

Landmarks of Science in Early India



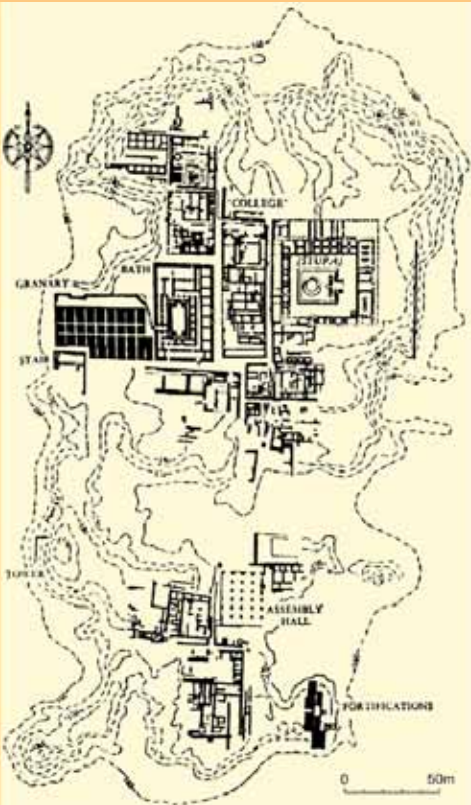
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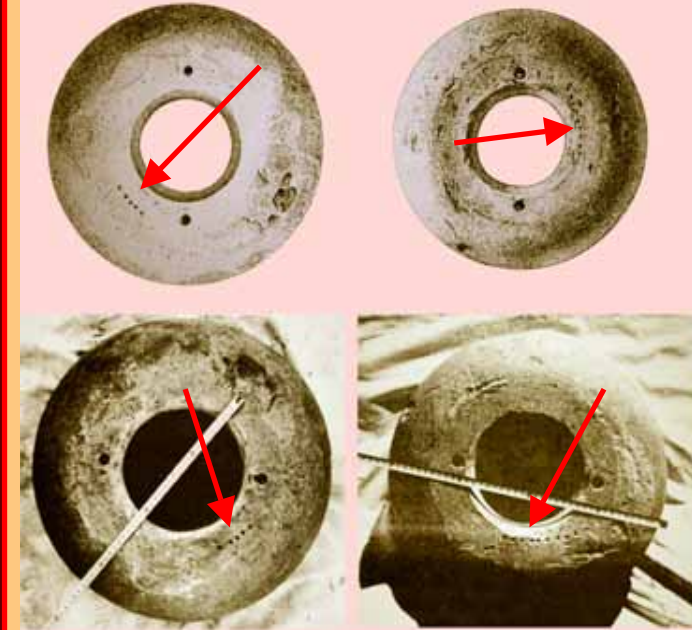
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1. Indus-Sarasvati Civilization

First steps of technology and science in the protohistoric era*



The east-west alignment of the main streets of Mohenjo-daro's "citadel" (or acropolis, *left*) was based on the Pleiades star cluster (*Krittika*), which rose due east at the time; it no longer does because of the precession of the equinoxes. (German archaeologist Holger Wanzke, "Axis systems and orientation at Mohenjo-daro", 1987)



The mystery of Mohenjo-daro's "ring stones" (*above right*): the small drilled holes (see red arrows), showed the stones were used to track the path of the sun through the year, as seen from Mohenjo-daro. Such evidences demonstrate the first steps in observational astronomy. There are other hints, such as possible astronomical symbolism on a few seals. (Finnish scholar Erkkä Maulan, "The Calendar Stones from Mohenjo-daro", 1984)

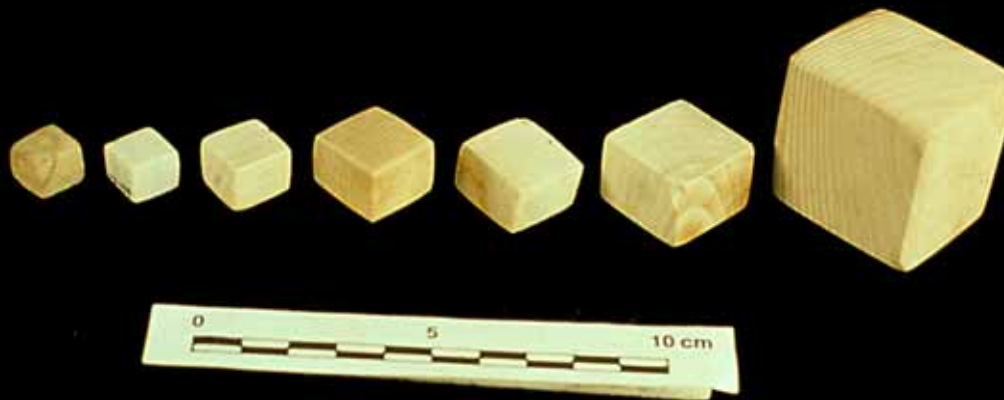
* For technology, please see separate pdf file on this civilization.

A rudimentary decimal system

The standardized Harappan system of weights followed a dual binary-decimal progression:

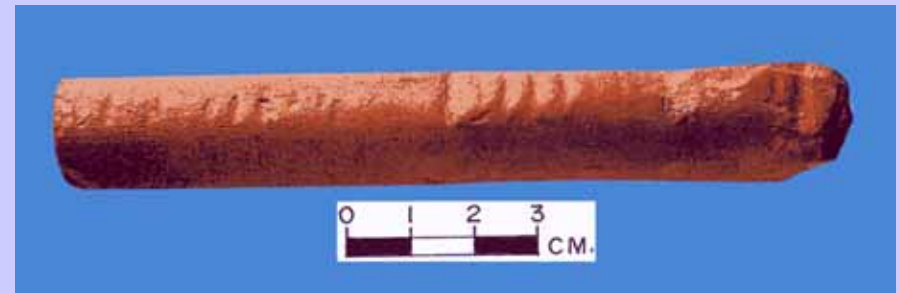
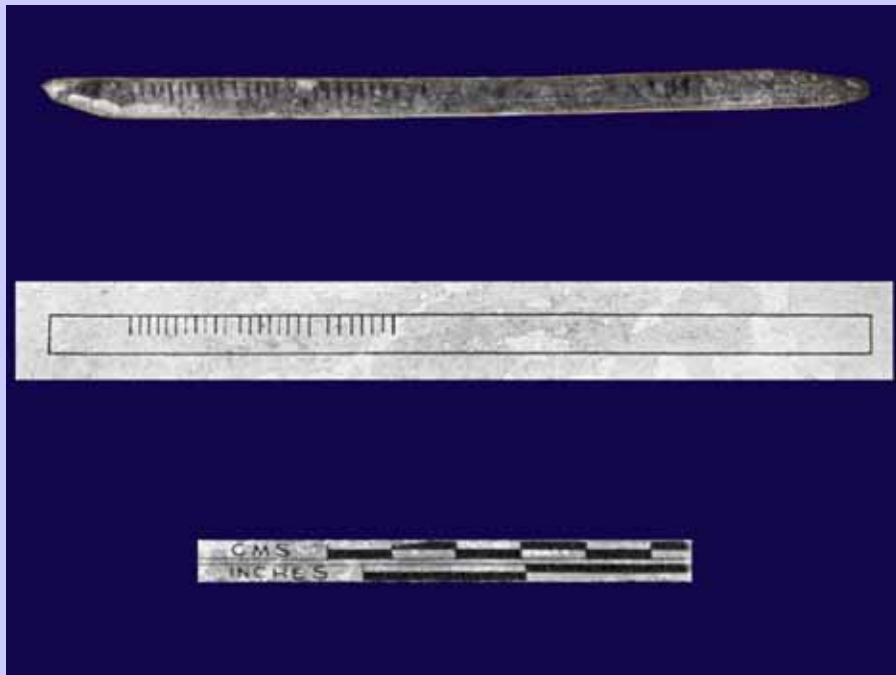
- 1 (= 0.86 g), 2, 4, 8, 16, 32, 64;
- then, instead of continuing with the geometric progression:
- 160, 200, 320, 640, 1,600, 3,200, 6,400, 8,000, 12,800;
- therefore tens, hundreds and thousands of previous units.

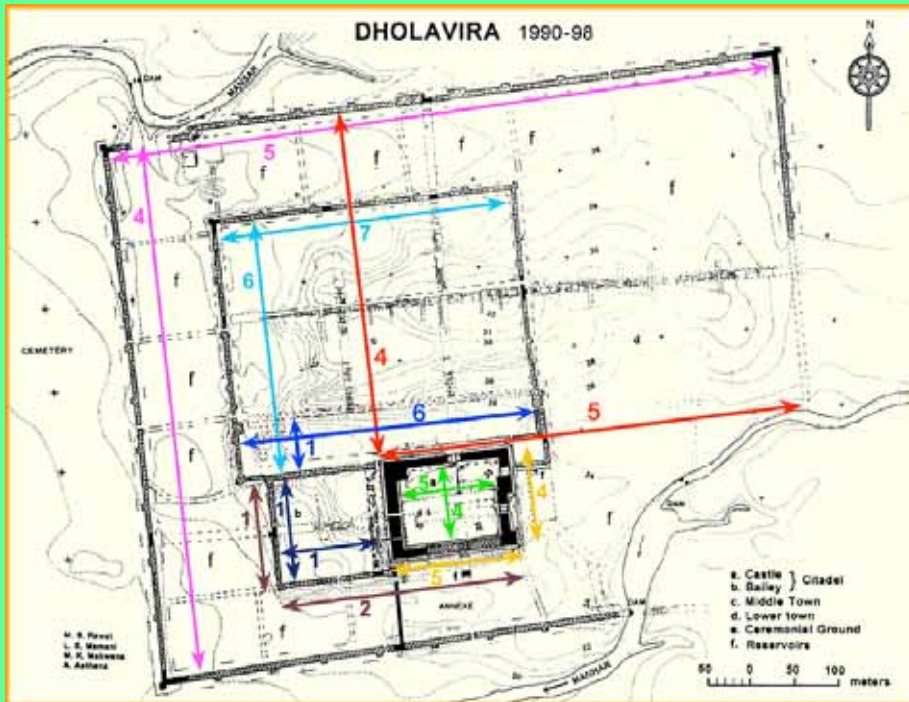
Note: This does not mean that the Harappans noted their numbers in a decimal manner (that is virtually impossible, as this development comes much later). Several other ancient civilizations also used multiples of 10 without a decimal system of numeral notations.



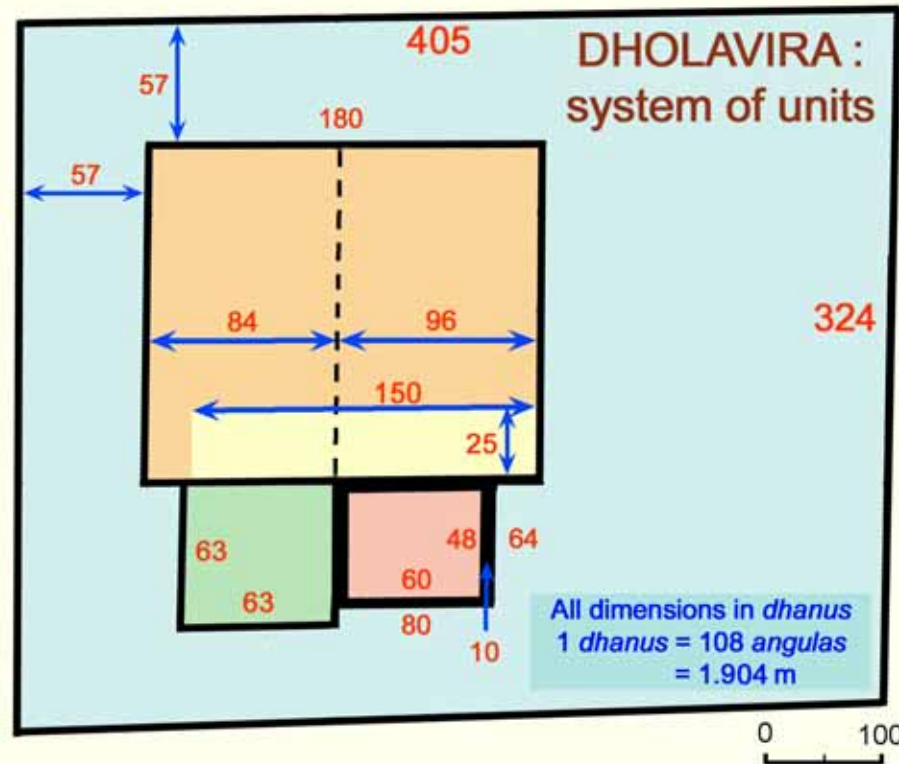
Linear units related to those of historical times

- Lothal's measuring scale (*bottom left*, 27 graduations spanning 46 mm):
1 unit = 1.77 mm.
- V. Mainkar in 1984: 10 Lothal units come close to the *Arthashastra's angula* or digit (1.778 cm in his estimate).
- Kalibangan's terracotta scale: a unit of 1.75 cm (analyzed by Prof. R. Balasubramaniam, IIT-Kanpur).
- In contemporary Egypt and Mesopotamia (later in China, Greece, Japan, or the Roman Empire), the traditional digit fluctuated between 1.6 and 1.9 cm .



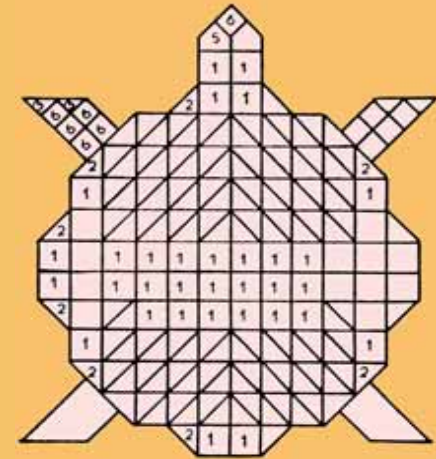


- Dholavira's successive enclosures follow strict ratios (*left*), evidence of specific concepts of auspiciousness, sacred proportions, etc.
- It is possible to calculate the unit of length used in planning Dholavira: 1.9 m (108 times 1.76 cm, the Harappan *angula*). (Research by Michel Danino.)

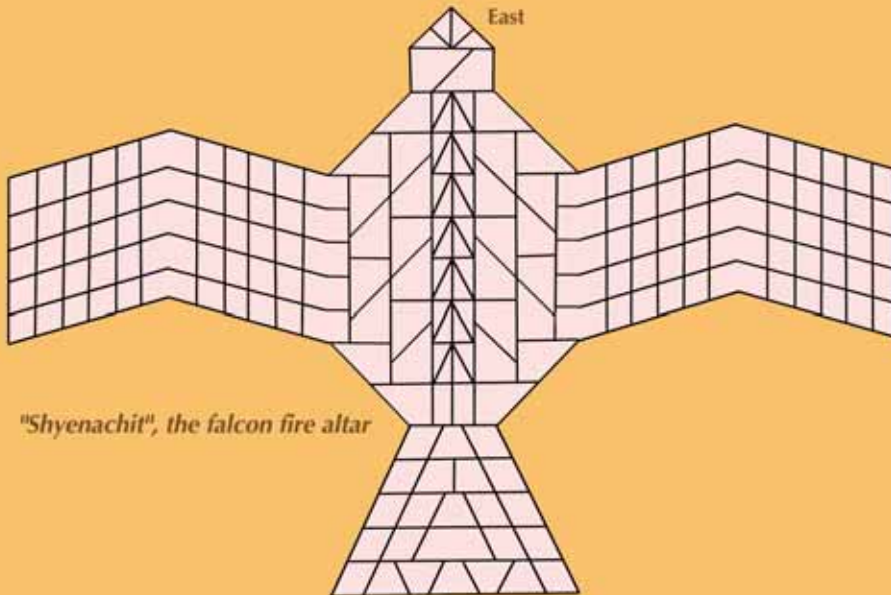


2. Historical Era: Pre-Siddhantic Period

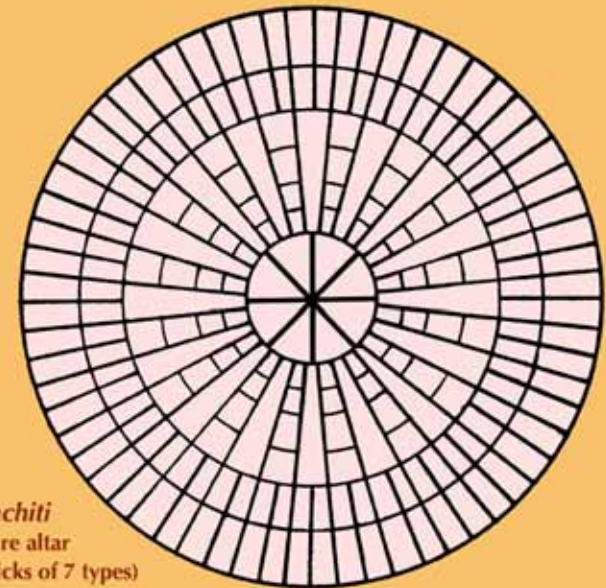
- Geometry of the *Shulbasûtras* (6th to 10th century BCE, possibly earlier): these four ancient texts deal with complex fire altars of various shapes constructed with bricks of specific shapes and area: the total area of the altar must always be carefully respected.
- This leads to precise but purely geometrical calculations (algebra does not exist yet).



Kurmachit fire altar - first layer
(with 6 types of bricks totalling 200)



"Shyenachit", the falcon fire altar

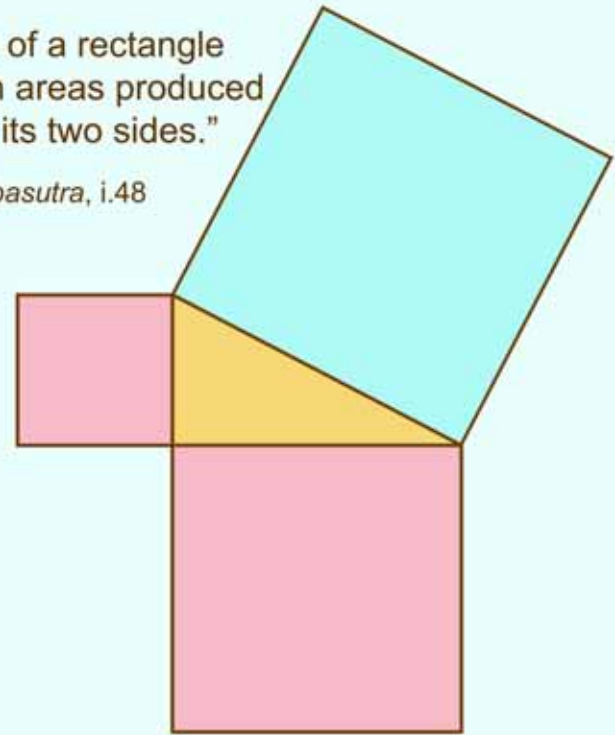


The *rathachakrachiti*
or chariot-wheel fire altar
(first layer, 200 bricks of 7 types)

- The *Shulbasûtras* give a precise geometric expression of the so-called “Pythagorean theorem.”
- Right angles were made by ropes marked to give the triads 3, 4, 5 and 5, 12, 13 ($3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$).
- We should rename this theorem the “Shulba theorem”!

“The diagonal of a rectangle produces both areas produced separately by its two sides.”

Baudhayana Sulbasutra, i.48



Examples of other geometric operations in the *Shulbasûtras*:

- “Squaring the circle” (and vice-versa): geometrically constructing a square having the same area as a given area.
- Adding or subtracting the areas of two squares (to produce a single square).
- Doubling the area of a square.
- In the last construction, $\sqrt{2}$ works out to 577/408 or 1.414215, correct to the 5th decimal. (Same precision with $\sqrt{3}$.)

1. 1 2 4 8 16 32	17. 1 2 4 8 16 32	33. 1 2 4 8 16 32	49. 1 2 4 8 16 32
2. 0-----	18. 0-----	34. 0-----	50. 0-----
3. -0-----	19. -0-----	35. -0-----	51. -0-----
4. 00-----	20. 00-----	36. 00-----	52. 00-----
5. --0-----	21. --0-----	37. --0-----	53. --0-----
6. 0-0-----	22. 0-0-----	38. 0-0-----	54. 0-0-----
7. -00-----	23. -00-----	39. -00-----	55. -00-----
8. 000-----	24. 000-----	40. 000-----	56. 000-----
9. ---0-----	25. ---0-----	41. ---0-----	57. ---0-----
10. 0---0-----	26. 0---0-----	42. 0---0-----	58. 0---0-----
11. --0-0-----	27. --0-0-----	43. --0-0-----	59. --0-0-----
12. 00-0-----	28. 00-0-----	44. 00-0-----	60. 00-0-----
13. -00-0-----	29. -00-0-----	45. -00-0-----	61. -00-0-----
14. 0-00-----	30. 0-00-----	46. 0-00-----	62. 0-00-----
15. --000-----	31. --000-----	47. --000-----	63. --000-----
16. 0000-----	32. 0000-----	48. 0000-----	64. 0000-----

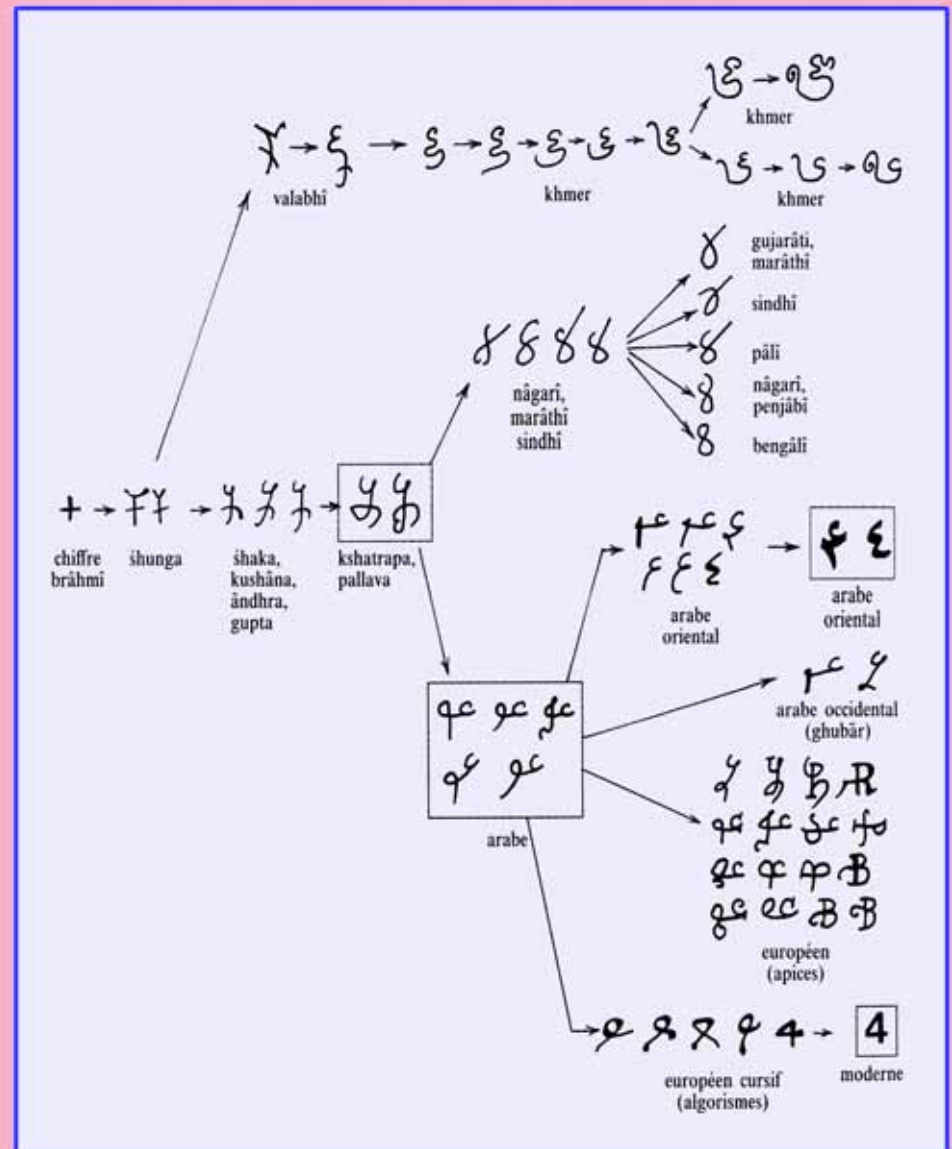
The *prastāra* for six syllables from Pingala's *Chandasāstra*

- Pingala's treatise *Chhandashāstra* or the science of verse meters (a few centuries BCE, Pingala being perhaps the brother of the famous grammarian Panini).
- Notation of verse meters for verses with various numbers of syllables (6 in this case).
- Syllables are light (*laghu*) or heavy (*guru*). The system of notation, spelling out every possible combination of light and heavy syllables, is identical to the modern binary notation of numbers used in computers.

Earliest inscriptions (first centuries BCE and AD): numerals without decimal place value. See for instance how in the first column, 40 is formed by repeating the symbol for 20 twice. There is also no symbol for 0 and no mathematical concept of zero.

	Kharoṣṭhī				Brāhmī				
	SAKA PARTHIAN KUṢĀNA	AŚOKA Inscriptions	NĀNĀCHĀT Inscriptions	NĀSIK Inscriptions	SAKA PARTHIAN KUṢĀNA	AŚOKA Inscriptions	NĀNĀCHĀT Inscriptions	NĀSIK Inscriptions	
1			—	—	80	3333		∞	
2			=	=	90				
3			≡	≡	100	∟1	21	∩	
4	X	+	∴∴	∴∴	200	∟∟	22	∩	
5	IX		∴∴	∴∴	300	∟∟∟	23		
6	X	∞∞	∴	∴	400		24		
7	X		?	?	500		25		
8	XX			∴∴	700		26		
9			?	?	1000		∟	∩	
10	∟		∞∞∞∞	∞∞	2000			∩	
20	3		0	0	3000			∩	
30					4000		∴	∩	
40	33			∟	6000		∴∴		
50	∟33	B.∞			8000			∩∴	
60	333		∟		10,000		∴∞		
70	∟333			∴	20,000		∴∞		

- It is well established that so-called “Arabic numerals” originated in India. Their evolution has been traced to the Brâhmî script of Mauryan times.
- The Webster dictionary gives the synonym of “Hindu-Arabic numerals.”
- All Indian numerals are also ultimately derived from Brâhmî numerals (except for Tamil, which had a different system using letters).

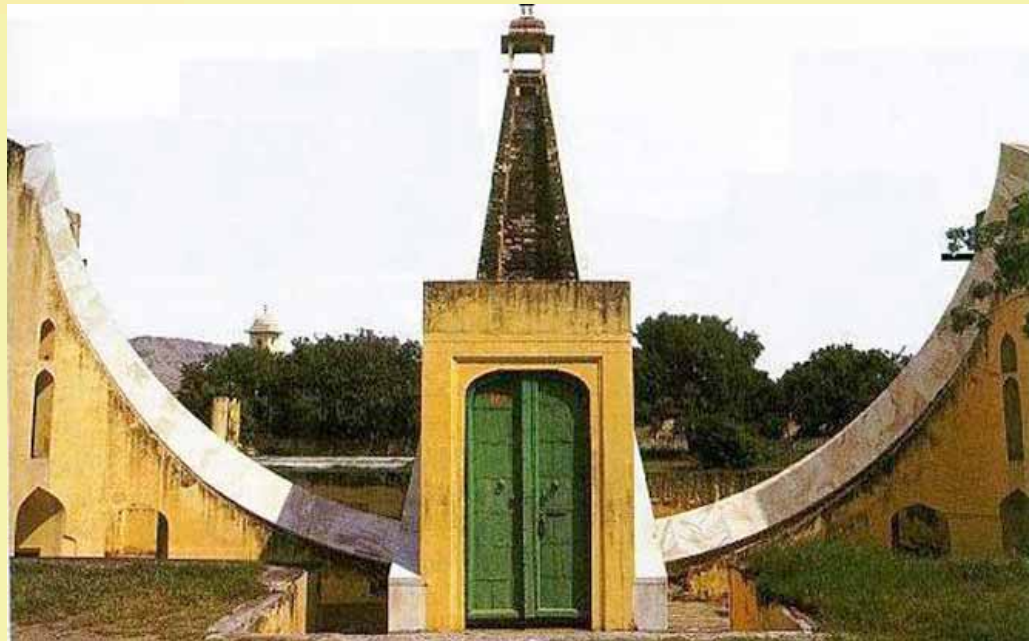


*Evolution of the numeral 4
from Brahmi to the modern
(by Georges Ifrah)*



Ghati yantra bowl floating in water

- *Ghati yantra*, a type of water clock: the bowl, with a small hole at its bottom, sinks after 24 mn (a unit of time called *ghati*, equal to $1/60^{\text{th}}$ of a day).
- Ancient texts refer to various other devices (gnomons, sun dials etc.) which have disappeared, but point to a long tradition of observation.
- (*Below:*) A sun dial (Jantar Mantar, Jaipur, 18th century). Such massive structures are found only in recent times; ancient observatories must have consisted of simple implements made of wood.



Indian time scale

← **compare** →

Judeo-Christian time scale

Satya: 1,728,000 years
Tretâ: 1,296,000 years
Dvâpara: 864,000 years
Kali: 432,000 years

Chaturyuga : 4,320,000 years
Duration of a “day of Brahmâ” =
one *kalpa* or 1,000 *chaturyuga* =
4.32 billion years.

Anno mundi (year of the
world’s creation):

- 3761 BC (Judaism)
- 4004 BC (Christianity)

“The Hindu religion is the only one of the world's great faiths dedicated to the idea that the Cosmos itself undergoes an immense, indeed an infinite, number of deaths and rebirths. It is the only religion in which the time scales correspond, no doubt by accident, to those of modern scientific cosmology. Its cycles run from our ordinary day and night to a day and night of Brahma, 8.64 billion years long. Longer than the age of the Earth or the Sun and about half the time since the Big Bang. And there are much longer time scales still.”

U.S. astronomer Carl Sagan, *Cosmos*

Ancient Indians conceived the infinity of time and space:

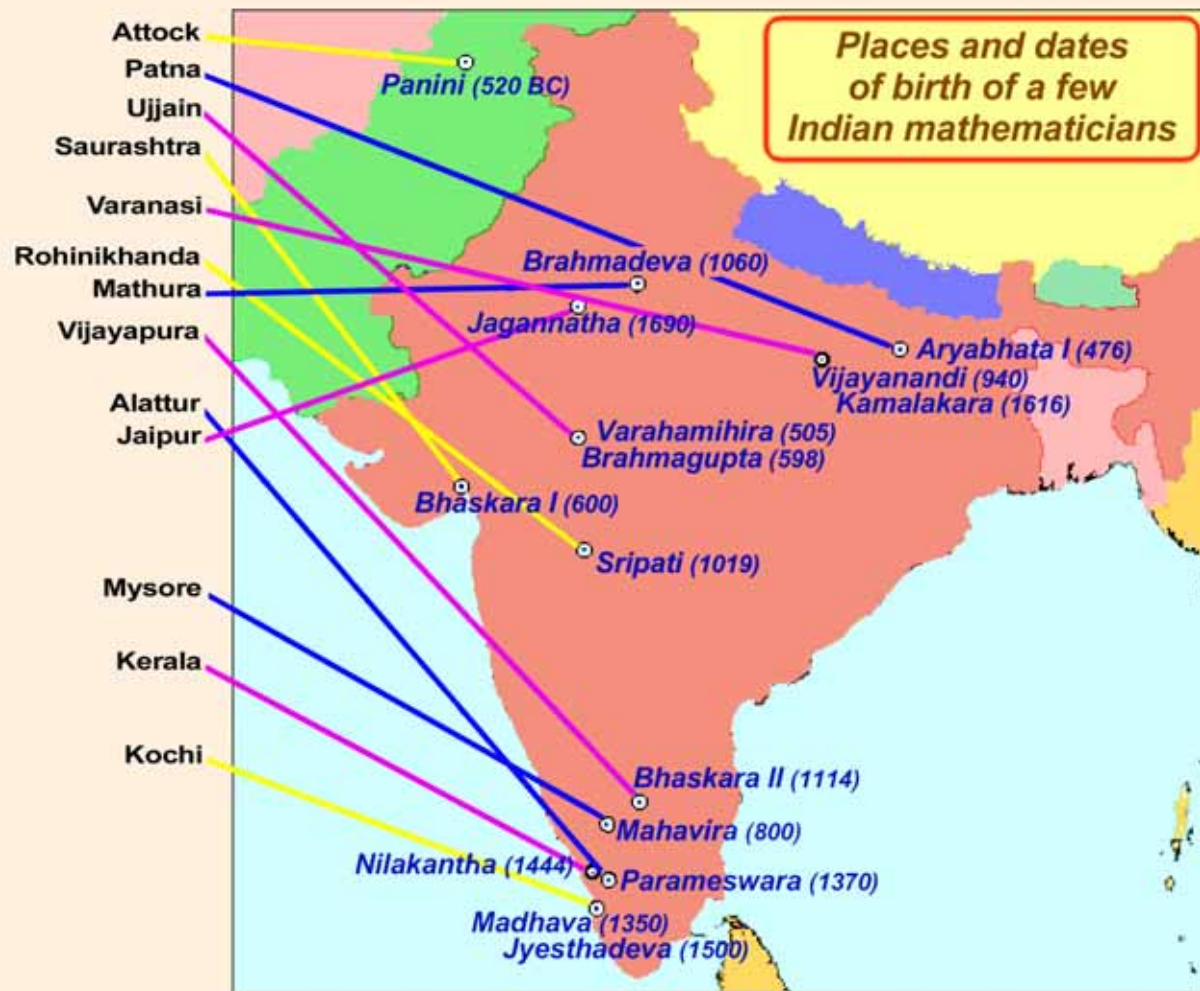
- **Cyclic time.**
- **Limitless space — Bhâskara I: “The sky is beyond limit; it is impossible to state its measure”.**
- **The concept of infinity underlies much of later Indian science: Brahmagupta first spelt out the mathematical definition of infinity.**
- **S. Ramanujan: “The man who knew infinity” is the title of one of his biographies.**



Concept of evolution:

- The notion of *Dashavatar* (10 incarnations of the divine consciousness) contains the seed of the concept of evolution: the first body is a fish, the second an amphibian, the third a mammal, the fourth half-man half-animal, the fifth a short man, etc. (later stages reflecting a spiritual evolution).
- A conceptualization or intuition of the truth expressed by Darwinian evolution.

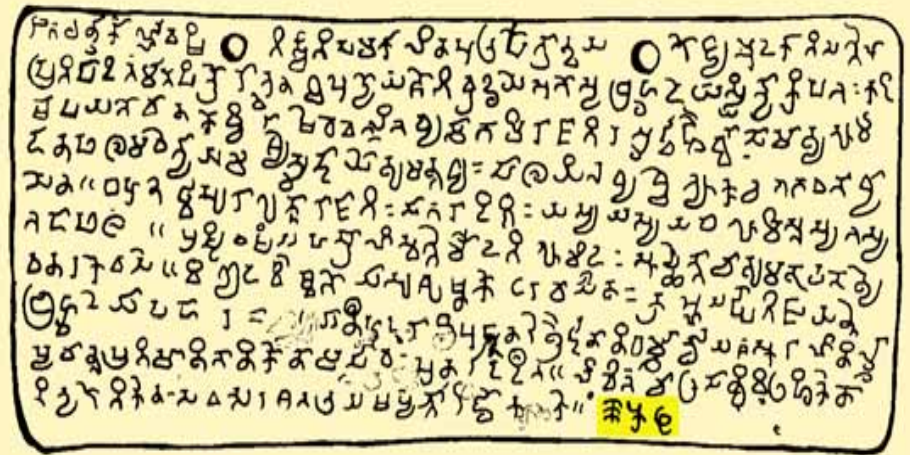
**3. Highlights of the Siddhantic Period
(from the 5th century CE):
the golden age of Indian mathematics
and astronomy**



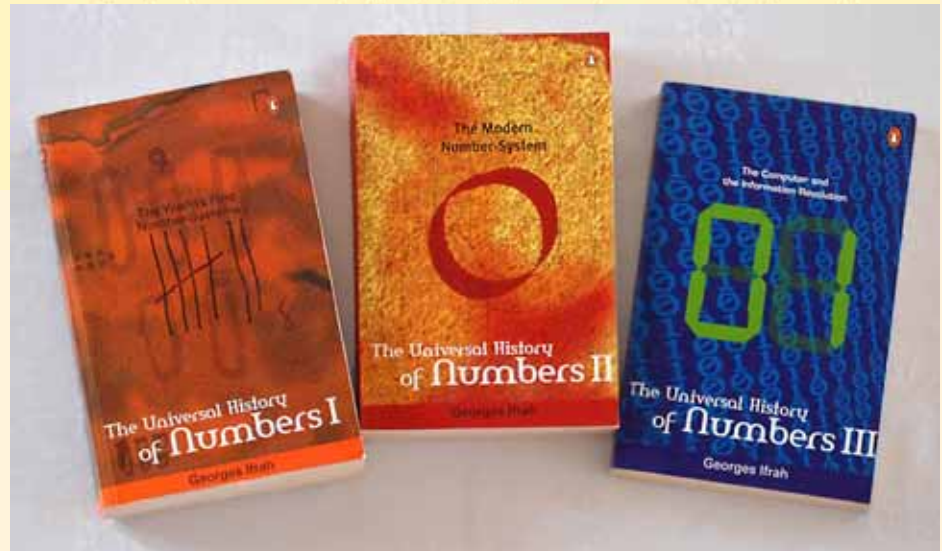
Early Indian scientists

- This map (adapted from the website of St. Andrews University, Scotland) lists the main figures of early Indian science. (The exact place or epoch of many of them remains uncertain).
- Note the shift to the South, especially Karnataka and Kerala, after the 12th century.

The first known inscription with a decimal place-value notation (Sankheda, Gujarat, dated “346” Chhedi era, or AD 594): for the first time, “3” stands for hundreds, “4” for tens and “6” for units.



Georges Ifrah: *The Universal History of Numbers*, in 3 volumes. Volume 2 is mostly about India’s contributions to mathematics.



Testimonies from two French mathematicians:

- “The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.” — Laplace (early 19th century)
- “The Indian mind has always had for calculations and the handling of numbers an extraordinary inclination, ease and power, such as no other civilization in history ever possessed to the same degree. So much so that Indian culture regarded the science of numbers as the noblest of its arts.... A thousand years ahead of Europeans, Indian savants knew that the zero and infinity were mutually inverse notions....” — Georges Ifrah (1994)

Âryabhata was a brilliant scientist who lived at Kusumapura (probably today's Patna). In 499 CE, he wrote the *Âryabhatîya*, a brief but extremely important treatise of mathematics and astronomy, at the age of 23! A few highlights:

Âryabhata about the earth:

- The earth is a rotating sphere: the stars do not move, it is the earth that rotates.
- Its diameter is 1,050 *yojanas*. Its circumference is therefore $1050 \times 13.6 \times \pi = 44,860$ km, about 12% off. (1 *yojana* = 8,000 human heights)



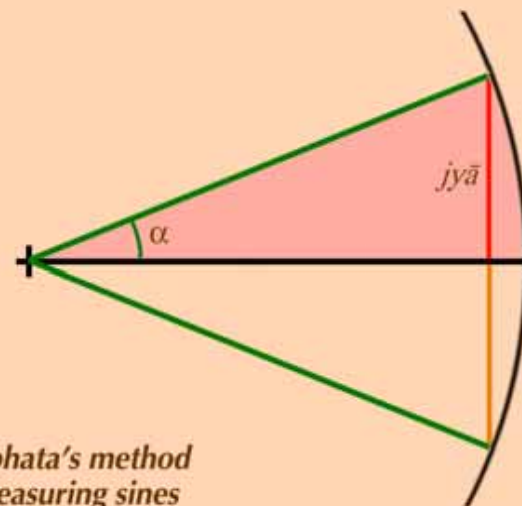
Âryabhata on eclipses:

“The moon eclipses the sun, and the great shadow of the earth eclipses the moon.”

(*Âryabhatîya*, IV.37)

*makhi bhakhi phakhi dhakhi ñakhi ñakhi
ñakhi hasjha skaki kiṣga ghakhi kighva |
ghlaki kigra hakya dhaki kica
sga śjha ñva kla pta pha cha kala-ardha-jyāḥ ||*

(Āryabhaṭīya, verse 12)



*Aryabhata's method
for measuring sines*

Āryabhata also ...

- Gave a table of sines (*above*): 24 values for the first quadrant in increments of 3.75° (all values correct to 3 or 4 significant figures).
- Proposed that $\pi = 62832 / 20000 = 3.1416$, adding that it was an “approximate” value.
- Gave for the first time the formula for the area of a triangle.
- Solved in integers linear indeterminate equations of the type $ax + c = by$ through the *kuttaka* or “pulverizing” method.

**“A thousand years ahead of Europeans,
Indian savants knew that the zero and
infinity were mutually inverse notions....”**
— Georges Ifrah

***Khachheda* means “divided by *kha*”;**

***Kha* (space) stands for zero;**

“Divided by zero” = infinity.

— Brahmagupta,

***Brahmasphuta Siddhânta* (628 CE)**

Foundations of modern algebra

- Solutions in integers for $Nx^2 + 1 = y^2$ were proposed by Brahmagupta (the *bhâvanâ* method).
- Mahâvira (9th century): approximate formulas for the area and circumference of an ellipse; work on permutations and combinations.
- Bhâskara II (12th century) developed the improved “cyclic method” (*chakravâla*); e.g., smallest solutions to $61x^2 + 1 = y^2$ are 226153980 & 1766319049. Lagrange reached the same solutions in the 18th century, but through a much longer method.
- Bhâskara II also worked on derivatives (of a sine function in relation to the velocity of planets).

The Kerala School

- Parameswara (1360-1455): detailed observations of eclipses over 55 years and consequent correction techniques; minute corrections for the position of planets after long periods of time.
- Infinite series, especially of trigonometric functions.
- Mâdhava (14th century): power series expansions for sine and cosine (correct to $1/3600^{\text{th}}$ of a degree).
- Infinite series of π (resulting in values with 10 correct decimals).
- Nîlakantha (15th century): formula for the sum of a convergent infinite geometric series. Concept of heliocentrism (building on Parameswara).

The calculus controversy:

“Indian mathematics had on Arabic mathematics, and ultimately, through Latin translations, on European mathematics, an influence that is considerably neglected.... If indeed it is true that transmission of ideas and results between Europe and Kerala occurred [about calculus], then the ‘role’ of later Indian mathematics is even more important than previously thought.... The work of Indian mathematicians has been severely neglected by Western historians.”

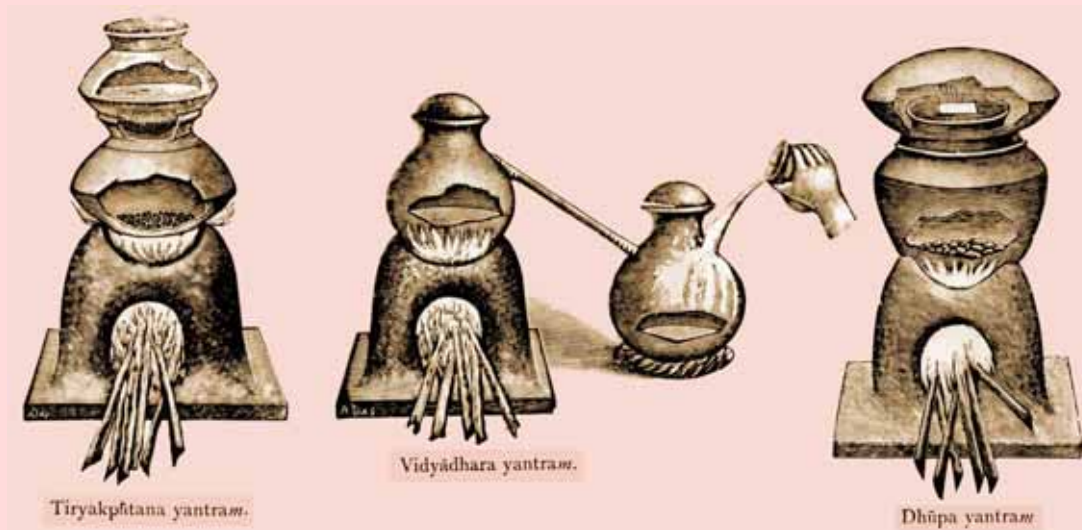
— British mathematician Ian Pearce

(www-history.mcs.st-andrews.ac.uk/history/Projects/Pearce/index.html)

Specificities of Indian scientific method

- Little interest in axiomatics, more in pragmatic methods.
- Nevertheless, a notion of proof (*upapatti*) does exist, especially in the commentaries (e.g. Jyesthadeva's *Yuktibhâsâ* of 1530 CE). Observed results must be validated.
- Great skill for developing efficient algorithms (the term's etymology has an Indian connection, through the Persian mathematician Al-Khwarizmi (800-847 CE). Especially visible in astronomical calculations.

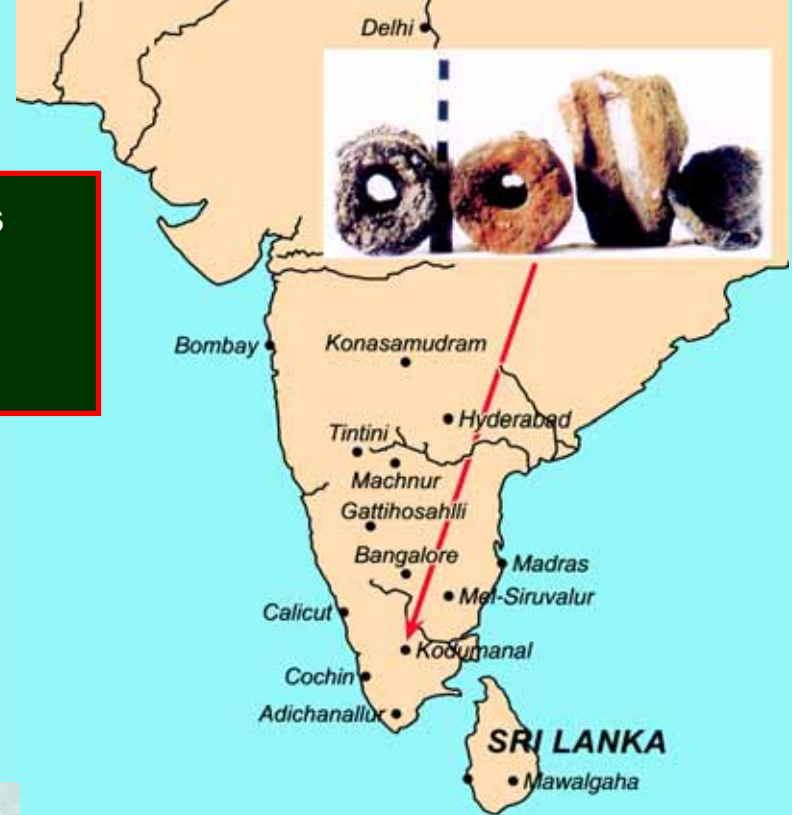
India was a pioneer in many technologies.



- Metallurgy (bronze, iron, wootz, zinc...)
- Pottery (ceramic, faience...)
- Pigments (painting, dyeing...)

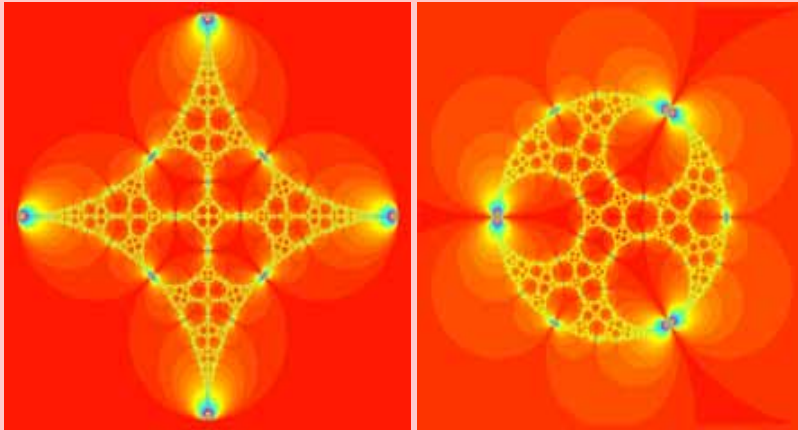
- Perfumes & cosmetics
- Medicines
- Chemistry and alchemy

Sites where wootz steel was prepared (note Kodumanal near Coimbatore).



The Delhi Iron Pillar: a thin layer of iron and phosphorus compound makes it rustproof, even after more than 1500 years.

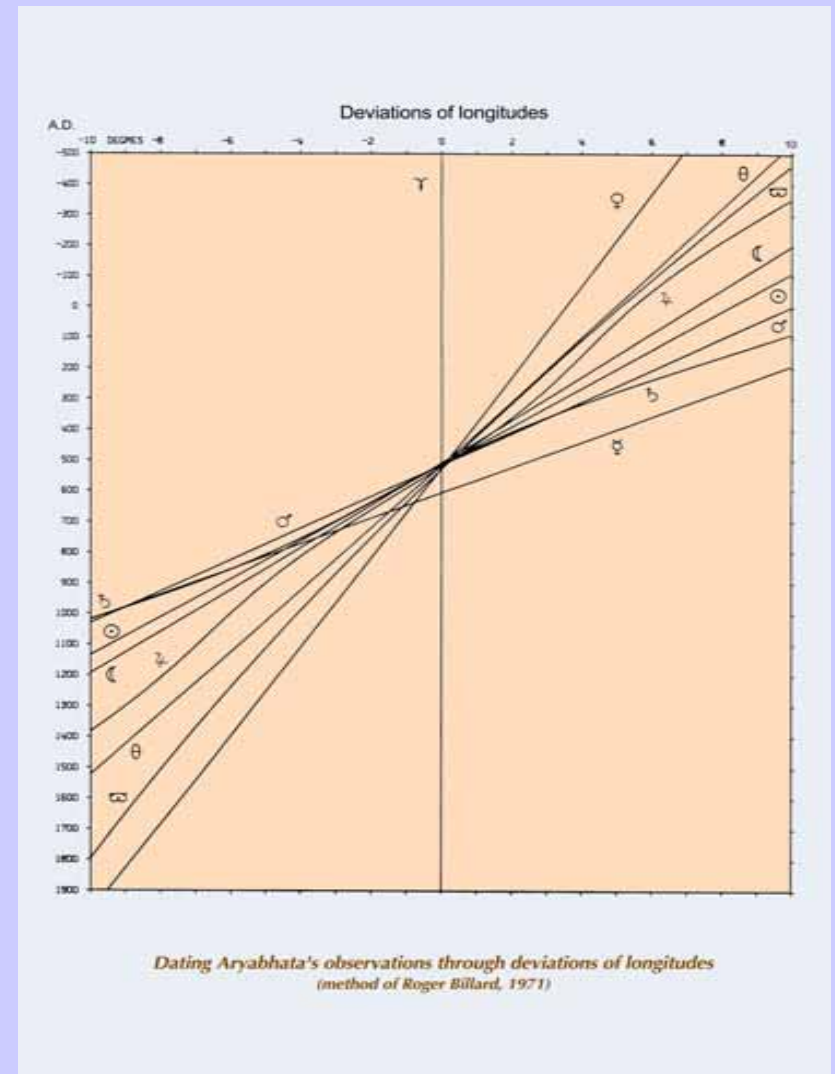
4. Using modern science to validate ancient traditions

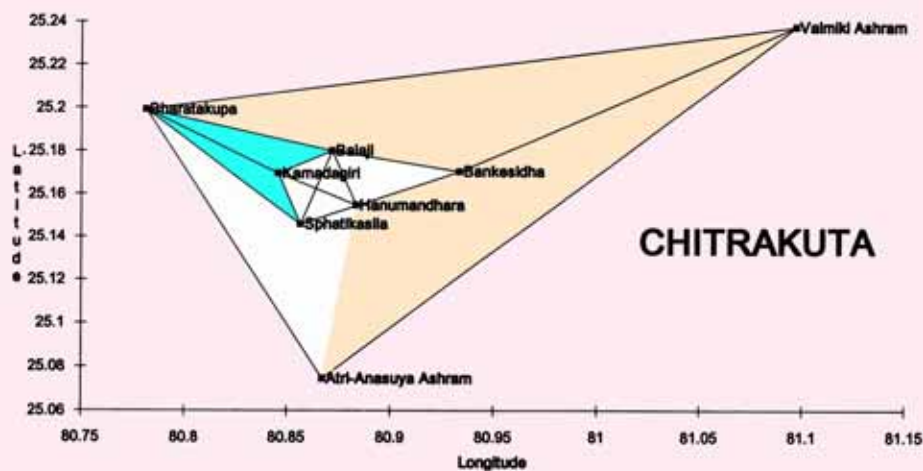


Indra's Pearls

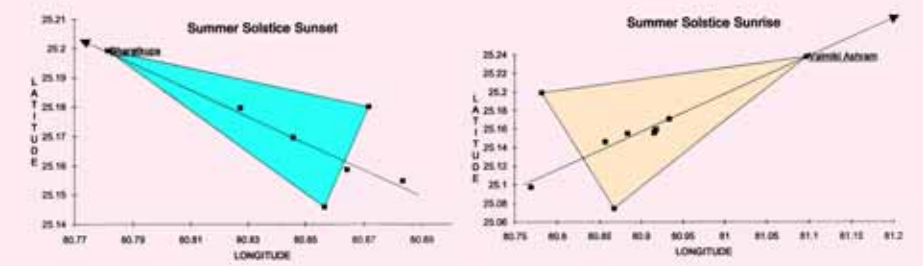
The *Avatamsaka Sûtra* (a Buddhist text, a few centuries BCE) depicts a network of pearls placed in heavens by Indra so that “in each pearl one can see the reflections of all the others, as well as the reflections within the reflections and so on.” Three U.S. mathematicians found that Indra’s pearls follow the arrangement of circles in a “Schottky group.” Two such structures are shown above.

In 1971 Roger Billard (a French mathematician and Sanskritist) did a statistical study of the deviations of longitudes of Âryabhata's observations of planets. He proved that the deviations were smallest around 500 CE, which is the date of the *Âryabhatîya*. This gave the lie to scholars who claimed that Âryabhata had borrowed his table of planetary positions from Babylonian astronomers.





U.S. astrophysicist McKim Malville, with Indian scholars, studied India's sacred geography: at Varanasi, Chitrakut, Vijayanagara and other sites, sacred sites (shrines, ashrams etc.) were oriented towards specific points of the sun's path across the year.



Chitrakut (associated with Lord Ram, who is often represented by a symbolic as an arrow): once mapped with GPS, ashrams form arrows that point to the sunrise and sunset on the summer solstice.

Varanasi: the 14 Aditya shrines precisely track the path of the sun through the year, month after month.



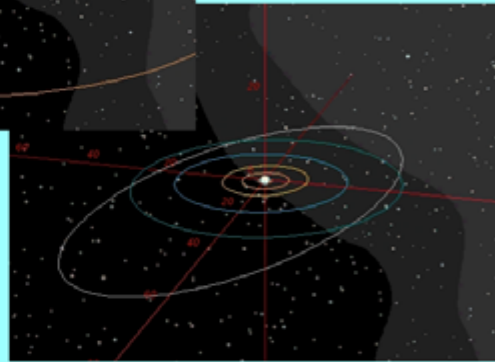
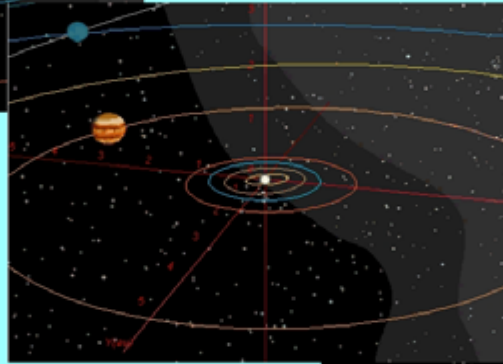
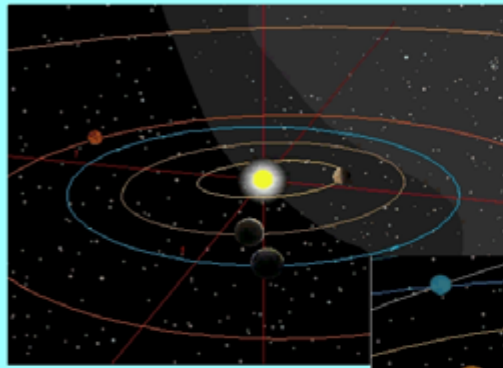
5. Unresolved Riddles

*“Swift and all beautiful art thou,
O Surya, maker of the light,
illuminating all the radiant realm”*
(Rig-Veda, 1.50.4)

Sayana comments thus on the above hymn :

*tathā ca smaryate yojanānām
sahasre dve dve sate dve ca yojane
ekena nimisārdhena kramamāna*
“Thus it is remembered :
[O Surya] you who traverse
2,202 *yojana* in half a *nimesa*.”

With a *yojana* of 13.6 km and a *nimesa* of $16/75^{\text{th}}$ of a second, this amounts to 280,755 km/s — just 6% from the speed of light (299,792 km/s): Coincidence? Intuition or inspired knowledge? Or some lost method of measurement? In any case, the fact should be noted.



- Āryabhata's "orbit of the sky" is 12,474,720,576,000 *yojanas* $\approx 16.8 \cdot 10^{13}$ km; therefore a diameter of $5.4 \cdot 10^{13}$ km, about 4,600 times our solar system's diameter.
- That is of the same order as 10 parsecs ($30 \cdot 10^{13}$ km), a distance where the Sun has a magnitude of 4.7, almost the limit of visibility to the human eye.

Āryabhata's "orbit of the sky"

Bhaskara I: "For us, the sky extends to as far as it is illumined by the rays of the Sun. Beyond that, the sky is immeasurable.... The sky is beyond limit; it is impossible to state its measure."

In other words, Āryabhata's "orbit of the sky" is of the same order as the distance "illumined by the sun". Very likely a coincidence; nevertheless, his conception of the scale of the universe deserves to be noted.

Conclusion:

The growth of a truly scientific spirit

- **Bhâskara II: “It is necessary to speak out the truth accurately before those who have implicit faith in tradition. It will be impossible to believe in whatever is said earlier unless every erroneous statement is criticized and condemned.”**
- **In Europe at the same time, science was strangled by religious dogmas (remember how Galileo was compelled to condemn the heliocentric Copernican system which he knew to be true: otherwise he would have been burned at the stake like Giordano Bruno).**
- **India’s scientific advances were relayed to Europe by the Arabs (who translated many Indian texts into Arabic and Persian) and contributed much to the birth of modern science. This contribution has not yet been fully assessed.**
- **In India, the study of ancient science remains neglected. For instance, a large number of ancient manuscripts in Kerala and Tamil Nadu remain untranslated and unpublished. History of science is not even recognized as a full-fledged academic discipline.**

A few Internet resources

- www-history.mcs.st-andrews.ac.uk/history/Indexes/Indians.html
(and all links on that page)
- http://en.wikipedia.org/wiki/Indian_mathematics
- http://en.wikipedia.org/wiki/List_of_Indian_mathematicians
- http://en.wikipedia.org/wiki/Indian_astronomy
- www.insa.ac.in/html/home.asp
- www-history.mcs.st-andrews.ac.uk/history/Projects/Pearce/index.html
- www.infinityfoundation.com/mandala/tks_essays_frameset.htm
- www.indianscience.org
(and in particular: www.indianscience.org/essays/essays.shtml)

A few suggested readings

(for those who wish to explore further)

General

- *A Concise History of Science in India*, ed. D. M. Bose, S. N. Sen & B. V. Subbarayappa (Indian National Science Academy, 1989)
- *Indian Science and Technology in the Eighteenth Century*, Dharampal (Other India Bookstore)
- *Encyclopaedia of Classical Indian Sciences*, ed. Helaine Selin & Roddam Narasimha (Universities Press, 2007)

Mathematics

- *Indian Mathematics and Astronomy – Some Landmarks*, S. Balachandra Rao (Jnana Deep Publications, 3rd ed. 2004)
- *The Universal History of Numbers: From Prehistory to the Invention of the Computer*, George Ifrah (Penguin Books, 2005, 3 vols.)
- *The Crest of the Peacock*, George Gheverghese Joseph (Penguin Books, 2000)
- *Geometry in Ancient and Medieval India*, T. A. Sarasvati Amma (Motilal Banarsidass, 1999)
- *Lilavati of Bhaskaracarya*, translators K. S. Patwardhan, S. A. Naimpally & S. L. Singh (Motilal Banarsidass, 2001)
- *Computing Science in Ancient India*, eds. T. R. N. Rao & Subhash Kak (Munshiram Manoharlal Publishers 2000)

Chemistry

- *A History of Hindu Chemistry*, Acharya Prafullachandra Ray (Shaibya Prakashan Bibhag, centenary edition 2002)
- *Chemistry and Chemical Techniques in India*, ed. B.V. Subbarayappa (Centre for Studies in Civilizations, vol. IV, part 1, 1999)

Astronomy

- *The Sūrya Siddhānta: a Textbook of Hindu Astronomy*, tr. & ed. by Ebenezer Burgess & Phanindralal Gangooly (Motilal Banarsidass, 2000)
- *History of Astronomy in India*, ed. S. N. Sen & K. S. Shukla (Indian National Science Academy, 1985)
- *Indian Astronomy – An Introduction*, S. Balachandra Rao (Universities Press, 2000)
- *Indian Astronomy – A Primer*, S. Balachandra Rao (Bhavan's Gandhi Centre of Science and Human Values, 2008)
- *Bhaskara I and His Astronomy*, S. Balachandra Rao (Rashtriya Sanskrit Vidyapeetha, 2003)
- *Aryabhata I and His Astronomy*, S. Balachandra Rao (Rashtriya Sanskrit Vidyapeetha, 2003)
- *Ancient Cities, Sacred Skies: Cosmic Geometries and City Planning in Ancient India*, J. McKim Malville & Lalit M. Gujral (IGNCA & Aryan Books International, 2000)
- *Jai Singh and His Astronomy*, Virendra Nath Sharma (Motilal Banarsidass, 1995)

Technology

- *History of Technology in India*, ed. A. K. Bag (Indian National Science Academy, 1997)
- *Delhi Iron Pillar: New Insights*, R. Balasubramaniam (Indian Institute of Advance Study & Aryan Books International, 2002)
- *Marvels of Indian Iron through the Ages*, R. Balasubramaniam (Rupa & Infinity Foundation, 2008)
- *The Rustless Wonder: A Study of the Iron Pillar at Delhi*, T. R. Anantharaman (Vigyan Prasar 1996)
- *India's Legendary Wootz Steel: An Advanced Material of the Ancient World*, Sharada Srinivasan & Srinivasa Ranganatha (NIAS & IISc, 2004, see <http://met.iisc.ernet.in/~rangu/>)
- *History of Iron Technology in India: From Beginning To Pre-Modern Times*, Vibha Tripathi (Rupa & Infinity Foundation, 2008)