

Rotation of Charged Quantum Systems

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I pass the lighted window of a shop
where perfume is sold. The window is
filled with pieces of colored glass,
tiny transparent bottles in delicate
colors, like bits of a shattered
rainbow.

Tom, in *The Glass Menagerie*
by Tennessee Williams

Preface

The work described in this dissertation was performed in the Cavendish Laboratory, Cambridge and in the laboratories of the National Bureau of Standards in Boulder, Colorado. It was supported in turn by the Science Research Council; Trinity College, Cambridge; and the U.S. Office of Naval Research. It has benefitted from lectures and practical experience gained at the CERN laboratories in Geneva, and by lectures given by the Department of Applied Mathematics and Theoretical Physics (DAMTP) in Cambridge. To all these institutions and the people they represent, my warmest thanks.

This dissertation had its origins in the Cavendish Laboratory, Cambridge, whilst I was a research student working under the supervision of Dr. J.R. Waldram on an experimental project to detect microwave radiation using superconducting weak links. In the Summer of 1979, Professor John Clarke visited from Berkeley. In a discussion he made the comment that, in order to be able to measure rotation velocity well using the London moment, one would need an 'electron' with a high mass. On December 27th, 1979 a solution presented itself as to how to achieve this: apply an electrostatic voltage. The electron mass is thereby increased by the relativistic mass-energy $-e V / c^2$. Robin Ball was the nucleus for wide-ranging discussions on this effect. When I brought to Professor A. Brian Pippard a proposal to use this effect to build a superconducting gyroscope with no moving parts, there emerged during our discussions the simple explanation for the operation of the gyroscope which is given at the beginning of chapter 3.

Results were obtained within a week of the start of preliminary experiments on the gyroscope, and I am grateful to Paul Booth and to Dave Swainston and the rest of the Low Temperature Physics workshop of the Cavendish Laboratory for the quality workmanship they invested in this and all experimental apparatus. The experiments on the gyroscope described in chapter 3 were performed between July 1981 and June 1982 in the Cryoelectronic Metrology Group of the National Bureau of Standards in Boulder, Colorado. The work has benefitted from the interest and

enthusiasm of the scientists in this group. The experiments were performed jointly by Dr. James E. Zimmerman and myself working in collaboration. I am most grateful to Jim for his support, his encouragement, for long and fruitful discussions on many topics, and for his infectious excitement to be working in physics. Much of my experience in experimental physics I owe to Jim. He has been my best teacher, and his lessons will never be forgotten.

Chapters 2 and 4 have their genesis in long, deep and enthusiastic discussions with Robin Rall in early 1980 on ideas based upon the thinking which led to the superconducting gyroscope. During discussions with Stuart Parkin about the electron's mass being modified by an electrostatic potential, he asked about the effect of the screening hole surrounding an electron. The search for an answer to his question led to the correction to the formula for the London moment of a rotating superconductor which is described in chapter 2. The dissertation has been influenced extensively by the penetrating comments of Professor Brian Josephson. My warmest thanks to all these scientists.

I am most grateful to those who read and made comments upon the text of this and earlier versions of this dissertation. Professors A.B. Pippard and G. Rickayzen have between them read the entirety of an earlier version, and the dissertation has been greatly influenced by their comments. John Waldram has also made comments on the earlier version and upon chapter 2 of this dissertation. Richard Kautz and Dave Bartlett have commented on chapter 3. John Gallop has made comments upon the entirety of this thesis.

Lastly I would like to record my thanks to colleagues in both Cambridge University and the National Bureau of Standards in Boulder, who have given me scientific and emotional support. It is impossible to name them all here. Olly Wright has kept the Low Temperature Physics group in the Cavendish sane for many years with his constant good humour and juggling skills. Sandy McCarthy, 'Boss and conference secretary', has likewise been the mainstay of the Cryoelectronics group at the Bureau of Standards. She has proven just how far one can ski if one

drinks enough. Finally, I wish to thank Peta Dunstan whose personal support and love of music and poetry have taught me so much.

This dissertation is written in my own words. Chapter 3 describes experiments performed jointly by Dr. James E. Zimmerman and myself working in collaboration. All of the other chapters describe my own work and contain nothing which is the outcome of work done in collaboration. The dissertation is not substantially the same as any other which I have submitted or am submitting for a degree or diploma or any other qualification at any other university.

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Summary

Rotation has a special place in mechanics. Unlike linear motion, which has meaning only in a relative sense, rotation has an absolute existence of its own. It is possible to determine one's angular velocity apparently without reference to systems outside; for example, the centrifugal or Coriolis forces could be measured. Rotation is the subject of study in this dissertation.

Chapter 1 of this dissertation develops a general formalism to describe the effects of rotation upon quantum-mechanical systems. The chapter brings out the close parallels which exist with the gauge theory of the electromagnetic interaction. In particular, a magnetic field is related to a rotation in the formalism.

Chapter 2 discusses a particular example of the relationship between a rotation and a magnetic field. The chapter gives a full quantum-mechanical analysis of the magnetic field (first discussed by London) which appears spontaneously when a sample of superconductor is set into rotation. There is a correction to London's accepted formula for the field. The correction involves the work-function of the superconductor.

Chapter 3 describes experiments on a superconducting gyroscope with no moving parts. The charges on a capacitor generate magnetic fields when set into rotation, and in operation the gyroscope measures these fields. The experiments were performed in collaboration with Dr. James E. Zimmerman, and were based upon proposals made earlier by the author. The experiments have verified the theory of the effect to an accuracy of 5%.

Chapter 4 discusses the possibility of building apparatus related to the superconducting gyroscope, but modified to be able to measure gravitational fields or gravity waves. It is demonstrated that, although there are effects which might be measured in principle, there are serious sources of noise which would mask the signal by several orders of magnitude in the practical instruments considered.

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To my parents