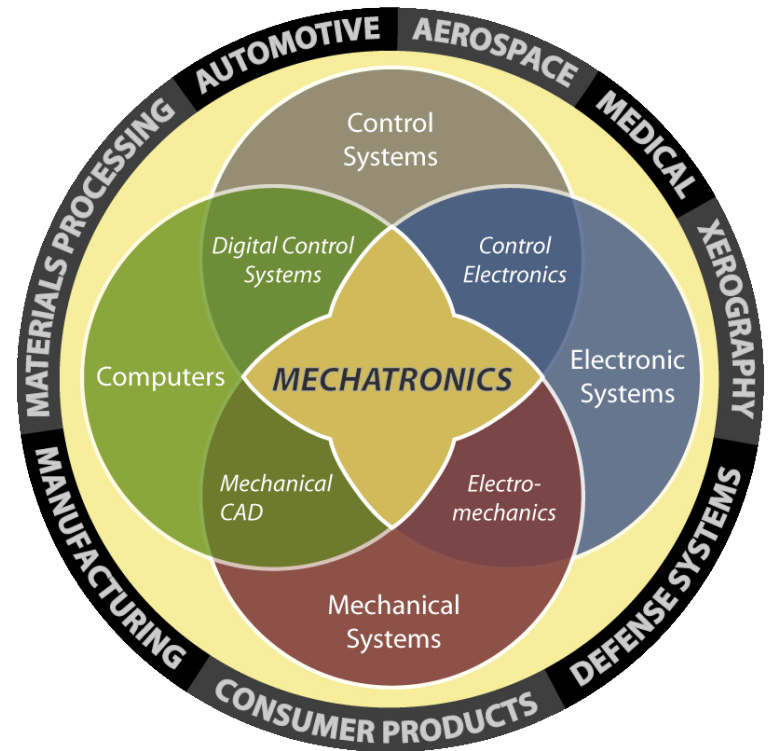
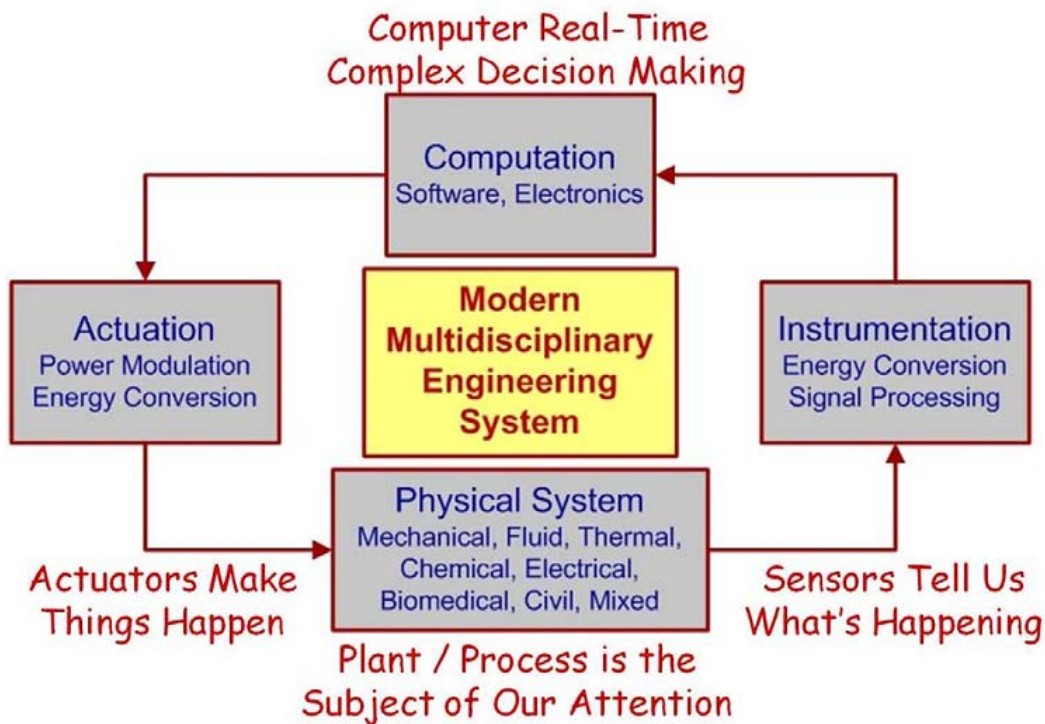


# Fundamental Principles of Mechanical Design



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# Introduction

- Precision machines are essential elements of an industrial society.
- A precision machine is an integrated system that relies on the attributes of one component to augment the weaknesses of another component.
- Here we emphasize the design of mechanical and structural precision machine components and their integration with sensor, actuator, and control systems to maximize performance.
- Design is a mixture of analysis and creative thought.
- Good designs are based on excellent concepts and properly designed details.

- Design engineers must be more creative than their competition and more observant of the world around them.
- Analogous to physical exercise, analysis is a form of mental exercise that trains the mind to be strong and swift. Many designs would never have even been conceived of if the design engineer did not understand the basic physics behind the process or machine that prompted the need for a new design.
- Design engineers must be good at identifying problems. Once a problem is identified, it will usually yield to an unending barrage of creative thought and analysis.
- In addition to identifying and solving problems, the design engineer must also learn to identify what the customer really needs, which is not necessarily what the customer thinks that he or she needs.

# Fundamental Principles

- Understand them. Associate images with them.
- When you look at a machine or structure, try to see the fundamental principles in either action or missing.
- With a deep knowledge of these fundamental principles, one can rapidly generate strategies and concepts with the greatest viability.
- With a deep understanding of fundamental principles, one can critically evaluate other machines and components.

- Occam's Razor
  - Keep things simple to start and add detail as the design develops.
- Simplicity vs. Complexity
  - Create designs that are explicitly simple. Keep complexity intrinsic.
- Laws of Nature
  - Fundamental Laws of Nature – the basics come before the details!
  - Newton's Laws, Laws of Thermodynamics, Maxwell's Equations
- Saint-Venant's Principle
  - When an object is to be controlled, sketch it being held at points several characteristic dimensions apart.

- Golden Rectangle
  - Don't know what size it should be? Start with a ratio of about 1.6:1.
- Independent Functions
  - Keep the functions of a design independent from one another.
- Abbe's Principle
  - Small angular deflections are amplified by distance to create large linear displacements.
- Maxwell & Reciprocity
  - Uncomfortable with a design? Try inverting it, or turning it on its side.

- Self-Principles
  - Use an object's geometry or other property to prevent a problem.
- Stability
  - Stable, neutrally stable, and unstable effects can help or hurt. Some things can be made neutrally stable and fast, while vibration can induce instability. Beware of buckling of compression members.
- Symmetry
  - Try a design that is symmetric, and then impose reciprocity to consider a design that is not symmetric.
- Parallel Axis Theorem
  - Add mass away from the neutral axis to increase strength and stiffness.



- Accuracy, Repeatability, and Resolution
  - Make your machine repeatable first, then tune for accuracy if you have fine enough resolution. Tell the same story each time, the correct story, and with enough detail so people can understand it.
- Sensitive Directions and Reference Features
  - Why pay for performance in a direction that is not needed? Establish reference datums from which you measure critical parameters.
- Triangulate for Stiffness
  - Triangulate parts and structures to make them stiffer.

- Load Paths
  - Plan the load paths in parts, structures, and assemblies.
  - Draw a line through the path that forces follow, and seek to minimize its length. If the path and shape changes significantly as the machine moves, then the machine will have limited accuracy and may have limited repeatability.
- Avoid Bending Stresses
  - Prefer tension and compression.
- Manage Friction
  - Avoid sliding friction. Use rolling element bearings.
- Free Body Diagrams & Superposition
  - To analyze a complex object, separate it into parts and label the forces and moments on each part that are imposed by other parts.

- Preload
  - Loose fits between objects mean you cannot predict where one object will be with respect to the other. Apply loads between the objects as part of manufacturing and assembly to take out the slack in the system.
- Centers of Action
  - If forces are applied through the centers of mass, stiffness, and friction, there will be no moments and hence minimal errors.
- Exact Constraint Design
  - The number of points at which a body is held or supported should be equal to the number of degrees of freedom that are to be constrained.

- Elastically Averaged Design
  - Hold or support a body with ten times (or more) compliant points than there are degrees of freedom to be restrained, such that the errors in the compliant support points will average out.
- Stick Figures
  - Initially sketch an idea using simple stick figures, which also denote where major coordinate systems are located in the design. The coordinate systems are to be used for modeling individual modules, e.g., for creating an error budget to predict a machine's repeatability and accuracy.

Let's discuss these in more detail!

# Occam's Razor

- William of Occam (1284-1347) – English philosopher and theologian
- “Plurality should not be assumed without necessity.”
- A problem should be stated in its most basic and simplest terms.
- The simplest theory that fits the facts of a problem is the one that should be selected.
- Limit Analysis should be used to check ideas.
- Use fundamental principles as catalysts to help keep things simple.

- Complexity is to be minimized in both design and manufacturing. A careful systems approach is required.
- Opportunities to Keep Things Simple
  - Overall Design Strategy
  - Design Concept
  - Formation of Modules
    - Modules have to be assembled and interfaced to each other.
  - Design and Selection of Components
    - Detailed design gets done at the component level.
    - Keep functional requirements of the design in mind.
    - Keep implied functional requirements of the components in mind.

- Goal is to minimize the number of components (which may lead to the specification of a few highly complex components) and balance component complexity with the number and complexity of the manufacturing processes and the quantity to be manufactured.

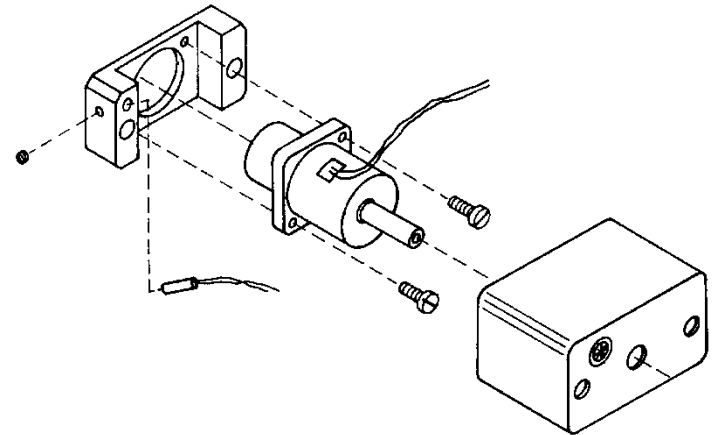
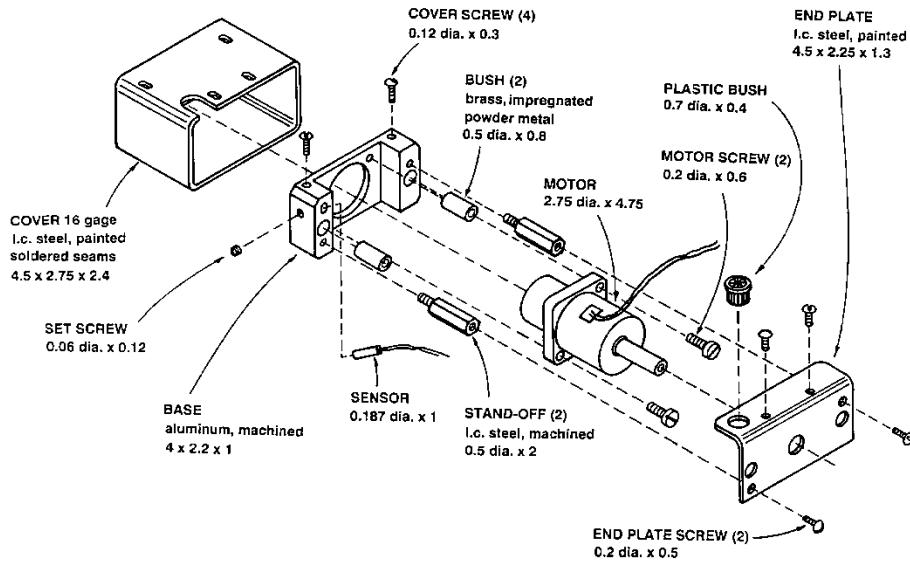
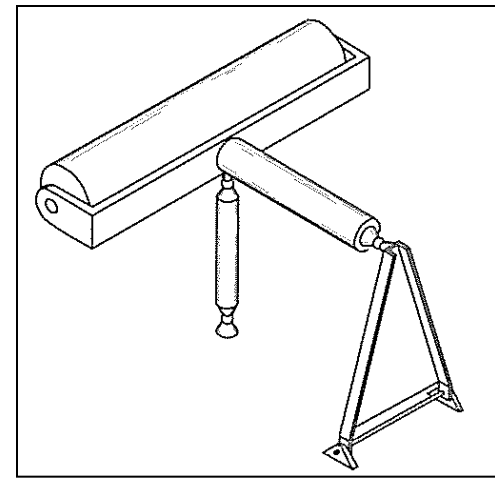
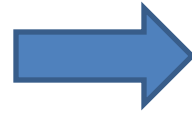
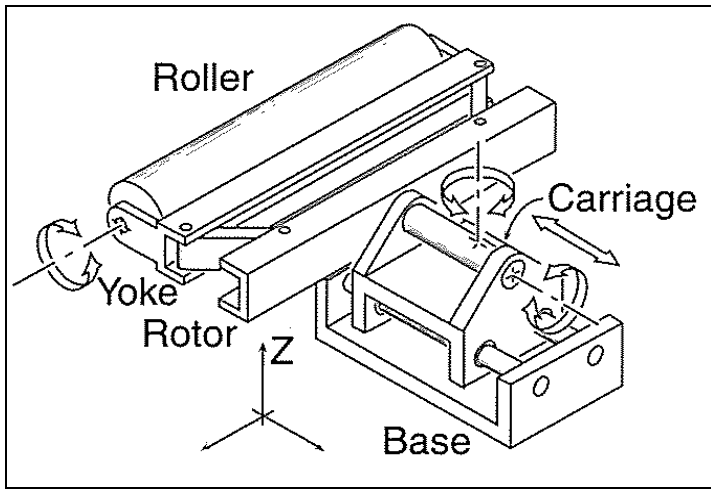
# Simplicity vs. Complexity

- Create designs that are explicitly simple. Keep complexity intrinsic.
- What makes a design simple? Can your intuition alone judge simplicity? Will you know it when you see it?
- The less thought and less knowledge a device requires for production, testing, and use, the simpler it is.
- How much thought? How much knowledge? Judge best by comparing one solution to another.
- Of course, it may take lots of thought and knowledge to get to a design requiring little of either – that is design!



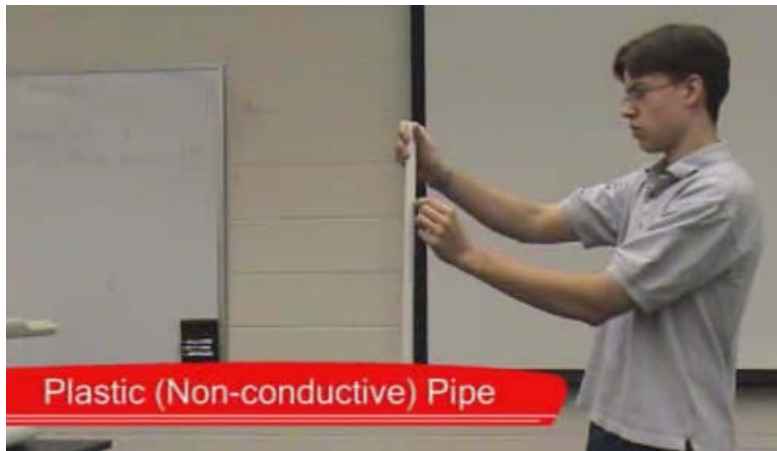
- What a simple design means is that everyone involved with its production and use sees nothing that looks complicated from his or her own perspective or convention.
- Complexity is buried and invisible. There is a hierarchy for knowledge and thought. Each hierarchical level may be intrinsically complex, yet the device remains simple if the complexity resides only within its own level.
  - Screw threads have abundant scientific and manufacturing complexity, yet that complexity is invisible to the designer. We just say  $\frac{1}{4}$  -20.
- Complex geometry no longer implies a complicated design. A part's geometry may be complicated, yet its production and use may be no more difficult than for one with much simpler geometry.

- Simplicity can be subtle. Design symmetry into components. If a part is asymmetric, knowledge and thought are needed to orient it for either the assembly or the use. Assembling or using symmetrical parts requires less knowledge and thought than asymmetric parts.
- Simplicity can be paradoxical. Symmetry adds information to the component part, thus adding cost to manufacture, but makes the part simpler to use e.g., the double-sided key.
- Two common techniques for keeping things simple by keeping complexity hidden are:
  - Purchasing rather than making components
  - Specifying components by standards

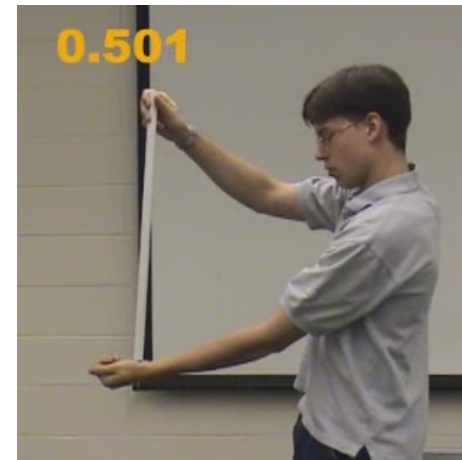


# Laws of Nature

- To develop a physical model of an existing system or of a system concept, we use engineering judgment and make simplifying assumptions.
- To develop a mathematical model, a model that can predict system dynamic behavior, we apply the Laws of Nature to the physical model.
- Drop a strong cylindrical magnet down a copper pipe with plenty of clearance between the magnet and the pipe wall. What happens? How many Laws of Nature are illustrated?
  - Newton's 2<sup>nd</sup> Law, Newton's Law of Gravitation, Ampere's Law, Faraday's Law, Lenz's Law



Plastic (Non-conductive) Pipe



0.501



Copper (Conductive) Pipe

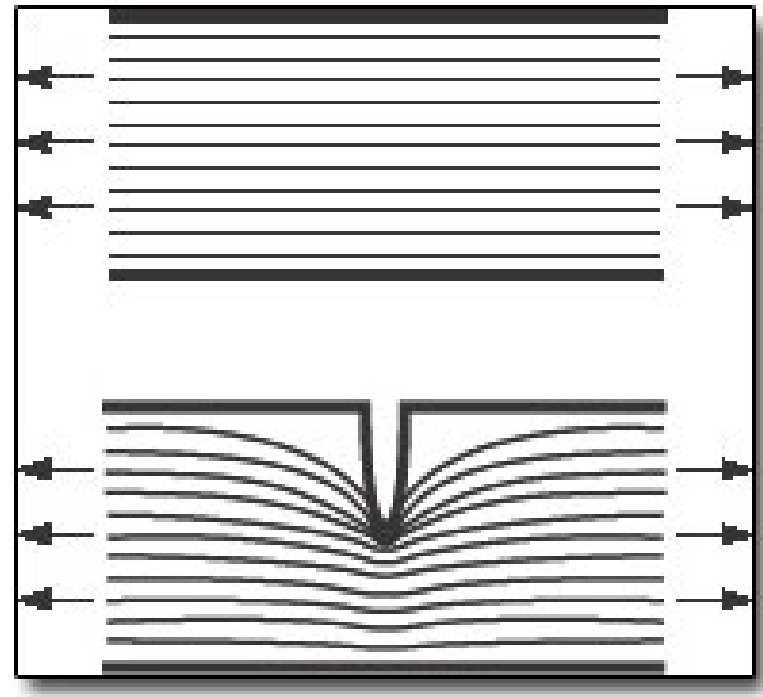
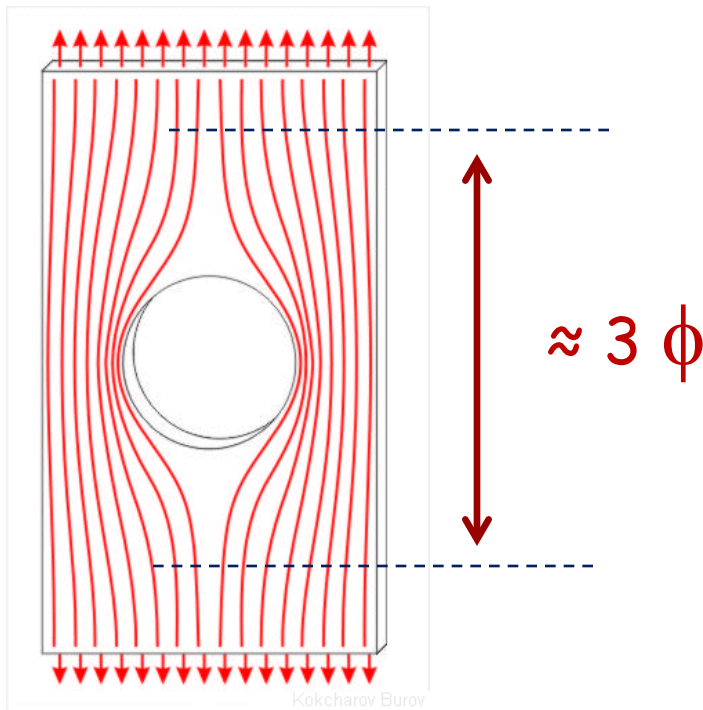


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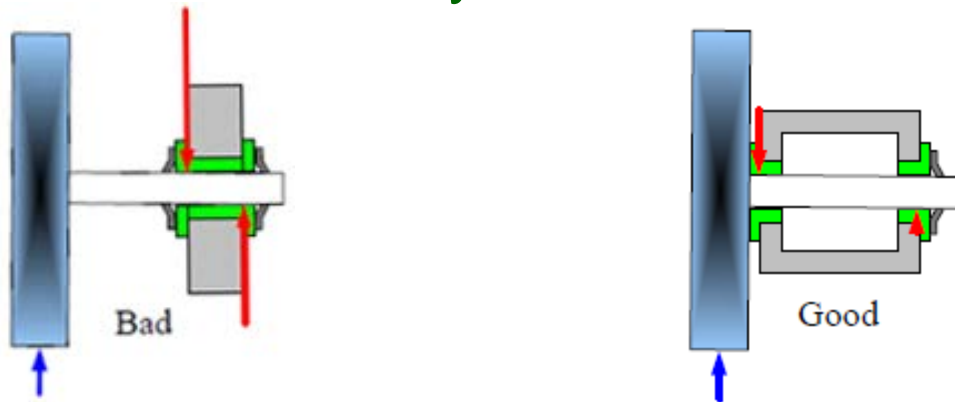
# Saint-Venant's Principle

- **Saint-Venant** was a strong proponent of coordinating experimental and practical work with theoretical study. His greatest contribution to engineering is his approach to modeling practical problems.
- *The principle says that several characteristic dimensions away from an effect, the effect is essentially dissipated. Or if an effect is to dominate a system, it must be applied over 3-5 characteristic dimensions of the system.*
  - Local effects of loading do not effect global strains.
  - For example, the bending strains at the root of a cantilever beam are not influenced by the local deformations of a point load applied to the end of a cantilever beam.

# Saint-Venant's Principle

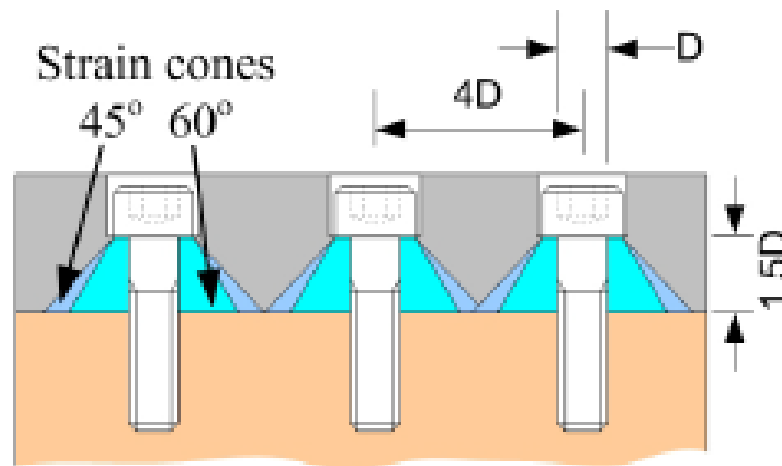


- The engineering applications of this observation are profound for the development of conceptual ideas and initial layouts of designs.
  - To not feel something's effects, be several characteristic dimensions away.
  - To dominate and control something, apply constraints over several characteristic dimensions.
- Application of Saint-Venant's Principle
  - When mounting bearings to support a shaft, the bearings should be spaced 3-5 shaft diameters apart if the bearings are to effectively resist moments applied to the shaft.



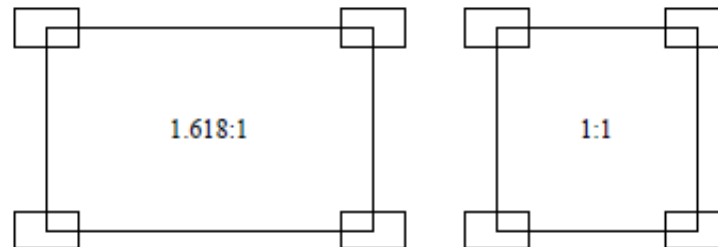
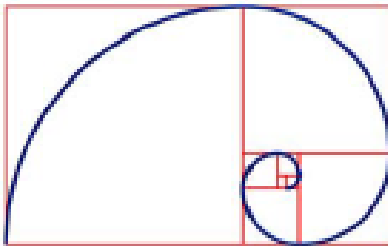


- In machine tool structure, if one is to minimize bending, the length of the structure should be no more than 3-5 times the depth of the beam.
- When bolting components together, in order to make the bolted joint act almost as it were welded together, the bolts' strain (stress) cones should overlap. The strain cone emanates from  $45^\circ$  to  $60^\circ$  under the bolt head. The strain cones typically overlap if the bolts are spaced less than 3-5 bolt diameters apart.



# Golden Rectangle

- The Golden Rectangle, discovered by Pythagoras, is a rectangle whose sides are in proportion such that when a square is cut from the rectangle, the remaining rectangle has the same proportions as the original rectangle, i.e.,  $a/b = 1.618$



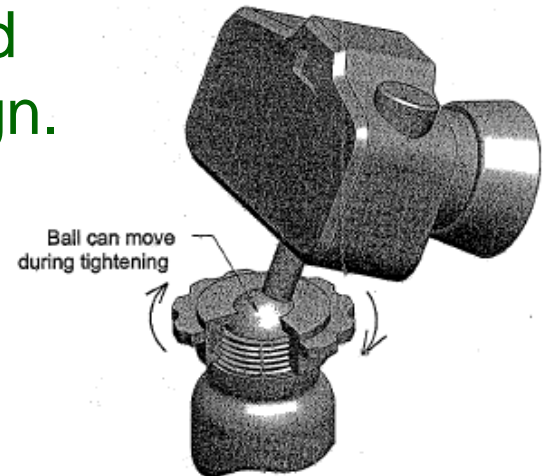
- If the pattern of subtracting squares continues and a spiral curve is drawn by linking together circular arcs whose centers are the corners of the squares, the spiral continues to infinity.

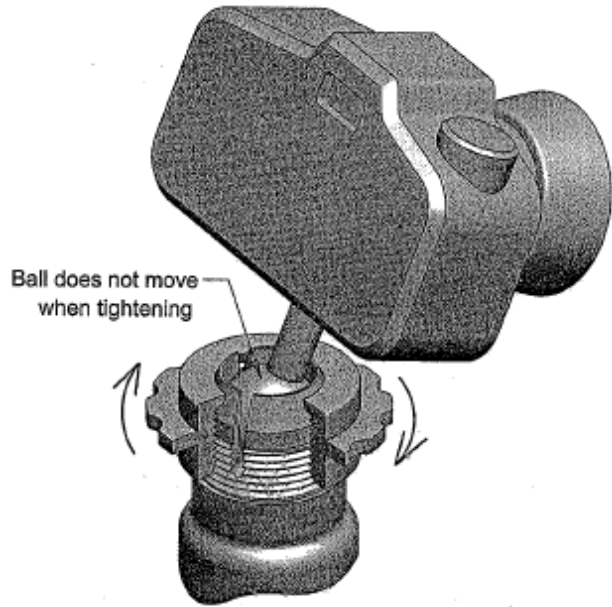
- The Golden Rectangle can help the engineer initially sketch concepts that have a greater chance of being realizable. When designing a machine, one can either:
  - start from the outside and work inwards, which might force the inner workings to be cramped and lead to degradation of performance, or
  - start from the inside with a critical module and design outwards, which might result in a machine that is too spacious.
- A compromise from the beginning can be achieved with either method by initially sketching concepts while keeping the proportions of the Golden Rectangle in mind. Saint-Venant's principle should also be considered when laying out proportions and spacing of machine elements.

# Independent Functions

- Keep the functions of a design independent from one another.
- During the concept development stage of the design process, you will decompose a device or system into basic functions. Keep those functions separate!
- This is not as easy as it sounds. There will be temptations to combine functions and features and thus compromise the design.

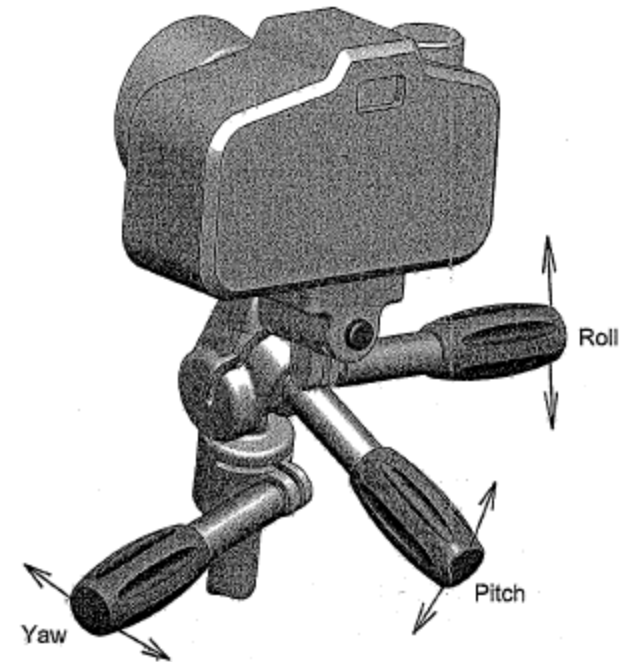
Consider the ball-and-socket tripod head for a camera. There is no functional independence of positioning and locking.





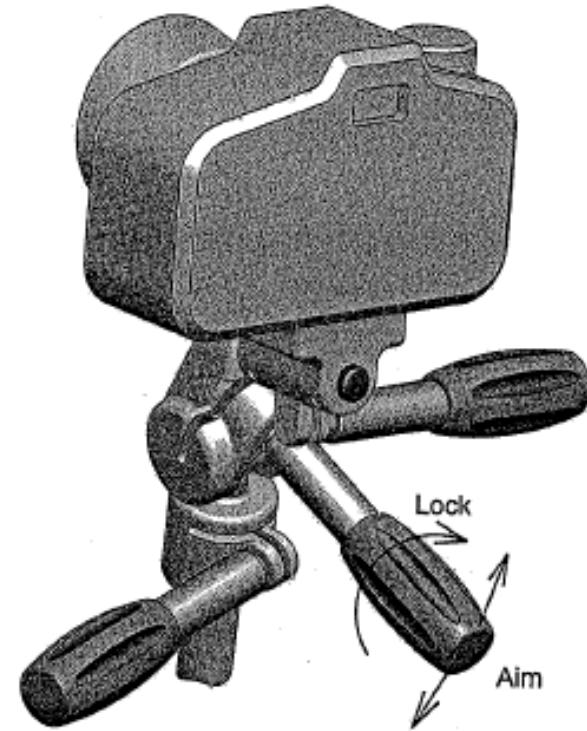
This is an improvement in separating functions. Here tightening the nut creates no rotational friction between nut and ball, but at a cost in manufacturing.

Multi-axis pan head camera mount has functional independence of all three rotational locking and positioning functions.



- Accept that everything in design, including independence of function, is a compromise. You need not take an unyielding posture to any rule of design, but always understand how and why you yield.

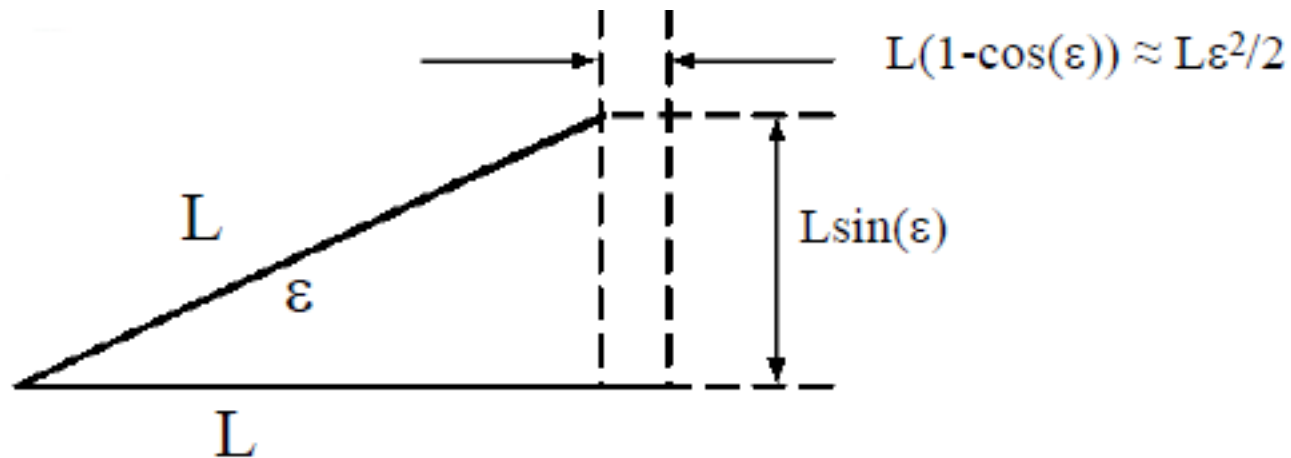
Seek independence of the functions of a device, but do not preclude combining functions within parts. Here, the handles are used for two functions, positioning and locking. Combined adjustment and locking functionality reduces part count and operates intuitively.



# Abbe's Principle

- The amplification of angular motion to create large translational motions is one of the foremost principles in the design of precision and robust machines.
- The Abbe Principle (Dr. Ernst Abbe 1840-1905) resulted from observations about measurement errors in the manufacture of microscopes.
  - *If errors in parallax are to be avoided, then the measuring system must be placed coaxially with the axis along which the displacement is to be measured on the workpiece.*

- When an angular error is amplified by a distance, e.g., to create an error in a machine's position, the strict definition of the error is a sine or cosine error.

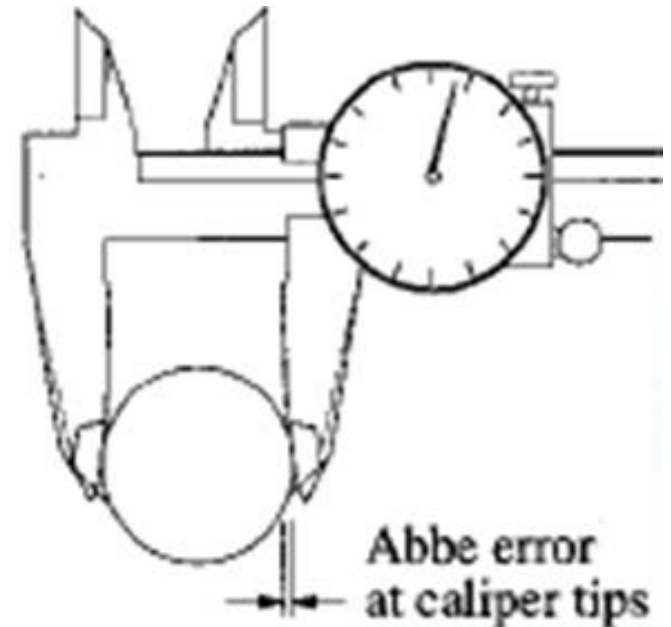


- The implications of this observation on the design of instruments and machines are profound:
  - Always try to place the measurement system as close to the line of action (the process) as possible.
  - Always try to place bearings and actuators as close to the line of action (the process) as possible.

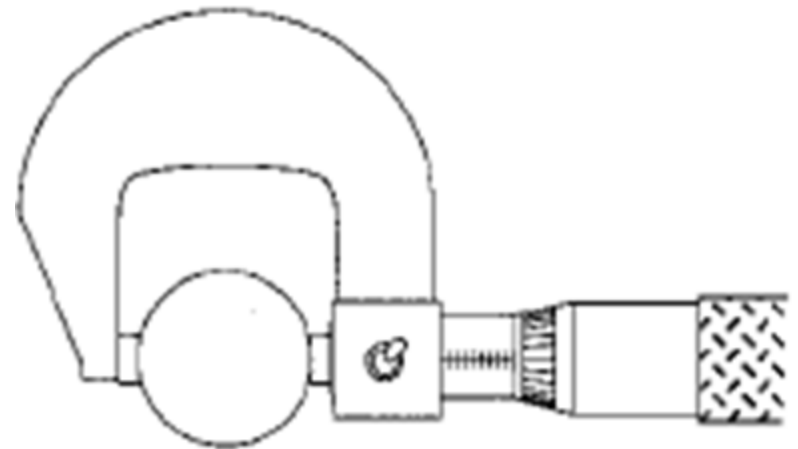


- The philosophy of Abbe's Principle also extends to other types of measurements.
  - When measuring temperature, it is important to place the temperature sensor as close as possible to the process to be measured.
  - The idea is the same for pressure, flow, voltage, current, etc.
  - In each case, the farther away you are from the process to be measured, the greater the chance for errors to reduce the accuracy of the measurement.
- Note that a small angular deflection in one part of a machine quickly grows as subsequent layers of a machine are stacked upon it.

Consider the task of measuring a shaft, where one can use a micrometer or a dial caliper. The dial caliper is quick and easy to use because with the simple action of your thumb, you can slide the caliper open so it can be placed around the object, and then you can slide it closed to make contact with the object and make the measurement. As can be felt with a cheap caliper, any rocking motion in the caliper head will cause a measurement error. In addition, forces on the caliper jaws cause deformations in the structure and the bearings that support the head. Fortunately, the long range of travel of the caliper head makes it a very versatile measuring instrument.



The micrometer, on the other hand, requires a much more laborious turning of the screw to adjust it to fit over the part, and you have to carefully position it to ensure that you are contacting the part properly. The micrometer, however, is probably 10x more accurate than the dial caliper for the simple reason that the mechanism of motion and measuring are completely in-line with the measurement to be made. In this case, the Abbe offset is zero. However, the range of measurement motion is very limited, making the micrometer a specialized instrument.

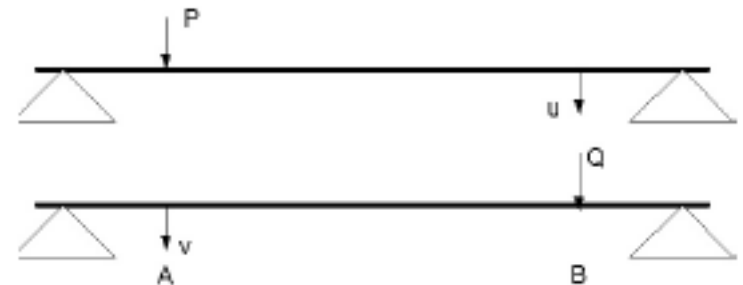


# Maxwell & Reciprocity

- James Clerk Maxwell's (1831-1879) theory of reciprocity is commonly applied to mechanics and precision measurement systems, and philosophically to how difficult problems can be creatively solved.
- Let A and B be any two points of an elastic system. Let the displacement of point B in any direction U due to a force P acting in any direction V at A be u. Let the displacement of A in the direction of V due to a force Q in the direction U at B be v. Then  $Pv = Qu$ .

Reversal  
Critical Thinking

$\frac{1}{\text{problem}} = \text{opportunity}$



# Self Principles

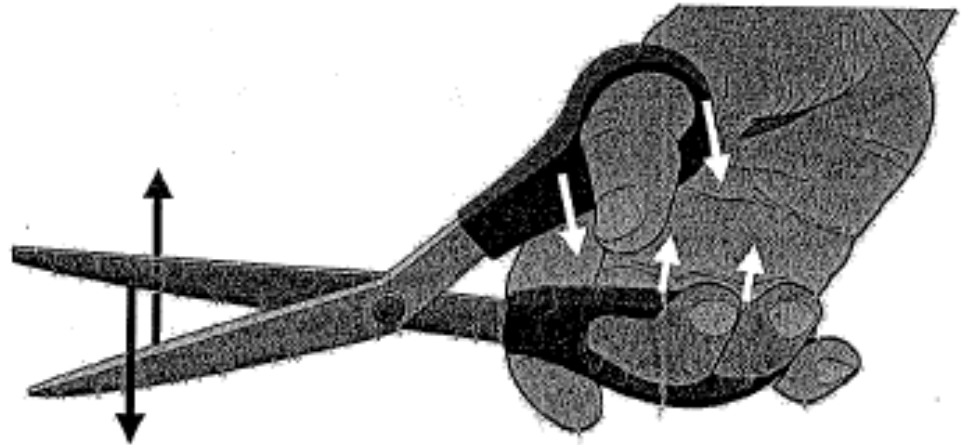
- Self principles utilize the phenomena the machine is trying to control to help control the phenomena.
- There are four basic types of self principles: self-help, self-balancing, self-protecting, and self-checking.
- Self-Help Systems
  - Self-help systems use the phenomena to be controlled as a means for controlling the phenomena. For example, some airplane door mechanisms swing the door out when opened, yet the door acts as a self-sealing tapered plug when closed. The modern tubeless tire is a result of careful structural design that utilizes the elastic nature of the tire to make a seal that allows the tire to be inflated.

- Forces applied to a structure or mechanism are used to great advantage when they:
  - Create new, useful forces
  - Transform or redirect themselves
  - Balance either themselves or existing loads
  - Help to distribute loads
- Self-help that Creates Forces
  - Pressure-formed seals such as O-rings and internally-mounted doors use self-help to create robust seals.
  - The capstan and windlass use self-help to pull in boat anchors by creating and multiplying friction through several turns of chain.

## – Self-help that Redirects Forces

- Left-handed scissors have little to do with how thumb and fingers fit, and lots to do with how hand action naturally forces the blades' cutting edges together.
- Left- and right-handed scissors are mirror images of each other, improving the scissors' shearing action for either user.

### Self-Help in Scissors

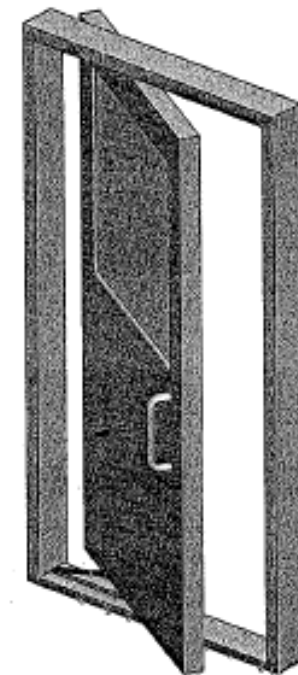
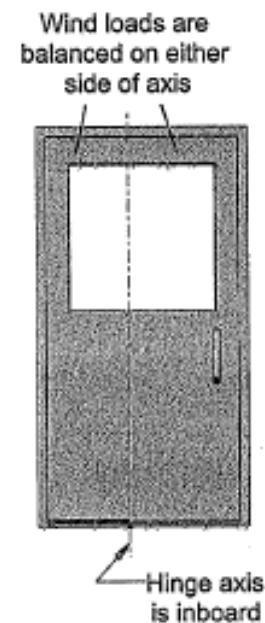


## – Self-help that Balances Forces

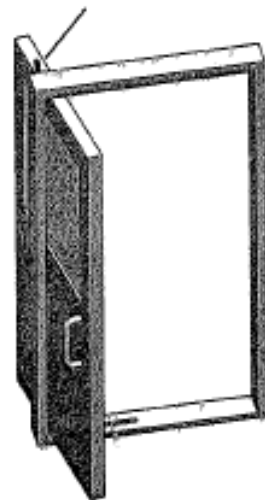
- Self-help can balance or neutralize, at least partially, an undesirable effect, e.g., counterweights on draw and lift bridges and in cable-operated elevators.

A balanced door with an articulated hinge exhibits self-help. Wind will not open this door, yet it opens and closes easily in strong winds.

The hinge point, rather than being at an outer edge, is inboard when the door is closed. Wind pressure on the door face is balanced on either side of the hinge axis. As the door opens, the pivot axis slides by mechanical action to the side to open the door fully.

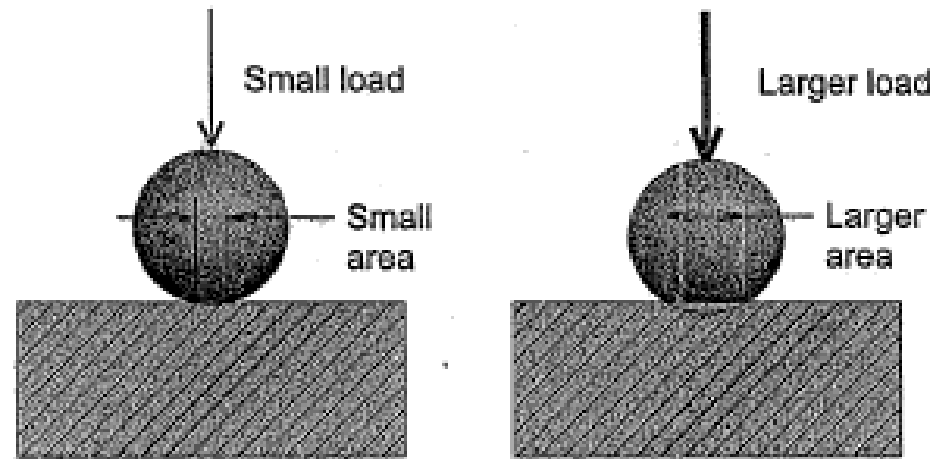


Articulated hinge slides door outward for full open



## – Self-help that Distributes Loads

- In this case, the load path changes with deformation, thereby improving performance.
- In Hertzian contact stress, the larger the load, the larger the contact area.



- Structures can also be designed to stiffen or strengthen with deflection. For example, beams become much stiffer with large deflections (if they don't yield) as the bending progresses to tension.



- Self-Balancing Systems

- Self-balancing systems utilize geometry to reduce an undesirable effect such as high stress.
- An example of this is the leaning of turbine blades. As the turbine spins, the shape of the blade pumps air through the turbine and creates large bending stresses on the blade. If the blade is tilted so that its length axis is not along a radius of the turbine, centrifugal forces can be used to create a bending moment opposite to that caused by the airflow. In this manner, cyclic tensile stresses can be minimized.

- Self-Protecting Systems

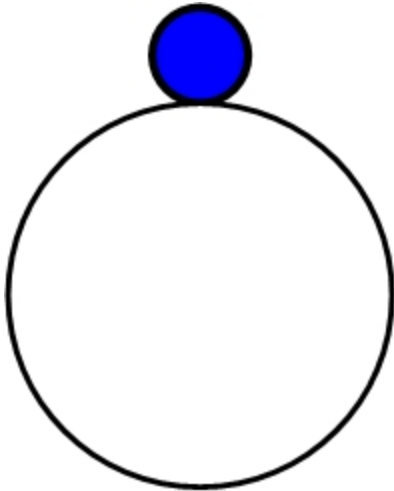
- Self-protecting systems utilize passive elements to ensure that the system is not overloaded to the point of permanent deformation. They do this by providing additional force transmission paths after a predetermined elastic deformation occurs.
- However, in protecting one component the design engineer must make sure that the effect does not ripple through the system and damage other components.
- Usually, self-protected systems can return to their normal operating range unaffected by overloads.
- The most common example of self-protecting systems are found in spring-actuated mechanisms.

- Self-Checking Systems

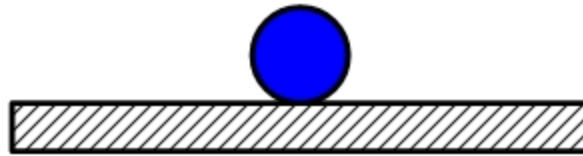
- Self-checking systems utilize the concept of symmetry to ensure geometric accuracy. This involves turning the device through  $180^\circ$  and comparing the measurements to the first set.
- For example, if a coordinate measuring machine (CMM) is very repeatable but not accurate, it can still be used to measure the straightness of a straightedge. The edge of the straightedge is placed vertical so that gravity does not create an error. The straightedge is measured with the CMM and then the straightedge is rotated  $180^\circ$  and measured again. The measurements are subtracted from each other, which cancels out the straightness error in the CMM.

# Stability

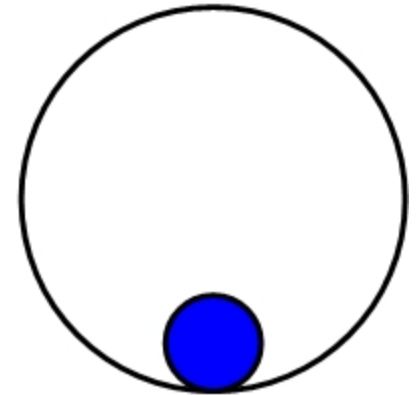
- Stability is of utmost importance in the design of structures, actuators, bearings, and control systems.
- All systems are either stable, marginally stable, or unstable.



**UNSTABLE**



**MARGINALLY  
STABLE**



**STABLE**

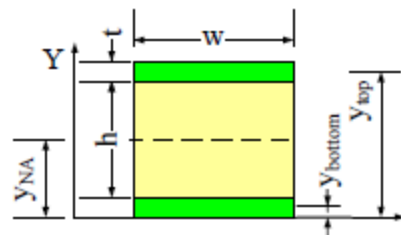
# Symmetry

- Can symmetry help or hurt a design?
- In general, symmetry is a good thing in a system, and thus a good starting point in a design.
- Symmetry is a principle that is closely related to stability. Symmetry can sometimes provide stability, sometimes not.
- In structures, symmetry may exist on an overall scale, such as between two sides of a machine, but within each side of the machine, symmetry takes second place to the desired formation of triangles so as to make a system well-braced and resistant to shearing.
- Structures are often symmetric to take loads from either side.

- In bearings, it is often the opposite, where on an overall scale one side of the system has bearings configured and mounted to restrain five degrees of freedom of a shaft, and at the other end of the shaft, the bearings only prevent radial motion of the shaft.
- List what is symmetric and what is not symmetric about each potential concept. Ask if symmetry helps or hurts. Study what would happen if what was symmetric was made non symmetric, and vice versa.

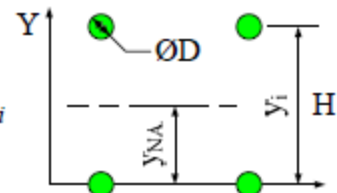
# Parallel Axis Theorem

- The Parallel Axis Theorem is a very powerful tool for calculating the area moments of inertia for complex objects.
  - Starting with basic shapes, whose areas and area moments of inertia about their local neutral axes are known, a design engineer first finds the location of the neutral axis of the composite structure, and then uses the parallel axis theorem to determine the area moment of inertia for the composite structure.



$$y_{NA} = \frac{\sum_{i=1}^N y_i A_i}{\sum_{i=1}^N A_i}$$

$$I = \sum_{i=1}^N I_i + \sum_{i=1}^N (y_i - y_{NA})^2 A_i$$



# Accuracy, Precision, & Resolution

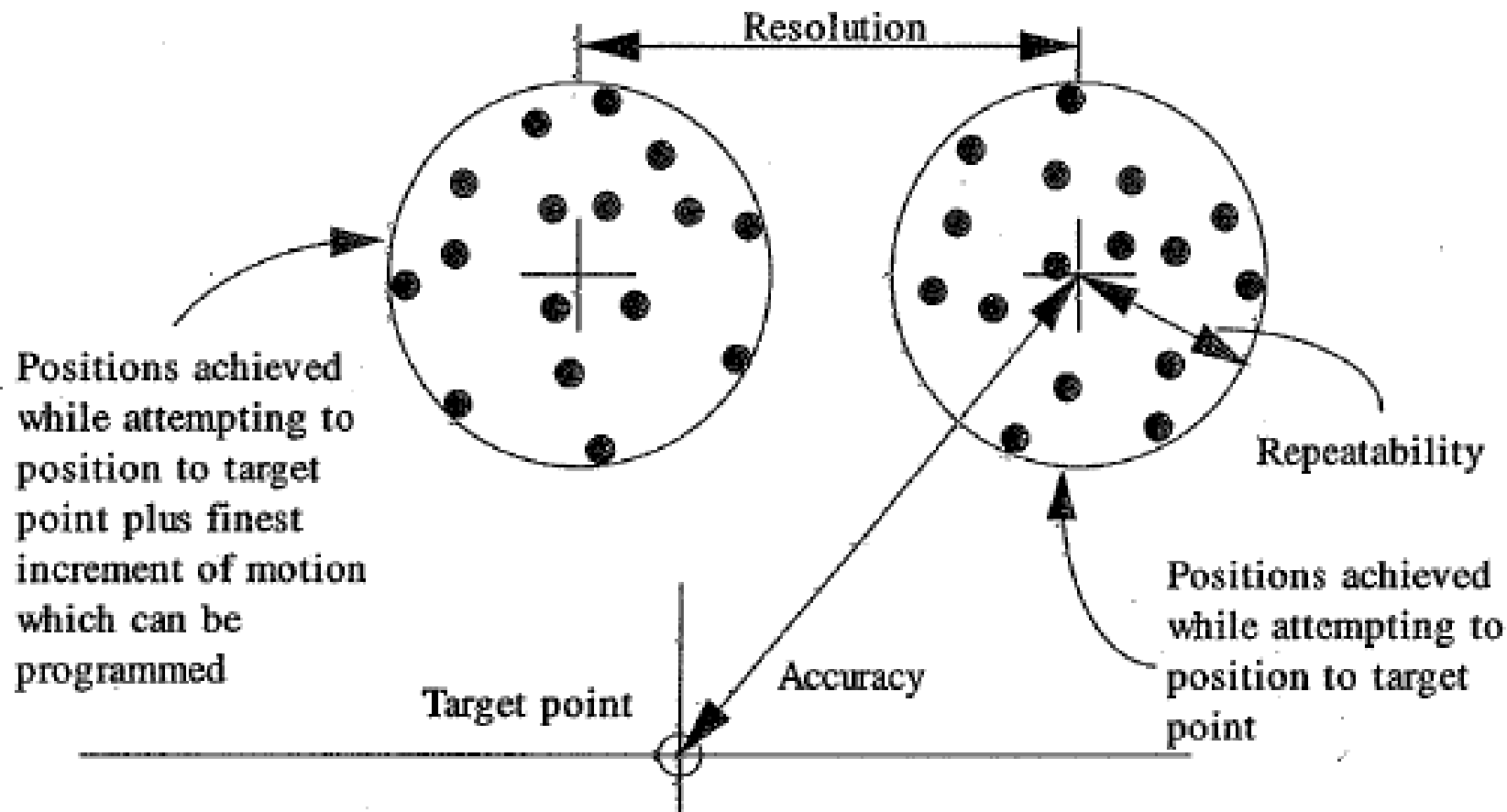
- A good machine designer understands and has developed an intuitive feeling for the factors that affect machine performance. Equally important, the design engineer must also understand the basic physics that characterize a machine component or system. This knowledge is essential for the development of good designs and the proper selection of components.
- The design of quality precision machines depends primarily on the ability of the design and manufacturing engineers to predict how the machine will perform before it is built.



- The most important factors affecting the quality of a machine are the accuracy, precision (repeatability), and resolution of its components and the manner in which they are combined. These factors are critical because they affect every one of the parts that will be manufactured using the machine.
- Minimizing machine cost and maximizing machine quality mandate predictability of accuracy, repeatability, and resolution. This allows the design engineer to optimize the choice of components and to specify manufacturing tolerances.
- Designing a machine that has good accuracy, repeatability, and resolution is not a black art. It appears to be a black art only when the observer lacks the time or resources to use scientific principles to discover the true nature of the phenomena.

- In the context of taking a deterministic approach to design, it is very important to understand the rules of the game. There are three basic definitions to remember with respect to how well a machine tool can position its axes: accuracy, precision (repeatability), and resolution.
- The diagram on the next slide represents these concepts.
- Accuracy is the ability to tell the truth.
- Precision is the ability to tell the same story over and over again.
- Resolution is how detailed your story is.
- These three concepts are often improperly used, so let's discuss them.

# Accuracy, Precision (repeatability), and Resolution



## Marksman's Target

- Accuracy is the ability of system to tell the truth.
  - Accuracy is a measure of how well a system can achieve or measure a state that is traceable to a national or international standard.
  - If one were to use a device to make multiple measurements of a known standard, the difference between the known measurement and the average of all measurements taken is the accuracy of the measurement.
  - Accuracy is the maximum translational or rotational error between any two points in the machine's work volume.
  - Accuracy can be represented as the difference between the root mean square radius of all the bullet holes in a target and the radius of the bull's eye.

- Linear, planar, and volumetric accuracy can all be similarly defined for a machine.
- Accuracy is affected by the shape and size of components, as well as thermal growth, backlash, wear, friction, deformations, and sensor and control system accuracy.

- Repeatability (precision) is the ability to tell the same story over and over again.
  - Repeatability tells how well a system can achieve or measure the same state each time.
  - Repeatability is the error between a number of successive attempts to move the machine to the same position.
  - If one were to use a device and make multiple measurements of a known standard, the standard deviation of the measurements would be the repeatability of the measurement, and the number of standard deviations would give a confidence level about the repeatability.
  - Repeatability is affected by thermal growth, backlash, wear, friction, deformations, and sensor and control system repeatability.

- Repeatability can be represented by the diameter of the circle which contains N% of the bullet holes in the target.
- Bidirectional repeatability is the repeatability achieved when the point is approached from two different directions.
- For a set of N data points with a normal (Gaussian) distribution the mean  $x_{\text{mean}}$  and standard deviation  $\sigma$  are defined as:

$$x_{\text{mean}} = \frac{1}{N} \sum_{i=1}^N x_i \quad \sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - x_{\text{mean}})^2}$$

- The standard deviation is used in the determination of the probability of occurrence of an event in a system that has a normal distribution.

- The percent chance of a value occurring within a number of standard deviations  $k$  of its expected value is given in the table.
- The probabilistic approach to a problem is only a tool to allow us to deal with variables that are too numerous or expensive to sort out properly by common sense and good metrology. Required use of this tool might be an indication that it is time to take a closer look at the system and see if the system can be changed to make it deterministic and therefore more controllable.

$k$	N%
1.0	68.2689
2.0	95.4500
3.0	99.7300
4.0	99.9937
5.0	99.9999
6.0	100.0000



- Often the key to repeatability is not within the machine itself, but in isolating the machine from variations in the environment.

- Resolution is how detailed your story is.
  - Resolution is the ability of a system to tell the fine details of the story or what is the minimum increment that can be detected or moved.
  - If one were to try and move to a desired position, and then measure the error, and then correct the position by the smallest achievable amount, it would be limited by the resolution of the system.
  - Resolution is the larger of the smallest programmable step or the smallest mechanical step the machine can make during point-to-point motion.
  - Resolution is important because it gives a lower bound on the repeatability that one could obtain if one really tried.
  - Resolution is affected by friction and also the resolution of the measurement and control system.

- General Comments
  - Of primary concern is the certainty of the measurements themselves used to characterize the accuracy, precision, and resolution of a machine, and what parameters these measurements themselves are functions of.
  - Classify each of the motions your concepts require in terms of the required accuracy, repeatability, and resolution.
  - How can the concepts be optimized to minimize required accuracy and instead rely more on repeatability?
  - Can you minimize friction so as to likely increase resolution?

- Accuracy, Repeatability, & Resolution Mapping
  - Mapping is the ability to measure or predict the repeatability or accuracy of a machine, and then use this knowledge of the error to compensate for it.
  - Thus, repeatability can sometimes substitute for accuracy. In the static sense, this means that if parts cannot be machined accurate enough to ensure that their assembly has the desired accuracy, then assemble them into the machine and then map the machine's performance. The machine's control system can use the map to compensate for the errors.

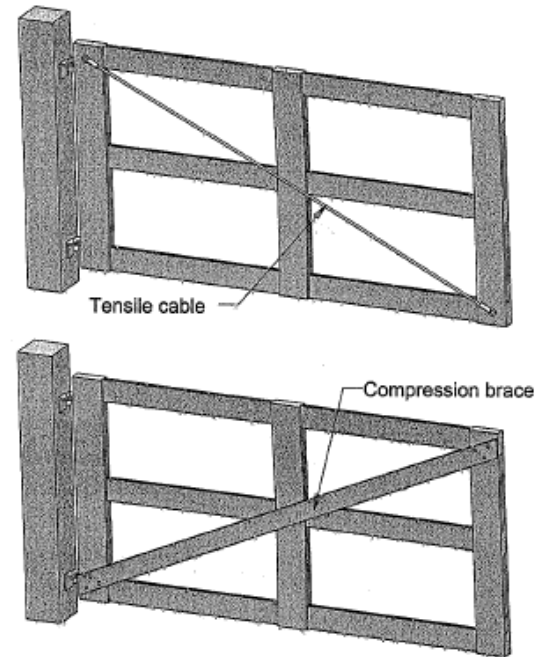
# Sensitive Directions

- Always identify the directions in which accuracy and repeatability are most important. These are the sensitive directions in a machine.
- In addition to the sensitive directions, there are also key features (real or virtual), which will not change even though many other features in the design change as it evolves.
- Put a lot of effort into accuracy for the directions in which you need it. Always be careful to think about where you need precision.

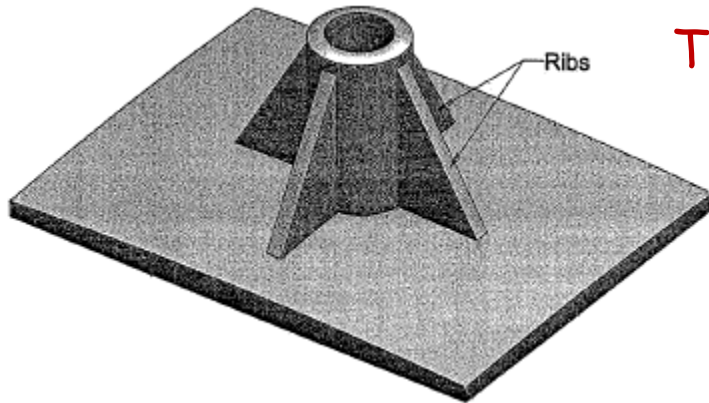
# Triangulate for Stiffness

- Triangulate parts and structures to make them stiffer.
- Triangulation applies to structures and structural elements. When components or structures need to be stiff, create triangles.

Consider a swinging fence gate. Triangulating members prevent, or correct, sagging. A tensile cable connects the lower outer corner to the upper inner corner, and transmits load to the gatepost. Alternately, a compression brace can be added to the diagonal to transmit the load to the gatepost.



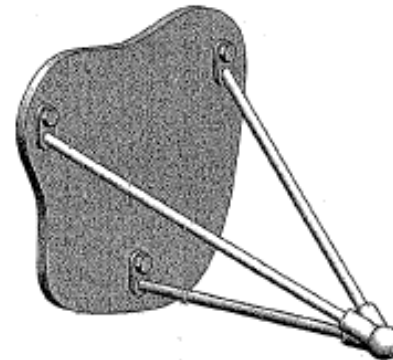
- Ribs are common in molded and cast parts. They efficiently stiffen the structure by using thin braces, which are required by many manufacturing processes.



Triangulating ribs in a molded part

- The three-dimensional equivalent of a triangle is a tetrahedron. Four triangles give three-dimensional rigidity.

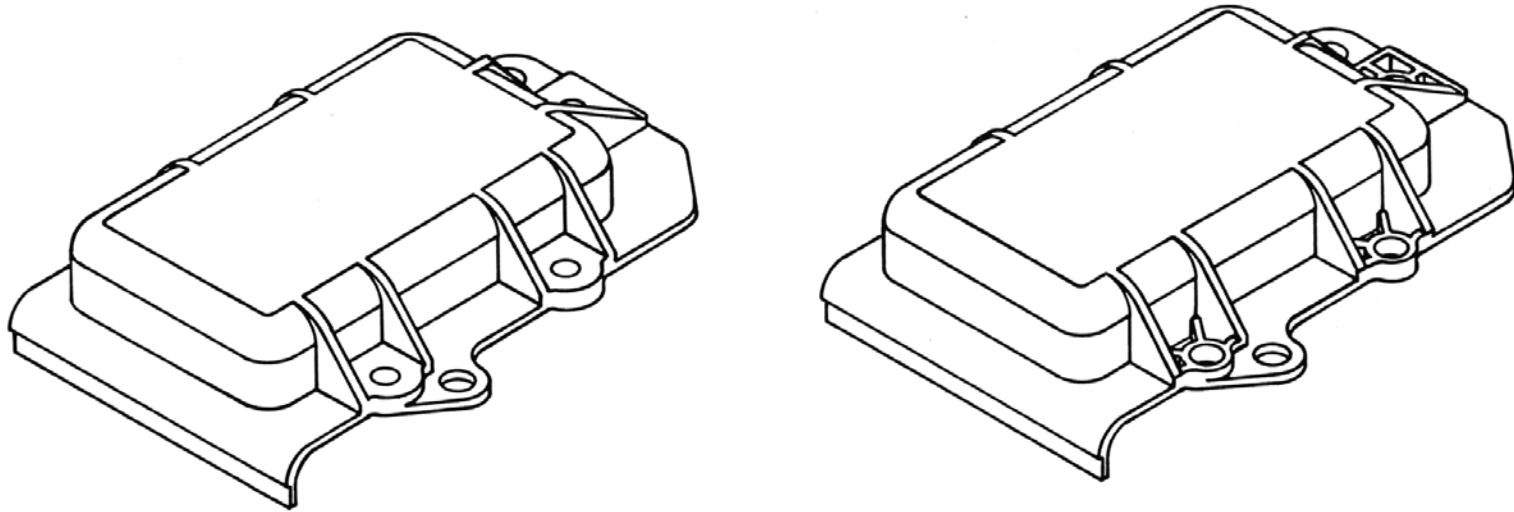
A tetrahedron:  
three-dimensional  
triangulation



- Elements that triangulate are webs, truss members, ribs, and flanges. All are tensile or compressive load-carrying members in the plane of deflection.
- To triangulate, first identify the plane in which the load and the undesirable stress or deflection lie, then add integrally attached ribs, webs, flanges, or truss elements in that plane.
- But beware! Stiffer does not always mean stronger or more robust. Stiffening transfers loads to a different place, a place that might be weaker or more susceptible to fracture, fatigue, corrosion, or whatever else leads to failure.



# Triangulate for Stiffness

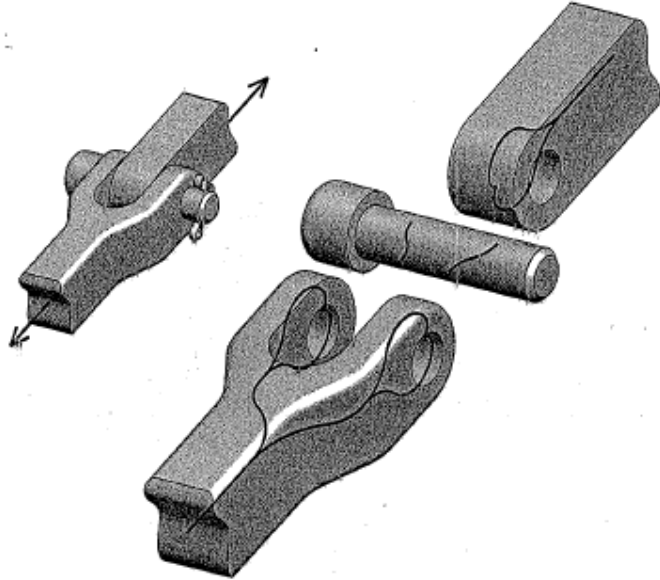


Ribbed holes maintain strength, while providing uniform wall thickness  
=> less cycle time + material, fewer defective parts during manufacturing

# Load Paths

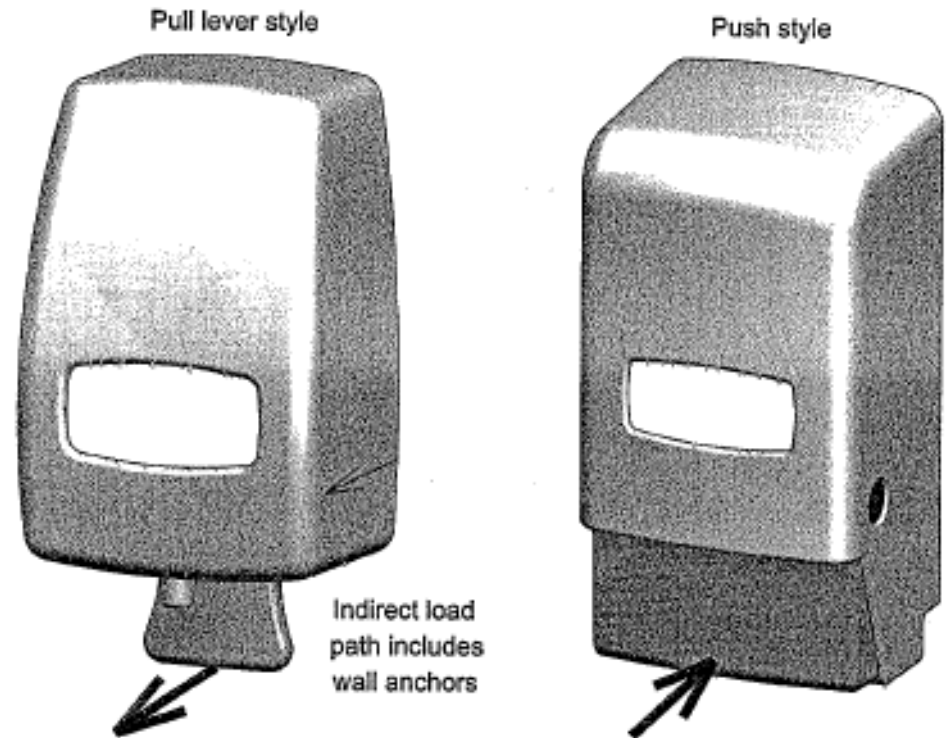
- Plan the load paths in parts, structures, and assemblies.
- Whenever you design a part or assembly, you should clearly define the paths of applied and internal loads.
- You want the load path to be:
  - Short
  - Direct
  - In a line or, barring that, in a plane
  - Symmetric
  - Non-redundant, or, barring that, elastic
  - Locally closed
  - Easily analyzed

- The first step in planning the load path is visualizing it.



Load paths through a pinned clevis connection

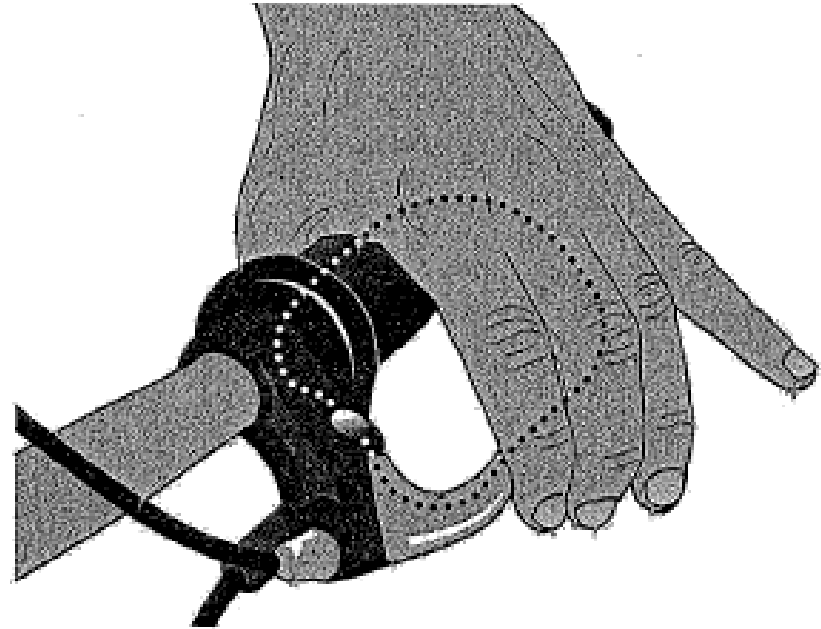
In the pull-lever style, load transfers through a lever stop, the housing, a wall plate, screws, wall anchors, and the wall. The push-type dispenser transfers the load more directly to the wall and over a large area, a much better load path.



Wall-mounted liquid soap dispensers

- Locally-closed paths are especially preferred. User-centered design benefits from locally-closed load paths. Always consider how user-generated forces are opposed.

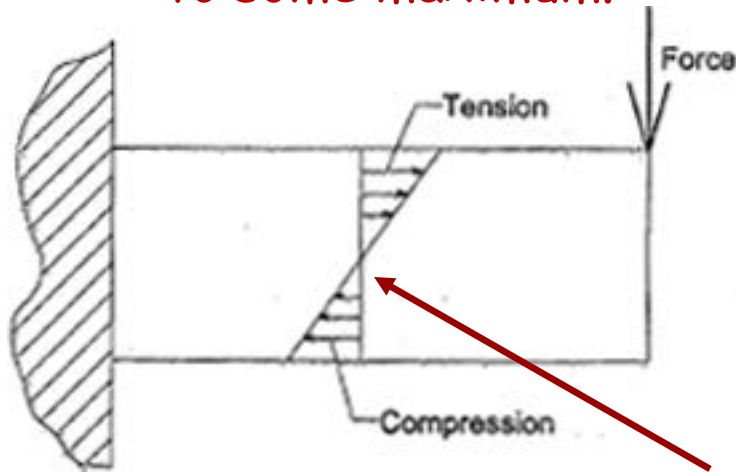
Bicycle hand brakes are a common example. A bicycle handbrake is squeezed rather than pulled or pushed. The load path is locally closed. Pulling, rather than squeezing the brake lever, would give a load path through the rider's arm and body, to the seat, through the frame, and, finally, to the handlebar. This load path is unstable and perilous.



# Avoid Bending Stresses

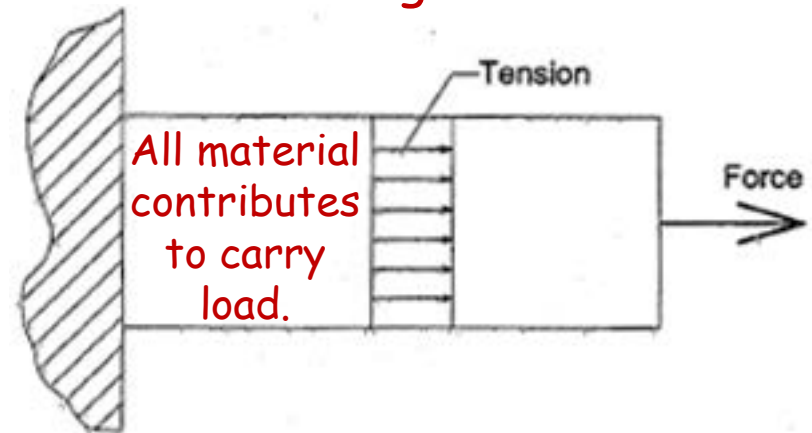
- Avoid bending stresses. Prefer tension and compression.
- As we have seen, triangulating members typically have tension and compression loads rather than bending loads.

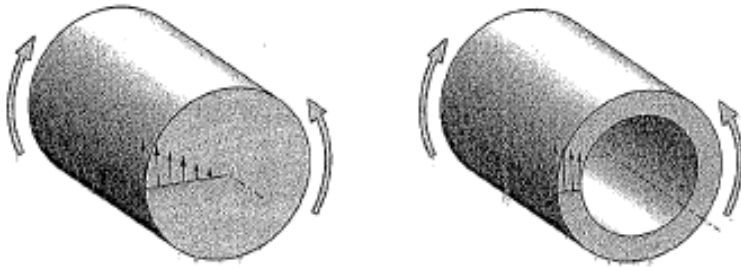
Bending produces stress distributions that vary from zero to some maximum.



Material is unstressed and contributes nothing to rigidity or strength.

Pure tension or compressive members have constant stress throughout.

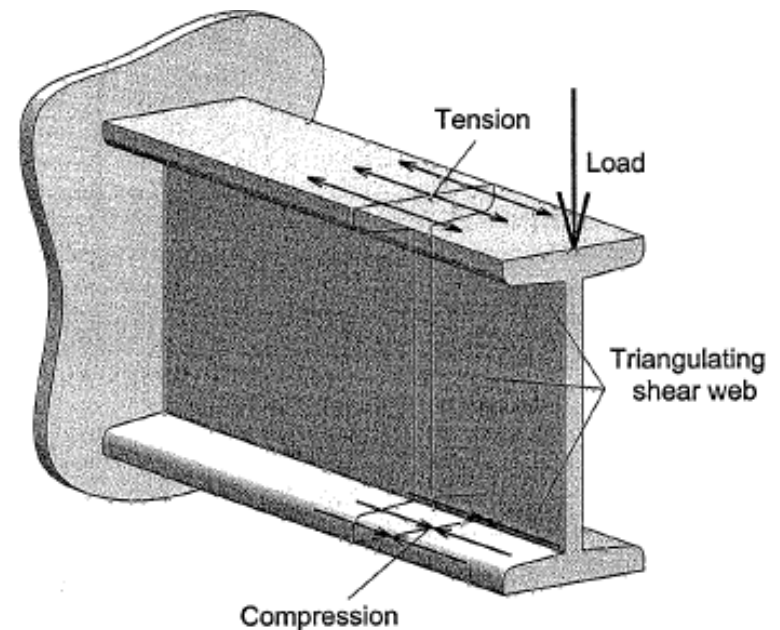




Shear stress distributions in solid and hollow torsion shafts are shown. Material near the center of a solid shaft carries no load. Material in a thin, hollow shaft has nearly constant shear.

- In bending and torsion, much of the material contributes nothing to carrying load. The highly stressed areas fail even while the neutral axis material remains unloaded. Clearly, bending and torsion loads are inefficiently borne by structural elements, whereas uniformly-loaded elements use material quite efficiently.
- **Design to get uniform stress.**

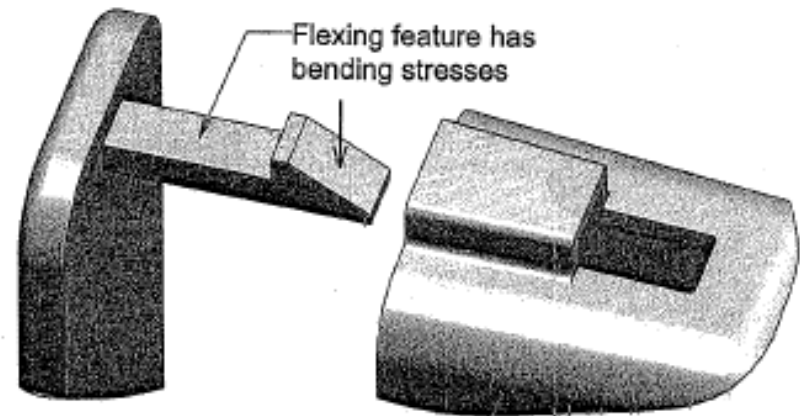
- I-beams are a practical way to use material efficiently to support bending deflections. Most of the material is at the maximum possible distance from the centroid. Thus, the material in an I-beam is nearly uniformly loaded, and at the maximum stress.
- Also note that the vertical portion of an I-beam is a triangulating shear web.



- Of course, you will sometimes want to take advantage of bending and non-uniform torsion.
- For example, torsional springs are cantilever beams loaded in bending.
- Consider snap-fit features; these are cantilever beams loaded in bending.

So an advantage of non-uniform stress is flexibility.

Any time you seek flexibility in a part or structure, bending or non-uniform torsional loads are the better answer.



Cantilever Snap-Fit

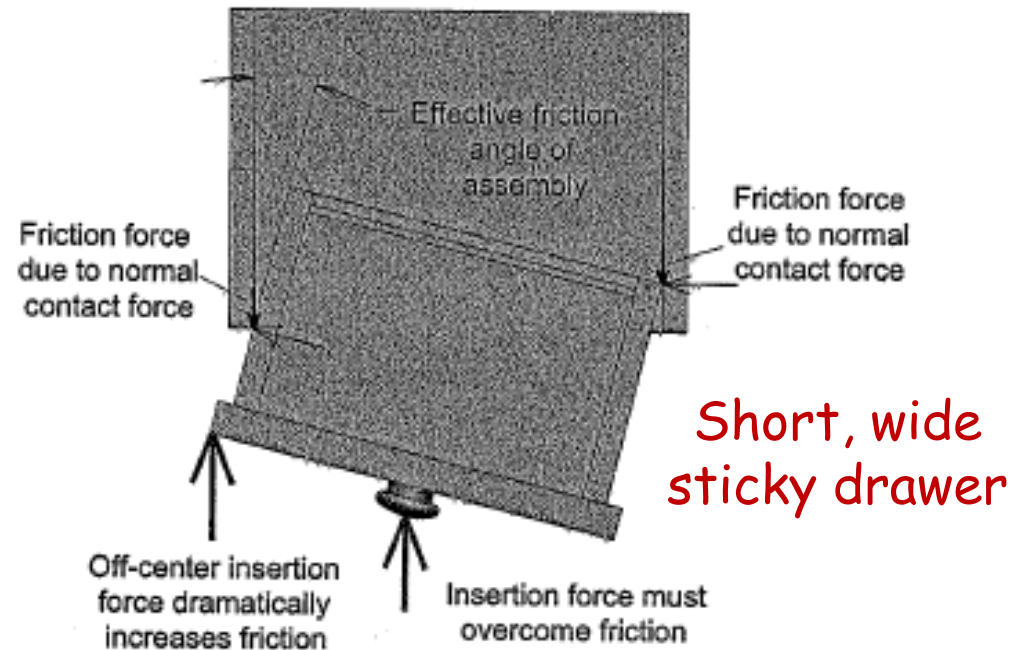


# Manage Friction

- Manage friction in mechanisms.
- The only thing certain about friction in mechanical design is that you will have it. How much, and its consequences, are uncertain.
- Never trust friction – manage it.
- Avoid Sliding Friction
  - Designers avoid sliding friction whenever possible.
  - Managing sliding friction is difficult: stick-slip behavior owing to the difference between static and dynamic friction, wide variation in friction coefficients, uncontrolled lubrication status, variable surface finishes, surface damage, and wear.

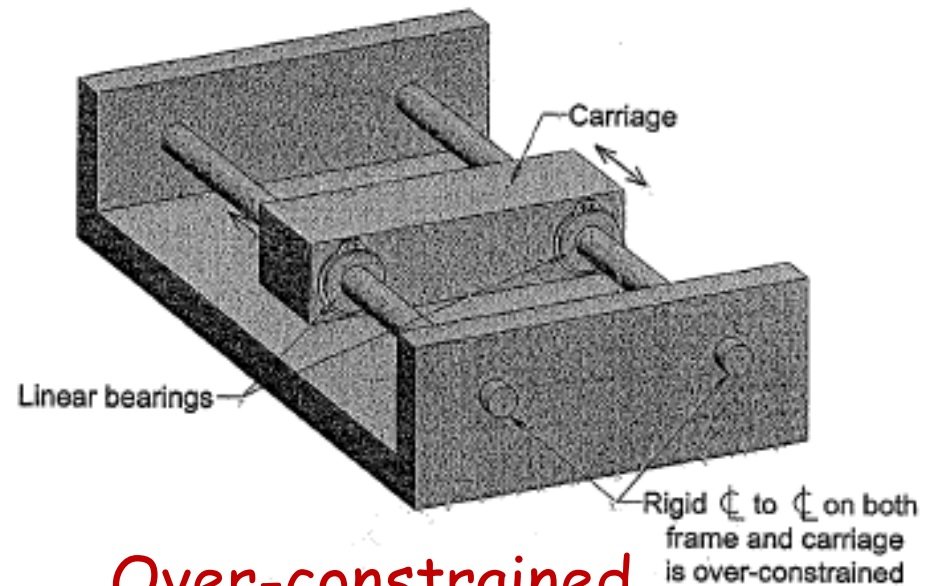
- If you do select sliding elements, select suitable materials, carefully design the guiding system, test beyond reason, and expect the worst.
- Maximize the Length of Linearly-Guided Components
  - Linearly-guided components jam when the insertion force is inadequate to overcome the friction of the guides' contacts.

Short, wide systems are especially susceptible not only because of the length-to-width ratio, but because the insertion force is too easily applied off center.



- A good minimum rule-of-thumb for length-to-width and for length-to-load arm is 1.5 to 1. And your design must guarantee contact only at the ends and not in the middle of the moving component.
- Also note that linear systems rigidly guided on two separate axes are over-constrained. These will jam and stick and wear out and be difficult to manufacture.

Eliminate the fixed center-to-center distance in either the carriage or the rails, or expect trouble.

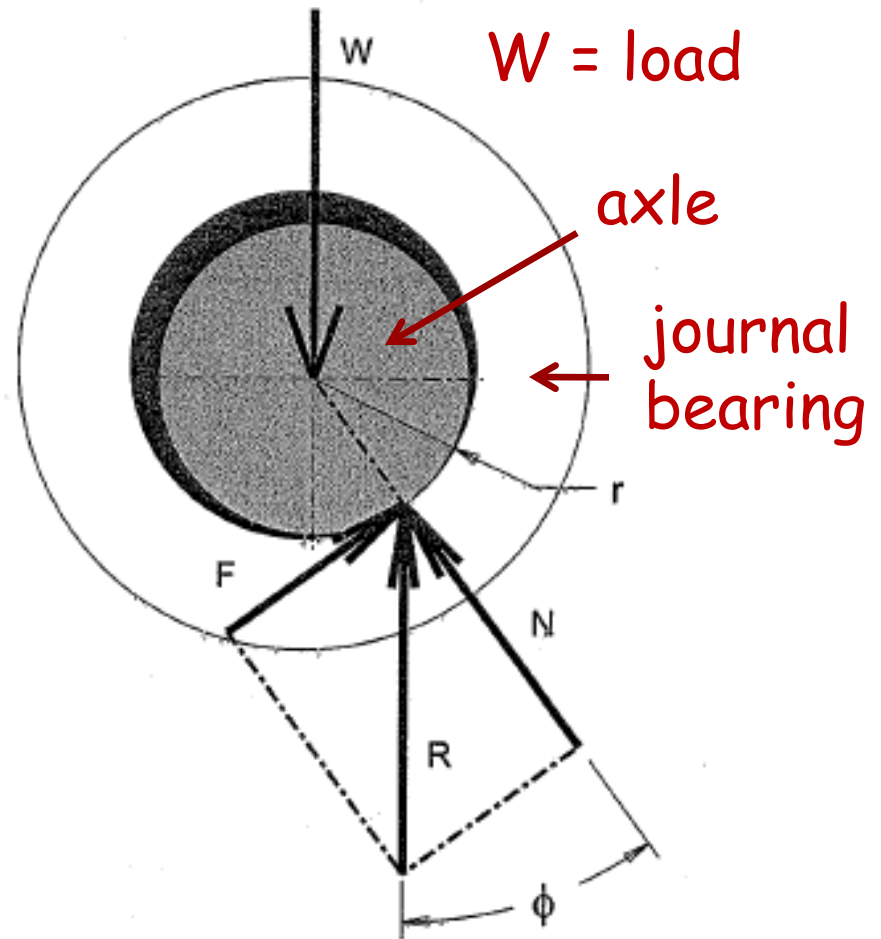


- Select Rotary Motion over Linear Motion
  - The journal bearing is why rotary motion is so much better than linear motion.

The moment  $M$  to turn the axle is given

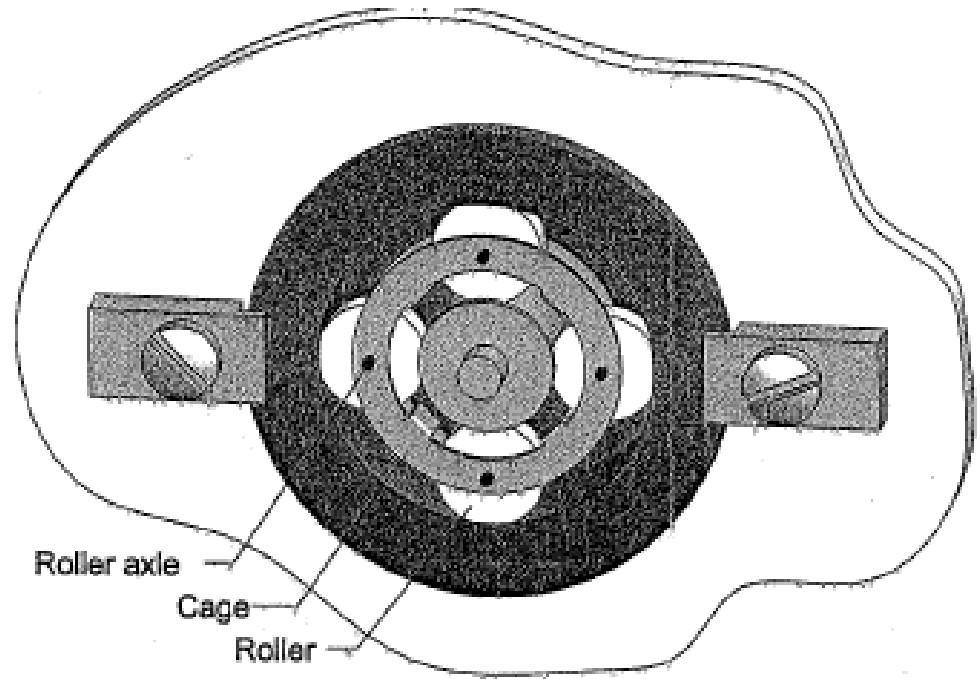
by:  $M = R \times r \times \sin \phi = F \times r$

While  $\phi$  is governed by the coefficient of friction and the use of lubricants, to reduce friction, use small diameter axles.



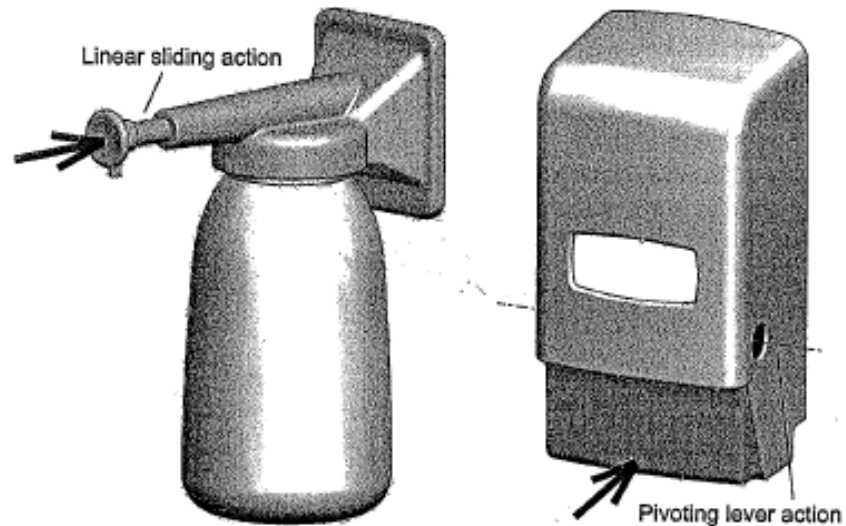
- Consider the caged roller bearing. Roller and ball bearings have advantageous rolling friction rather than sliding friction (unless the rollers or balls touch each other). A cage separates the rolling elements in modern bearings.

Rotational sliding friction between cage and roller is still unavoidable, but the miniature axles on the rollers manage friction by using the power of small radius.



- Rotary motion is a much better selection over linear motion in mechanical design. Linear motion works with proper guiding and with good linear bearings. Also you need space, and lots of it, for linear guidance.
- Rotating bearings, of whatever form, offer stable, compact guidance to the moving components of a mechanism in a minimum space.

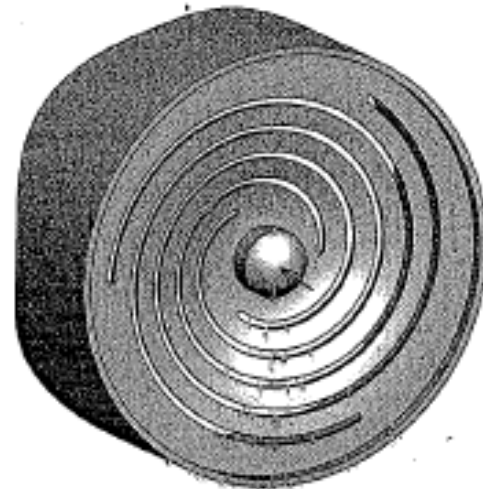
Plunger-style vs. lever-style soap dispenser. The lever-style has a natural advantage for managing friction.



- Use Rolling Element Bearings Whenever Possible.
  - A reliable way to manage friction is to use rolling element bearings. Rolling elements also often eliminate wear.
  - A great advantage of rolling element bearings is that they help to keep it simple. The complexity of ball bearings is intrinsic and doesn't normally concern engineers.
  - However, with bearings you are walking a tightrope above over-constraint. Simple ball bearings have the inherent advantage of small rotational degrees of freedom to accommodate some misalignment. Beyond that, keep a watchful eye out for over-constraint and mitigate with self-aligning and adjustable features.

- Use Flexures to Eliminate Friction.
  - For a completely frictionless system, you cannot do better than flexures.
  - They are uncommon and are typically reserved for precision instruments.
  - Nevertheless, consider using flexures for precise, frictionless motion over a limited deflection.
  - The lack of friction means that flexure-defined motion is predictable and reliable.

Triple spiral frictionless flexure for guiding axial displacement





# Preload

- Components that move relative to one another generally have tolerances that leave clearances between their mating features. These clearances result in backlash which is difficult to control.
- There are two strategies that can be used to limit or prevent backlash.
  - Increase the accuracy of components and decrease clearances. This can increase system accuracy, but will increase cost and can decrease system robustness.
  - Preload components against each other. This is not difficult to accomplish, increases system reliability, and is very robust.

- Preload involves using a spring, or compliance in the mechanism itself, to force components together so there is no clearance between elements.
- However, the compliance in the preload method itself must be chosen such that it locally can deform to accommodate component errors without causing large increases in the forces between components.
- The challenge to the design engineer is to develop clever methods of creating a compliant member which deforms when subject to local distortions while not introducing undesirable deflections in other directions.
- Also, to reduce the variation in the preload force on the system, the amount of deflection used to create the preload by compressing the compliant element should be substantially larger than any expected local deformation.

# Centers of Action

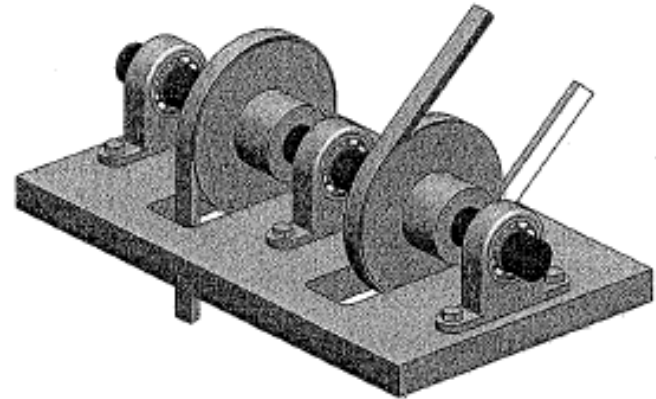
- Minimizing moments on a system minimizes angular motion, which minimizes sine errors and thus has the potential to enhance the robustness of a design.
- Centers of Action are virtual points within a body such that forces applied through these points generate no moments on the body.
  - Center of Mass
  - Center of Stiffness
  - Center of Friction
  - Center of Thermal Expansion

# Exact Constraint Design

- Use exact constraint when designing structures and mechanisms – never over-constrain a design.
- If you rigidly constrain a component at more places than are needed, you will start a fight between these places. This is over-constraint.
- Exactly constrained designs are statically determinate, whereas over-constrained or under-constrained designs are statically indeterminate.
- Exact constraint means applying just enough constraints to define a position or motion – no more, no less.

- Three bearings on one shaft do not work. It is not luck you need in trying to fit the shaft through three bearings, it's sympathy – it won't go!

3 bearings on one shaft:  
an over-constrained design

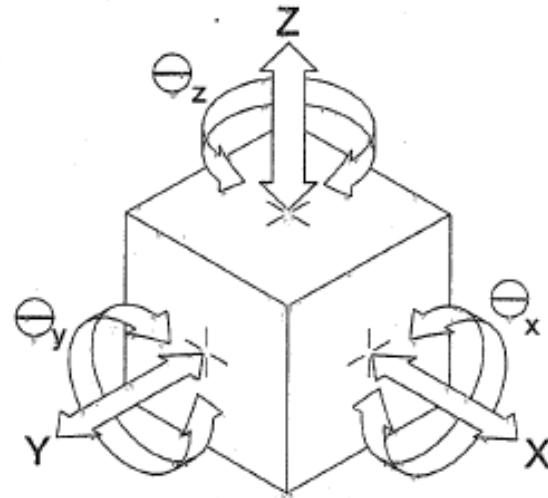


- Strictly speaking, even two rigidly-fixed ball bearings on one shaft is an over-constraint if the shaft cannot slide axially in one or the other inner races.
- Advantages of exactly constrained designs compared to over-constrained designs are: no binding, no play, repeatable position, no internal stress, loose-tolerance parts, easy assembly, and robustness to wear and environment.

- Basic Theory of Exact Constraint

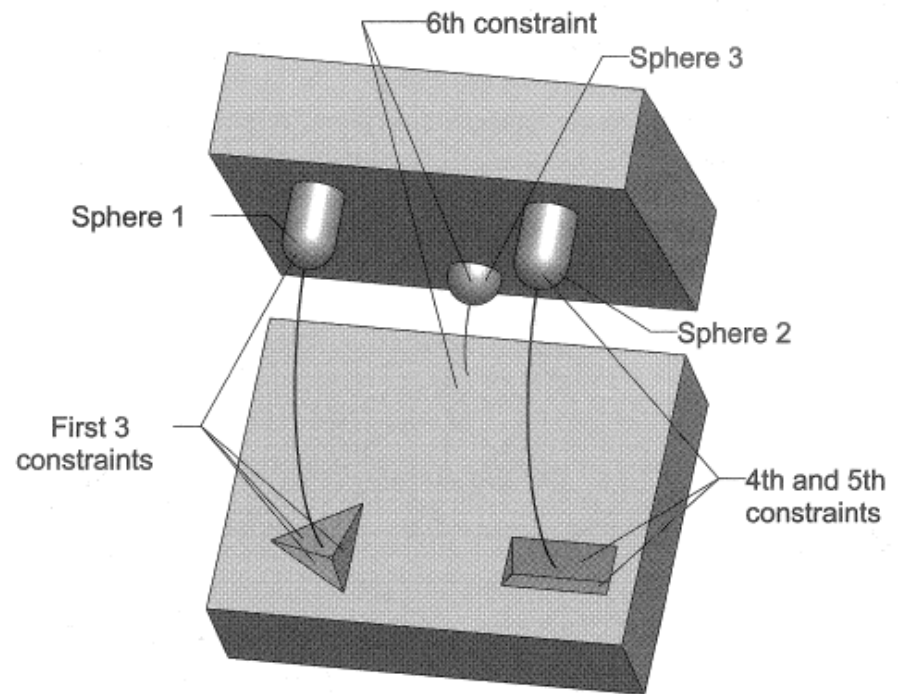
- A three-dimensional object has 6 degrees of freedom: 3 translations and 3 rotations. Selectively constrain these degrees of freedom to obtain the desired motion or structure.
- The degrees of freedom is the number of independent coordinates needed to completely specify the motion of a mechanical system. This depends on the system itself and is independent of the coordinate system chosen.

Six degrees of freedom



- A common example for illustrating exact constraint is a kinematic connection which clearly shows all six constraints.

The first three constraints come from sphere 1 contacting three surfaces in a trihedral receptacle. These are the three translation constraints. The V-shaped groove with sphere 2 supplies two rotational constraints, and the plate surface with sphere 3, the final. You also need a nesting force, the plate's weight, for example. Remove the plate and it returns to the base in exactly the same position. No precision dimensions are required.



A kinematic connection:  
perfect 3D constraint

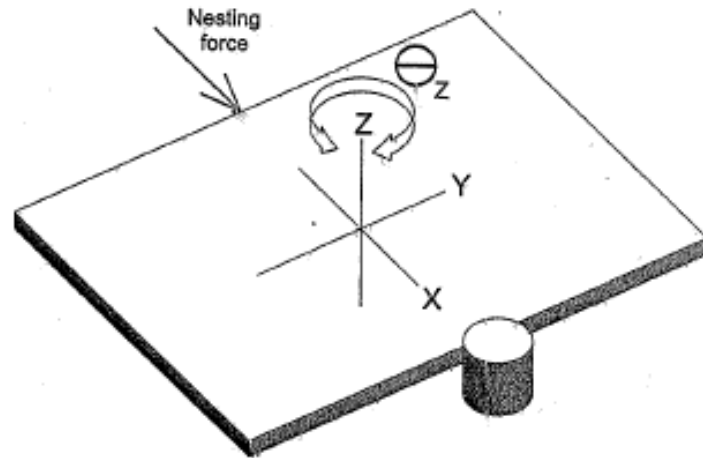
- Precision (of position), also called repeatability, is the degree to which a part, or feature on a part, will return to exactly the same position time after time. When a machine is designed so that its parts are exactly constrained, extraordinary precision is automatically obtained.
- Accuracy (of position) is the degree to which the location of a part or feature exactly coincides with its desired or intended location.
- In general, precision is achievable without accuracy, but accuracy is not achievable without precision. By designing exactly constrained connections, we can achieve high-precision machines using ordinary low-cost, inaccurate parts. Then, through the use of adjustments or fixturing techniques, accuracy can be achieved.



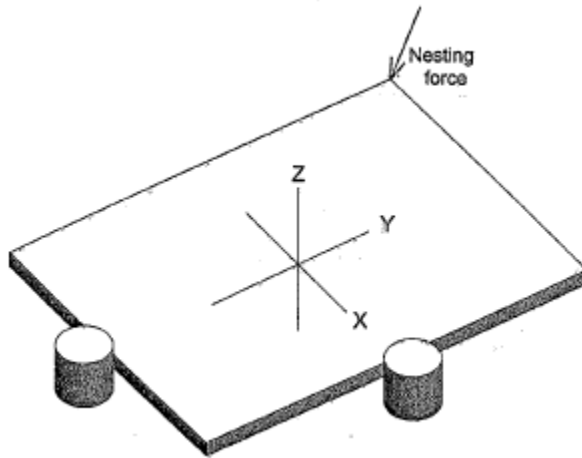
- Exact Constraint in Two Dimensions

- Exact constraint is easier to picture in two dimensions than three. The principles are the same, but in two dimensions there are three degrees of freedom: two translation and one rotation.
- A constraint is a point of contact maintained by a nesting force wherein the nesting force vector goes through the contact point normal to the surface of contact.

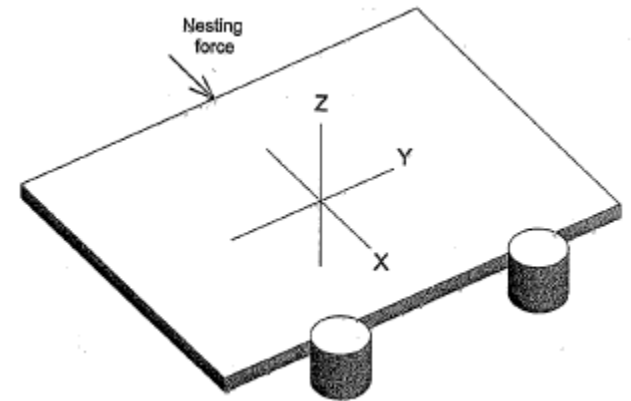
one constraint:  
x motion



- Two posts can be used to constrain against translation in two dimensions or rotation and one translation.



two constraints:  
x motion and rotation



two constraints:  
x motion and y motion

- Add a third post, and the plate has a single, unambiguous position in 2D space. The nesting force, of course, has to be able to resist any applied forces.

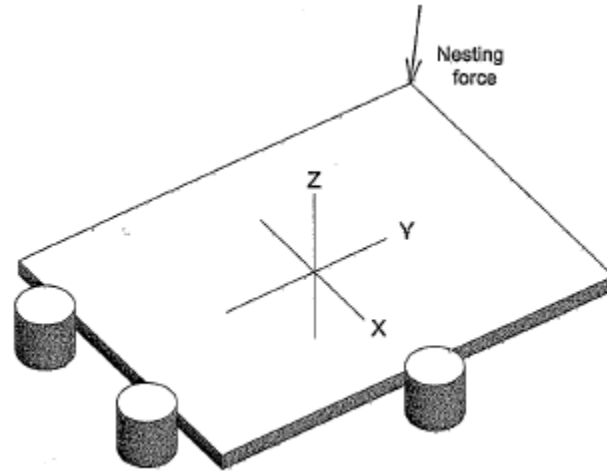
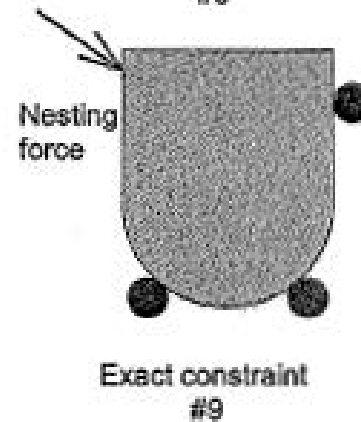
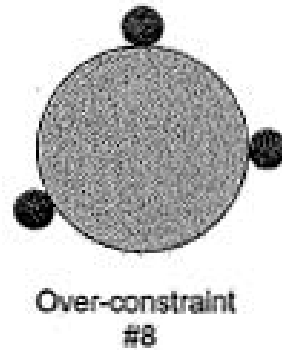
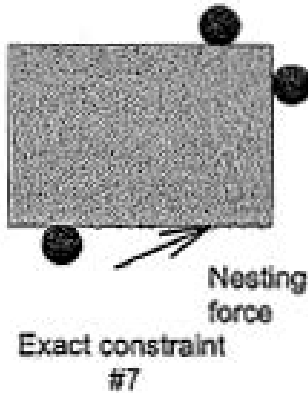
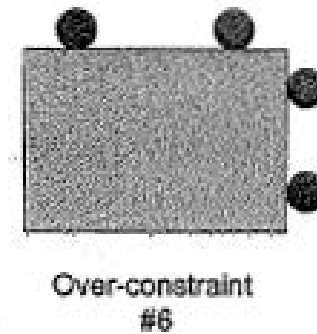
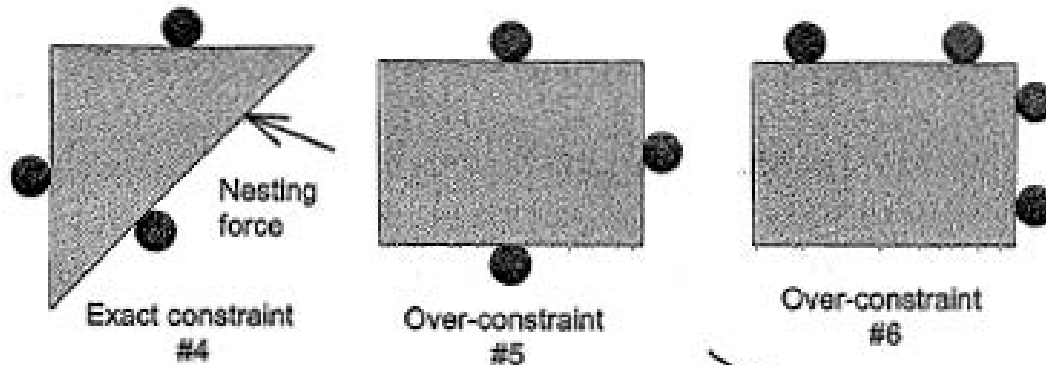
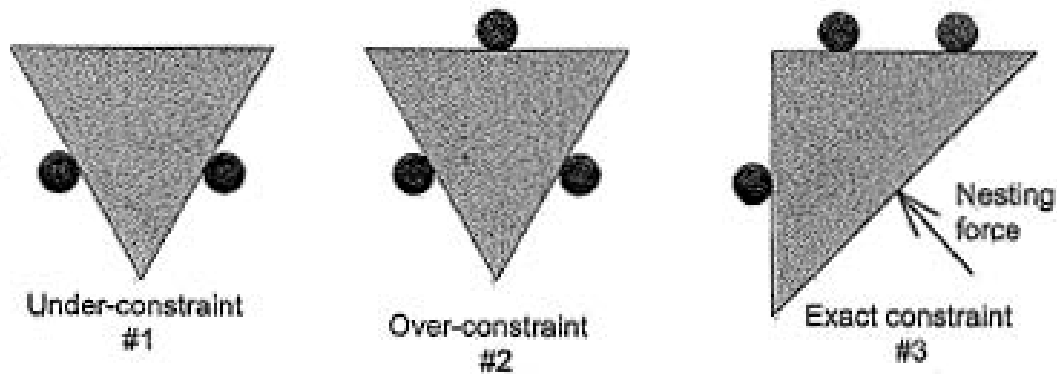


plate fully constrained  
in two dimensions

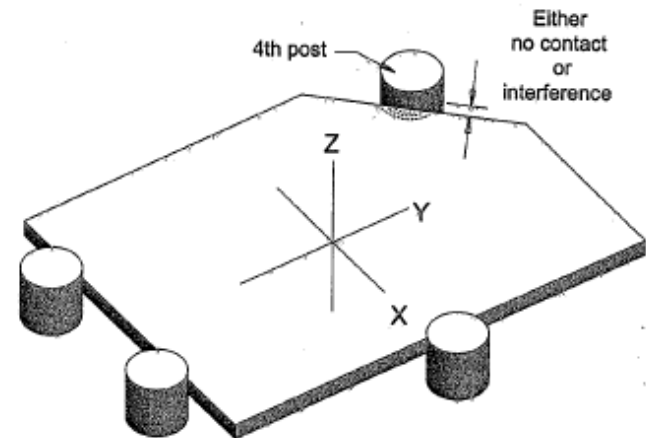


## Two-dimensional constraint condition examples

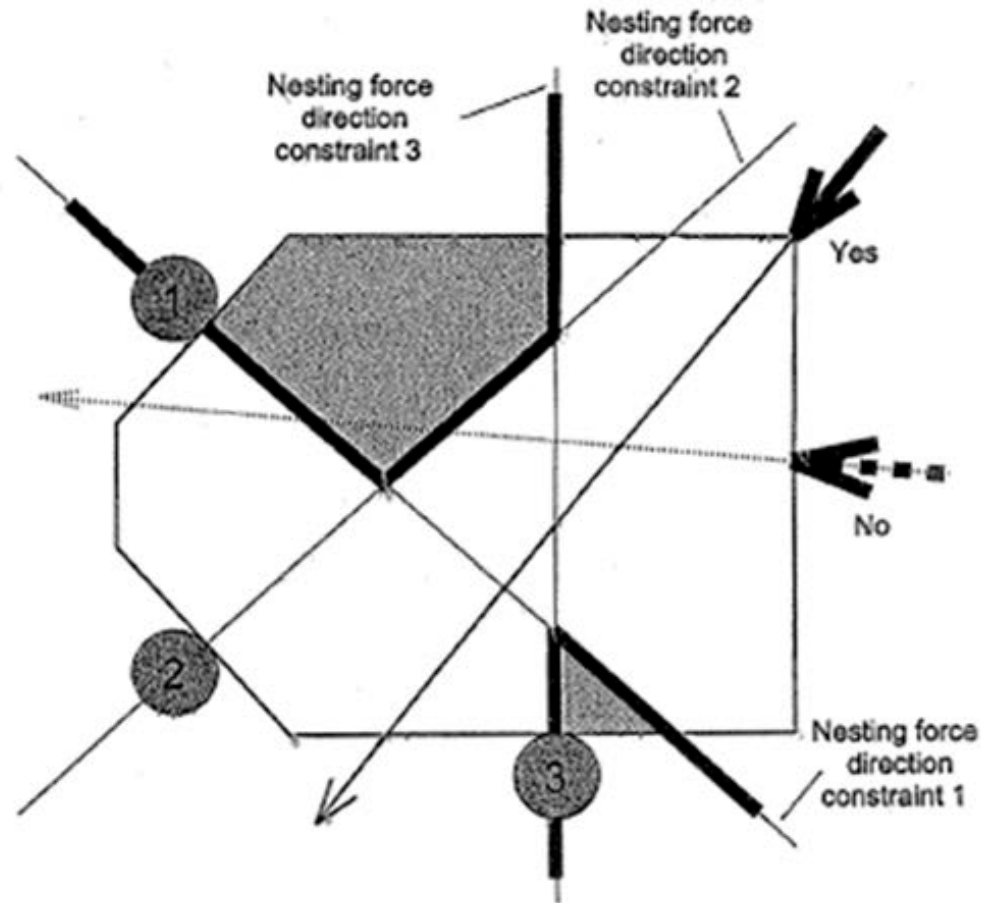
- Nesting Forces

- You cannot just replace the nesting force with a fourth post. Dimensions are never perfect, and the post either interferes with or does not touch the plate.
- Nesting forces (one for each contact point) can be combined vectorially into a single force.
- In practice, nesting forces are created, for example, by weight, cams, wedges, springs, and screws, and counteract externally applied loads.

A fourth post is over-constrained and does not replace the nesting force.



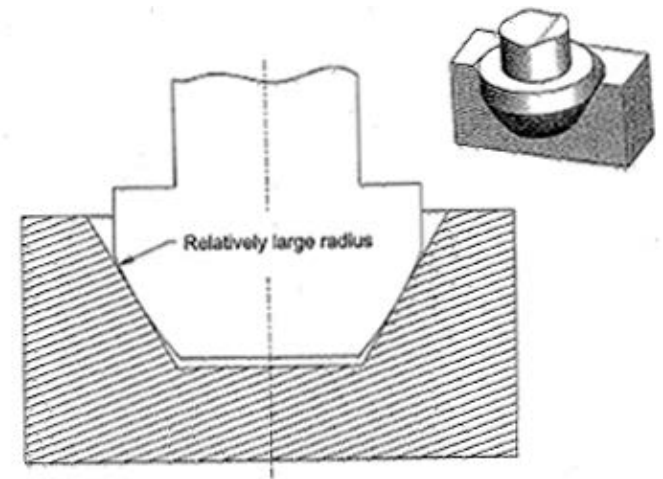
- There is a nesting force “window” outside of which any nesting force will tumble the component out of the stable, nested position.



- Constraint Theory in Practice

- If materials were inelastic and unyielding, and, therefore, components neither deformed nor failed, we could design everything with perfect exact constraint. But they are not, so we compromise.
- Never over-constrain a design, unless you know how.
- Two examples are curvature matching and elastic constraint design.

The trihedron receptacle exactly constrains three translations, but creates infinite stresses at points of contact. Instead, use a conical hole for circle contact to better distribute the load and for ease of machining. Curvature or surface matching compromises exact constraint, but improves load-carrying.



- To use elastic constraint, combine a flexing feature with the proper nesting forces, and the redundant constraint then also contacts its mating surface.
- If you must overconstrain a design to increase load-carrying or improve stability, you must mitigate that over-constraint with elastic deformation.
- Increase rigidity by adding flexibility? What? But you apply the flexibility only in the direction needed to take up variations such as manufacturing tolerances.

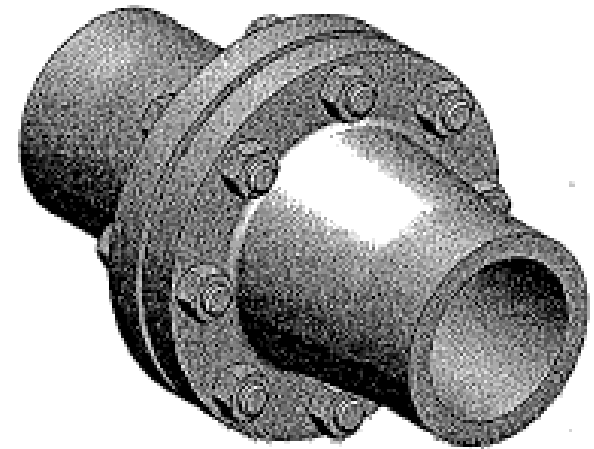
Office chairs never have just three casters. Recent popular designs have five which adds stability and makes them safe to lean back in. Despite the pedestal arrangement, the feet flex enough for all five to contact the floor.





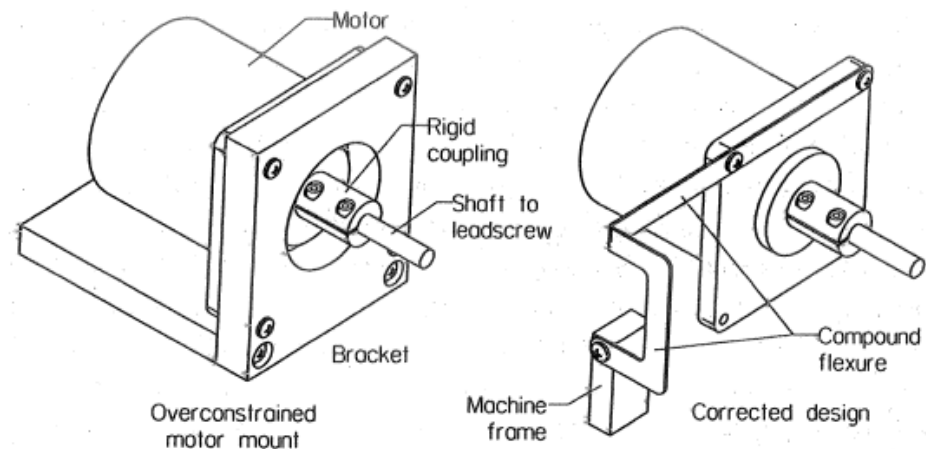
- Combine surface matching and elastic constraint design and you get a bolted flange joint, which is a frictional constraint.
- Large clearance holes avoid obvious over-constraint, even with multiple bolts, and manufacturing the two flat surfaces to adequate accuracy is not difficult. The bolts easily draw in any inaccuracy.

Flanged joint: an elastic, surface-matching, frictional constraint



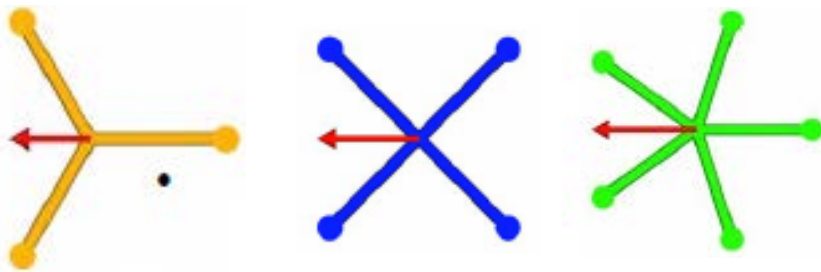
- Real-World Example

- Linear drives for scanning often consist of a motor attached to a leadscrew, which in turn drives the scanning carriage. The first design (figure, left) showed a cyclic error in the linear motion. More accurate machining, better part alignment, and a more expensive leadscrew were all tried with no success. A constraint analysis then showed that the design was over-constrained. With the motor rigidly attached to the leadscrew, the motor requires only a single constraint, i.e., prevent rotation of its housing. The compound flexure (figure, right) eliminated the problem; the expensive leadscrew, accurate machining, and assembly steps were unnecessary.

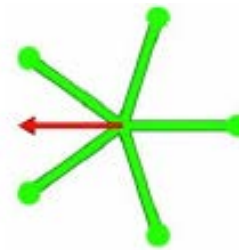
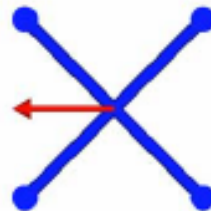
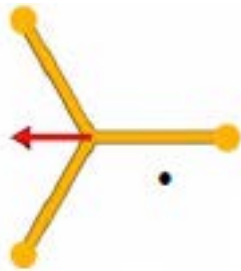


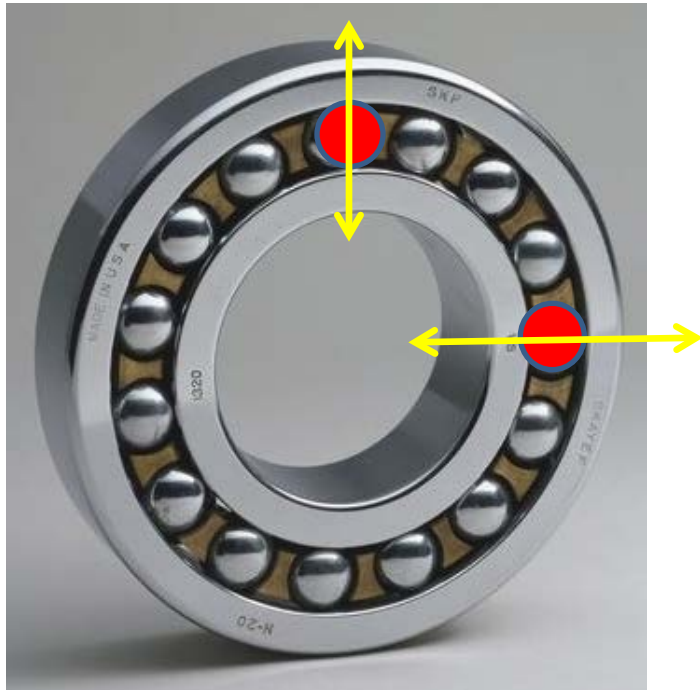
# Elastically Averaged Design

- Controlled Compliance can make an over-constrained four- or five-legged chair not wobble. Elastic deformation compensates for geometric errors. However, the forces are managed so yielding does not occur.
- When there are many compliant elements, each of which locally deforms to accommodate an error, in total they can form a very rigid and accurate system.



The red arrow shows the minimum radius from the chair center of stiffness to the edge of the supports for stability - the chair will tip if the CG shifts outside this radius. All the legs have to be compliant.





Exact Constraint Design dictates the use of only two balls between the Inner Race and Outer Race to constrain the two in-plane translations.

Elastic Averaging enables the use of multiple balls, thus greatly improving load bearing characteristics.

This places greater emphasis on manufacturing uniformity.

Gears, bearings, screws, flexures all make use of elastic averaging.

