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Electric Circuit Analysis Using a Simple Assumption-Based Technique

Mousaab M. Nahas*

Electrical and Computer Engineering Department, Faculty of Engineering, University of Jeddah, Jeddah, Saudi Arabia
*Corresponding author: mnahas1@uj.edu.sa

Abstract This paper demonstrates a simple technique to analyze linear electric circuits. The technique is based on assuming some voltage or current values according to the given resistance/impedance values such that a relative circuit performance is obtained. The actual circuit results are achieved directly by normalizing the assumed values using a specific formula. In practice, this method either reduces the number of simultaneous equations or simplifies the mathematical formulas; hence the overall analysis procedure is reasonably accelerated. It is highly recommended that this method is taught in electric circuit courses especially in the beginning chapters so that students can have more variety of circuit analysis techniques. It is also possible to apply such a method to verify an existing solution or vice versa.

Keywords: *electric circuits, circuit analysis techniques, circuit theory, DC and AC circuits, methods of analysis*

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1. Introduction

It is already recognized that the electric circuit theory is one of the main fundamental theories on which all branches of electrical engineering are based. Therefore, electric circuit analysis techniques are very important to be taught to electrical engineering students at the beginning of electrical engineering program [1]. They are also useful for students in other specializations such as physics and applied mathematics [2,3,4]. It is well-known that there are different circuit analysis techniques which are all based on three main laws: Ohm's law, KCL and KVL. The techniques can involve combining resistors/impedances in series or parallel, voltage division or current division. For more complex circuits, nodal or mesh analysis are advised to be used where the solution procedure is significantly facilitated [1,5-12]. In fact, the above techniques along with the Y-Δ transformations are fairly sufficient to solve all linear electric circuits where the appropriate procedure is decided according to the given circuit design.

In this research, we propose additional simple technique that is never presented elsewhere such that a student can have more options either to analyze a given circuit or to verify an existing solution. This technique is again based on the three famous laws mentioned above but it works backwards since we begin by assuming circuit values for some elements located far from the source. After complete analysis, these values are normalized to obtain the actual circuit results. This is unlike original methods in which the analysis usually starts from the given source and moves forwards to reach the actual circuit values. The technique proposed here can apply easily to most linear circuit designs including resistive DC circuits, charging *RC*, *RL* and *RLC* circuits, and AC circuits. It is not claimed that this technique can replace the traditional methods, especially the nodal and mesh techniques, but in many cases it can be the best option where the number or complexity of equations is reasonably reduced.

For simplicity and as done in all references, we start with simple resistive DC circuits upon which our proposed method is fully explained. Various cases (examples) are given to ensure the validity of the technique over different designs. These examples are mostly taken from one reference [1] just for consistency and rapid verification. Other circuits including AC ones are then presented in the later sections.

2. Simple Resistive Circuits

As known, simple resistive circuits are composed of resistors that are connected in series, parallel or both. The new technique is mainly based on Ohm's law which states that the voltage across a resistor is directly proportional to the current passing through the resistor [5-12]. Mathematically,

$$v = Ri \tag{1}$$

It is therefore possible to say that the voltage across a resistor is directly proportional to the given resistance assuming constant current. On the other hand, the current through a resistor is inversely proportional to the given resistance assuming constant voltage. This would imply that, for any resistor, the voltage value can be initially assumed where it is either a multiple or fraction of the given resistance. This is of course assuming that i is the proportionality constant in (1). For simplicity, the voltage and resistance can be assumed to be equal where i is assumed to be 1. Therefore, equation (1) temporarily becomes

$$V = R \tag{2}$$

where capital letter is used to distinguish from the actual voltage value.

In contrast, if the voltage is fixed, it is possible to predict a current value according to the resistance where the proportionality constant is the voltage in this case.

The above two situations are applicable in case that there are several resistors in the circuit. However, both situations cannot be applied simultaneously since they are based on assumptions hence relative results, and dual assumption would corrupt the relativity. Therefore, the choice of the right approach must be dependent on the circuit design which mostly involves series and/or parallel connections. The following sections discuss the different possible options and their application.

2.1. Series Resistors

Suppose we have resistors connected in series as shown in Figure 1. The original approach is to combine them into one equivalent resistor thus the current i is obtained directly by Ohm's law hence v_1 and v_2. Alternatively, the voltage division principle can be used where the source voltage v is divided among the resistors in direct proportion to their resistances [5,6,8].

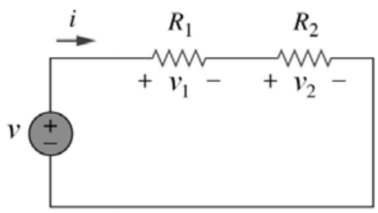

Figure 1. Series resistors

In our proposed method, since the current is already constant through both resistors, it is possible to apply equation (2) as

$$V_1 = R_1, V_2 = R_2$$

where V_1 and V_2 are relative voltages assuming $i = 1$.

According to KVL, the total assumed voltage V is

$$V = V_1 + V_2 = R_1 + R_2 \tag{3}$$

Certainly, this value is relative and is most probably a multiple of the given source voltage v. Thus it is possible to define a *multiplicity factor* (m) that is the ratio of the relative voltage obtained by (3) to the actual given voltage, i.e.

$$m = \frac{V}{v} \tag{4}$$

Therefore, the actual voltages across the resistors are

$$v_1 = \frac{V_1}{m}, v_2 = \frac{V_2}{m} \tag{5}$$

In practice, the above solution procedure can be done very fast where simple calculations are involved.

The unit prefix of the assumed values must be compatible with the given source unit. Typically, the assumed voltages are in V while the currents are either in A or mA. Basically, if the resistances are given in kΩ, the currents are assumed in mA.

2.2. Parallel Resistors

Suppose we have resistors connected in parallel as shown in Figure 2. Again, the original approach is to combine them into one equivalent resistor thus the current i is obtained directly by (1). The sub-currents i_1 and i_2 are obtained either by (1) or by current division where the total current i is divided among the resistors in direct proportion to the other resistance in parallel [6,8,10].

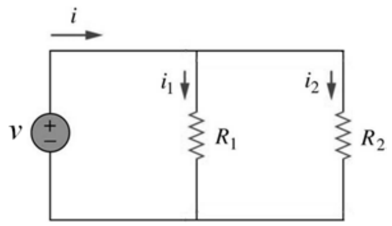

Figure 2. Parallel resistors

In our approach, since the voltage is already constant across both resistors and the current through each resistor is directly proportional to the other resistance, it is possible to assume the current values as

$$I_1 = R_2, I_2 = R_1 \tag{6}$$

According to KCL, the total assumed current I is

$$I = I_1 + I_2 = R_2 + R_1 \tag{7}$$

and

$$V = V_1 = V_2 = R_1 I_2 = R_2 I_1$$

From (6) and (7), the voltage is then

$$V = R_1 R_2 \tag{8}$$

Obviously, the current and voltage values obtained above are all relative and are multiples (or fractions) of the actual currents and voltages, respectively. The actual values are then achieved by dividing all assumed values by the multiplicity factor m defined in equation (4). Therefore,

$$v = \frac{V}{m}, i = \frac{I}{m}, i_1 = \frac{I_1}{m}, i_2 = \frac{I_2}{m}. \tag{9}$$

If the circuit has a current source rather than a voltage source, the multiplicity factor m is then defined as the ratio of the relative current obtained by (7) to the actual current i given in the circuit. That is

$$m = \frac{I}{i}$$

In general, the multiplicity factor can be re-defined as the ratio of the assumed value to the actual value and is determined based on the source type given in the circuit, i.e.

$$m = \frac{\text{Assumed}}{\text{Actual}} \tag{10}$$

3. The Technique Application

3.1. Series Resistors Example

Consider that we are required to find v_1, v_2 and i in the circuit shown in Figure 3.

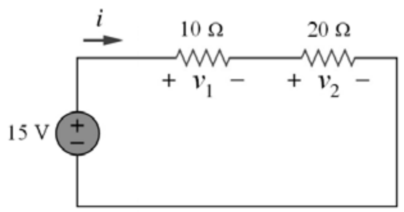

Figure 3. Series resistors example

From (2), we assume that

$$V_1 = 10\text{V}, V_2 = 20\text{V}$$

Thus, the total relative voltage is

$$V = 30 \text{ V}$$

From (4), the multiplicity factor is

$$m = \frac{30}{15} = 2$$

Therefore,

$$v_1 = \frac{10}{2} = 5\text{V}, v_2 = \frac{20}{2} = 10\text{V}.$$

Hence

$$i = 0.5\text{A}.$$

3.2. Parallel Resistors Example

Consider we are required to determine all currents and voltages in the circuit shown in Figure 4 [1].

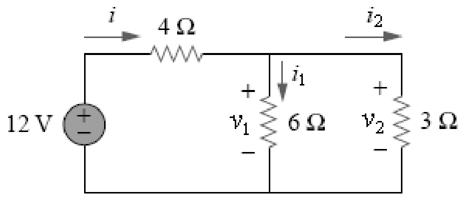

Figure 4. Parallel resistors example

From (6), we can assume that

$$I_1 = 3\text{A}, I_2 = 6\text{A}.$$

Thus, the total relative current is

$$I = 9\text{A}$$

and the voltages are

$$V_1 = V_2 = 3 \times 6 = 18\text{V}.$$

Now, the total relative voltage of the circuit is the sum of V_1 and the 4-Ω resistor's relative voltage which is basically $4I$ that is 36 V. Therefore,

$$V = 36 + 18 = 54\text{V}.$$

Thus, the multiplicity factor is

$$m = \frac{54}{12} = 4.5$$

As a result,

$$v_1 = v_2 = \frac{18}{4.5} = 4\text{V}, i = \frac{9}{4.5} = 2\text{A},$$

$$i_1 = \frac{3}{4.5} = 0.67\text{A}, i_2 = \frac{6}{4.5} = 1.33\text{A}.$$

Obviously, the above solutions are excessively detailed and can be shortened into fewer steps if the solver is familiar with the technique.

3.3. Series Resistors with Dependent Source Example

Consider the circuit shown in Figure 5 [1], in which we need to find i and v_o.

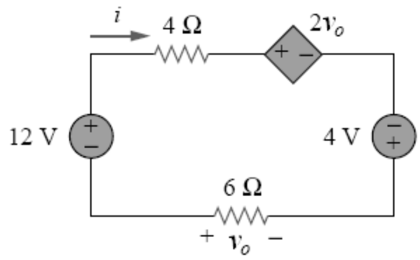

Figure 5. Series resistors with dependent source

The total actual supplied voltage is

$$v = 12 + 4 = 16 \text{ V}.$$

The assumed voltages are $V_{4\Omega} = 4$ V and $V_o = -6$ V (according to the given polarity)

Since, the dependent source has a function of v_o which is assumed to be V_o, it is added to the relative voltages, thus equation (3) becomes

$$V = |V_{4\Omega}| + |V_o| + 2V_o = 4 + 6 + 2(-6) = -2\text{V} \quad (11)$$

Note that the resistor voltages are always positive in (11) as passive sign convention is always satisfied. However, the dependent source voltage is not.

The multiplicity factor is

$$m = \frac{-2}{16} = -\frac{1}{8}$$

Hence

$$i = \frac{1}{-1/8} = -8\text{A}, v_2 = \frac{-6}{-1/8} = 48\text{V}.$$

3.4. Parallel Resistors with Dependent Source Example

Consider that we want to determine i_o and v_o in the parallel circuit shown in Figure 6 [1].

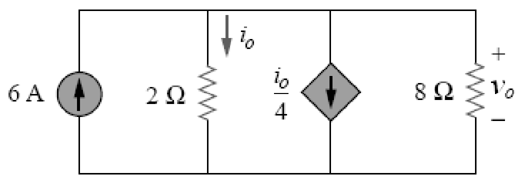

Figure 6. Parallel resistors with dependent source

According to (6), the relative (assumed) values are

$$I_o = 8\text{A},\ I_{8\Omega} = 2\text{A} \Rightarrow V_o = 2\times 8 = 16\text{V}.$$

Since, the dependent source is a function of i_o which is assumed to be I_o, it is added to the relative currents, thus equation (7) becomes

$$I = I_o + I_{8\Omega} + \frac{I_o}{4} = 8 + 2 + \frac{8}{4} = 12\text{A}. \qquad (12)$$

Note that if the dependent source was in the opposite direction, it would be subtracted in (12).

However, the multiplicity factor is

$$m = \frac{12}{6} = 2$$

and hence

$$i_o = \frac{8}{2} = 4\text{A}, v_o = \frac{16}{2} = 8\text{V}.$$

In fact, the solution of the above circuits in Figure 5 and Figure 6 is much shorter than that obtained by traditional approaches.

3.5. Charging *RLC* Circuit Example

Consider the circuit in Figure 7 [1]. We would like to find i, v_C and i_L under dc conditions.

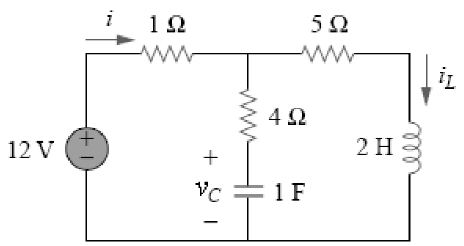

Figure 7. Charging *RLC* circuit

Under dc conditions, the circuit becomes as shown in Figure 8, which is a simple resistive circuit with series connection.

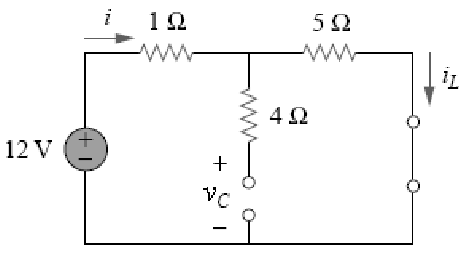

Figure 8. *RLC* circuit under dc conditions

We assume

$$V_{1\Omega} = 1\text{V}, V_{5\Omega} = 5\text{V} \Rightarrow I = 1\text{A}.$$

Using equation (3),

$$V = 6\text{V}.$$

Thus,

$$m = \frac{6}{12} = \frac{1}{2}$$

As a result,

$$i = i_L = \frac{1}{1/2} = 2\text{A}, v_C = \frac{5}{1/2} = 10\text{V}.$$

4. AC Circuits

As known, AC circuits are usually composed of impedances that are connected in series, parallel or both. These impedances are related to resistors, capacitors and inductors. Since Ohm's and Kirchhoff's laws work in phasor domain with all kinds of passive elements in the same way used with DC circuit resistors, it is possible to apply our technique on AC circuits as well. To do so, we shall now base on Ohm's law in phasor form which states that the phasor voltage across a passive element is directly proportional to the phasor current passing through the element [5-12]. Mathematically,

$$\mathbf{V} = \mathbf{ZI} \qquad (13)$$

where **Z** is the element impedance. Again, it is possible to state that the phasor voltage is directly proportional to the given impedance assuming constant current. Also, the phasor current through an element is inversely proportional to its impedance assuming constant voltage. This would imply that the phasor voltage value can be initially assumed as long as it is in direct proportion to **Z**, or the phasor current is assumed such that it is inversely proportional to **Z**. Doing this would allow finding a relative circuit values that are eventually normalized to reach the actual values. However, the application of the above two assumptions is based on the AC circuit design as discussed in the next sections.

4.1. Series Impedances

Suppose we have AC circuit with passive elements connected in series as shown in Figure 9. The traditional approach is to combine the impedances into one equivalent impedance thus the phasor current **I** is obtained directly by (13) hence $\mathbf{V}_1$ and $\mathbf{V}_2$. Alternatively, the voltage division principle can be used where the source voltage **V** is divided among the elements in direct proportion to their impedances [1,7,8,9].

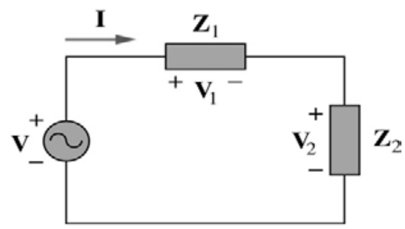

Figure 9. Series impedances

In our technique, we solve the circuit by assuming the element voltage value in direct proportion to its given impedance as mentioned above. For simplicity, we can assume that the voltage across any element is

$$\mathbf{V}' = \mathbf{Z} \qquad (14)$$

where the current is assumed to be 1, and prime is used to distinguish from the real voltage value.

Applying (14) on both elements, we get the relative voltages as

$$\mathbf{V}_1' = \mathbf{Z}_1, \mathbf{V}_2' = \mathbf{Z}_2$$

According to KVL, the total assumed voltage $\mathbf{V}'$ is

$$\mathbf{V}' = \mathbf{V}_1' + \mathbf{V}_2' = \mathbf{Z}_1 + \mathbf{Z}_2 \qquad (15)$$

Clearly, this value is relative and is simply a multiple or fraction of the given source voltage **V**. Thus it is possible to define the multiplicity factor in phasor domain as the ratio of the relative phasor value (whether voltage or current) to the source phasor value given in the circuit. In this case,

$$\mathbf{M} = \frac{\mathbf{V}'}{\mathbf{V}} \tag{16}$$

Note that the multiplicity factor is now complex, and that is why it is represented by capital-bold letter.

Therefore, the actual phasor voltages across the elements are basically

$$\mathbf{V}_1 = \frac{\mathbf{V}_1'}{\mathbf{M}}, \mathbf{V}_2 = \frac{\mathbf{V}_2'}{\mathbf{M}}. \tag{17}$$

We believe that the above procedure is simpler than the conventional procedure for such circuit.

4.2. Parallel Impedances

Suppose we have impedances connected in parallel as shown in Figure 10. Again, the conventional approach is to combine them into one impedance thus the current **I** is obtained directly by Ohm's law hence $\mathbf{I}_1$ and $\mathbf{I}_2$. Alternatively, the current division principle can be used where the total phasor current **I** is divided among the elements in direct proportion to the other impedance in parallel [6,9,12].

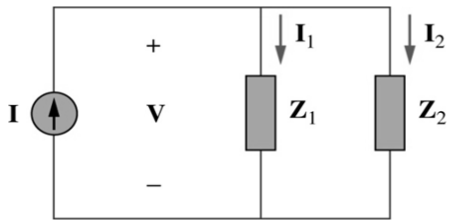

Figure 10. Parallel impedances

In our approach, since the voltage is constant across both elements and the current through each one is directly proportional to the other impedance, it is possible to assume the phasor current values as

$$\mathbf{I}_1' = \mathbf{Z}_2, \mathbf{I}_2' = \mathbf{Z}_1 \tag{18}$$

According to KCL, the total assumed current $\mathbf{I}'$ is

$$\mathbf{I}' = \mathbf{I}_1' + \mathbf{I}_2' = \mathbf{Z}_2 + \mathbf{Z}_1 \tag{19}$$

and the voltage across both elements is

$$\mathbf{V}' = \mathbf{V}_1' = \mathbf{V}_2' = \mathbf{Z}_1\mathbf{I}_2' = \mathbf{Z}_2\mathbf{I}_1'$$

Using (18), the parallel voltage is then

$$\mathbf{V}' = \mathbf{Z}_1\mathbf{Z}_2 \tag{20}$$

The phasor current and voltage values obtained by (19) and (20) are obviously relative and are again multiples or fractions of the actual current **I** and voltage **V**, respectively. The actual values are then achieved by dividing all assumed values by the multiplicity factor **M** which is in this case

$$\mathbf{M} = \frac{\mathbf{I}'}{\mathbf{I}} \tag{21}$$

Therefore,

$$\mathbf{I} = \frac{\mathbf{I}'}{\mathbf{M}}, \mathbf{V} = \frac{\mathbf{V}'}{\mathbf{M}}, \mathbf{I}_1 = \frac{\mathbf{I}_1'}{\mathbf{M}}, \mathbf{I}_2 = \frac{\mathbf{I}_2'}{\mathbf{M}}. \tag{22}$$

5. The Technique Application in AC Circuits

5.1. Series Impedances Example

Assume we need to find $v(t)$ and $i(t)$ in the circuit shown in Figure 11 [1].

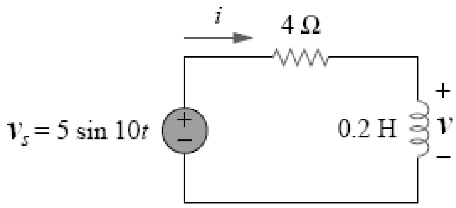

Figure 11. Series impedances example

$$\mathbf{V}_S = 5\angle -90°\text{V}, \omega = 10$$

From (15), the total assumed (relative) AC voltage and current are

$$\mathbf{V}_S' = 4 + j2\text{V}, \ \mathbf{I}' = 1\text{A}$$

Thus,

$$\mathbf{M} = \frac{\mathbf{V}_S'}{\mathbf{V}_S} = \frac{4 + j2}{5\angle\text{-}90°} = 0.894\angle 116.57°$$

Therefore, the actual phasor voltage and current are

$$\mathbf{V} = \frac{j2}{0.894\angle 116.57°} = 2.236\angle\text{-}26.57°\text{V}$$

$$\mathbf{I} = \frac{1}{0.894\angle 116.57°} = 1.118\angle\text{-}116.57°\text{A}$$

Hence,

$$v(t) = 2.236\sin(10t + 63.43°)\text{V}$$

$$i(t) = 1.118\sin(10t - 26.57°)\text{V}$$

5.2. Parallel Impedances Example

Consider we are required to determine $v_o(t)$ in the circuit shown in Figure 12 [1].

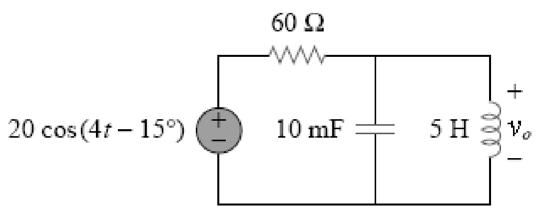

Figure 12. Parallel impedances example

Since the inductor and capacitor are in parallel, we can use (18) to assume that

$$\mathbf{I}_L' = \mathbf{Z}_C, \ \mathbf{I}_C' = \mathbf{Z}_L$$

That is

$$\mathbf{I}'_L = -j25\text{A}, \mathbf{I}'_C = j20\text{A}.$$

Therefore, the total assumed current according to (19) is

$$\mathbf{I}' = \mathbf{I}'_L + \mathbf{I}'_C = -j5\text{A}$$

and the relative voltage across the inductor and capacitor according to (20) is

$$\mathbf{V}'_o = \mathbf{I}'_L \times \mathbf{I}'_C = 500 \text{ V}.$$

From the above, the relative voltage across the 60-Ω resistor can be obtained using 60**I′** which is −j300 V. Therefore, the total relative voltage for the whole circuit is

$$\mathbf{V}' = 500 - j300.$$

The multiplicity factor is

$$\mathbf{M} = \frac{500 - j300}{20\angle\text{-}15°} = 29.15\angle\text{-}15.96°$$

As a result,

$$\mathbf{V_o} = \frac{500}{29.15\angle\text{-}15.96°} = 17.15\angle 15.96°\text{V}$$

In time domain,

$$v_o(t) = 17.15\cos(4t + 15.96°)\text{V}.$$

6. Conclusion

In this paper, we introduce a simple technique to analyze linear DC and AC electric circuits. The technique is based on assuming element values according to the given resistances/impedances such that a relative circuit results are attained. The actual performance is achieved by normalizing the assumed values using a specific formula. It is practically shown that this technique significantly facilitates the solution procedure in many cases where it either simplifies the calculations or reduces the number of equations. However, it is not claimed that such a method is proposed to replace the conventional methods, but it is highly recommended to be considered for circuit analysis where some complex steps can effectively be saved. In addition, the technique is advised to be used with the Y-Δ transformations and the circuit theorems including superposition, source transformation, Thevenin's and Norton's theorems. Furthermore, we suggest that this method is taught in electric circuit courses especially in the introductory chapters to give the students more variety of circuit solving techniques.

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