A problem is that one cannot select a period with un-continuous time processes in the empirical data sets because such periods have not exist in the equity market, by meaning "period" it refer to one week or longer. Next conclusion is if one can select time points in the series and create a realistic un-continuous time series. Let us say an un-continuous simulation on Company A is wanted. The empirical data of Company A never had any periods with very low liquidity than one or two days together, so it is impossible to just pick-out periods and simulate them. But suppose there are several separate days on a period of six years historical data with very low liquidity. From here one can pick the twenty-five worst illiquid days and with bootstrapping randomly select them into an un-continuous time process. It seems very nice, but it is only possible if the daily log-return is independent-identically-distributed (i.i.d). To check if the daily log-return is i.i.d, the six year data should look independent and equally distributed. A sample of the Swedish stock index OMXS30 log-returns I shown in figure 4 bellow.

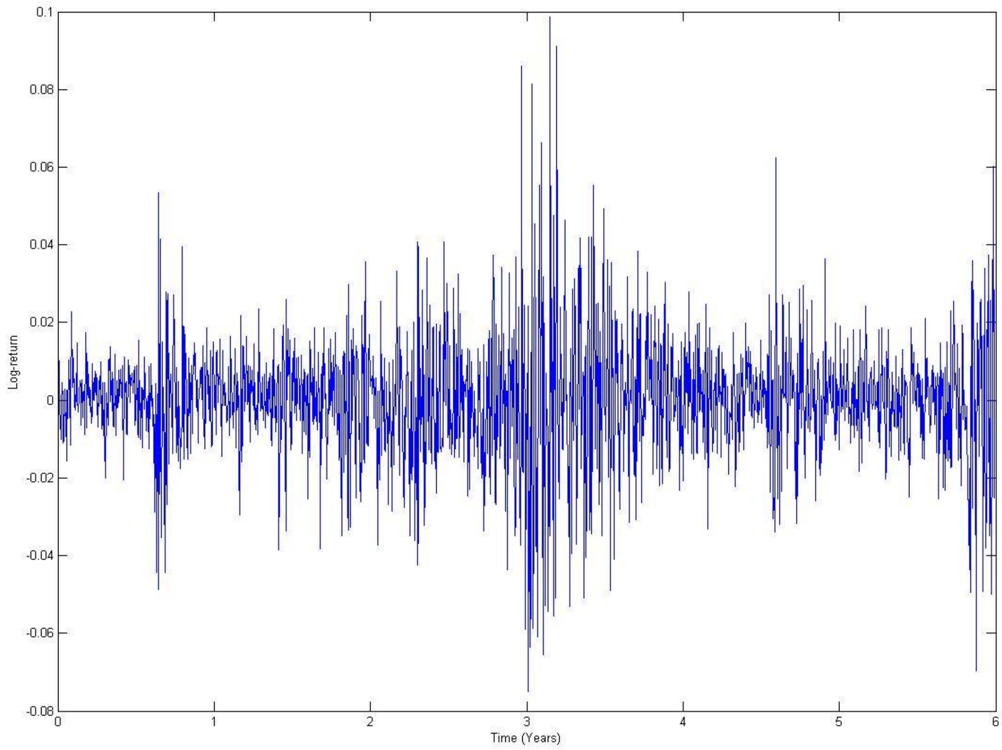

Figure 4, OMXS30 log-returns during last six years.

Figure 4 not seams so random and i.i.d, there are several parts where it does not look i.i.d. and one cannot even approximate this to i.i.d. Suppose there are time series of log-returns that can be approximate as i.i.d. it may be periods when no major market events occurs, as the financial crisis and so on, is it smart to use them?

Let us suppose there is possible to approximate the log-returns as i.i.d, one can now choose a spectrum of different illiquid days, and then simulate an un-continuous time process. Even this will not be perfect, as said above there are not so many days it is happens. For a 99 percent liquidity decrease some equities can generate up to thirty samples, but some generate bare one or two samples in eight years of historical data. Even if all stocks can generate thirty samples it is not going to get any good simulations, simply because the expected events has to be re-simulated at least couple of hundred times over again to generate a clear distribution. If there then are only twenty or thirty samples, the samples will be re-simulated over and over again, which create a jagged distribution with similar profit and losses. And a second problem is that some samples might come from crisis which means mostly days with heavily losses [3], and other might come from just boom cycle days with randomly low liquidity. If the issuer has a expected view of the market in the future and wants to simulate the ad-hoc event, it is not possible with the historical data because the samples are from the past. With rather few samples, and data sets of past events, empirical data is not so reliable and a further look for other time series estimation is needed.

Parametric Data

Another approach is using parametric data. The parametric data is created from a distribution that fits for the stocks and stock-indexes. The parametric distribution can create infinity number of samples, and no re-simulations on similar samples will happen frequently, and the profit and loss distribution will have a smoother shape when using parametric data. Another advantage with parametric data is the possibility of choosing expected volatility and expected movements in the underlying. In the empirical case one has to rely on the historical data whatever future expectations, and is especially bad if one has quite different view of the future. If the issuer believes in high or low volatility or some expected market movements, one will not have any simulation problems. So far one can say that simulations with parametric data give the issuer more alternative and clearer results.

The problem is to find a distribution that fits for most common equities and equity-indexes, a distribution that give realistic values in both low or high volatility for a thin or fat distribution. Student's *t*-distribution has a structure that fits well for a random chosen equity or equity-indexes, in both low and high volatility. When using the Student's *t*-distribution one has to give an input parameter of ν, ν is the number of degrees of freedoms. A relationship between ν and the variance is shown bellow

$$\sigma^2 = \frac{\nu}{\nu-2}, \tag{10.1}$$

which gives

$$\nu = \frac{2\sigma^2}{\sigma^2-1}. \tag{10.2}$$

If the simulator has an expected value of the underlying volatility on the underlying asset, by using equation (3.1) a value of ν is given, and a distribution of log-returns is given. A graphic illustration of Student's *t*-distribution with different values on ν is shown in figure 5 bellow.

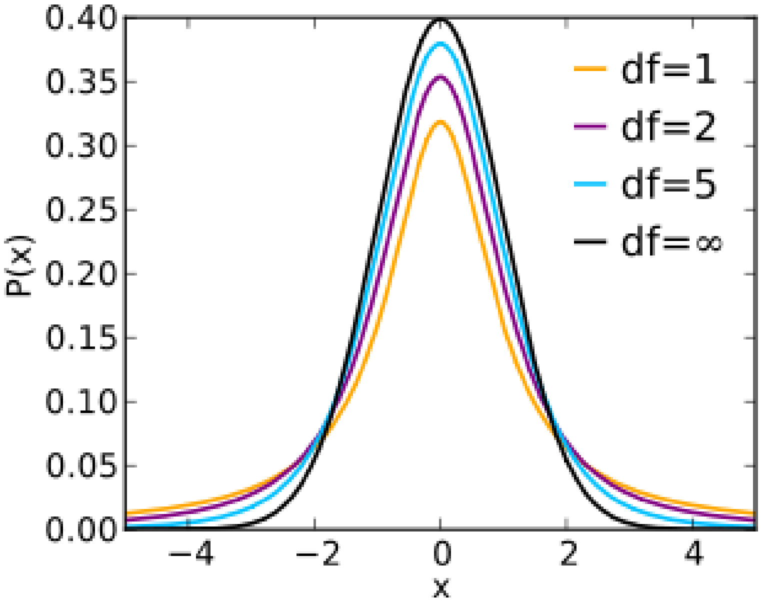

Figure 5, Student's t-distribution on four different degrees of freedom.

Figure 5 shows the Student's *t*-distribution with one, two, five and infinity degrees of freedom. Normally equities or equity-indexes have between two and five degrees of freedoms, depending how volatile the underlying market is.

So instead of using historical data the simulation program uses a parametric view. But this does not mean one can use historical data. The simulator can of course use both methods, but need to know the strengths and weakens of the two methods. A smart way before simulate an event with parametric data is by look on old illiquid data sets and combine the input with the future expectations.

Volatility Surface

The volatility surface shows the relative expensive an option has compared to another option with different strikes and maturities. It simply says, different options have different implied volatilities, and it is the difference in implied volatility between the options that describes the relative expense. The reason why using volatility surfaces is because the *Black-Scholes* model is based on a log-normal distribution, and chapters above concluded that equities and equity-indexes does not have a log-normal distribution. So instead of creating different distributions for different underlings, options issuers and traders use the *Black-Scholes* model with different volatilities for different options, and the volatility surface is then created [8]. The volatility surface will then "transform" on the log-normal distribution to the correct distribution of log-returns. The volatility surface is created by making a reverse calculation on the *Black-Scholes* model with respect on the volatility. By collecting current spot prices on an underlying, let us say Company A outstanding options, the implied volatilities is then generated. The implied volatilities with different strikes and maturities create the surface. Usually people talking about the volatility smile or volatility skew. The smile or skew is created by plotting options with similar maturity but with different strike prices. Figure 6 gives an illustration of this.

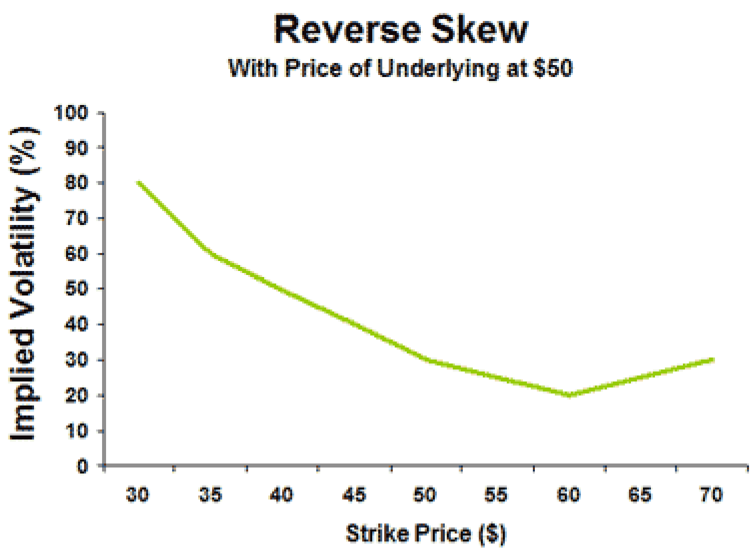

Figure 6, Represent volatility skew or smile.

Figure 6 describing the relative expenses between the strikes on Company A options with similar maturity. Or one could say the difference in demand for different strike prices with similar maturity. The longer time period to maturity the shape in figure 6 tends to look like a soft sloping line from left to right, the skew. And closer to maturity the more of a smile is created, as figure 6 start to show.

The volatility surface is similar for both call and put options. That means a deeply ITM call option and a deeply OTM put option is priced relative much if there strikes and maturities are equal. The absolute prices between the call and put option will of course differ a lot.

The claim that a call and put option with same strike and maturity is priced relative much, can be shown with the put-call parity bellow

$$p + Se^{-qt} = c + Ke^{-rt} \, . \tag{11.1}$$

Where *p* is the price of a put option and *c* is the price of a call option. Suppose the p_{bs} and c_{bs} are calculated put and call prices from the *Black-Scholes* model and that the p_m and c_m are the market spot prices on the options. And the put call parity holds of course for *Black-Scholes* model, and then the following must hold

$$p_{bs} + Se^{-qt} = c_{bs} + Ke^{-rt} \, , \tag{11.2}$$

and of course the arbitrage opportunities must also hold for the market spot prices, so that

$$p_m + Se^{-qt} = c_m + Ke^{-rt} \, , \tag{11.3}$$

subtracting equation (11.2) and equation (4.3), one gets

$$p_{bs} - p_m = c_{bs} - c_m \, . \tag{11.4}$$

Equation (11.4) shows that the pricing error when the *Black-Scholes* model is used to price a European put option it should be exactly the same as the pricing error it is used to price a European call option with the same strike price and maturity, and the claim is proved.

The volatility surface is also important when the issuers hedging their outstanding options. In equation (2.1) one sees that the delta is depending on the implied volatility, an implied volatility that is used from the volatility surface. Suppose an issuer believes in an illiquid market near in the future and wants to simulate the profit and loss distribution. The issuer then uses the volatility surfaces that are generated recently from the options spot prices. However the implied volatilities are of course a belief in the future expectations. If the current market expectations are not similar with the simulated market events the volatility surface will not do an optimal delta hedge in the simulation, either if the simulations rely on empirical or parametric data. The volatility surface needs to be change so it fits for the simulated data samples. This is a hard problem to solve, create a realistic volatility surface which produce a realistic profit and loss distribution. To obtain this one has to change the volatility surface and compare the different results to find the optimal surface, which needs a powerful computer and time. But there are some shortcuts one can use. First of all if the simulated volatility is gather than the current underlying one, the whole volatility surface will shifts up, or down if the simulated volatility is lower. Then if the simulated market movements is bigger or lower than the expected, than the skew or smile will have steeper or flatter shape in the ends. These shortcuts will of course not give an exact delta hedge but it is still better than use the current volatility surface.

A deeper look and testes has not been done for the change in volatility surface due to shortage of time and lack of computer power. A smaller review of the subject is described in the last chapter.

The Price of Illiquidity

Now a first look into simulated events and result can be done with the knowledge from chapters above, present how different events affect the profit and loss distribution. We will see the value of a continuous delta hedge, the value to operating in a perfect option market and to obtain a optimal vega hedge.

Let us first look when an issuer hedging in a normal period, when the delta hedge is a continuous process and have possibility to hedge vega. Suppose Company A spot price is 100 and there is an issued vanilla call option with strike price of 120 and there are 60 days to maturity, and we expects an underlying volatility of 30 percent and an expected return of 130 at maturity. Company A have circa 2500 shifts per day with high volume in each shift which gives the issuer the possibility to follow each shift without any market impact [1, 2, 3]. The time continuous process would look like figure 1 with 2500 daily shifts. By using Student's *t*-distribution with number of degrees of freedom that gives a fitted underlying volatility for Company A, and re-simulated 1000 times to obtain a comprehensible profit and loss distribution. Figure 8 shows the result.

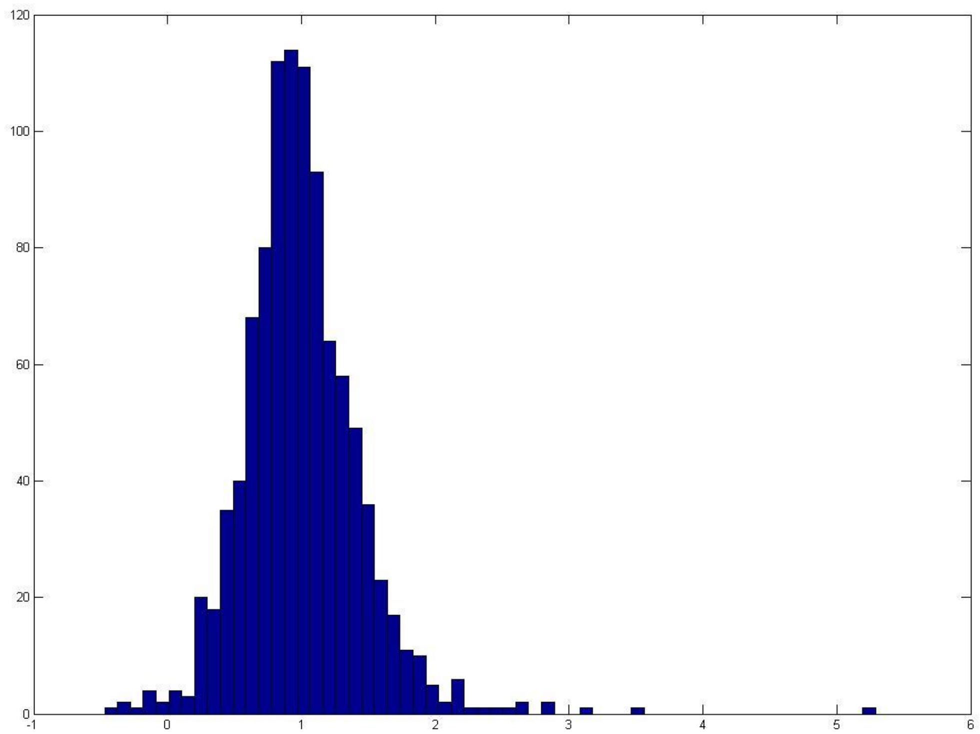

Figure 8, Profit and loss for Company A's vanilla call option from now to maturity.

From figure 8 one sees a light tail distribution but more of a spike with thin and short tails. And also mean which is positive. This is of course what the issuer wants and wants to expect. High control of the expected profit and loss and a positive gain as result of the service that the institution supplies. But let us simulate the vanilla call option on Company A again with similar conditions as for figure 8 but only with 25 possible delta hedges per day. Figure 9 shows the simulated event.

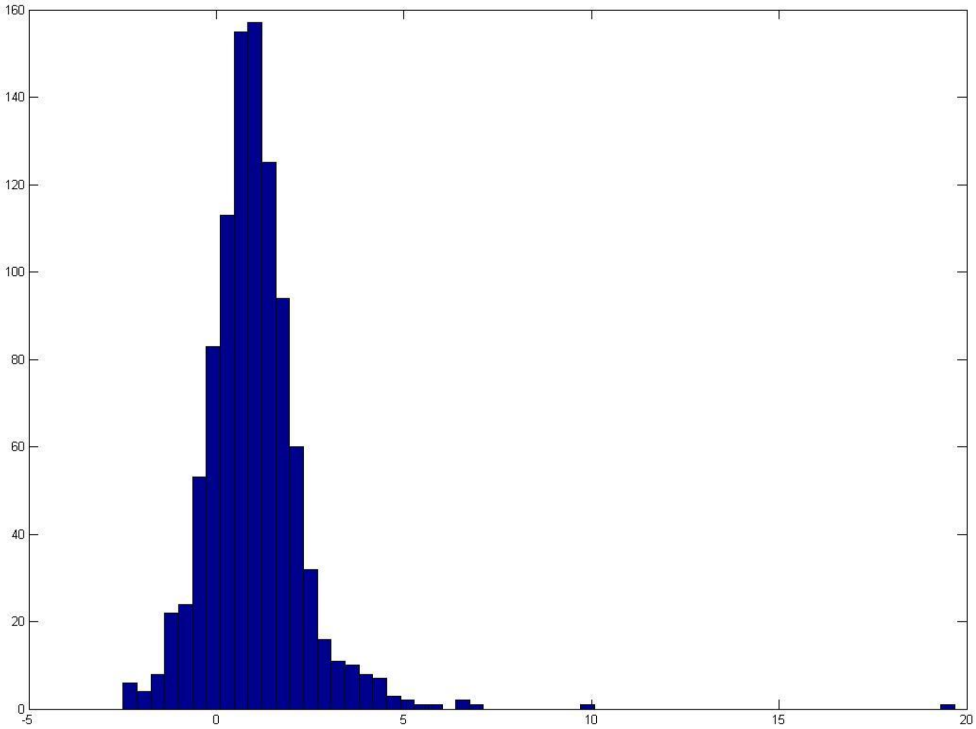

Figure 9, Profit and loss distribution for Company A's vanilla call option from now to maturity.

The difference between figures 8 and 9 is distinct. First, the mean or gain is lower in figure 9 and the tails reaches far more away in the profit and loss distribution. Hence, the issuer would now consider outstanding options would be worth it, or if the pricing models are correct. But one thing is self evident, weaker profitability and less control of the expected profitability. The profitability is going to decrease and the control of the profitability becomes harder to expect as the continuous delta hedge time process is more un-continuous.

Let us look at the event when the issuer is not able to delta hedge at all, and does not have any earlier delta neutral positions. The simulated vanilla call option in figure 10 is equal as in figures 8 and 9. And the vanilla call option was simulated with equivalent conditions as in figures 8 and 9 except non delta hedging. Figure 10 is shown bellow

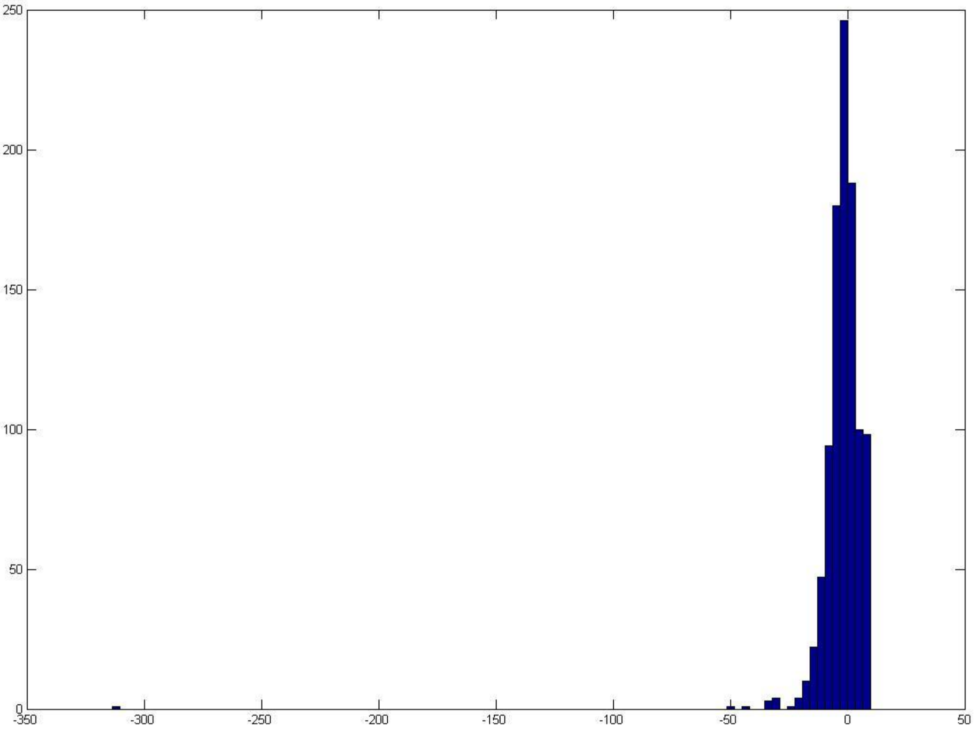

Figure 10, Shows the difference when no delta hedge is done.

This is how the distribution would look like if there is no delta hedge at all. The mean is however quite similar as figure 8 and 9, but the shape of the distribution is devastating, 10 percent of the mass is a loss of 10 times the revenues or more. The importance of time continuous process in vital for the institutions when hedging their issues instruments when one studies figure 10. The simulated conditions are of course simulated so these outcomes would appear, but the simulated conditions are not by itself particular different or extreme for 60 days for a randomly underlying asset in OMXS30.

The Price of Volatility

In chapter "Vega Hedging" one mention that the option market is considered not available, which makes the vega hedging not feasible and the impact should be obvious. Let us make a simulation with the option from figure 8, but with a higher simulated underlying volatility. This means the current value of the option should be to low due to our view of the future, and only higher cost is to expect. Figure 11 show the simulation.

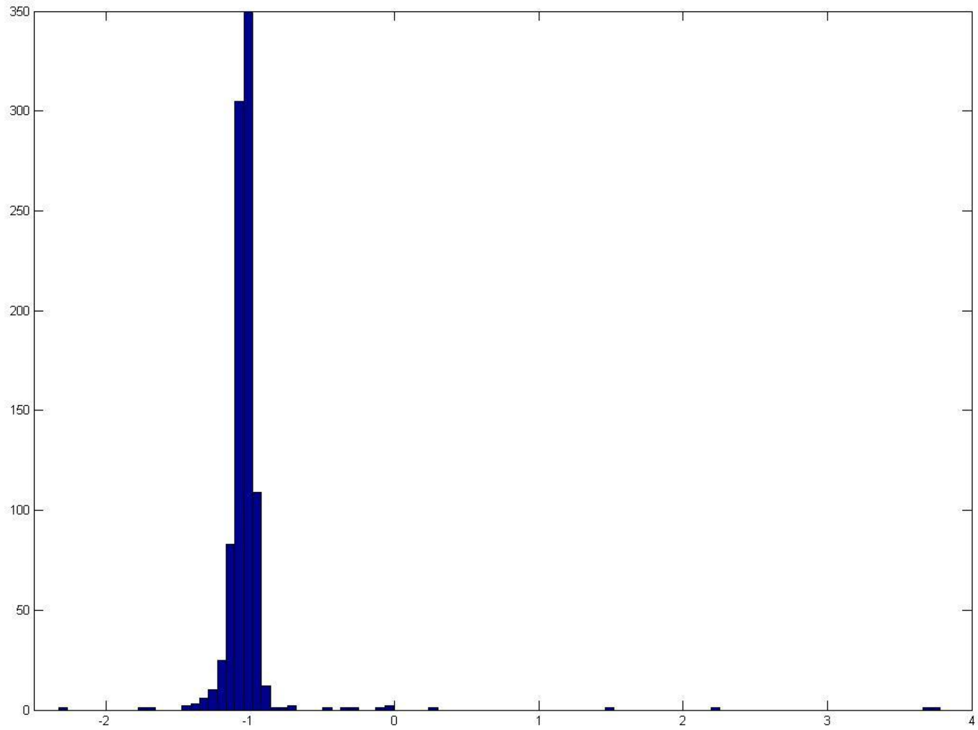

Figure 11, Similar conditions for the vanilla call option as in figure 8 but with higher underlying volatility.

The difference between figure 8 and 11 is distinct. The profit is much lesser and the mean is actually negative now from before but with a quite similar shape of distribution. Overall one can say that the profit and loss distribution from figure 8 has only removed slightly to the left. So a frozen option market will create an impact with higher losses if the underlying volatility raises relative the implied volatility when the instrument is issued. But there will probably be no visible difference in the variance.

The Price of Exotic Options

The most important type of instruments when talking about the hedging cost is exotic options. As explained in earlier chapters exotic options can produce higher delta values than 1 and larger values for gamma than for vanilla options. A smaller demonstration when simulating an Up & In barrier option for un-continuous time processes will be shown graphic. Considered the following, an Up & In call barrier option is issued on Company A with a current spot price of 100 and the strike price at 120 and 60 days to maturity with a barrier set to 130. Now consider one expects a future spot price of 140 at maturity. This will give that the expected simulated spot price reaches the barrier and knocks-in to a vanilla call option. The simulation also considered a time processes where only 5 delta hedges is possible per trading day with no market impact. Figure 12 show the simulated barrier option.

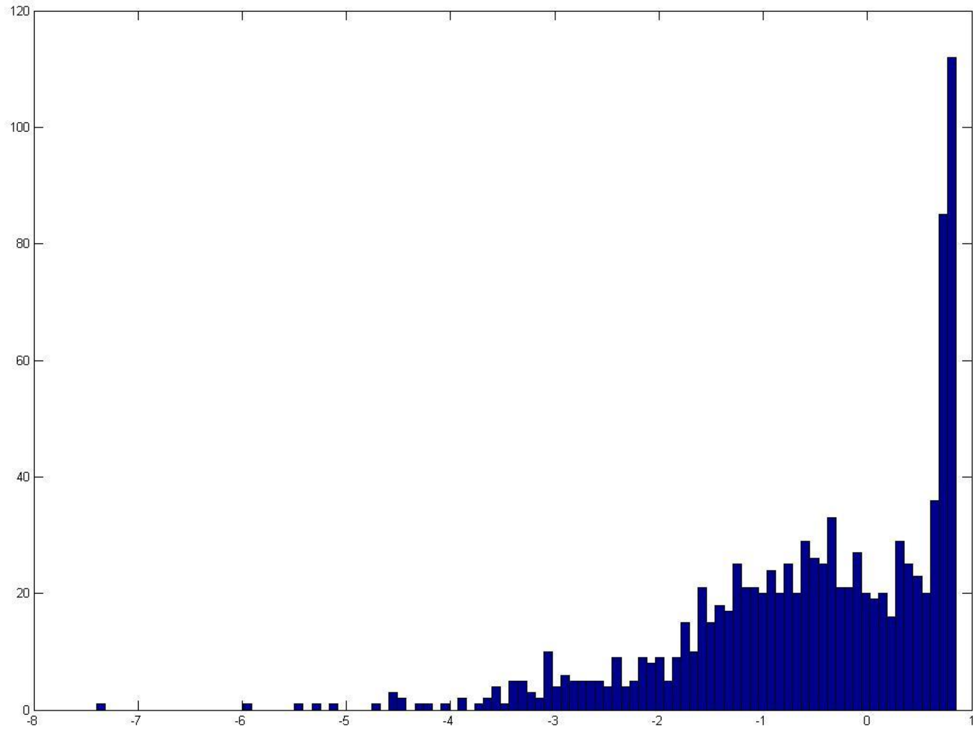

Figure 12, Shows the profit and loss distribution of an Up & In barrier option.

Figure 12 one will see that there are some positive gains roughly one third, but then there are some simulations that pass away to the far loss end. An important mention is that if the barrier option in figure 12 was simulated with time continuous processes and considered vega hedging and son on the shape if the distribution would look like as figure 8. Figure 12 compared with the above figures 8 to 11 one can concluded that exotic options is a bigger problem in un-continuous time processes than vanilla options. But to compare it with the examples of vanilla options above is not alright, there are actually two different types of options even if the pay-off is similar. But the thought is to show the increasing hedging cost the exotics can accomplish and non symmetric shape of distributions.

Future Topics

During the thesis, there were some topics that never been studied deep enough to achieve better results, due to lack of time and other circumstances. Two of them were the volatility surface and simulation time.

The volatility surface was one of the bigger challenges that shown up during the thesis. When the future expectation in the simulation is different from the current, one will not receive the exact profit and loss distribution. Some shortcuts where introduced, but still it is not going to give a perfect match. A further project is to create a program that simulated different volatility surface and see which gives the best fits. If the volatility surface is not accurate the calculated delta is not the optimal, however the changes may well not be the largest if the implied volatility does not rises to high, but it stills something to continue to work with.

A second topic that came up to solve was the simulation time in the program, especially if one chooses to simulate all issued instruments, the simulation cloud take several days. If there is no possibility to use a powerful computer, a continue would be to find approximations to calculate the total hedging cost if there is no possibility to let the computer run for several day in a row.

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