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ARISTOTLE'S THEORY OF DEFINITION

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Aristotle's theory of definition is expounded in a group of statements scattered throughout many of his works. I will try to gather the most important of these somewhat disconnected statements and give an axiomatic exposition of them. (I am indebted to a discussion with the late Professor Anders Wedberg of Stockholm University on certain passages in Aristotle's writings.) I will reduce some fundamental notions employed by Aristotle to one primitive notion, also used by him, and select a group of statements, some of which involve a further primitive notion, as postulates from which the other statements can readily be derived. Some lemmata needed for the proof of the important theorem T9 will also be explicitly stated and proved.

§ 1. Aristotle on substances and universals

Let us take the notion of nearest genus (ἐγγυτάτω γένος, Top. 143 a 18-24, Met. 1033 b 34-1034 a 1; ἐπάνω γένος, Top. 122 a 3-6) as primitive and introduce the following abbreviated notation:

$G\alpha\beta$ =def. α is a nearest genus of β.

The Greek letters 'α', 'β', 'γ', with or without subscripts, will be used to refer to arbitrary classes of individuals. If $G\alpha\beta$ holds true, then β may be said to be a nearest species of α.

An ordered pair $\langle\alpha,\beta\rangle$ such that $G\alpha\beta$ holds may be called a 'G-pair'. We may now say that a genus is the first member of a G-pair:

D1 α is a genus =def. there is a β such that $G\alpha\beta$.

Similarly, a species (εἶδος) is the second member of a G-pair:

<u>D2</u> α is a species =def. there is a β such that $G\beta\alpha$.

A class which is the first or second member of a G-pair is called '<u>secondary substance</u>' (δεύτερα οὐσία):

<u>D3</u> (<u>Cat</u>. 2 a 11-18)

α is a secondary substance =def. α is a genus or α is a species.

Aristotle's secondary substances correspond to Plato's natural classes or natural kinds.

Even Plato's implicit notion of a <u>highest genus</u> occurs explicitly in Aristotle's philosophy.

<u>D4</u> (<u>Met</u>. 1014 b 9-14; <u>An. post</u>. 100 b 1-3)

α is a highest genus =def. α is a genus and α is not a species.

The analogous notion of a <u>lowest species</u> (ἄτομον τῷ εἴδει, ἀδιάφορα) is also distinguished:

<u>D5</u> (<u>An. post</u>. 96 b 15-21; cf. also 97 b 31 and <u>Cat</u>. 2 b 22-24)

α is a lowest species =def. α is a species and α is not a genus.

We also need a certain derivative of the relation G, viz., the proper <u>ancestral</u> of G, denoted by 'G*'. When $G^*\alpha\beta$ holds, β is said to be <u>subordinate</u> (ὑπ' ἀλλήλων) to α in the genus-species hierarchy.

<u>D6</u> (<u>Cat</u>. 1 b 20-24; <u>Top</u>. 107 a 18-23)

$G^*\alpha\beta$ =def. there are $\gamma_1, \gamma_2, \ldots, \gamma_n$ such that $G\gamma_1\gamma_2$, $G\gamma_2\gamma_3, \ldots, G\gamma_{n-1}\gamma_n$ and such that $\gamma_1 = \alpha$, $\gamma_n = \beta$, for $n \geq 2$.

The corresponding ordinary ancestral *G, then, is the relational sum of G* and identity.

A highest genus and a lowest species exemplify two extreme types of secondary substance. Another kind of class, outside of the hierarchy of secondary substances (cf. <u>T9</u> below), is constituted by what the scholastics called the '<u>differentiae</u>' (διαφορά):

D7 (Met. 1058 a 6-8; An. post. 96 b 25 - 97 b 6)

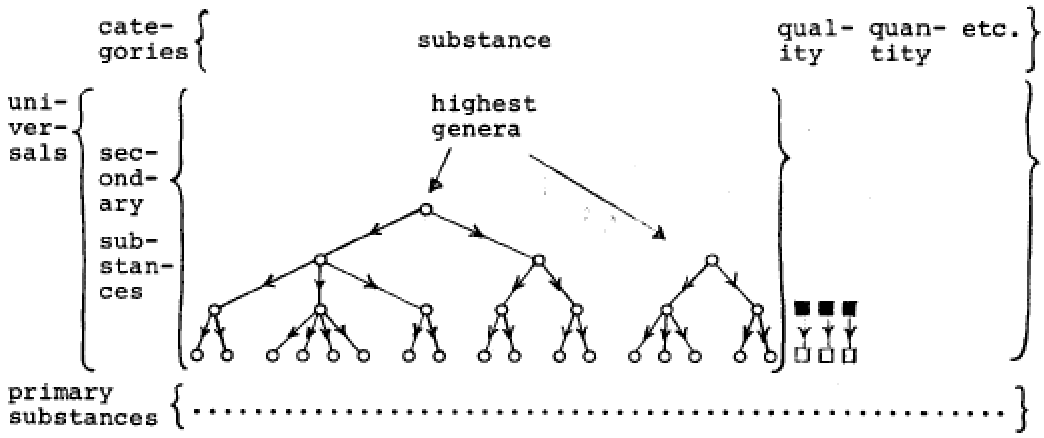
> α is a differentia =def. there are secondary substances β and γ such that $G\beta\gamma$ and $\alpha \neq \beta$ and $\alpha \neq \gamma$ and γ = the intersection of α and β.

Aristotle assumes that for each species of a genus there is a unique differentia (Top. 143 b 8 f., 145 a 6 f.). The genus together with the differentia of a species constitutes the essence (τί ἐστι) of the species (An. post. 97 a 23-28).

Plato's distinction between sensible things and abstract Ideas corresponds to Aristotle's distinction between primary substances (πρώτη οὐσία) and universals (καθόλου). The primary substances are individual objects, and only these have concrete existence in space and time. The notion of primary substance will be taken here as the second primitive concept in the axiomatization of Aristotle's theory of definition.

Among the universals there are the secondary substances, which are the "natural" classes of primary substances. Among the "non-natural" classes of primary substances there are the extensions (in our sense) of qualities denoted by quality-words (such as 'white'). The class of these qualities forms the extension (in our sense) of the unanalyzable category of quality. Other such universals are the categories of quantity, relation, place, time, etc. The qualities of primary substances coming under these categories can be denoted - in accordance with the exposition of Aristotle - respectively by such words as 'two cubits long', 'double', 'in the Lýkeion', 'yesterday', etc.

I believe that a logically relevant part of Aristotle's ontology can be summarized in the following diagram:



Here 'o' and '□' symbolize "natural" and "non-natural" classes respectively and '■' symbolizes abstract entities which are not classes. The combination '■→□' means that □ is the extension of ■, and '④→①' means that G④① if ④ is immediately above ①.

According to this interpretation, the Aristotelian hierarchies of secondary substances generated by means of the G-relation constitute a system of supremum-semilattices with respect to the ordinary ancestral *G.

§ 2. <u>The classification of primary substances</u>

First let us regard some assumptions concerning the lowest species. Aristotle supposes that all lowest species are nonempty and mutually exclusive:

<u>P1</u> (<u>An. post</u>. 83 b 28-31)

> If α is a lowest species, then there is a primary substance which is a member of α.

<u>P2</u> (<u>De part. an</u>. 642 b 30-32)

> If α and β are lowest species and $\alpha \neq \beta$, then there is no primary substance which is a member of both α and β.

Next we have an assumption which says, in combination with the definition of species (<u>D2</u>), that every species has exactly one nearest genus:

<u>P3</u> (<u>Top</u>. 121 b 29 f., 144 a 11-13)

> If $G\alpha\beta$ and $G\alpha\gamma$, then $\alpha = \gamma$.

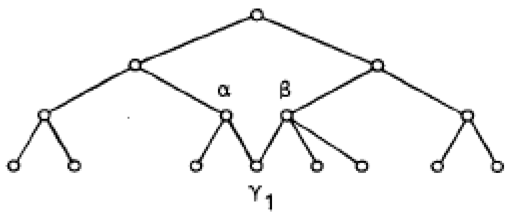
Aristotle also considers (<u>Top</u>. 121 b 30-37) but seems to dismiss (<u>Top</u>. 121 b 32 f.) the following (cf. <u>T6</u> below):

> $G^*\alpha\gamma$ and $G^*\beta\gamma$ if and only if $\alpha = \beta$ or $G^*\alpha\beta$ or $G^*\beta\alpha$ or there is a γ_1 such that $G^*\gamma_1\alpha$ and $G^*\gamma_1\beta$.

A weaker interpretation of Aristotle's text would be:

> If $G^*\alpha\gamma$ and $G^*\beta\gamma$, then $\alpha = \beta$ or $G^*\alpha\beta$ or $G^*\beta\alpha$ or there is a γ_1 such that $G^*\gamma_1\alpha$ and $G^*\gamma_1\beta$.

This statement, however, would follow from our theorem <u>T6</u> and hence is of no interest in the present context. The following structure is an example of the fourth possibility of the consequent:



The following four postulates P4-P7 state that every genus is exhaustively divided into at least two mutually exclusive species:

P4 (Top. 111 a 33 f., 121 a 27-35)

> If α is a genus and x is a member of α, then there is a β such that Gαβ holds and x is a member of β.

P5 (Top. 121 a 25 f.)

> If Gαβ, then β is a subclass of α.

P6 (Top. 123 a 30-33)

> If Gαβ, then there is at least one γ distinct from β such that Gαγ.

P7 (Top. 127 a 24; An. post. 97 a 20-22)

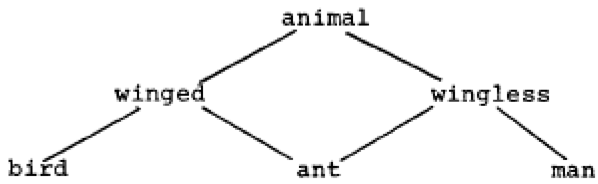
> If Gαβ and Gαγ and β ≠ γ, then there is no primary substance which is a member of both β and γ.

Note that P2 does not follow from P7.

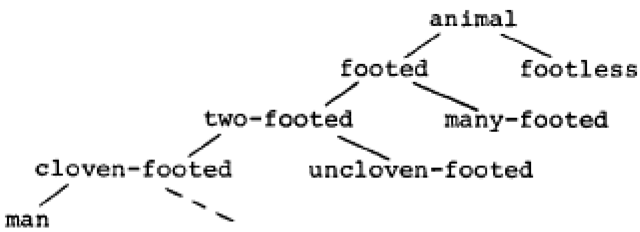
Aristotle criticizes at length (De. part. an. 642 b 5 - 644 b 21) a principle which could be formulated thus:

> If Gαβ, then there is exactly one γ distinct from β such that Gαγ.

The method of dichotomy based on this principle is defective according to Aristotle, since it could easily lead to erroneous results. For example, a classification of the following kind (De part. an. 642 b 33-35) would contradict P3:



And a classification of the following kind (<u>De part. an</u>. 642 b 7-8, 644 a 4-6) would contradict <u>P6</u>:



For according to Aristotle, both 'footless animal' and 'uncloven-footed two-footed animal' are predicates without a secondary substance as extension (<u>ib</u>. 642 b 22-24).

Regarding the relation G* of subordination, we have to state two assumptions to the effect that there are no infinite descending or ascending G*-chains.

<u>P8</u> (<u>An. post</u>.81 b 30 - 82 a 35, 83 b 1-7, 24-31, 84 a 25-28)

> There is no infinite sequence $\langle\alpha_1,\alpha_2,\ldots\rangle$ such that $G\alpha_i\alpha_{i+1}$ holds for all $i \geq 1$.

In other words, a downward path from a genus to one of its nearest species and further down to one of the nearest species of this species considered as a genus and so on always ends in a lowest species.

<u>P9</u> (<u>ib</u>.)

> There is no infinite sequence $\langle\alpha_1,\alpha_2,\ldots\rangle$ such that $G\alpha_{i+1}\alpha_i$ for all $i \geq 1$.

In other words, the upward path from a species to its nearest genus and to the nearest genus of this genus considered as a species and so on always ends in a highest genus.

Finally, we have to add the assumption that there is more than one highest genus:

<u>P10</u> (<u>Met</u>. 1014 b 9-14; <u>An. post</u>. 100 b 1-3)

> There are at least one α and one β such that $\alpha \neq \beta$ and such that α is a highest genus and β is a highest genus.

This postulate expresses one of the chief differences between the Platonic and the Aristotelian theories of "natural" classification.

§ 3. <u>Some consequences of the assumptions on classification</u>

From the postulates <u>P1</u>-<u>P10</u> we may derive some other interesting statements which are pertinent to Aristotle's theory of definition. (The proofs thereof are, however, not supplied by Aristotle.)

<u>T1</u> (<u>Cat</u>. 1 b 20-22; <u>Top</u>. 121 b 11-13)

> If $G^*\alpha\beta$, then β is a proper subclass of α.

<u>Proof</u>: <u>Case 1</u>. Assume that $G\alpha\beta$ holds. Then by <u>P5</u>, β is a subclass of α. By <u>P1</u> and <u>P6</u>-<u>P7</u>, there is a nonempty species γ such that $\gamma \neq \beta$ and $G\alpha\gamma$. Hence, β is a proper subclass of α. <u>Case 2</u>. Assume that there is a number $n \geq 3$ such that $G\gamma_1\gamma_2, G\gamma_2\gamma_3, \ldots, G\gamma_{n-1}\gamma_n$, where $\alpha = \gamma_1$ and $\beta = \gamma_n$. Then by Case 1, γ_n is a proper subclass of γ_{n-1}, γ_{n-1} is a proper subclass of $\gamma_{n-2}, \ldots, \gamma_2$ is a proper subclass of γ_1. Therefore, β is a proper subclass of α.

<u>T2</u> (<u>Cat</u>. 2 b 5 f., 15 f.)

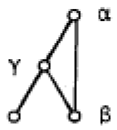
> If α is a secondary substance, then there is a primary substance which is a member of α.

<u>Proof</u>: <u>Case 1</u>. α is a lowest species. Then by <u>P1</u>, α is nonempty. <u>Case 2</u>. α is not a lowest species. Then by <u>P8</u>, there is a lowest species β such that $G^*\alpha\beta$. Therefore, α is nonempty by virtue of <u>P1</u> and <u>T1</u>.

<u>T3</u> If $G\alpha\beta$, then there is no γ such that $G\alpha\gamma$ and $G\gamma\beta$.

<u>Proof</u>. Assume $G\alpha\beta$, $G\alpha\gamma$, and $G\gamma\beta$. Then $\alpha = \gamma$ by <u>P3</u>. Therefore by <u>T1</u>, not $G\alpha\gamma$, which contradicts the assumption.

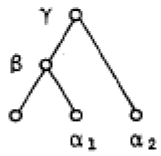
Hence, a structure of the following type is excluded:



It is an open problem whether the following principle can be substantiated in Aristotle's writings:

> If α_1 and α_2 are different lowest species and $G\beta\alpha_1$ holds, then there is no γ such that $G\gamma\beta$ and $G\gamma\alpha_2$.

Such a principle would exclude structures of the following type:



<u>T4</u> If α is a genus and x is a primary substance which is a member of α, then there is a lowest species β such that $G^*\alpha\beta$ holds and x is a member of β.

<u>Proof</u>: Assume the antecedent. Then by <u>P4</u>, there is a β_1 such that $G\alpha\beta_1$ and x is a member of β_1. If β_1 is a lowest species, the theorem holds. If β_1 is not a lowest species, then there is a β_2 such that $G\beta_1\beta_2$ and x is a member of β_2. And so on. By <u>P8</u>, we thus obtain a finite sequence $G\alpha\beta_1$, $G\beta_1\beta_2, \ldots, G\beta_{n-1}\beta_n$ such that β_n is a lowest species and x is a member of each β_i.

<u>T5</u> If α is a genus and β is a lowest species and if there is a primary substance which is a member of both α and β, then $G^*\alpha\beta$ holds.

<u>Proof</u>: Let α be a genus and β a lowest species and let the individual x be a member of both α and β. Then by <u>T4</u>, there is a lowest species γ such that $G^*\alpha\gamma$ and x is a member of γ. By <u>P2</u>, $\gamma = \beta$ and therefore $G^*\alpha\beta$.

<u>T6</u> (<u>Top</u>. 107 a 18-31)

> If $G^*\alpha\gamma$ and $G^*\beta\gamma$, then $\alpha = \beta$ or $G^*\alpha\beta$ or $G^*\beta\alpha$.

<u>Proof</u>: The antecedent implies that $G\alpha_n\alpha_{n-1}, \ldots, G\alpha_2\alpha_1$, whereby

$\alpha_n = \alpha$ and $\alpha_1 = \gamma$ for some n, and further that $G\beta_m\beta_{m-1}, \ldots,$ $G\beta_2\beta_1$, whereby $\beta_m = \beta$ and $\beta_1 = \gamma$ for some m. Case 1. m = n. Then $\alpha = \beta$, by P3. Case 2. m < n. Then by Case 1, $\alpha_m = \beta_m$, and therefore $G^*\alpha\beta$. Case 3. m > n. Then by Case 1, $\alpha_n = \beta_n$, and therefore $G^*\beta\alpha$.

T7 If α and β are genera and if there is a primary substance which is a member of both α and β, then there is a γ such that $G^*\alpha\gamma$ and $G^*\beta\gamma$.

Proof: Let α and β be genera and let the individual x be a member of both α and β. Then by T4, there is a lowest species γ such that $G^*\alpha\gamma$ and x is a member of γ. By T5, $G^*\beta\gamma$ also holds.

T8 If α and β are secondary substances, then $\alpha = \beta$ or $G^*\alpha\beta$ or $G^*\beta\alpha$ or α and β are mutually exclusive.

Proof: Case 1. α and β are genera. Then the consequent follows by T2, T6, and T7. Case 2. α is a genus and β is a lowest species. Then by T5, $G^*\alpha\beta$ or α and β are mutually exclusive. Case 3. α and β are lowest species. Then by P2, $\alpha = \beta$ or α and β are mutually exclusive.

T9 (Top. 122 b 20-23)

If α is a differentia, then α is not a secondary substance.

Proof: By D7, the antecedent implies that there are secondary substances β and γ such that (i) $G\beta\gamma$, (ii) $\alpha \neq \beta$ and $\alpha \neq \gamma$, (iii) γ is the intersection of α and β. Assume: (iv) $G^*\alpha\gamma$. By (i), (ii), and T6, $G^*\alpha\beta$ or $G^*\beta\alpha$. If $G^*\alpha\beta$, then β is a proper subclass of α, by T1. Hence by (iii), $\gamma = \beta$, which contradicts (i). On the other hand, if $G^*\beta\alpha$, then $\alpha = \beta$ or $\alpha = \gamma$, by (i), (iv), and T3. But this contradicts (ii). Therefore, we have (v) not $G^*\alpha\gamma$. Furthermore by (iii), α is not a subclass of γ. Hence by T1, (vi) not $G^*\gamma\alpha$. Finally by (iii) and T2, it holds that (vii) α has an element in common with γ. Now (ii), (v), (vi), and (vii) contradict all four possibilities in the consequent of T8. Hence, α and γ are not both secondary substances. But γ is a secondary substance. Therefore, α is not a secondary substance.

T10 (Met. 998 b 22; Top. 127 a 26-38; An. post. 92 b 13 f.)

A universal predicate (such as 'being' or 'unity') does not denote a genus.

Proof: By P10.

§ 4. Definition and classification

To define a species means to indicate its essence (Top. 103 b 9 f.). Hence, an Aristotelian definition is a statement in the form of 'α =def. β', where α is a species and β is a combination of a nearest genus and a differentia corresponding to α (Top. 103 b 14-16). (We let the substituends of 'α' and 'β' denote the classes α and β, respectively.) For example (Politics 1253 a 10): Man =def. Animal [which is] Speech-possessing. Here 'Animal' denotes the genus and 'Speech-possessing' (λόγον) denotes the differentia of the concept of Man.

From Aristotle's theory of classification there follow a number of theorems on definitions. For example, by D2 every species is definable in Aristotle's sense:

T11 (Met. 1030 a 11-13; Top. 141 b 29-31)

For every species α there is an Aristotelian definition with a name of α as definiendum.

Furthermore, a species has only one definition:

T12 (Top. 141 a 35 - 141 b 1, 154 a 10 f.)

If α is a species for which there are β and γ such that α =def. β and α =def. γ are Aristotelian definitions, then $\beta = \gamma$.

For there is a one-to-one correspondence between any species and its differentia, and by P3, every species has exactly one nearest genus.

It also follows that a differentia or a highest genus is not definable in the sense of Aristotle.

T13 (Top. 107 b 33, 122 b 18-24)

> If α is a differentia, then there is no Aristotelian definition with a name of α as definiendum.

For by T9, α is not a species in this case.

T14 (Met. 1014 b 9 f.; Top. 121 a 12-19)

> If α is a highest genus, then there is no Aristotelian definition with a name of α as definiendum.

For by D4, α is not a species in this case.

It is also clear that a primary substance is not definable in this sense. A particular can only be said to belong to such and such a species. Hence, only species are definable in Aristotle's sense.

By a chain of Aristotelian definitions I mean a sequence of Aristotelian definitions $D_1, D_2, \ldots$ such that D_i, $i \geq 2$, is in the form of 'α_i =def. β_i' and α_i is the genus which is part of the essence of β_{i-1}. Hence, a chain of Aristotelian definitions follows an upward G*-path in the hierarchy of species and genera. Then by P9, we have:

T15 (An. post. 83 b 24-27)

> There is no infinite chain of Aristotelian definitions.

§ 5. The conditions of adequacy of an Aristotelian definition

In the Topics, Aristotle considers various relations between subject and predicate in statements of the form of 'All α are β', where α is a species. Such statements may be subdivided with respect to the convertibility of the substituends of 'α' and 'β', i.e., whether statements of the form of 'All β are α' are true or not (Top. 103 b 7-8).

The first of these subclasses, the class of convertible statements, may be further subdivided with regard to whether the substituend of 'β' indicates the essence of α or not, where the essence of α is the intersection of the genus of α and the differentia of α. In the first case, β is called the 'definiens' (ὅρος) of α (Top. 101 b 38, 103 b 10). In the second case, β is called a 'proprium' (ἴδιον) of α (Top. 102 a 18 f., 103 b 10).

In the second main subclass of statements of the form of 'All α are β', the substituends of 'α' and 'β' are not convertible, i.e., some β is not an α. This class may be subdivided with regard to whether β is one of the components of the essence of α or not. In the first case, β is either the genus or the differentia of α (Top. 103 b 12-16). In the second case, β is called an 'accident' (συμβεβηκὸς) of α (Top. 102 b 4-7, 103 b 16-19).

An important modification of the Platonic conception of definition develops from the above distinction between the so-called predicables of definiens and proprium. That β is a proprium of α means that all α are β, that all β are α, and that β is not the essence of α. Hence, the substituends of 'α' and 'β' having the same extension (in our sense) does not suffice to make statements of the form of 'α =def. β' an adequate definition in Aristotle's sense. In this respect, Aristotle's requirements for an adequate definition are stronger than Plato's.