Applied Mathematics: Statistics
Advanced Higher

Statistical Formulae and Tables

Formerly known as *Mathematical Formulae and Statistical Tables*

November 2004

November 2004

Price: £1.50

Publication code: BB2406

ISBN: 1 85969 552 3

Published by the Scottish Qualifications Authority
Hanover House, 24 Douglas Street, Glasgow G2 7NQ, and Ironmills Road, Dalkeith, Midlothian EH22 1LE

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STATISTICAL FORMULAE

Binomial distribution Bin(n, p):

$$\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0,\ 1,\ 2,\ \ldots,\ n$$

$$\mathrm{E}(X) = np, \qquad \mathrm{V}(X) = np(1-p)$$

Poisson distribution Poi(μ):

$$\mathrm{P}(X = x) = \frac{e^{-\mu}\mu^x}{x!}, \qquad x = 0,\ 1,\ 2,\ \ldots$$

$$\mathrm{E}(X) = \mu, \qquad \mathrm{V}(X) = \mu$$

Sample standard deviation $s = \sqrt{\dfrac{\sum (x_i - \overline{x})^2}{n-1}} = \sqrt{\dfrac{\sum x_i^2 - (\sum x_i)^2 / n}{n-1}}$

where n is the sample size.

Sums of squares and products:

$$S_{xx} = \sum (x_i - \overline{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{yy} = \sum (y_i - \overline{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xy} = \sum (x_i - \overline{x})(y_i - \overline{y}) = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n}$$

Product moment correlation coefficient $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

To test the null hypothesis that the population product moment correlation coefficient $\rho = 0$ use the test statistic $t = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}}$

Linear regression:

The linear model is $Y_i = \alpha + \beta x_i + \varepsilon_i$,
where ε_i are independent, $\mathrm{E}(\varepsilon_i) = 0$ and $\mathrm{V}(\varepsilon_i) = \sigma^2$.

Least squares estimates for α and β are a and b respectively and are given by

$$b = \frac{S_{xy}}{S_{xx}} \quad \text{and} \quad a = \overline{y} - b\overline{x}$$

The sum of squared residuals is $SSR = S_{yy} - \dfrac{(S_{xy})^2}{S_{xx}}$

An estimate for σ^2 is $s^2 = \dfrac{SSR}{n-2}$

To test the null hypothesis that the slope parameter $\beta = 0$
use the test statistic $t = \dfrac{b}{\dfrac{s}{\sqrt{S_{xx}}}}$

If $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$ then

a $100(1-\alpha)\%$ prediction interval for Y_i / x_i is given by $\hat{Y}_i \pm t_{n-2,1-\alpha/2}\, s\sqrt{1 + \dfrac{1}{n} + \dfrac{(x_i - \overline{x})^2}{S_{xx}}}$

and a $100(1-\alpha)\%$ confidence inverval for $\mathrm{E}(Y_i / x_i)$ is given by $\hat{Y}_i \pm t_{n-2,1-\alpha/2}\, s\sqrt{\dfrac{1}{n} + \dfrac{(x_i - \overline{x})^2}{S_{xx}}}$

Chi-squared statistic: $\displaystyle\sum \frac{(O_i - E_i)^2}{E_i}$

Mann-Whitney statistic W:

$$\mathrm{E}(W) = \frac{1}{2}n(n+m+1) \quad \text{and} \quad \mathrm{V}(W) = \frac{1}{12}nm(n+m+1)$$

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $F(x) = \mathrm{P}(X \leq x)$ where X has the Binomial distribution Bin(n, p).
Omitted entries to the left and right of tabulated values are 1·0000 and 0·0000 respectively, to four decimal places.

p		0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 4$	$x =$ 0	0·8145	0·6561	0·5220	0·4096	0·3164	0·2401	0·1785	0·1296	0·0915	0·0625
	1	0·9860	0·9477	0·8905	0·8192	0·7383	0·6517	0·5630	0·4752	0·3910	0·3125
	2	0·9995	0·9963	0·9880	0·9728	0·9492	0·9163	0·8735	0·8208	0·7585	0·6875
	3		0·9999	0·9995	0·9984	0·9961	0·9919	0·9850	0·9744	0·9590	0·9375
$n = 6$	$x =$ 0	0·7351	0·5314	0·3771	0·2621	0·1780	0·1176	0·0754	0·0467	0·0277	0·0156
	1	0·9672	0·8857	0·7765	0·6554	0·5339	0·4202	0·3191	0·2333	0·1636	0·1094
	2	0·9978	0·9842	0·9527	0·9011	0·8306	0·7443	0·6471	0·5443	0·4415	0·3438
	3	0·9999	0·9987	0·9941	0·9830	0·9624	0·9295	0·8826	0·8208	0·7447	0·6563
	4		0·9999	0·9996	0·9984	0·9954	0·9891	0·9777	0·9590	0·9308	0·8906
	5				0·9999	0·9998	0·9993	0·9982	0·9959	0·9917	0·9844
$n = 8$	$x =$ 0	0·6634	0·4305	0·2725	0·1678	0·1001	0·0576	0·0319	0·0168	0·0084	0·0039
	1	0·9428	0·8131	0·6572	0·5033	0·3671	0·2553	0·1691	0·1064	0·0632	0·0352
	2	0·9942	0·9619	0·8948	0·7969	0·6785	0·5518	0·4278	0·3154	0·2201	0·1445
	3	0·9996	0·9950	0·9786	0·9437	0·8862	0·8059	0·7064	0·5941	0·4770	0·3633
	4		0·9996	0·9971	0·9896	0·9727	0·9420	0·8939	0·8263	0·7396	0·6367
	5			0·9998	0·9988	0·9958	0·9887	0·9747	0·9502	0·9115	0·8555
	6				0·9999	0·9996	0·9987	0·9964	0·9915	0·9819	0·9648
	7						0·9999	0·9998	0·9993	0·9983	0·9961
$n = 10$	$x =$ 0	0·5987	0·3487	0·1969	0·1074	0·0563	0·0282	0·0135	0·0060	0·0025	0·0010
	1	0·9139	0·7361	0·5443	0·3758	0·2440	0·1493	0·0860	0·0464	0·0233	0·0107
	2	0·9885	0·9298	0·8202	0·6778	0·5256	0·3828	0·2616	0·1673	0·0996	0·0547
	3	0·9990	0·9872	0·9500	0·8791	0·7759	0·6496	0·5138	0·3823	0·2660	0·1719
	4	0·9999	0·9984	0·9901	0·9672	0·9219	0·8497	0·7515	0·6331	0·5044	0·3770
	5		0·9999	0·9986	0·9936	0·9803	0·9527	0·9051	0·8338	0·7384	0·6230
	6			0·9999	0·9991	0·9965	0·9894	0·9740	0·9452	0·8980	0·8281
	7				0·9999	0·9996	0·9984	0·9952	0·9877	0·9726	0·9453
	8						0·9999	0·9995	0·9983	0·9955	0·9893
	9								0·9999	0·9997	0·9990

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION (continued)

p		0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 12$	$x =$ 0	0·5404	0·2824	0·1422	0·0687	0·0317	0·0138	0·0057	0·0022	0·0008	0·0002
	1	0·8816	0·6590	0·4435	0·2749	0·1584	0·0850	0·0424	0·0196	0·0083	0·0032
	2	0·9804	0·8891	0·7358	0·5583	0·3907	0·2528	0·1513	0·0834	0·0421	0·0193
	3	0·9978	0·9744	0·9078	0·7946	0·6488	0·4925	0·3467	0·2253	0·1345	0·0730
	4	0·9998	0·9957	0·9761	0·9274	0·8424	0·7237	0·5833	0·4382	0·3044	0·1938
	5		0·9995	0·9954	0·9806	0·9456	0·8822	0·7873	0·6652	0·5269	0·3872
	6		0·9999	0·9993	0·9961	0·9857	0·9614	0·9154	0·8418	0·7393	0·6128
	7			0·9999	0·9994	0·9972	0·9905	0·9745	0·9427	0·8883	0·8062
	8				0·9999	0·9996	0·9983	0·9944	0·9847	0·9644	0·9270
	9						0·9998	0·9992	0·9972	0·9921	0·9807
	10							0·9999	0·9997	0·9989	0·9968
	11									0·9999	0·9998
$n = 14$	$x =$ 0	0·4877	0·2288	0·1028	0·0440	0·0178	0·0068	0·0024	0·0008	0·0002	0·0001
	1	0·8470	0·5846	0·3567	0·1979	0·1010	0·0475	0·0205	0·0081	0·0029	0·0009
	2	0·9699	0·8416	0·6479	0·4481	0·2811	0·1608	0·0839	0·0398	0·0170	0·0065
	3	0·9958	0·9559	0·8535	0·6982	0·5213	0·3552	0·2205	0·1243	0·0632	0·0287
	4	0·9996	0·9908	0·9533	0·8702	0·7415	0·5842	0·4227	0·2793	0·1672	0·0898
	5		0·9985	0·9885	0·9561	0·8883	0·7805	0·6405	0·4859	0·3373	0·2120
	6		0·9998	0·9978	0·9884	0·9617	0·9067	0·8164	0·6925	0·5461	0·3953
	7			0·9997	0·9976	0·9897	0·9685	0·9247	0·8499	0·7414	0·6047
	8				0·9996	0·9978	0·9917	0·9757	0·9417	0·8811	0·7880
	9					0·9997	0·9983	0·9940	0·9825	0·9574	0·9102
	10						0·9998	0·9989	0·9961	0·9886	0·9713
	11							0·9999	0·9994	0·9978	0·9935
	12								0·9999	0·9997	0·9991
	13										0·9999
$n = 16$	$x =$ 0	0·4401	0·1853	0·0743	0·0281	0·0100	0·0033	0·0010	0·0003	0·0001	
	1	0·8108	0·5147	0·2839	0·1407	0·0635	0·0261	0·0098	0·0033	0·0010	0·0003
	2	0·9571	0·7892	0·5614	0·3518	0·1971	0·0994	0·0451	0·0183	0·0066	0·0021
	3	0·9930	0·9316	0·7899	0·5981	0·4050	0·2459	0·1339	0·0651	0·0281	0·0106
	4	0·9991	0·9830	0·9209	0·7982	0·6302	0·4499	0·2892	0·1666	0·0853	0·0384
	5	0·9999	0·9967	0·9765	0·9183	0·8103	0·6598	0·4900	0·3288	0·1976	0·1051
	6		0·9995	0·9944	0·9733	0·9204	0·8247	0·6881	0·5272	0·3660	0·2272
	7		0·9999	0·9989	0·9930	0·9729	0·9256	0·8406	0·7161	0·5629	0·4018
	8			0·9998	0·9985	0·9925	0·9743	0·9329	0·8577	0·7441	0·5982
	9				0·9998	0·9984	0·9929	0·9771	0·9417	0·8759	0·7728
	10					0·9997	0·9984	0·9938	0·9809	0·9514	0·8949
	11						0·9997	0·9987	0·9951	0·9851	0·9616
	12							0·9998	0·9991	0·9965	0·9894
	13								0·9999	0·9994	0·9979
	14									0·9999	0·9997

TABLE 1: BINOMIAL CUMULATIVE DISTRIBUTION FUNCTION (continued)

p	0·05	0·10	0·15	0·20	0·25	0·30	0·35	0·40	0·45	0·50
$n = 18$ $x =$ 0	0·3972	0·1501	0·0536	0·0180	0·0056	0·0016	0·0004	0·0001		
1	0·7735	0·4503	0·2241	0·0991	0·0395	0·0142	0·0046	0·0013	0·0003	0·0001
2	0·9419	0·7338	0·4797	0·2713	0·1353	0·0600	0·0236	0·0082	0·0025	0·0007
3	0·9891	0·9018	0·7202	0·5010	0·3057	0·1646	0·0783	0·0328	0·0120	0·0038
4	0·9985	0·9718	0·8794	0·7164	0·5187	0·3327	0·1886	0·0942	0·0411	0·0154
5	0·9998	0·9936	0·9581	0·8671	0·7175	0·5344	0·3550	0·2088	0·1077	0·0481
6		0·9988	0·9882	0·9487	0·8610	0·7217	0·5491	0·3743	0·2258	0·1189
7		0·9998	0·9973	0·9837	0·9431	0·8593	0·7283	0·5634	0·3915	0·2403
8			0·9995	0·9957	0·9807	0·9404	0·8609	0·7368	0·5778	0·4073
9			0·9999	0·9991	0·9946	0·9790	0·9403	0·8653	0·7473	0·5927
10				0·9998	0·9988	0·9939	0·9788	0·9424	0·8720	0·7597
11					0·9998	0·9986	0·9938	0·9797	0·9463	0·8811
12						0·9997	0·9986	0·9942	0·9817	0·9519
13							0·9997	0·9987	0·9951	0·9846
14								0·9998	0·9990	0·9962
15									0·9999	0·9993
16										0·9999
$n = 20$ $x =$ 0	0·3585	0·1216	0·0388	0·0115	0·0032	0·0008	0·0002			
1	0·7358	0·3917	0·1756	0·0692	0·0243	0·0076	0·0021	0·0005	0·0001	
2	0·9245	0·6769	0·4049	0·2061	0·0913	0·0355	0·0121	0·0036	0·0009	0·0002
3	0·9841	0·8670	0·6477	0·4114	0·2252	0·1071	0·0444	0·0160	0·0049	0·0013
4	0·9974	0·9568	0·8298	0·6296	0·4148	0·2375	0·1182	0·0510	0·0189	0·0059
5	0·9997	0·9887	0·9327	0·8042	0·6172	0·4164	0·2454	0·1256	0·0553	0·0207
6		0·9976	0·9781	0·9133	0·7858	0·6080	0·4166	0·2500	0·1299	0·0577
7		0·9996	0·9941	0·9679	0·8982	0·7723	0·6010	0·4159	0·2520	0·1316
8		0·9999	0·9987	0·9900	0·9591	0·8867	0·7624	0·5956	0·4143	0·2517
9			0·9998	0·9974	0·9861	0·9520	0·8782	0·7553	0·5914	0·4119
10				0·9994	0·9961	0·9829	0·9468	0·8725	0·7507	0·5881
11				0·9999	0·9991	0·9949	0·9804	0·9435	0·8692	0·7483
12					0·9998	0·9987	0·9940	0·9790	0·9420	0·8684
13						0·9997	0·9985	0·9935	0·9786	0·9423
14							0·9997	0·9984	0·9936	0·9793
15								0·9997	0·9985	0·9941
16									0·9997	0·9987
17										0·9998

TABLE 2: POISSON CUMULATIVE DISTRIBUTION FUNCTION

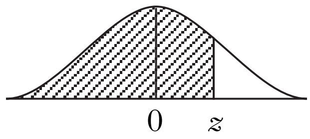
The tabulated value is $F(x) = P(X \le x)$ where X has the Poisson distribution Poi(μ).
Omitted entries to the left and right of tabulated values are 1·0000 and 0·0000 respectively, to four decimal places.

μ =	0·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0	4·5	5·0
x = 0	0·6065	0·3679	0·2231	0·1353	0·0821	0·0498	0·0302	0·0183	0·0111	0·0067
1	0·9098	0·7358	0·5578	0·4060	0·2873	0·1991	0·1359	0·0916	0·0611	0·0404
2	0·9856	0·9197	0·8088	0·6767	0·5438	0·4232	0·3208	0·2381	0·1736	0·1247
3	0·9982	0·9810	0·9344	0·8571	0·7576	0·6472	0·5366	0·4335	0·3423	0·2650
4	0·9998	0·9963	0·9814	0·9473	0·8912	0·8153	0·7254	0·6288	0·5321	0·4405
5		0·9994	0·9955	0·9834	0·9580	0·9161	0·8576	0·7851	0·7029	0·6160
6		0·9999	0·9991	0·9955	0·9858	0·9665	0·9347	0·8893	0·8311	0·7622
7			0·9998	0·9989	0·9958	0·9881	0·9733	0·9489	0·9134	0·8666
8				0·9998	0·9989	0·9962	0·9901	0·9786	0·9597	0·9319
9					0·9997	0·9989	0·9967	0·9919	0·9829	0·9682
10					0·9999	0·9997	0·9990	0·9972	0·9933	0·9863
11						0·9999	0·9997	0·9991	0·9976	0·9945
12							0·9999	0·9997	0·9992	0·9980
13								0·9999	0·9997	0·9993
14									0·9999	0·9998
15										0·9999

μ =	5·5	6·0	6·5	7·0	7·5	8·0	8·5	9·0	9·5	10·0
x = 0	0·0041	0·0025	0·0015	0·0009	0·0006	0·0003	0·0002	0·0001	0·0001	
1	0·0266	0·0174	0·0113	0·0073	0·0047	0·0030	0·0019	0·0012	0·0008	0·0005
2	0·0884	0·0620	0·0430	0·0296	0·0203	0·0138	0·0093	0·0062	0·0042	0·0028
3	0·2017	0·1512	0·1118	0·0818	0·0591	0·0424	0·0301	0·0212	0·0149	0·0103
4	0·3575	0·2851	0·2237	0·1730	0·1321	0·0996	0·0744	0·0550	0·0403	0·0293
5	0·5289	0·4457	0·3690	0·3007	0·2414	0·1912	0·1496	0·1157	0·0885	0·0671
6	0·6860	0·6063	0·5265	0·4497	0·3782	0·3134	0·2562	0·2068	0·1649	0·1301
7	0·8095	0·7440	0·6728	0·5987	0·5246	0·4530	0·3856	0·3239	0·2687	0·2202
8	0·8944	0·8472	0·7916	0·7291	0·6620	0·5925	0·5231	0·4557	0·3918	0·3328
9	0·9462	0·9161	0·8774	0·8305	0·7764	0·7166	0·6530	0·5874	0·5218	0·4579
10	0·9747	0·9574	0·9332	0·9015	0·8622	0·8159	0·7634	0·7060	0·6453	0·5830
11	0·9890	0·9799	0·9661	0·9467	0·9208	0·8881	0·8487	0·8030	0·7520	0·6968
12	0·9955	0·9912	0·9840	0·9730	0·9573	0·9362	0·9091	0·8758	0·8364	0·7916
13	0·9983	0·9964	0·9929	0·9872	0·9784	0·9658	0·9486	0·9261	0·8981	0·8645
14	0·9994	0·9986	0·9970	0·9943	0·9897	0·9827	0·9726	0·9585	0·9400	0·9165
15	0·9998	0·9995	0·9988	0·9976	0·9954	0·9918	0·9862	0·9780	0·9665	0·9513
16	0·9999	0·9998	0·9996	0·9990	0·9980	0·9963	0·9934	0·9889	0·9823	0·9730
17		0·9999	0·9998	0·9996	0·9992	0·9984	0·9970	0·9947	0·9911	0·9857
18			0·9999	0·9999	0·9997	0·9993	0·9987	0·9976	0·9957	0·9928
19					0·9999	0·9997	0·9995	0·9989	0·9980	0·9965
20						0·9999	0·9998	0·9996	0·9991	0·9984
21							0·9999	0·9998	0·9996	0·9993
22								0·9999	0·9999	0·9997
23									0·9999	0·9999

TABLE 3: STANDARD NORMAL CUMULATIVE DISTRIBUTION FUNCTION

The tabulated value is $\Phi(z) = P(Z \leq z)$ where Z has the Standard Normal distribution N(0, 1).



z	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
0·0	0·5000	0·5040	0·5080	0·5120	0·5160	0·5199	0·5239	0·5279	0·5319	0·5359
0·1	0·5398	0·5438	0·5478	0·5517	0·5557	0·5596	0·5636	0·5675	0·5714	0·5753
0·2	0·5793	0·5832	0·5871	0·5910	0·5948	0·5987	0·6026	0·6064	0·6103	0·6141
0·3	0·6179	0·6217	0·6255	0·6293	0·6331	0·6368	0·6406	0·6443	0·6480	0·6517
0·4	0·6554	0·6591	0·6628	0·6664	0·6700	0·6736	0·6772	0·6808	0·6844	0·6879
0·5	0·6915	0·6950	0·6985	0·7019	0·7054	0·7088	0·7123	0·7157	0·7190	0·7224
0·6	0·7257	0·7291	0·7324	0·7357	0·7389	0·7422	0·7454	0·7486	0·7517	0·7549
0·7	0·7580	0·7611	0·7642	0·7673	0·7704	0·7734	0·7764	0·7794	0·7823	0·7852
0·8	0·7881	0·7910	0·7939	0·7967	0·7995	0·8023	0·8051	0·8078	0·8106	0·8133
0·9	0·8159	0·8186	0·8212	0·8238	0·8264	0·8289	0·8315	0·8340	0·8365	0·8389
1·0	0·8413	0·8438	0·8461	0·8485	0·8508	0·8531	0·8554	0·8577	0·8599	0·8621
1·1	0·8643	0·8665	0·8686	0·8708	0·8729	0·8749	0·8770	0·8790	0·8810	0·8830
1·2	0·8849	0·8869	0·8888	0·8907	0·8925	0·8944	0·8962	0·8980	0·8997	0·9015
1·3	0·9032	0·9049	0·9066	0·9082	0·9099	0·9115	0·9131	0·9147	0·9162	0·9177
1·4	0·9192	0·9207	0·9222	0·9236	0·9251	0·9265	0·9279	0·9292	0·9306	0·9319
1·5	0·9332	0·9345	0·9357	0·9370	0·9382	0·9394	0·9406	0·9418	0·9429	0·9441
1·6	0·9452	0·9463	0·9474	0·9484	0·9495	0·9505	0·9515	0·9525	0·9535	0·9545
1·7	0·9554	0·9564	0·9573	0·9582	0·9591	0·9599	0·9608	0·9616	0·9625	0·9633
1·8	0·9641	0·9649	0·9656	0·9664	0·9671	0·9678	0·9686	0·9693	0·9699	0·9706
1·9	0·9713	0·9719	0·9726	0·9732	0·9738	0·9744	0·9750	0·9756	0·9761	0·9767
2·0	0·9772	0·9778	0·9783	0·9788	0·9793	0·9798	0·9803	0·9808	0·9812	0·9817
2·1	0·9821	0·9826	0·9830	0·9834	0·9838	0·9842	0·9846	0·9850	0·9854	0·9857
2·2	0·9861	0·9864	0·9868	0·9871	0·9875	0·9878	0·9881	0·9884	0·9887	0·9890
2·3	0·9893	0·9896	0·9898	0·9901	0·9904	0·9906	0·9909	0·9911	0·9913	0·9916
2·4	0·9918	0·9920	0·9922	0·9925	0·9927	0·9929	0·9931	0·9932	0·9934	0·9936
2·5	0·9938	0·9940	0·9941	0·9943	0·9945	0·9946	0·9948	0·9949	0·9951	0·9952
2·6	0·9953	0·9955	0·9956	0·9957	0·9959	0·9960	0·9961	0·9962	0·9963	0·9964
2·7	0·9965	0·9966	0·9967	0·9968	0·9969	0·9970	0·9971	0·9972	0·9973	0·9974
2·8	0·9974	0·9975	0·9976	0·9977	0·9977	0·9978	0·9979	0·9979	0·9980	0·9981
2·9	0·9981	0·9982	0·9982	0·9983	0·9984	0·9984	0·9985	0·9985	0·9986	0·9986
3·0	0·9987	0·9987	0·9987	0·9988	0·9988	0·9989	0·9989	0·9989	0·9990	0·9990
3·1	0·9990	0·9991	0·9991	0·9991	0·9992	0·9992	0·9992	0·9992	0·9993	0·9993
3·2	0·9993	0·9993	0·9994	0·9994	0·9994	0·9994	0·9994	0·9995	0·9995	0·9995
3·3	0·9995	0·9995	0·9995	0·9996	0·9996	0·9996	0·9996	0·9996	0·9996	0·9997
3·4	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9997	0·9998
3·5	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998	0·9998
3·6	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999	0·9999

TABLE 4: PERCENTAGE POINTS OF THE STANDARD NORMAL DISTRIBUTION

The entries in the table are such that for the Standard Normal distribution $P(Z > z_p) = p$.

p	z_p
0·500	0·00
0·250	0·67
0·100	1·28
0·050	1·64
0·025	1·96
0·010	2·33
0·005	2·58
0·001	3·09
0·0005	3·29

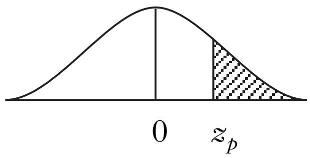
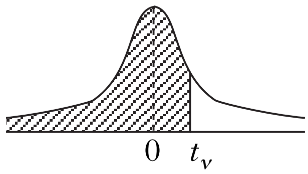


TABLE 5: THE STUDENT t DISTRIBUTION

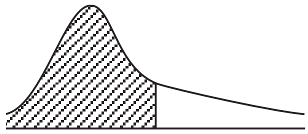
The table entry t_ν is such that $P(T \le t_\nu) = q$ where T has the Student t distribution with ν degrees of freedom.



$q =$	0·900	0·950	0·975	0·990	0·995	0·999	0·9995
ν = 1	3·078	6·314	12·706	31·821	63·656	318·289	636·578
2	1·886	2·920	4·303	6·965	9·925	22·328	31·600
3	1·638	2·353	3·182	4·541	5·841	10·214	12·924
4	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	1·476	2·015	2·571	3·365	4·032	5·894	6·869
6	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	1·314	1·703	2·052	2·473	2·771	3·421	3·689
28	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	1·311	1·699	2·045	2·462	2·756	3·396	3·660
30	1·310	1·697	2·042	2·457	2·750	3·385	3·646
31	1·309	1·696	2·040	2·453	2·744	3·375	3·633
32	1·309	1·694	2·037	2·449	2·738	3·365	3·622
33	1·308	1·692	2·035	2·445	2·733	3·356	3·611
34	1·307	1·691	2·032	2·441	2·728	3·348	3·601
35	1·306	1·690	2·030	2·438	2·724	3·340	3·591
36	1·306	1·688	2·028	2·434	2·719	3·333	3·582
37	1·305	1·687	2·026	2·431	2·715	3·326	3·574
38	1·304	1·686	2·024	2·429	2·712	3·319	3·566
39	1·304	1·685	2·023	2·426	2·708	3·313	3·558
40	1·303	1·684	2·021	2·423	2·704	3·307	3·551
∞	1·282	1·645	1·960	2·327	2·576	3·091	3·291

TABLE 6: THE CHI-SQUARED DISTRIBUTION

The table entry χ_ν^2 is such that $P\left(X_\nu^2 \le \chi_\nu^2\right) = q$ where X_ν^2 has the Chi-squared distribution with ν degrees of freedom.



$q =$	0·900	0·950	0·975	0·990	0·995	0·999	0·9995
ν = 1	2·706	3·841	5·024	6·635	7·879	10·827	12·115
2	4·605	5·991	7·378	9·210	10·597	13·815	15·201
3	6·251	7·815	9·348	11·345	12·838	16·266	17·731
4	7·779	9·488	11·143	13·277	14·860	18·466	19·998
5	9·236	11·070	12·832	15·086	16·750	20·515	22·106
6	10·645	12·592	14·449	16·812	18·548	22·457	24·102
7	12·017	14·067	16·013	18·475	20·278	24·321	26·018
8	13·362	15·507	17·535	20·090	21·955	26·124	27·867
9	14·684	16·919	19·023	21·666	23·589	27·877	29·667
10	15·987	18·307	20·483	23·209	25·188	29·588	31·419
11	17·275	19·675	21·920	24·725	26·757	31·264	33·138
12	18·549	21·026	23·337	26·217	28·300	32·909	34·821
13	19·812	22·362	24·736	27·688	29·819	34·527	36·477
14	21·064	23·685	26·119	29·141	31·319	36·124	38·109
15	22·307	24·996	27·488	30·578	32·801	37·698	39·717
16	23·542	26·296	28·845	32·000	34·267	39·252	41·308
17	24·769	27·587	30·191	33·409	35·718	40·791	42·881
18	25·989	28·869	31·526	34·805	37·156	42·312	44·434
19	27·204	30·144	32·852	36·191	38·582	43·819	45·974
20	28·412	31·410	34·170	37·566	39·997	45·314	47·498
21	29·615	32·671	35·479	38·932	41·401	46·796	49·010
22	30·813	33·924	36·781	40·289	42·796	48·268	50·510
23	32·007	35·172	38·076	41·638	44·181	49·728	51·999
24	33·196	36·415	39·364	42·980	45·558	51·179	53·478
25	34·382	37·652	40·646	44·314	46·928	52·619	54·948

TABLE 7: MANN-WHITNEY

The table relates to two independent samples of sizes n and m ($n \le m$). W is the sum of the ranks of the observations in the sample of size n.

There are $\binom{n+m}{n}$ possible sets of n ranks which can occur. Each column of the main table gives the number of such sets for which $W - \frac{1}{2}n(n+1)$ takes a value less than or equal to that stated at the top of the column.

n	m	$\binom{n+m}{n}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	3	20	1	2	4	7	10	13	16	18	19	20											
3	4	35	1	2	4	7	11	15	20	24	28	31	33	34	35								
4	4	70	1	2	4	7	12	17	24	31	39	46	53	58	63	66	68	69	70				
3	5	56	1	2	4	7	11	16	22	28	34	40	45	49	52	54	55	56					
4	5	126	1	2	4	7	12	18	26	35	46	57	69	80	91	100	108	114	119	122	124	125	126
5	5	252	1	2	4	7	12	19	28	39	53	69	87	106	126	146	165	183	199	213	224	233	240
3	6	84	1	2	4	7	11	16	23	30	38	46	54	61	68	73	77	80	82	83	84		
4	6	210	1	2	4	7	12	18	27	37	50	64	80	96	114	130	146	160	173	183	192	198	203
5	6	462	1	2	4	7	12	19	29	41	57	76	99	124	153	183	215	247	279	309	338	363	386
6	6	924	1	2	4	7	12	19	30	43	61	83	111	143	182	224	272	323	378	433	491	546	601
3	7	120	1	2	4	7	11	16	23	31	40	50	60	70	80	89	97	104	109	113	116	118	119
4	7	330	1	2	4	7	12	18	27	38	52	68	87	107	130	153	177	200	223	243	262	278	292
5	7	792	1	2	4	7	12	19	29	42	59	80	106	136	171	210	253	299	347	396	445	493	539
6	7	1716	1	2	4	7	12	19	30	44	63	87	118	155	201	253	314	382	458	539	627	717	811
7	7	3432	1	2	4	7	12	19	30	45	65	91	125	167	220	283	358	445	545	657	782	918	1064
3	8	165	1	2	4	7	11	16	23	31	41	52	64	76	89	101	113	124	134	142	149	154	158
4	8	495	1	2	4	7	12	18	27	38	53	70	91	114	141	169	200	231	264	295	326	354	381
5	8	1287	1	2	4	7	12	19	29	42	60	82	110	143	183	228	280	337	400	466	536	607	680
6	8	3003	1	2	4	7	12	19	30	44	64	89	122	162	213	272	343	424	518	621	737	860	994
7	8	6435	1	2	4	7	12	19	30	45	66	93	129	174	232	302	388	489	609	746	904	1080	1277
8	8	12870	1	2	4	7	12	19	30	45	67	95	133	181	244	321	418	534	675	839	1033	1254	1509