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Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model

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Abstract

Owing to the sporadic nature of demand for aircraft maintenance repair parts, airline operators perceive difficulties in forecasting and are still looking for superior forecasting methods. This paper deals with techniques applicable to predicting spare parts demand for airline fleets. The experimental results of 13 forecasting methods, including those used by aviation companies, are examined and clarified through statistical analysis. The general linear model approach is used to explain the variation attributable to different experimental factors and their interactions. Actual historical data for hard-time and condition-monitoring components from an airlines operator are used, in order to compare different forecasting methods when facing intermittent demand. The results confirm the continued superiority of the weighted moving average, Holt and Croston method for intermittent demand, whereas most commonly used methods by airlines are found to be questionable, consistently producing poor forecasting performance. We have, however, devised a new approach to forecasting evaluation, a predictive error-forecasting model which compares and evaluates forecasting methods based on their factor levels when faced with intermittent demand. A simple example is presented to illustrate the performance of the mathematical model. It is suggested that these findings may be applicable to other industrial sectors, which have similar demand patterns to those of airlines.

Scope and purpose

Demand forecasting is one of the most crucial issues of inventory management. Forecasts, which form the basis for the planning of inventory levels, are probably the biggest challenge in the repair and overhaul industry, as the one common problem facing airlines throughout the world is the need to know the short-term part demand forecast with the highest possible degree of accuracy. The high cost of modern aircraft and the expense of such repairable spares as aircraft engines and avionics constitute a large part of the total investment of many airline operators. These parts, though low in demand, are critical to operations and their unavailability can lead to excessive down time costs. Most airline materials managers deal with intermittent demand, which tends to be random and has a large proportion of zero values. In an effort to achieve this, the study has presented a model that could be of great benefit to airline operators and other maintenance service

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organisations. It will enable them to select in advance the appropriate forecasting method that better meets their cyclical demand for parts. This approach is consistent with the purpose of this study, which aims to compare different forecasting methods when faced with intermittent demand. © 2003 Elsevier Ltd. All rights reserved.

Keywords: Aircraft parts inventory; Forecasting intermittent demand; Aircraft maintenance engineering

1. Introduction

Demand forecasting is one of the most crucial issues of inventory management. Forecasts, which form the basis for the planning of inventory levels, are probably the biggest challenge in the repair and overhaul industry. One common problem facing airlines throughout the world is the need to forecast short-term part demand with the highest possible degree of accuracy. The high cost of modern aircraft and the expense of such repairable spares as aircraft engines and avionics contribute greatly to the considerable total investment of many airline operators. These parts, though low in demand, are critical to operations and their unavailability can lead to excessive down time costs. Most airline materials managers have to deal with intermittent demand, which tends to be random in time and of quantity, and has a large proportion of zero values. This topic has received extremely limited study within the aviation industry.

Forecasting the demand for parts with highly variable demand patterns is one of our main objectives since some of the traditional forecasting methods generate results with large error margins which result in many stock-outs. In our recent survey [1,2] of *airline operators* and *maintenance service organisations*, several common associated problems were identified in relation to developing service part forecasts. Firstly, it showed that most companies felt the service part forecasts they received were never realistic and as such they tried to outguess the forecast. Secondly, for those companies which did implement the material requirements planning (MRP) system, the service part forecast was loaded directly into the system without any review of forecasting error. Finally, the most commonly cited problems by firms in the development of reliable forecasts is the relatively high percentage of items which have experienced erratic or lumpy demand.

The main specific objectives in this research study are:

- To analyse the behaviour of different forecasting methods when dealing with lumpy and uncertain demand. We argue that the performance of a forecasting method should vary with the level and type of lumpiness (i.e., with the sources of lumpiness).
- Based on the forecast accuracy measurements and the results of their statistical analysis, a predictive model is developed successfully for each of the 13 forecasting methods analysed.

2. Literature review

This section involved two distinct focuses which will be presented separately. The first part will consider the variety of literature relating to the forecasting of intermittent demand. The second will discuss the practicalities of coping with such irregularities in demand in the airline industry.

2.1. Related research

In order to determine suitable spare part inventory levels, one must know about maintenance schedules and parts forecasting that feed into the MRP system. However, forecasting demand is reported as a major problem by some companies [1-3] which implement the MRP system. This is due to the nature of demand pattern variation in the airline sector, where such an intermittent demand produces a series of random values that appear at random intervals, leaving many time periods with no demand. The literature, that includes a relatively small number of proposed forecasting solutions to this demand uncertainty problem, can be found in [4-7]. Watson [8] found that the increase in average annual inventory cost resulted from the fluctuations in the forecast demand parameters of several lumpy demand patterns. The single exponential smoothing and the Croston methods are the most frequently used methods for forecasting low and intermittent demands [5,7]. In practice, the standard method for forecasting intermittent demand is the single exponential smoothing method, although some production management texts suggest the lesser-known alternative of the Croston method [5]. In their experimental study, Johnston and Boylan [9], after using a wide range of simulated conditions, observed an improvement in forecast performance using the Croston method when compared with the straight Holt (EWMA) method. On the other hand, Bartezzaghi et al. [4] in their experimental simulation found that EWMA appears applicable only with low levels of lumpiness. Willemain et al. [7] concluded that the Croston method is significantly superior to exponential smoothing under intermittent demand conditions. In addition, other methods such as the Wilcox [10] and Cox Process [11] methods were also used for forecasting intermittent demand. Both methods were shown to produce poor and unreliable forecasting results after being tested on the current research data and for that reason neither is included in the study. Zhao and Lee [12] concluded in their study that forecasting errors significantly increases total costs and reduce the service level within MRP systems. They argued that the selection of the forecasting methods has a significant impact on system performance. Their results showed that forecasting errors increase as variations in the demand increase. The fact that the existence of forecasting error increases the total cost of MRP systems has been reported in several other studies [12-15].

2.2. Airline forecasting systems review

Demand for air transport varies with time, as for many other goods. There are variations in daily, weekly, and annual demand which result in peaks at popular times. The competitive market in which most operators now work results in their trying to meet these peaks as far as reasonably possible. Aircraft availability has, therefore, to be maximised at these peaks and the maintenance fitted into a time slot when the planes are not required for commercial activities [16]. The context for this type of inventory forecasting in this review was based on our recent airline survey [1,2] and the findings were potentially helpful for carrying out further studies.

As it is general within the aviation industry that the usage patterns for most parts are unpredictable, and the forecasting of future demand was made by considering available maintenance contract information and looking at scheduled maintenance plans. Some companies prepare manual forecasts for expensive parts (rotables/repairables). Forecasts are generally based on past usage patterns such as flying hours or parts demand. On the other hand, the annual budgets for all departments in the technical division are taken into account, along with the number of forecasted flight hours/cycles, the number and type of checks planned for every aircraft, and the fleet size. With this data, the *purchasing department* tries to determine the quantity of stock necessary for the particular period. Alternatively, when new types of aircraft are introduced, the airframe and engine manufacturers normally provide a recommended spares provisioning list, based on the projected annual flying hours, which includes forecast usage information on new aircraft. Also the original equipment manufacturers provide overhaul manuals for the components fitted to the aircraft which enable an assessment of piece parts required based on reliability information and the specified components' operational and life limits.

Forecasting systems generally depend on the category of part used. This is due to aircraft parts being defined either as hard-time (HT), *predictable*, or condition-monitored (CM), *unpredictable* (both of these terms will be explained later in this paper). So our survey indicated that forecasting is a major problem, which could result in an inability to stock accurately, or the experience of a lack of "tie-in" to forecasts, especially in those companies that operate and support several major aircraft types and have large fleets.

3. Experimental framework

Thirteen forecasting methods, including those used by aviation companies have been considered in this study. Table 1 briefly summarises the philosophy of these techniques. The sample data used in this study consists of Fokker, BAe and ATR aircraft repairable parts which are unpredictable. The airline operator participating in this research kept records of weekly demand levels for each component which were then grouped in monthly and quarterly intervals of demand usage, a list of these is shown in Table 2. We limited the sample to parts that had valid demands (zero is a valid demand; missing is not). Only recurring demands, *hard-time* and *condition-monitoring*, which could be expected to occur routinely as a result of aircraft utilisation, were considered in this study. However, the data employed within this study exhibited trend, seasonal and irregular random fluctuation characteristics.

In this study we found it appropriate to use a *Microsoft Excel* spreadsheet, shown in previous studies [17,18], as a practical and sufficient tool for a limited budget. The time series data set was divided into an "initialisation" set and a "test" set. The initialisation set was then used to estimate any parameters and to initialise the method. Forecasts were then made for the test set. This procedure continued over the entire forecasting horizon. Accuracy measures are computed for the errors in the test set only.

Before making predictions using a forecasting method, we want to identify optimal values for *smoothing constant parameters* (α , β , and γ) that minimise the forecasting accuracy measures based on Theil's *U*-statistic range rules [19], therefore we applied the optimisation tool known as *solver*. The demand fluctuations are typically random and sporadic in this study, so choosing the right smoothing values was of vital importance. The issue of *smoothing constant parameters* itself is beyond the scope of this research study, however, for further explanation of this term refer to [19] as this investigation was based on crossed experimental factors rather than nested factors.

Table 1

A summary of selected forecasting methods

No.	Method	Abbreviation	Reference	Description
1	Additive winter	AW	[19,29]	Assumes that the seasonal effects are of con- stant size.
2	Multiplicative winter	MW	[19,29]	Assumes that the seasonal effects are pro- portional in size to the local de-seasonalized mean level.
3	Seasonal regression model	SRM	[30]	Is used in time series for modelling data with seasonal effects.
4	Component service life (replacement)	MTBR	[31]	Estimates of the service life characteristics of the part (MTBR & MTBO), derived from historical data (flying hours or number of landings).
5	Weighted calculation of demand rates	WCDR	[32]	The total demand for a given part during an experience period is divided by the total ac- tivity of the aircraft during the same period to give an average forecast rate.
6	Weighted regression demand forecasters	WRDF	[32]	Considers forecasts based on moving regres- sions in terms of flying hours.
7	Croston	Croston	[5]	Forecasting in circumstances of low and intermittent demand.
8	Single exponential smoothing	SES	[19]	Forecasting in circumstances of low and intermittent demand.
9	Exponentially weighted moving average	EWMA, Holt	[19,33]	An effective forecasting tool for time series data that exhibit a linear trend.
10	Trend adjusted exponential smoothing	TAES	[34]	Forecasting time series data that have a linear trend.
11	Weighted moving averages	WMA	[19]	A simple variation on the moving average technique that allows for just such weighting to be assigned to the data being averaged.
12	Double exponential smoothing	DES	[35]	Forecasting time series data that have a linear trend.
13	Adaptive-response-rate single exponential smoothing	ARRSES	[19]	Has an advantage over SES in that it allows the value of α to be modified in a controlled manner as changes in the pattern of data occur.

The following four environmental factors were included in the experiment: the seasonal period length, *SPL*; primary maintenance process, *PMP*; square coefficient of variation of demand, CV^2 and the average inter-demand interval, *ADI*. Since actual values for CV^2 and *ADI* were used, they are taken as covariate factors, whereas *SPL* and *PMP* are selected as categorical factors. The factor levels may be different from those of other research studies since, as this study is concerned with aviation maintenance, most variables were covariates rather than categorical variables. Table 3 summarises the four factors and a description follows.

A SL	A Summary of INLIN-UN WORKING A	•								
No.	Component description	Part number	Aircraft type	Quantity per aircraft	Fleet size	Maintenance processes	Period MTBO	Period MTBR	Repair TAT	Time series
-	Air conditioning unit	2203480-2	Fokker 100	2	17	HT	2000 FHs	854 FHs	20 MHs	95–99
0	Alternator unit	No 406-3	Fokker 27	2	16	HT	2500 FHs	1250 FHs	20 MHs	92–94
б	Battery—ultra pure	4078-8	Fokker 50	2	6	HT	1000 FHs	12 Weeks	5 MHs	95 - 00
4	Battery-distilled water	4608-1	Fokker 100	2	17	HT	1000 FHs	12 Weeks	5 MHs	94 - 00
S	Battery—lead acid	40678-2	ATR-72	1	S	HT	1000 FHs	12 Weeks	5 MHs	98-00
9	Brake assembly (heat pack)	AH 52220	Fokker 27	4	16	CM	750 FLs	700 FLs	20 MHs	89-95
7	Brake assembly (brake unit)	AH 52220	Fokker 27	4	16	CM	5000 FLs	1200 FLs	25 MHs	89-95
8	Brake assembly unit	AHA 2174-5	BAe 146	4	13	HT	9600 FLs	1500 FLs	24 MHs	66 - 06
6	Brake assembly unit	5011809-2	Fokker 100	4	17	CM	2500 FLs	2500 FLs	24 MHs	94–99
10	Brake assembly unit	5007996-1	Fokker 50	4	6	HT	3600 FLs	1200 FLs	18 MHs	95–99
11	Brake control valve	AC 61348	Fokker 27	2	16	HT	6600 FHs	3420 FHs	10 MHs	89–94
12	Combustion chamber	RK 49159A	Fokker 27	2×7	16	CM	1200 FHs	1200 FHs	28 MHs	92–95
13	DC Generator	30E02-21G1	Fokker 27	2	16	HT	2500 FHs	1104 FHs	28 MHs	92–94
14	Drag strut unit	200261001	Fokker 27	2	16	HT	12000 FLs	3620 FLs	30 MHs	89 - 94
15	Inverter assembly	1518-8-C	Fokker 27	2	16	HT	2700 FHs	617 FHs	17 MHs	92–94
16	Lock strut unit	200260001	Fokker 27	2	16	HT	12000 FLs	4510 FLs	45 MHs	89 - 94
17	Main undercarriage unit	200223001	Fokker 27	2	16	HT	12000 FLs	2882 FLs	250 MHs	89 - 94
18	Main wheel overhauled	5008131-5	Fokker 100	4	17	HT	2500 FLs	1007 FLs	12 MHs	92 - 00
19	Main wheel tyre changed	5008131-5	Fokker 100	4	17	CM	500 FLs	226 FLs	11 MHs	92 - 00
20	Main wheel overhauled	5007995-1	Fokker 50	4	6	HT	3500 FLs	1516 FLs	7.75 MHs	95 - 00
21	Main wheel tyre changed	5007995-1	Fokker 50	4	6	CM	700 FLs	316 FLs	4.45 MHs	95 - 00
22	Main wheel overhauled	AHA 1489	BAe 146	4	13	HT	1600 FLs	1245 FLs	10 MHs	00 - 06
23	Main wheel tyre changed	AHA 1489	BAe 146	4	13	CM	400 FLs	229^{a} FLs	4 MHs	00 - 06
24	Main wheel overhauled	AHA 1890	ATR-72	4	5	HT	1800 FLs	1600 FLs	6 MHs	96–96
25	Main wheel tyre changed	AHA 1890	ATR-72	4	5	CM	450 FLs	133 FLs	6 MHs	98-00
26	Maxaret anti-skid unit	AC 63538	Fokker 27	2×2	16	HT	4000 FLs	4000 FLs	16 MHs	89–94
27	Nose undercarriage unit	200490001	Fokker 27	1	16	HT	12000 FLs	3588 FLs	250 MHs	89 - 94
28	Nose undercarriage unit	201071001-3	Fokker 100	1	17	HT	20000 FLs	11495 FLs	220 MHs	00 - 96
29	Nose wheel overhauled	5008133-1	Fokker 100	2	17	HT	1250 FLs	1000 FLs	4.5 MHs	93-00
30	Nose wheel tyre changed	5008133-1	Fokker 100	2	17	CM	250 FLs	122 FLs	3.5 MHs	93 - 00
31	Nose wheel overhauled	5007998	Fokker 50	2	6	HT	2500 FLs	906^{a} FLs	3.5 MHs	95 - 00
32	Nose wheel tyre changed	5007998	Fokker 50	2	6	CM	500 FLs	232 FLs	2.5 MHs	95 - 00
33	Nose wheel overhauled	AHA 1349	BAe 146	2	13	HT	1100 FLs	1100 FLs	5 MHs	00 - 06
34	Nose wheel tyre changed	AHA 1349	BAe 146	2	13	CM	275 FLs	161 FLs	2 MHs	00 - 06
35	Nose wheel overhauled	AH 54474	ATR-72	7	5	HT	1200 FLs	527 FLs	6 MHs	98-00
36	Nose wheel tyre changed	AH 54474	ATR-72	2	5	CM	300 FLs	135 FLs	3 MHs	98-00
H	HT, hard-time; CM, condition mc	monitored; FHs, flying hours; FLs, flying landings; MHs, man hours; TAT, turn around time.	ying hours; FL	s, flying land	ings; M	Hs, man hours	; TAT, turn a	round time.		

^aOverhaul at every 5th tyre change; MTBO, mean time between overhaul; MTBR, mean time between removal.

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Table 2

Table 3 Environmental factors, (---) indicates covariates factor

Factor	Description	Levels	Values	Units
SPL	Seasonal period length	3	4,12,52	Quarters, months, weeks
PMP	Primary maintenance processes	2	HT, CM	Flight hours or landings
CV^2	Square coefficient of variation	_	0.0-1.65	
ADI	Average inter-demand interval		1.0 - 20.83	_

3.1. SPL

This is the number of periods for which the demand pattern is forecasted (Table 4). The most popular seasonal period length used by aviation companies [1,2] is either a monthly or a quarterly one. The longer the time horizon of the forecasts in terms of the number of time buckets, ¹ whereby a weekly *SPL* has more, i.e. is longer than, e.g. a monthly *SPL*, the greater the chance that established patterns and relationships will change, thereby invalidating forecasts. Thus forecasting accuracy decreases as the time horizon as thus defined increases [20].

3.2. PMP

The three Primary Maintenance Processes recognised by the UK CAA [21] are hard-time, on-condition, and condition-monitoring. In general terms, both the first two involve actions directly concerned with preventing failure, whereas the last does not. The condition-monitoring process is expected to lead to preventative action if necessary. The categories of component maintenance are as follows:

Hard-time, *HT*: This is defined as a preventive process in which the known deterioration of an item is restored to an acceptable level by the maintenance actions carried out at periods related to time in service. This time may be calendar time, number of cycles, or number of landings. The prescribed actions normally include servicing, full or partial overhaul, or replacement.

On-condition, *OC*: This is a preventive Primary Maintenance Process. It requires that an appliance or part be periodically inspected or checked against some appropriate physical standard to determine whether it can continue in service. The purpose of the standard is to remove the unit from service before failure during normal operation. These standards may be adjusted based on operating experience or tests, as appropriate, in accordance with a carrier's approved reliability program or maintenance manual.

Condition-monitoring, CM: This is not a preventive process, having neither hard-time nor on-condition elements, but one in which information on items, obtained by taking relevant measures on condition-related variables, is analysed, and interpreted on a continuing basis as a means of implementing corrective procedures. Models of decision aspects of condition-monitoring have concentrated upon cases where a direct measure of wear was available, such as the thickness of a brake

¹Time bucket refers to the units of time into which the planning horizon is divided, and is usually represented in weeks, days or months.

ADI Cr^3 ADI Cr^3 ADI Cr^3 ADI Cr^3 1 Air conditioning unit Fokker 10 25545 0.4466 Erratic 11111 0.5023 Iummy 1000 0.3987 Iummy 3 Battery-distilled water Fokker 20 1.5534 0.4305 Erratic 11067 0.000 0.3987 Iummy 3 Battery-distilled water Fokker 10 1.9314 0.25545 Erratic 1000 0.3987 Iummy 10000 0.3987 Iummy 4 Battery-distilled water Fokker 10 1.9314 0.2355 Erratic 1000 0.3987 Iummy 10000 0.1315 Iummy 6 Brake assembly (mate unit) Fokker 70 2.9300 0.2435 Erratic 1.0379 0.3081 Iummy 10000 0.3515 Iummy 10000 0.3515 Iummy 7 Brake ontrol valve Fokker 70 2.9300 0.3735 Erratic 1.0493 5556 <	No. Component description	Aircraft type	Weekly period	period	Demand categorisation	Monthly	Monthly period	Demand categorisation	Quarter	Quarterly period	Demand categorisation
Air conditioning unit Fokker 1000 2.5545 0.4466 Erratic 1.111 0.5029 Immitternt 1.0000 0.3987 1 Batteryulits pure Fokker 70 0.5363 Lumpy 1.0000 0.3987 1 Batteryulits pure Fokker 100 0.3140 Lumpy 1.0000 0.3987 1 Batteryulits pure Fokker 1000 1.314 0.2385 Erratic 1.0665 0.3864 Lumpy 1.0000 0.3861 Brake assembly unit Fokker 1.000 2.5617 0.1401 Erratic 1.2000 0.3861 Lumpy 1.0000 0.3861 Brake assembly unit Fokker 1.000 2.593 0.2333 Erratic 1.2000 0.3861 Lumpy 1.0000 0.1311 1.0000 0.3861 Lumpy 1.0000 0.1311 Erratic 1.2000 0.3861 Lumpy 1.0000 0.1311 Erratic 1.2000 0.3861 Lumpy 1.0000 0.1311 Erra			ADI	CV^2		IDI	CV^2		ADI	CV^2	
$\label{eq:active} To the the text of text of$	1 Air conditioning unit	Fokker 100	2.5545	0.4466	Erratic	1.1111	0.5029	Intermittent	1.0000	0.3987	Lumpy
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	2 Alternator unit	Fokker 27	2.7636	0.3905	Erratic	1.1667	0.3653	Lumpy	1.0000	0.1886	Lumpy
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	3 Battery—ultra pure	Fokker 50	1.6358	0.2140	Erratic	1.0000	0.2397	Lumpy	1.0000	0.0321	Lumpy
Batteylead acid ATR-72 19697 0.2655 Erratic 11000 0.1281 1 Batteylead acid ATR-72 19697 0.2655 Erratic 11000 0.1381 1 Brake assembly unit Fokker 27 5.353 0.1401 Erratic 112000 0.3561 Lumpy 10000 0.2353 Brake assembly unit Fokker 27 5.353 0.2343 Erratic 112000 0.3701 11701 0.0303 0.1711 10000 0.2353 11711 10000 0.2353 11714 10000 0.1711 10000 0.2353 11701 0.2353 11714 10000 0.1711 10000 0.1711 10000 0.1711 10000 0.1711 10000 0.1711 10000 0.1711 10000 0.1711 1172 0.2375 Erratic 117000 0.1711 1172 1172 0.2375 Erratic 117000 0.1711 1172 1172 0.2375 Erratic 117000 0.1711 1172 1171 <td>4 Battery-distilled water</td> <td>Fokker 100</td> <td>1.9314</td> <td>0.2851</td> <td>Erratic</td> <td>1.0685</td> <td>0.3084</td> <td>Lumpy</td> <td>1.0000</td> <td>0.1156</td> <td>Lumpy</td>	4 Battery-distilled water	Fokker 100	1.9314	0.2851	Erratic	1.0685	0.3084	Lumpy	1.0000	0.1156	Lumpy
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Brake assembly unit BAe 146 2.8108 0.2479 Erratic 1.2766 0.3516 Lumpy 1.0400 0.3333 I Brake assembly unit Fokker 50 0.3564 Bratic 1.2000 0.3902 Lumpy 1.0400 0.3333 I <td< td=""><td></td><td>Fokker 27</td><td>2.7923</td><td>0.2325</td><td>Erratic</td><td>1.2000</td><td>0.3661</td><td>Lumpy</td><td>1.0000</td><td>0.2285</td><td>Lumpy</td></td<>		Fokker 27	2.7923	0.2325	Erratic	1.2000	0.3661	Lumpy	1.0000	0.2285	Lumpy
		BAe 146	2.8108	0.2479	Erratic	1.2766	0.3516	Lumpy	1.0500	0.3233	Lumpy
$ \begin{array}{llllllllllllllllllllllllllllllllllll$		Fokker 100	2.5966	0.2564	Erratic	1.2000	0.4711	Lumpy	1.0400	0.2370	Lumpy
	_	Fokker 50	1.9037	0.3348	Erratic	1.0300	0.3902	Lumpy	1.0000	0.1711	Lumpy
$ \begin{array}{cccc} Combustion chamber Fokker 27 2.9420 0.3789 Erratic 1.3429 0.5074 Smooth 1.0667 0.4676 1 \\ DC Generator Fokker 27 2.5172 0.3050 Erratic 1.0000 0.182 Lumpy 1.0000 0.0533 1 \\ Inverter assembly unit Fokker 27 5.3793 0.4257 Erratic 1.0000 0.182 Lumpy 1.0000 0.1353 1 \\ Lock strut unit Fokker 27 5.3793 0.461 Erratic 1.9714 0.5212 Smooth 1.1000 0.5263 1 \\ Main wheel overhauled Fokker 10 2.0000 0.4763 Erratic 1.9714 0.5212 Smooth 1.1000 0.1353 1 \\ Lock strut unit Fokker 27 5.3000 0.4661 Erratic 1.9714 0.5212 Smooth 1.1000 0.1353 1 \\ Main wheel overhauled Fokker 100 1.1720 0.4609 Lumpy 10010 0.2202 Lumpy 1.0000 0.1353 1 \\ Main wheel overhauled Fokker 100 1.1720 0.4609 Lumpy 1.0000 0.5203 1 \\ Main wheel overhauled Fokker 50 1.3119 0.4063 Lumpy 1.0000 0.5204 Lumpy 1.0000 0.1352 1 \\ Main wheel overhauled BAe 146 1.1720 0.3032 Erratic 1.0200 0.2004 Lumpy 1.0000 0.1352 1 \\ Main wheel overhauled Fokker 50 1.3119 0.4063 Lumpy 1.0000 0.2004 Lumpy 1.0000 0.1352 1 \\ Main wheel verchauled Fokker 50 1.3119 0.4063 Lumpy 1.0000 0.2004 Lumpy 1.0000 0.1352 1 \\ Main wheel verchauled Fokker 10 1.1720 0.4503 Lumpy 1.0000 0.3022 Lumpy 1.0000 0.1766 1 \\ Main wheel verchauled Fokker 10 1.2303 0.3027 Erratic 1.0246 0.2302 Lumpy 1.0000 0.1796 1 \\ Main wheel verchauled Fokker 10 1.2303 0.3027 Erratic 1.2348 0.3478 1 \\ Nose undercarriage unit Fokker 27 7.7250 0.3542 Erratic 2.3226 0.3186 Erratic 1.2632 0.3030 1 \\ Nose wheel verchauled Fokker 10 1.2307 0.4357 Lumpy 1.0000 0.1796 1 \\ Nose wheel verchauled Fokker 10 1.2307 0.4357 Erratic 2.3226 0.3186 Erratic 1.3344 0.3478 1 \\ Nose wheel verchauled Fokker 10 1.2500 0.3542 Erratic 2.3226 0.3186 Erratic 1.3640 0.4397 1 \\ Nose wheel verchauled Fokker 10 1.2500 0.3542 Erratic 2.3226 0.3186 Erratic 1.3348 0.3478 1 \\ Nose wheel verchauled Fokker 10 1.2500 0.3542 Erratic 2.3226 0.3186 Erratic 1.3640 0.4397 1 \\ Nose wheel verchauled Fokker 50 1.3252 Erratic 2.0300 0.3629 Lumpy 1.0000 0.0439 1 \\ Nose wheel verchauled Fokker 50 1.3254 0.3186 Erratic 1.0200 0.2399 Lumpy 1.0000 0.0439 1 \\ Nose wheel $	11 Brake control valve	Fokker 27	4.1644	0.4633	Erratic	1.4792	0.6414	Smooth	1.0000	0.8090	Intermittent
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	12 Combustion chamber	Fokker 27	2.9420	0.3789	Erratic	1.3429	0.5074	Smooth	1.0667	0.4676	Lumpy
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	13 DC Generator		2.5172	0.3050	Erratic	1.0625	0.3517	Lumpy	1.0000	0.0539	Lumpy
Inverter assembly unit Fokker 27 1.3220 0.2873 Erratic 1.0000 0.2182 Lumpy 1.0000 0.1353 1 Lock strut unit. Fokker 27 1.3220 0.24061 Erratic 1.9714 0.5212 Smooth 1.1000 0.5263 1 Main wheel overhauled Fokker 100 1.1720 0.44061 Erratic 1.7561 0.4598 Erratic 1.0900 0.4301 1.1000 0.5263 1 Main wheel overhauled Fokker 100 1.1720 0.44063 Erratic 1.9714 0.5212 Lumpy 1.0000 0.1152 1 Main wheel verhauled Fokker 50 1.3119 0.4063 Erratic 1.4667 0.4498 Erratic 1.0000 0.1152 1 Main wheel verhauled Fokker 50 1.3119 0.4403 Lumpy 1.0100 0.2202 Lumpy 1.0000 0.1152 1 Main wheel verhauled Fokker 50 1.3119 0.4063 Lumpy 1.0000 0.2004 Lumpy 1.0000 0.1173 1 Main wheel verhauled BAc 146 1.1793 0.5031 Internittent 1.0246 0.2037 Lumpy 1.0000 0.1173 1 Main wheel verhauled BAc 146 1.1793 0.5031 Internittent 1.0246 0.2372 Lumpy 1.0000 0.1178 1 Main wheel verhauled BAc 146 1.1793 0.5031 Internittent 1.0246 0.3032 Lumpy 1.0000 0.1178 1 Main wheel verhauled BAc 146 1.1793 0.5031 Internittent 1.0246 0.3032 Lumpy 1.0000 0.1178 1 Nose undercarriage unit Fokker 27 7.725 0.3342 Erratic 2.3260 0.3332 Lumpy 1.0000 0.1766 1 Nose undercarriage unit Fokker 100 7.3125 0.0307 Erratic 1.0200 0.3332 Lumpy 1.0000 0.1766 1 Nose wheel verhauled Fokker 100 7.3125 0.14010 0.3332 Lumpy 1.0000 0.1766 1 Nose wheel verhauled Fokker 100 1.2500 0.4597 Lumpy 1.0100 0.2333 Erratic 1.346 0.3478 Nose wheel verhauled Fokker 100 1.2500 0.3542 Erratic 2.1600 0.1633 Pratic 1.346 0.3478 Nose wheel verhauled Fokker 100 1.2500 0.3523 Erratic 2.1600 0.2639 Lumpy 1.0000 0.2033 Nose wheel verhauled Fokker 50 1.5594 0.3458 Nooth 1.9149 0.6111 Smooth 1.3043 0.4276 Nose wheel verhauled Fokker 50 1.5594 0.3458 Nooth 1.9149 0.6111 Smooth 1.3043 0.4276 Nose wheel verhauled Fokker 50 1.5594 0.3458 Nooth 1.9149 0.6111 Smooth 1.3043 0.4276 Nose wheel verhauled Fokker 50 1.5594 0.3458 Nooth 1.9149 0.6111 Smooth 1.3043 0.4276 Nose wheel verhauled Fokker 50 1.5594 0.3458 Nooth 1.9149 0.6111 Smooth 1.3043 0.4276 Nose wheel verhauled Fokker 50 1.5594 0.3552 Erratic 1.0200 0.2599 Lumpy 1			5.3793	0.4727	Erratic	1.8000	0.9444	Smooth	1.2000	0.6233	Intermittent
Lock strut unit Fokker 27 5.9600 0.4061 Erratic 1.9714 0.5212 Smooth 1.1000 0.5263 1 Main wheel overhauled Fokker 27 5.9600 0.4605 Erratic 1.9714 0.5212 Smooth 1.1000 0.5263 1 Main wheel overhauled Fokker 100 2.0000 0.4763 Erratic 1.0900 0.5297 Intermittent 1.0000 0.4328 Main wheel byre changed Fokker 50 3.3256 0.3080 Erratic 1.0100 0.2024 Lumpy 1.0000 0.4328 1.000 0.4751 1.000 0.4751 1.000 0.3325 1.0000 0.3325 1.0000 0.3325 1.0000 0.3325 1.0000 0.3752 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.3765 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.3769 1.0000 0.476		Fokker 27	1.3220	0.2873	Erratic	1.0000	0.2182	Lumpy	1.0000	0.1353	Lumpy
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Nose undercarriage unit Fokker 100 7.3125 0.0710 Erratic 2.1600 0.1633 Erratic 1.3846 0.3478 Nose wheel overhauled Fokker 100 7.3125 0.0710 Erratic 2.1600 0.1633 Erratic 1.3846 0.3478 Nose wheel overhauled Fokker 100 1.2500 0.4597 Lumpy 1.0100 0.2839 Lumpy 1.0000 0.2033 Nose wheel overhauled Fokker 50 1.2500 0.4597 Lumpy 1.0100 0.2839 Lumpy 1.0000 0.2033 Nose wheel overhauled Fokker 50 1.5204 0.3522 Erratic 1.0100 0.2839 Lumpy 1.0000 0.6439 Nose wheel tyre changed Fokker 50 1.5294 0.3552 Erratic 1.0200 0.2559 Lumpy 1.0000 0.0439 Nose wheel tyre changed BAe 146 1.5548 0.4560 Erratic 1.0200 0.2559 Lumpy 1.0000 1.6522 Nose wheel tyre changed BAe 146 1.5648		Fokker 27	7.7250	0.3542	Erratic	2.3226	0.3186	Erratic	1.2632	0.3030	Lumpy
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Nose wheel overhauled BAe 146 7.2297 0.5148 Smooth 2.8182 1.2321 Smooth 1.6800 1.6522 5 Nose wheel tyre changed BAe 146 1.5648 0.4660 Erratic 1.0413 0.3253 Lumpy 1.0000 0.1281 1 Nose wheel overhauled ATR-72 20.8333 0.0000 Erratic 5.8000 0.1111 Erratic 2.5000 0.1111 1 Nose wheel tyre changed ATR-72 2.5000 0.3185 Erratic 5.8000 0.1111 Erratic 2.5000 0.2636 1		Fokker 50	1.5294	0.3522	Erratic	1.0200	0.2599	Lumpy	1.0000	0.0439	Lumpy
Nose wheel tyre changed BAe 146 1.5648 0.4660 Erratic 1.0413 0.3253 Lumpy 1.0000 0.1281 Nose wheel overhauled ATR-72 20.8333 0.0000 Erratic 5.8000 0.1111 Erratic 2.5000 0.1111 Nose wheel tyre changed ATR-72 2.5000 0.3185 Erratic 1.2083 0.3387 Lumpy 1.0000 0.2636	32 Nose wheel overhauled	BAe 146	7.2297	0.5148	Smooth	2.8182	1.2321	Smooth	1.6800	1.6522	Smooth
Nose wheel overhauled ATR-72 20.8333 0.0000 Erratic 5.8000 0.1111 Erratic 2.5000 0.1111 Nose wheel tyre changed ATR-72 2.5000 0.3185 Erratic 1.2083 0.3387 Lumpy 1.0000 0.2636		BAe 146	1.5648	0.4660	Erratic	1.0413	0.3253	Lumpy	1.0000	0.1281	Lumpy
ATR-72 2.5000 0.3185 Erratic 1.2083 0.3387 Lumpy 1.0000 0.2636		ATR-72	20.8333	0.0000	Erratic	5.8000	0.1111	Erratic	2.5000	0.1111	Erratic
	35 Nose wheel tyre changed	ATR-72	2.5000	0.3185	Erratic	1.2083	0.3387	Lumpy	1.0000	0.2636	Lumpy

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pad in a braking system [22], those measurements are related stochastically to the condition of the component.

3.3. Demand size and average time interval factors, CV^2 , ADI

Demand pattern classification, another distinguishing feature of this study, is when the time series vary systematically according to their inherent variability. In this study, the data demand patterns explicitly consider both the demand pattern and the size of demand when it occurs, They are classified into four categories [23] based on modified Williams' criteria [24]. In this case, the categorisation schemes have the following characteristics:

The " $ADI \leq x, CV^2 \leq y$ " condition tries effectively to test for stock keeping units, which are not very intermittent and erratic (i.e. faster moving parts or parts whose demand pattern does not raise any significant forecasting or inventory control difficulties).

The " $ADI > x, CV^2 \le y$ " condition tests for low demand items or intermittent demand patterns with constant, or more generally, no highly variable demand sizes (i.e. not very erratic).

The " $ADI > x, CV^2 > y$ " condition tests for lumpy demand items, lumpy demand may be defined as a demand with great differences between each period's requirements and with a great number of periods with zero requests.

The " $ADI \le x, CV^2 > y$ " condition tests for erratic (irregular) demand items with rather frequent demand occurrences (i.e. not very intermittent).

In all these cases, x denotes the average inter-demand interval, cut-off value (ADI = 1.32) which measures the average number of time periods between two successive demands and y, the corresponding square coefficient of variation, cut-off value ($CV^2 = 0.49$), that is equal to the standard deviation of period requirements divided by the average period requirements.

4. Experimental results and analysis

Analysis of variance, ANOVA, is used to explain the variation attributable to the various experimental factors and their interactions. Table 5 presents a summary of the general linear model, GLM, results and reports the *p*-values for each of the main factors and their two-way interactions. For all methods, the third- and fourth-order interactions were found to be insignificant and as such were eliminated from this analysis. A natural logarithm transformation of the dependent variable and some independent variables were used to overcome the problem of non-constancy of error variance in linear models, see [25].

Owing to space limitations, the MAPE technique will be the only method reported in this paper due to its advantageous performance with intermittent demand [26,27]. GLM output results for all accuracy measuring techniques are not presented here. However, they are available upon request from the authors.

In this study, we have devised a new approach to forecasting evaluation. This new model compares and evaluates the forecasting methods based on their factor levels. The description and function of the model will be discussed later in this paper. Firstly, however, the experimental results need further clarification through an ANOVA of the experimental factor-design employing the forecast errors (measured in terms of MAPE) as the dependent criterion, as shown in Table 5, for the overall

Table 5 A summary of unbalanced ANOVA (GLM) results for forecasting factors (*p*-values) $\sim \log MAPE$

Factors	ARRSES	AW	Croston	DES	Holt	MTBR	MW	SES	SRM	TAES	WCDR	WMA	WRDF
SPL	0.000 ¹	0.006 ¹	0.000 ¹	0.000^{1}	0.000^{1}	0.000^{1}	0.000 ¹	0.000^{1}	0.017 ⁵	0.000 ¹	0.000^{1}	0.000^{1}	0.000 ¹
PMP	0.000^{1}	0.004^{1}	0.000^{1}	0.000^{1}	0.001 ¹	0.000^{1}	0.129	0.000^{1}	0.001^{1}	0.000^{1}	0.000^{1}	0.001^{1}	0.000^{1}
CV^2	0.076	0.049 ⁵	0.051^{5}	0.022^{5}	0.089	0.038^{5}	0.000^{1}	0.018^{5}	0.124	0.017^{5}	0.090	0.002^{1}	0.016^{5}
ADI	0.000^{1}	0.0215	0.000^{1}	0.000^{1}	0.000^{1}	0.000^{1}	0.000^{1}	0.000^{1}	0.051^{5}	0.000^{1}	0.000^{1}	0.000^{1}	0.000^{1}
$SPL \times PMP$	0.261	0.461	0.079	0.049 ⁵	0.250	0.351	0.969	0.063	0.297	0.019 ⁵	0.433	0.239	0.025^{5}
$SPL \times CV^2$	0.286	0.726	0.063	0.326	0.089	0.216	0.686	0.125	0.464	0.096	0.187	0.275	0.265
$SPL \times ADI$	0.000^{1}	0.0165	0.000^{1}	0.000^{1}	0.000^{1}	0.001^{1}	0.000^{1}	0.000^{1}	0.039 ⁵	0.000^{1}	0.000^{1}	0.000^{1}	0.000^{1}
$PMP \times CV^2$	0.017^{5}	0.0215	0.001^{1}	0.001^{1}	0.012^{5}	0.000^{1}	0.032^{5}	0.001^{1}	0.0135	0.000^{1}	0.001^{1}	0.008^{1}	0.001^{1}
$PMP \times ADI$	0.006^{1}	0.418	0.002^{1}	0.000^{1}	0.063	0.010^{5}	0.260	0.001^{1}	0.037^{5}	0.001 ¹	0.004^{1}	0.032 ⁵	0.001^{1}
$CV^2 \times ADI$	0.246	0.591	0.072	0.043 ⁵	0.176	0.119	0.000^{1}	0.139	0.573	0.088	0.100	0.646	0.263

Superscript¹ indicates significance at the 0.01 level. Superscript⁵ indicates significance at the 0.05 level. No superscript denotes a lack of significance at both levels (at 0.01 and 0.05 level).

experimental, main factor and two-way interaction effects. In addition, Table 6a-c give the significant coefficients of the fitted GLMs.

Generally, in Table 5 all factors, except for *PMP* and CV^2 , have significant main effects on all methods in terms of the *MAPE* accuracy measure. *PMP* is significant for most methods, except MW which is not significant (p = 0.129). CV^2 was also found to be significant for most methods, for ARRSES, Holt, WCDR and SRM (p = 0.076, 0.089, 0.090 and 0.124, respectively), however, they are not significant. This shows that all the experimental factors, *SPL*, *PMP*, CV^2 and *ADI*, have a significant effect on the *MAPE* measure of the forecasting methods' error accuracy.

Table 5 also indicates that the interaction $SPL \times PMP$ for all methods is not significant except for the DES, TAES and WRDF methods where they are found to be significant at the 0.05 level. The $SPL \times CV^2$ is not significant for all methods, while the interaction $SPL \times ADI$ is significant for all methods at the 0.01 levels except for the AW and SRM methods which were at the 0.05 level. The interaction $PMP \times CV^2$ was significant for all methods too, at 0.01 level for methods (ARRSES, AW, Holt, MW and SRM) the rest of the methods were at 0.01 levels. The $PMP \times ADI$ interaction was not significant for the AW and MW methods, however, it was marginally significant (p=0.063) for the Holt method while for all other methods it was significant for either level.

Finally, the interaction of $CV^2 \times ADI$ was found to be only significant with the two methods DES and MW, while for all other methods it was insignificant.

As reported above, the significant terms were similar for most methods except AW, DES, Holt and MW. In the next few sections, we examine, for each method, the effects of each factor on the accuracy of the MAPE measurement.

4.1. The effect of seasonal period length

The coefficients of *SPL* increase with seasonal period length (quarterly, monthly and weekly respectively) as expected (refer to Section 3.1). Quarterly *SPL* reduces the forecasting error on average for all methods, compared with a monthly *SPL* (Table 6a). The interaction of $SPL \times PMP$

Table 6

Methods	CV^2	2 fficient	ADI Coefficient	Seasonal	period lengt	th coeffici	ent levels	PMP coef	ficients
	Coe	meient	Coefficient	SPL = 4	SPL	= 12	SPL = 52	HT	СМ
. ,				ed by MAPE t					
ARRSES	0.74		0.56057	-1.1206		0162	1.1044	0.5024	-0.502
AW	1.41		0.53050	-1.0565	-0.0		1.0935	0.4721	-0.472
Croston	0.70	87	0.57775	-1.1567	-0.0		1.1647	0.5454	-0.545
DES	0.93		0.54884	-1.1020		0823	1.0197	0.6264	-0.626
Holt	0.67		0.46677	-1.0382	-0.0		1.1317	0.4431	-0.443
MTBR	0.91		0.41238	-0.8178		0282	0.7896	0.7171	-0.717
MW	5.80		3.08110	-2.9568	-0.8		3.8309	0.7837	-0.783
SES	0.92		0.52308	-0.9888	-0.0		1.0544	0.5763	-0.576
SRM	1.10		0.24350	-0.6740	-0.0		0.7349	0.5535	-0.553
TAES	0.96		0.60377	-1.1090		097	1.0993	0.6476	-0.647
WCDR	0.73		0.55559	-1.1078)239	1.0839	0.6445	-0.644
WMA	1.25		0.43977	-1.2226		1537	1.0689	0.4444	-0.444
WRDF	0.99	07	0.57909	-1.0286	0.0)467	0.9819	0.5858	-0.5853
Methods	$SPL \times PMP$)					SPL imes 0	CV^2	
	SPL = 4		SPL = 12		SPL = 52				
	HT	СМ	HT	СМ	HT	CM	SPL = 4	SPL = 12	SPL = 52
(b) Coeffic	ients of the fit	ted models M	APE						
ARRSES	-0.09472	0.09472	-0.01096	0.01096	0.10568	-0.10	568 0.4447	0.2449	-0.6896
AW	-0.02867	0.02867	-0.06592	0.06592	0.09459	-0.09	0.2345	0.3191	-0.5536
Croston	-0.11346	0.11346	-0.01245	0.01245	0.12591	-0.12	591 0.5553	0.3522	-0.9075
DES	-0.12285	0.12285	-0.04211	0.04211	0.16496	-0.16	496 0.3558	0.2938	-0.6496
Holt	-0.06814	0.06814	-0.04321	0.04321	0.11135	-0.11	0.5189	0.4330	-0.9519
MTBR	-0.08958	0.08958	-0.00487	0.00487	0.09445	-0.09	0.5423	0.2007	-0.7430
MW	-0.03050	0.03050	-0.03360	0.03360	0.06410	-0.06	410 0.9360	0.9800	-1.9160
SES	-0.11804	0.11804	-0.03021	0.03021	0.14825	-0.14	825 0.4762	0.3767	-0.8529
SRM	-0.11628	0.11628	0.01389	-0.01389	0.10239	-0.102	0.5260	0.2184	-0.7444
TAES	-0.16313	0.16313	-0.00152	0.00152	0.16465	-0.16	465 0.5664	0.3498	-0.9162
WCDR	-0.04181	0.04181	-0.05031	0.05031	0.09212	-0.092	0.5115	0.3283	-0.8398
WMA	-0.08706	0.08706	-0.02345	0.02345	0.11051	-0.11	0.2965	0.3788	-0.6753
WRDF	-0.16215	0.16215	0.00564	-0.00564	0.15651	-0.15	651 0.4699	0.1540	-0.6239
Methods	$SPL \times AL$	DI		PMP	$\times CV^2$		$PMP \times ADI$		$CV^2 \times AD$
	SPL = 4	SPL = 12	SPL =	52 <i>HT</i>	С	'M	HT	СМ	
(c) Coeffic	ients of the fit	ted models M	APE						
ARRSES	0.5570	-0.14659		041 -0.71	191 0.	.7191	-0.13735	0.13735	0.12300
AW	0.5929	-0.12110				.8247	-0.05192	0.05192	-0.16590
Croston	0.5998	-0.16728				.8967	-0.13635	0.13635	0.16576
DES	0.6248	-0.21030				.9427	-0.18090	0.18090	0.20820
Holt	0.4685	-0.09842				.7066	-0.08697	0.08697	0.13504
MTBR	0.3807	-0.10488				.4015	-0.13481	0.13481	0.17340
MW	2.1575	0.4226				.9702	-0.49100	0.49100	-4.03490
SES	0.4440	-0.0837				.9353	-0.15025	0.15025	0.14544
SRM	0.3139	-0.08784				.9347	-0.12494	0.12494	0.14760
TAES	0.6169	-0.18018				.1749	-0.15855	0.15855	0.17320
WCDR	0.5394	-0.14253				.0067	-0.15086	0.15086	0.18040
WMA	0.7095	-0.27225				.7667	-0.10272	0.10272	-0.04670
WRDF	0.5519	-0.14560				.9377	-0.15726	0.15726	0.11480

Significant interactions shown in bold.

is found to be only significant with the DES, TAES and WRDF methods. Table 6b shows that for a quarterly, *SPL* the effect of *PMP* on accuracy measurement will be reduced for the Hard-Time and increased by Condition-Monitoring components and similarly for DES and TAES methods on a monthly *SPL* basis. Conversely, for WRDF method the effect of *PMP* on accuracy measuring will be increased for the Hard-Time and reduced by Condition-Monitoring components. Finally, for a weekly *SPL*, the effect of *PMP* on accuracy measuring will be increased for the Hard-Time and reduced by Condition-Monitoring components for these methods.

4.2. The effect of Primary Maintenance Processes

Primary Maintenance Processes, *PMPs*, show that hard-time *HT* components have more effect in increasing the forecasting error measured by *MAPE*, compared with condition-monitoring *CM* (Table 6a). The interaction of *PMP* with both CV^2 and *ADI* was found to be significant for most methods, Table 6c indicating that as CV^2 increases the effect of *PMP* on *MAPE* will be reduced for the Hard-Time and increased by condition-monitoring components for all methods. And it is a similar story for *ADI* as the average inter-demand interval increases the effect of *PMP* on *MAPE* will be reduced with *HT* and increased for *CM* components for all methods except for the AW, Holt and MW methods.

4.3. The effect of squared coefficient of variation on demand

From Table 6a the coefficient of CV^2 is positive and similar for all methods. However, Table 6c shows that for the DES method this positive coefficient is augmented by a positive coefficient of $CV^2 \times ADI$, i.e. as ADI increases this effect increases, while for the MW method this positive coefficient is offset by a negative coefficient of $CV^2 \times ADI$, i.e. as ADI increases this effect reduces. This can be explained by saying that as ADI increases thus the number of time periods with zero demand will increase, in this case the performance of MW will be reduced.

4.4. The effect of average inter-demand interval

In this study, the coefficient of ADI is positive, i.e. as ADI increases the impact of reducing the forecasting method performance will be higher (higher MAPE). Table 6a shows the coefficient effect to be much higher for the MW (3.08) while on average (0.50) for the rest of the methods. The interaction of $SPL \times ADI$ shows that the coefficient of ADI decreases as SPL increases (Table 6c), but the benefit is usually less as ADI increases, indicating that for very lumpy demand, the advantage of a quarterly SPL is reduced, i.e. with weekly SPL, the ADI improves the forecasting methods' performance. On the other hand, with a quarterly SPL, the ADI displays a minor impact on the forecasting performance. Hence, the relevance of ADI depends on the seasonal period length selected.

5. Predictive error-forecasting model, PEFM

In trying to establish which forecast method is best in any particular situation, it is necessary to have statistical information available, particularly with regard to the size of the forecasting errors.

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5		Forecasting Techniques	MAPE	Forecast Error Predicted							
6		WMA	4.43	84.07	EVALUATIN	IG FORECAST M	IETHOD A	CCURACY		? ×	
7		SRM	4.60	99.29							
8		DES	4.67	106.49		ccuracy Measures PE, ME, MPE, MSE, I	DMCE CCEY	MAPE		OK	
9		Holt	4.81	122.26	(1140) 114	· c, mc, mc, mc, moc, i	KHOC, 55C)				
10		MTBR	4.81	122.89		eriod Length		12		Cancel	
11		WRDF	4.83	124.78	(4, 12, 52)						
12		TAES	4.84	125.91	Primary Ma	intenance Processa	45	HT			
13		WCDR	4.84	126.38	(HT, CM)			l			
14		ARRSES	4.85	127.43							
15		Croston	4.85	127.87	Square Co	efficient of Variation	n on Demand	0.39			
16		SES	4.86	129.05							
17		AW	4.87	130.32	Average In	nter-demand Interv	al	2.76			
18 19		MW	INVALID DATA	•							
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Fig. 1. Proposed predictive error forecasting model, dialog box.

The predictive error forecasting model, as its name suggests, is thus a model whereby the forecast error predicted for any component selected together with its most efficient parameters is found. By entering in the prepared dialog-box, the forecasting factors of a specific item [e.g. *SPL*, *PMP*, CV^2 and *ADI*, (see Fig. 1)], the adapted Visual Basic for Applications² runs through a series of complex calculations of prepared coefficients. In order to proceed with the explanation of the predictive error-forecasting model and its relevant example, we first start by defining the GLM mathematical model definition.

5.1. Models' mathematical properties derivations

The GLM is a straightforward extension of simple and multiple regression allowing for more than one independent variable. The objective of GLM is the same as that of multiple regression; that is, we want to use the relationship between a response (dependent) variable and factor (independent) variables to predict or explain the behaviour of the response variable. In multiple regression, the

 $^{^{2}}$ The programme and the table of routines contained in the VBA functions module are available upon request from the authors.

coefficient attached with each independent variable should measure the effect and average change in the response variable associated with changes in that independent variable, while all other independent variables remain fixed. This is the standard interpretation for a regression coefficient in a multiple regression model. So this means that all models have been "regression" models where the response variable is related to quantitative independent variables. The multiple linear regression is specified as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_m x_m + \varepsilon, \tag{1}$$

where y is the dependent variable, x_j , j = 1, 2, ..., m, represent m different independent variables, β_0 is the intercept (value when all the independent variables are 0), β_j , j = 1, 2, ..., m, represent the corresponding m regression coefficients, ε is the random error, usually assumed to be normally distributed with mean zero and variance σ^2 .

The basic model, a generalised general linear model, has the form of Eq. (1), which is then further specified by the use of two key properties:

- *Property one*: Unequal cell, the standard analysis of variance calculations for multifactor designs can only be used if we have *balanced data*, which occurs if the number of observations for all factor-level combinations, usually called *cells*, are all equal.
- *Property two*: Models including both dummy and interval-independent variables (continuous variables) sometimes called the covariate are a simple extension.

The obvious difference between this model and a regression model (Eq. (1)) is the absence of *independent variables*. This is where dummy variables come in. The model using dummy variables is

$$y_{ij} = \mu z_0 + \alpha_1 z_1 + \alpha_2 z_2 + \dots + \alpha_t z_t + \varepsilon_{ij}$$

$$i = 1, 2, \dots, t, \ j = 1, 2, \dots, n_i,$$
(2)

where n_i is the number of observations in each factor level, t is the number of such factor levels, μ is the overall mean, α_i are the specific factor-level effects, subject to the restriction $\sum \alpha_i = 0$, which means that the "average" factor-level effect is zero. Note that implementing this restriction requires that only (t - 1) of the $\hat{\alpha}$ estimate need be estimated because any one parameter is simply the negative of the sum of all the others, i.e. coefficient of the final level is the sum of minus the other coefficients. z_i are the dummy variables indicating the presence or absence of certain conditions for observations.

It is quite obvious from this that the GLM is more cumbersome than the usual standard analysis of variance calculations. However, it cannot be used in all applications and it becomes necessary to use the general linear model when analysing factorial data with unequal cell frequencies.

5.2. Illustrative example

In this study, the linear statistical procedure GLM is fitted to the data for each method, allowing estimation of any forecasting accuracy measurement for any given set of factors and covariates included. The coefficients used are slightly different to those given in Table 6a–c as non-significant terms are eliminated one by one. These coefficients are presented in tabular form for each of the

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13 forecasting methods. The predictive model then selects the most appropriate forecasting method based on the lowest forecast errors (e.g. measured in terms of MAPE selected). The whole process takes less than a second. This process is demonstrated by the following example:

For this example, the DES method is selected, and assumes factor values of: SPL = 12 (monthly), PMP = HT, $CV^2 = 0.39$, ADI = 2.76 and in addition to those factors we select the *MAPE* technique as a forecast accuracy measurement.

Table 5 reminds us that the DES method has five significant interactions which are then reduced to four after non-significant terms are eliminated: PMP and CV^2 , all interact with SPL. This means that PMP and CV^2 factors will all depend on the level of the SPL (quarterly, monthly or weekly). PMP interacts with both CV^2 and ADI and both factors will also depend on the level of PMP (HT or CM). Based on the model interaction given above, the GLM equation should read as follows:

$$y_{ij} = \mu + \delta_i + \lambda_j + (\delta\lambda)_{ij} + \beta_1(CV^2) + \beta_2(ADI) + \varepsilon_{ij},$$

where y_{ij} is the response (*MAPE*) in the *j*th *PMP*, j = 1, 2, in the *i*th *SPL*, $i = 1, 2, 3, \mu$ is the mean (or intercept), δ_i is the effect of the *i*th *SPL*, λ_j is the effect of the *j*th *PMP*, $(\delta\lambda)_{ij}$ is the interaction between *SPL* and *PMP*, β_1, β_2 are the regression coefficients for CV^2 and *ADI*.

Note that μ , δ_i , and λ_j are parameters describing factor levels, whereas the β_i are regression coefficients. For simplicity, the general linear model equation could alternatively be written as follows:

MAPE = constant + coefficient of SPL level + coefficient of PMP level + CV^2 coefficient + ADI coefficient + coefficient for SPL interacts with PMP + ADI coefficient interacts with SPL level + CV^2 coefficient interacts with PMP level + ADI coefficient interacts with PMP level.

The constant term in the fitted GLM is 3.0789, the other estimated coefficients follow.

Effect of SPL = 12 is given by 0.1671. Effect of PMP = HT is given by 0.6458. Coefficient of CV^2 is 1.5031. Coefficient of ADI is 0.62223. Coefficient for SPL interacts with PMP combination = -0.07015. Additional coefficient of ADI for SPL of 12 = -0.20068. Additional coefficient of CV^2 for PMP of HT = -0.7919. Additional coefficient of ADI for PMP of HT = -0.21538.

The estimated forecast error measured in terms of MAPE is then given as

$$\log MAPE = 3.0789 + (0.1671) + (0.6458) + (1.5031 \times 0.39) + (0.62223 \times 2.76) + (-0.07015)$$

 $+(-0.20068 \times 2.76) + (-0.7919 \times 0.39) + (-0.21538 \times 2.76) = 4.67$

by taking the natural logarithm. Hence MAPE is equal to 106.49.

The *MAPE* estimated in this way for these input variables using various forecasting methods are displayed in ascending order, as shown in Fig. 1.

6. Discussion and conclusions

Accurate forecasting is critical for the airline operators as the price of not having the right part available at the right time in the right place is steep. An aircraft operator can incur costs of more than \$50,000 for each hour if a plane is on the ground. However, it was recognised from the start that demands for aircraft spares exhibited unexpectedly high variation and a large number of airline companies still used earlier methods [1, 2] specifically SES and MTBR, with little or no appreciation of the other forecasting methods used in this study. The results of this study show the use of the SES and MTBR methods to be questionable as they consistently create poor forecasting performance which remains poor as the demand variability increases. Accordingly, it is recommended that companies reconsider using them.

The evaluations made in the study were made for aircraft parts which had previously received little attention. They clearly show that traditional forecasting techniques mentioned above are based on assumptions that are inappropriate for parts with sporadic demand. Croston [5] demonstrated that using simple exponential smoothing forecast methods to set inventory levels can lead to excessive stocking. The analysis of results in this study concludes that the forecasting demand methods are clearly dominated by the weighted moving average and its superiority increases with the increase of *SPL*. Weighted moving averages is much superior to exponential smoothing and could provide tangible benefits to *airline operators* and *maintenance service organisations* forecasting intermittent demand. The highest forecasting error occurs when Winter's method forecasts demand with high variation. This conclusion contradicts previous research of forecasting intermittent demand, particularly [7,28]. With the exception of the results of the run of the model shown in Fig. 1, for all others the WMA, Holt and Croston methods were superior.

This research has shown that the level of appropriate factors has an effect on the forecasting performance. The results indicate that the impact of demand variability, such as CV^2 and ADI, on forecast errors (measured in terms of MAPE) is significant, and that as demand variability (CV^2 and ADI) increases, so the MAPE increases. ADI has more effect on a quarterly SPL than with a monthly and weekly SPL. This was observed in most methods except for the MW. Further, to determine if there are isolatable conditions or characteristics according to aircraft component type or their associated parts which may cause certain forecasting methods to predict more accurately, PMP was tested, and was shown to have a significant effect in terms of forecasting performance. Again this was for most methods except for the MW. Generally, hard-time HT components have more effect in increasing accuracy measuring MAPE, compared with condition-monitoring CM.

The study has presented a model that could be of great benefit to airline operators and other maintenance service organisations. It will enable them to select in advance the appropriate forecasting method that better meets their cyclical demand for parts. This approach is consistent with our objectives to compare different forecasting methods when faced with intermittent demand.

Finally, given the consistent results obtained in this study, it is believed these results are robust, especially in the light of their congruence with theoretical arguments appearing in the literature. This study has taken a step in the direction of defining the relationship between the accuracy of forecasting measurement and their factors. Although we have used data from one particular airline operator, it is suggested that these findings may be applicable elsewhere as other industrial sectors have similar demand patterns to airlines.

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