# A Simple World Model Appropriate to the Geometric Theory of Fields 

Ulrich E. Bruchholz<br>Wurzen, Germany<br>www.bruchholz-acoustics.de<br>E-mail: Ulrich.Bruchholz@t-online.de

In memoriam Paul Marmet (1932-2005)


#### Abstract

A kind of de Sitter world is derived natural way. The result is supported by recent observations. It is symmetric in time, what involves also a negative time arrow. This negative time arrow lets understand the nature of antimatter, which is not directly visible. Thus, dark matter could consist of antimatter.


Keywords: General relativity, Geometric field theory, World model
The proposed solution is a special de Sitter world and could be known. However, it is worthwhile to derive a natural and simple as possible solution, for the consequences told in this paper. The solution is supported by recent observations. Lehnert, who sees the universe from a fundamentally different view (Lehnert, 2009, v.2), recapitulates these observations. The formulae in present paper involve just the widely flat but nevertheless expanding universe. It is not proper to the Standard model (because it precludes any Big Bang), but it harmonizes with the Geometric theory of fields (Bruchholz, 2009, v. 2 and v.4).

Taking into consideration a constant part of the Riemannian curvatures $K_{\circ}$, one has to extend each component of the Riemann (curvature) tensor for

$$
\begin{equation*}
R_{\mu \nu \sigma \tau} \longrightarrow R_{\mu v \sigma \tau}-K_{\circ} \cdot\left(g_{\mu \sigma} g_{\nu \tau}-g_{\mu \tau} g_{\nu \sigma}\right) \tag{1}
\end{equation*}
$$

(Eisenhart, 1949). All field equations keep valid that way.
The geometric theory of fields says that distributed masses and charges do not exist. If we disregard electromagnetism, Einstein's gravitation equations are simplified to

$$
\begin{equation*}
R_{\mu}^{v}=-3 K_{\circ} \delta_{\mu}^{v} \tag{2}
\end{equation*}
$$

With spherical coordinates

$$
\begin{equation*}
x^{1}=r \quad, \quad x^{2}=\theta \quad, \quad x^{3}=\varphi \quad, \quad x^{4}=\mathrm{j} c t \tag{3}
\end{equation*}
$$

the time-independent and central symmetrical solutions with $K_{\circ}$ become

$$
\begin{equation*}
g_{22}=\frac{g_{33}}{\sin ^{2} \theta}=r^{* 2} \quad, \quad g_{44}=1-K_{\circ} r^{* 2} \quad, \quad g_{11}=\frac{\left(\frac{\partial r^{*}}{\partial r}\right)^{2}}{1-K_{\circ} r^{* 2}} \tag{4}
\end{equation*}
$$

the rest 0 . As well, $r^{*}$ is an arbitrary function of $r$ within the limit that $r^{*} \rightarrow r$ around the observer.
If we demand isotropic spatial hypersurfaces, i.e.

$$
\begin{equation*}
g_{11}=\frac{g_{22}}{r^{2}}=\frac{r^{* 2}}{r^{2}} \tag{5}
\end{equation*}
$$

follows

$$
\begin{equation*}
r^{*}=\frac{r}{1+\frac{K_{0}}{4} r^{2}} \tag{6}
\end{equation*}
$$

respectively

$$
\begin{equation*}
g_{11}=\frac{g_{22}}{r^{2}}=\frac{g_{33}}{r^{2} \sin ^{2} \theta}=\frac{1}{\left(1+\frac{K_{0}}{4} r^{2}\right)^{2}} \quad, \quad g_{44}=\left(\frac{1-\frac{K_{0}}{4} r^{2}}{1+\frac{K_{0}}{4} r^{2}}\right)^{2} . \tag{7}
\end{equation*}
$$

As first consequence follows $K_{\circ} \geq 0$, that means the constant curvature cannot be negative, respectively, the space-time has a space-like curvature radius. With it (according to Eisenhart), the second consequence consists in it, that an isotropic hypersurface (not the space-time !) is close in itself, like the surface of a sphere.

Now, we select isotropic hypersurfaces, that may change in time but shall have the same clock course at all places, with the transformation conditions

$$
\begin{gather*}
g_{11}^{\prime}=\frac{\left(\frac{\partial r^{*}}{\partial r^{\prime}}\right)^{2}}{1-K_{\circ} r^{* 2}}+\left(\frac{\partial x^{4}}{\partial r^{\prime}}\right)^{2} \cdot\left(1-K_{\circ} r^{* 2}\right)=\left(\frac{r^{*}}{r^{\prime}}\right)^{2} \\
g_{14}^{\prime}=\frac{\frac{\partial r^{*}}{\partial r^{\prime}} \frac{\partial r^{*}}{\partial x^{4}}}{1-K_{\circ} r^{* 2}}+\frac{\partial x^{4}}{\partial r^{\prime}} \frac{\partial x^{4}}{\partial x^{4}} \cdot\left(1-K_{\circ} r^{* 2}\right)=0 \\
g_{44}^{\prime}=\frac{\left(\frac{\partial r^{*}}{\partial x^{4}}\right)^{2}}{1-K_{\circ} r^{* 2}}+\left(\frac{\partial x^{4}}{\partial x^{4^{\prime}}}\right)^{2} \cdot\left(1-K_{\circ} r^{* 2}\right)=1 \tag{8}
\end{gather*}
$$

in which $r^{\prime}, x^{4^{\prime}}$ are coordinates for the new hypersurfaces.
These conditions lead to the simple partial differential equation

$$
\begin{equation*}
1-K_{\circ} r^{* 2}-\left(\frac{r^{\prime}}{r^{*}}\right)^{2}\left(\frac{\partial r^{*}}{\partial r^{\prime}}\right)^{2}=\left(\frac{\partial r^{*}}{\partial x^{4^{\prime}}}\right)^{2} \tag{9}
\end{equation*}
$$

The third consequence is $\frac{\partial r^{*}}{\partial x^{4}} \neq 0$, that means such hypersurface must change in time. Conversely, a time-independent hypersurface involves different clock courses.

The setup

$$
\begin{equation*}
r^{*}=p\left(r^{\prime}\right) \cdot q\left(x^{4^{\prime}}\right) \tag{10}
\end{equation*}
$$

results in

$$
\begin{equation*}
\mathrm{d} x^{4^{\prime}}=\frac{\mathrm{d} q}{\left[-K_{\circ} q^{2}+\frac{1}{p^{2}}-\frac{r^{\prime 2}}{p^{4}}\left(\frac{\partial p}{\partial r^{\prime}}\right)^{2}\right]^{\frac{1}{2}}} \tag{11}
\end{equation*}
$$

Under the condition

$$
1-\frac{r^{\prime 2}}{p^{2}}\left(\frac{\partial p}{\partial r^{\prime}}\right)^{2}>0
$$

follows

$$
\begin{equation*}
\frac{K_{\circ}^{\frac{1}{2}} r^{*}}{\left[1-\left(\frac{r^{\prime}}{p}\right)^{2}\left(\frac{\partial p}{\partial r^{\prime}}\right)^{2}\right]^{\frac{1}{2}}}=\cosh \left[\left(-K_{\circ}\right)^{\frac{1}{2}}\left(x^{4^{\prime}}-x_{\circ}^{4^{\prime}}\right)\right] \tag{12}
\end{equation*}
$$

With

$$
\begin{equation*}
p=\frac{r^{\prime}}{1+\frac{K_{0}}{4} r^{\prime 2}} \quad \text { and } \quad x_{\circ}^{4^{\prime}}=0 \tag{13}
\end{equation*}
$$

(close hypersurfaces) follows

$$
\begin{equation*}
r^{*}=\frac{r^{\prime}}{1+\frac{K_{o}}{4} r^{\prime 2}} \cosh \left(K_{\circ}^{\frac{1}{2}} c t\right) \tag{14}
\end{equation*}
$$

With it, the universe expands exponentially. Any superposed gravitation could affect this result. However, recent observations contradict a relevant influence by gravitation.

## Solution from observer's view

The last notes refer to the coordinates at zero-time. However, the observer with his scales does not increase with the hypersurface. With it, we have to introduce observer-related coordinates. That are with the solution according to Equ. (14)

$$
\begin{equation*}
r^{\prime \prime}=r^{\prime} \cdot \cosh \left(K_{o}^{\frac{1}{2}} c t\right) \quad, \quad x^{4^{\prime \prime}}=x^{4^{\prime}}=\mathrm{j} c t \tag{15}
\end{equation*}
$$

With it, coordinates and metrics become

$$
\begin{gather*}
r^{\prime}=\frac{r^{\prime \prime}}{\cosh \left(K_{o}^{\frac{1}{2}} c t\right)}, \quad r^{*}=\frac{r^{\prime \prime}}{1+\frac{K_{0}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)}},  \tag{16}\\
\frac{\partial r^{\prime}}{\partial x^{4^{\prime \prime}}}=\frac{j K_{o}^{\frac{1}{2}} r^{\prime \prime} \sinh \left(K_{\circ}^{\frac{1}{2}} c t\right)}{\cosh ^{2}\left(K_{o}^{\frac{1}{2}} c t\right)}, \quad \frac{\partial r^{\prime}}{\partial r^{\prime \prime}}=\frac{1}{\cosh \left(K_{\circ}^{\frac{1}{2}} c t\right)}  \tag{17}\\
\frac{r^{*}}{r^{\prime}}=\frac{\cosh \left(K_{\circ}^{\frac{1}{2}} c t\right)}{1+\frac{K_{o}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)}} \tag{18}
\end{gather*}
$$

$$
\begin{gather*}
g_{44}^{\prime \prime}=1+\left(\frac{\partial r^{\prime}}{\partial x^{4^{\prime \prime}}}\right)^{2} \cdot\left(\frac{r^{*}}{r^{\prime}}\right)^{2}=1-\frac{K_{0} r^{\prime \prime 2} \tanh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)}{\left(1+\frac{K_{0}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(\left(\sigma_{0}^{\frac{1}{2}} c t\right)\right.}\right)^{2}},  \tag{19}\\
g_{11}^{\prime \prime}=\left(\frac{\partial r^{\prime}}{\partial r^{\prime \prime}}\right)^{2} \cdot\left(\frac{r^{*}}{r^{\prime}}\right)^{2}=\frac{1}{\left(1+\frac{K_{0}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)^{2}}\right)^{2}},  \tag{20}\\
g_{22}^{\prime \prime}=\frac{g_{33}^{\prime \prime}}{\sin ^{2} \theta}=r^{* 2}=\frac{r^{\prime \prime 2}}{\left(1+\frac{K_{0}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)}\right)^{2}},  \tag{21}\\
g_{14}^{\prime \prime}=\frac{\partial r^{\prime}}{\partial r^{\prime \prime}} \frac{\partial r^{\prime}}{\partial x^{4^{\prime \prime}}} \cdot\left(\frac{r^{*}}{r^{\prime}}\right)^{2}=\frac{j K_{0}^{\frac{1}{2}} r^{\prime \prime} \tanh \left(K_{2}^{\frac{1}{2}} c t\right)}{\left(1+\frac{K_{0}}{4} \frac{r^{\prime \prime 2}}{\cosh ^{2}\left(K_{0}^{\frac{1}{2}} c t\right)}\right)^{2}} . \tag{22}
\end{gather*}
$$

What have the formulae to say?
First, the observer sees that the curvature radius of the hypersurface increases with the time. This special curvature radius is minimal and identical with the curvature radius of the space-time at $t=0$. However, there is a "horizon" asymptotically approximating to $r^{\prime \prime}=K_{o}^{-\frac{1}{2}}$, that is the curvature radius of the space-time. The area "behind the horizon" is invisible. In return, the observer sees the symmetry state with the minimal hypersurface $(t=0)$ at the "horizon". With it, the visible area does not increase, and the included "matter" vanishes more and more. It looks as though the space did move from observer to "horizon", see Equ. (22), but it is the included matter. (Space is not defined to move.) Moreover, the visible area becomes more and more flat.
With Equ. (16), the curvature vector of the unmoved body's world-line results around the observer approximately in

$$
\begin{equation*}
k^{1} \approx K_{\circ} r^{\prime \prime} \quad, \quad k^{2}=k^{3}=0 \quad, \quad k^{4} \approx 0 \tag{23}
\end{equation*}
$$

That means a force to the (held) body with its mass away from observer. This is to take into consideration for equations of motion. However, it is irrelevant for energy questions, because one had to fasten the bodies. Who should do it?
This world model supports the assumption that antimatter has a negative time arrow, because it is symmetrical. The case $t<0$ is the complementary world. That means, antimatter is invisible in a direct way. Since gravitation is unipolar (i.e. there is no negative mass), antimatter should be observed via gravitation. Could not dark matter consist of antimatter ? This would solve the symmetry problem.

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