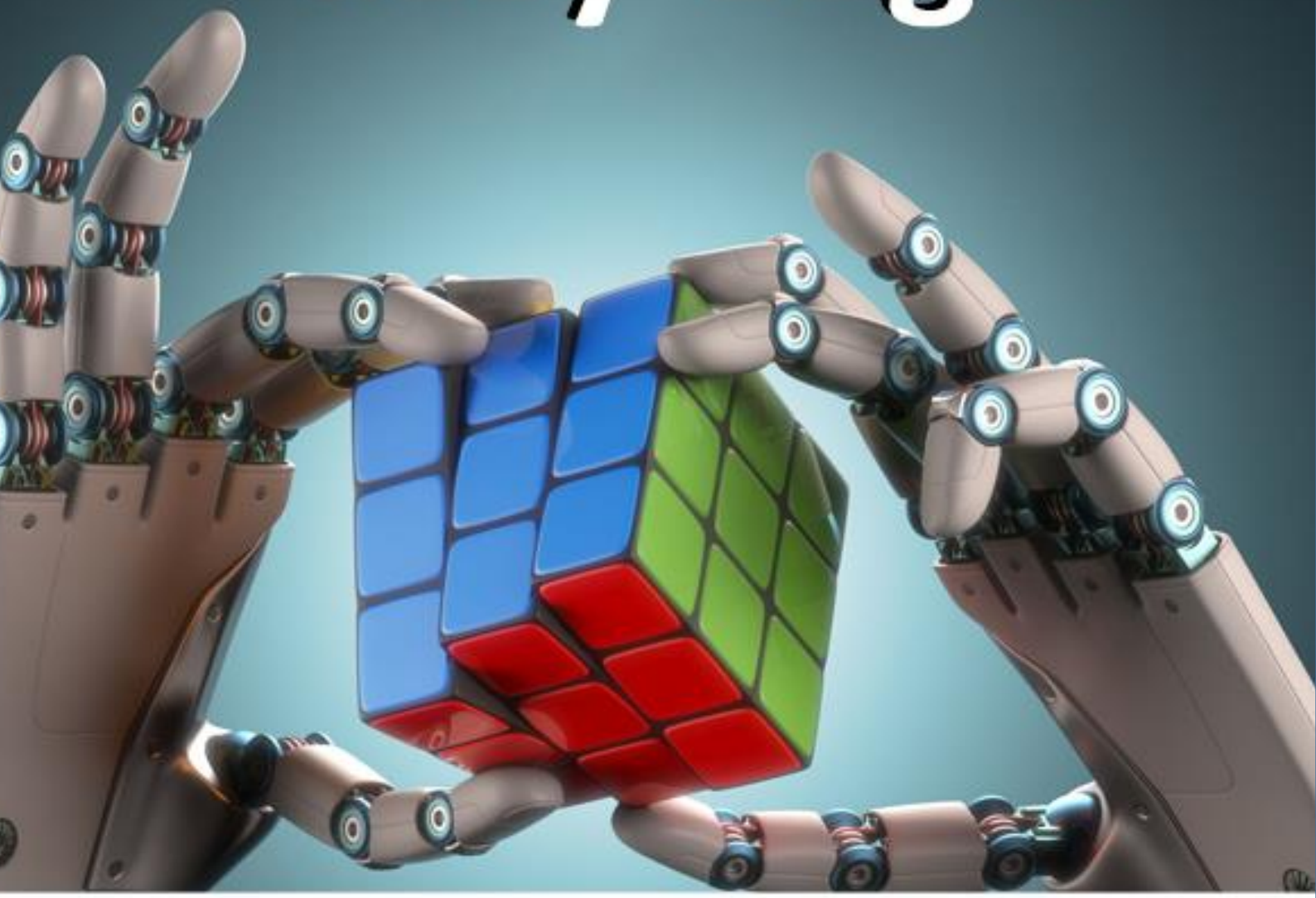
Fuzzy Logic



tutorialspoint

SIMPLY EASY LEARNING

www.tutorialspoint.com

https://www.facebook.com/tutorialspointindia https://twitter.com/tutorialspoint

About the Tutorial

Fuzzy Logic resembles the human decision-making methodology and deals with vague and imprecise information. This is a very small tutorial that touches upon the very basic concepts of Fuzzy Logic.

Audience

This tutorial will be useful for graduates, post-graduates, and research students who either have an interest in this subject or have this subject as a part of their curriculum. The reader can be a beginner or an advanced learner.

Prerequisites

Fuzzy Logic is an advanced topic, so we assume that the readers of this tutorial have preliminary knowledge of Set Theory, Logic, and Engineering Mathematics.

Disclaimer& Copyright

© Copyright 2017 by Tutorials Point (I) Pvt. Ltd.

All the content and graphics published in this e-book are the property of Tutorials Point (I) Pvt. Ltd. The user of this e-book is prohibited to reuse, retain, copy, distribute or republish any contents or a part of contents of this e-book in any manner without written consent of the publisher.

We strive to update the contents of our website and tutorials as timely and as precisely as possible, however, the contents may contain inaccuracies or errors. Tutorials Point (I) Pvt. Ltd. provides no guarantee regarding the accuracy, timeliness or completeness of our website or its contents including this tutorial. If you discover any errors on our website or in this tutorial, please notify us at contact@tutorialspoint.com.

tutorialspoint
SIMPLYEASYLEARNING

Table of Contents

About the Tutorial i

Audience i

Prerequisites i

Disclaimer& Copyright i

Table of Contents ii

1. FUZZY LOGIC – INTRODUCTION 1

2. FUZZY LOGIC – CLASSICAL SET THEORY 2

Mathematical Representation of a Set 2

Types of Sets 3

Operations on Classical Sets 5

Properties of Classical Sets 7

3. FUZZY LOGIC – FUZZY SET THEORY 9

Mathematical Concept 9

Representation of fuzzy set 9

Operations on Fuzzy Sets 10

Properties of Fuzzy Sets 11

4. FUZZY LOGIC – MEMBERSHIP FUNCTION 13

Mathematical Notation 13

Features of Membership Functions 14

Fuzzification 15

Defuzzification 15

5. FUZZY LOGIC – TRADITIONAL FUZZY REFRESHER 17

Quantifiers 20

Nested Quantifiers 20

6. FUZZY LOGIC – APPROXIMATE REASONING 21

Fuzzy Logic Rule Base 22

Interpretations of Fuzzy IF-THEN Rules 22

Linguistic Variable 23

Propositions in Fuzzy Logic 23

Fuzzy Qualifiers 24

7. FUZZY LOGIC – FUZZY INFERENCE SYSTEM 25

Characteristics of Fuzzy Inference System 25

Functional Blocks of FIS 25

Working of FIS 26

Methods of FIS 26

Mamdani Fuzzy Inference System 26

Takagi-Sugeno Fuzzy Model (TS Method) 27

Comparison between the two methods 28

8. FUZZY LOGIC – FUZZY DATABASE AND QUERIES 29

9. FUZZY LOGIC – FUZZY QUANTIFICATION 30

10. FUZZY LOGIC – FUZZY DECISION MAKING 31

Steps for Decision Making 31

Types of Decision Making 31

11. FUZZY LOGIC – FUZZY LOGIC CONTROL SYSTEM 33

Why Use Fuzzy Logic in Control Systems 33

Assumptions in Fuzzy Logic Control (FLC) Design 33

Architecture of Fuzzy Logic Control 34

Major Components of FLC 34

Steps in Designing FLC 34

tutorialspoint
SIMPLYEASYLEARNING

Advantages of Fuzzy Logic Control 35

Disadvantages of Fuzzy Logic Control 35

12. FUZZY LOGIC – ADAPTIVE FUZZY CONTROLLER 36

Basic Steps for Implementing Adaptive Algorithm 36

Operational Concepts 36

Parameterization of System 37

Mechanism Adjustment 37

Parameters for selecting an Adaptive Fuzzy Controller 38

13. FUZZY LOGIC – FUZZINESS IN NEURAL NETWORKS 39

Why to use Fuzzy Logic in Neural Network 39

Neural-Trained Fuzzy Logic 40

Examples of Neural-Trained Fuzzy system 40

14. FUZZY LOGIC – FUZZY LOGIC APPLICATIONS 41

tutorialspoint
SIMPLYEASYLEARNING

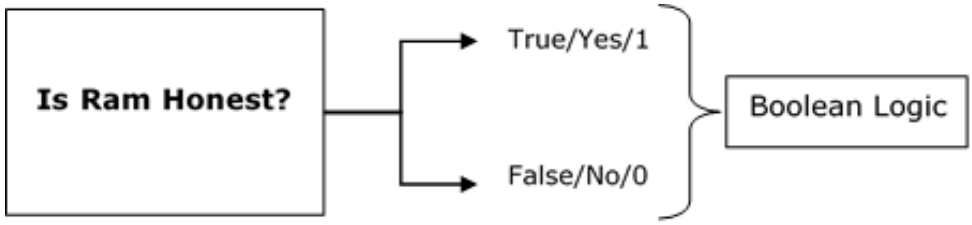
1. Fuzzy Logic – Introduction

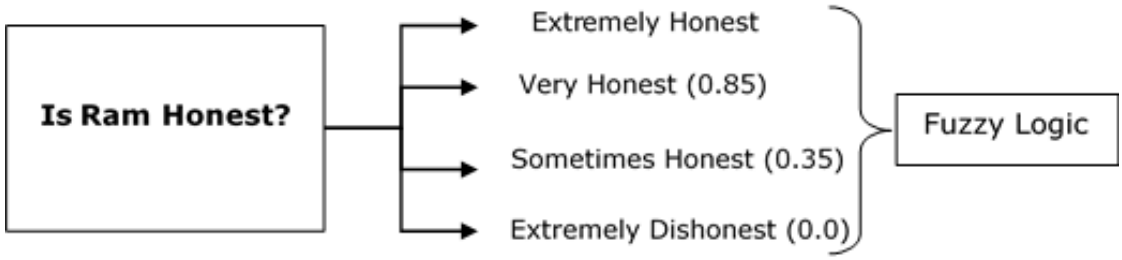
The word **fuzzy** refers to things which are not clear or are vague. Any event, process, or function that is changing continuously cannot always be defined as either **true** or **false**, which means that we need to define such activities in a Fuzzy manner.

What is Fuzzy Logic?

Fuzzy Logic resembles the human decision-making methodology. It deals with vague and imprecise information. This is gross oversimplification of the real-world problems and based on degrees of truth rather than usual true/false or 1/0 like Boolean logic.

Take a look at the following diagram. It shows that in fuzzy systems, the values are indicated by a number in the range from 0 to 1. Here 1.0 represents **absolute truth** and 0.0 represents **absolute falseness**. The number which indicates the value in fuzzy systems is called the **truth value**.





In other words, we can say that fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness. There can be numerous other examples like this with the help of which we can understand the concept of fuzzy logic.

Fuzzy Logic was introduced in 1965 by Lofti A. Zadeh in his research paper "Fuzzy Sets". He is considered as the father of Fuzzy Logic.

tutorialspoint
SIMPLYEASYLEARNING

2. Fuzzy Logic – Classical Set Theory

A **set** is an unordered collection of different elements. It can be written explicitly by listing its elements using the set bracket. If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.

Example

- A set of all positive integers.
- A set of all the planets in the solar system.
- A set of all the states in India.
- A set of all the lowercase letters of the alphabet.

Mathematical Representation of a Set

Sets can be represented in two ways –

Roster or Tabular Form

In this form, a set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.

Following are the examples of set in Roster or Tabular Form:

- Set of vowels in English alphabet, $A = \{a, e, i, o, u\}$
- Set of odd numbers less than 10, $B = \{1,3,5,7,9\}$

Set Builder Notation

In this form, the set is defined by specifying a property that elements of the set have in common. The set is described as A={x:p(x)}

Example 1 – The set $\{a, e, i, o, u\}$ is written as –

$$A = \{x: x \text{ is a vowel in English alphabet}\}$$

Example 2 – The set {1,3,5,7,9} is written as –

$$B = \{x: 1 \leq x < 10 \text{ and } (x\%2) \neq 0\}$$

If an element x is a member of any set S, it is denoted by x∈S and if an element y is not a member of set S, it is denoted by y∉S.

Example – If $S = \{1,1.2,1.7,2\}, 1 \in S$ but $1.5 \notin S$

tutorialspoint
SIMPLYEASYLEARNING

Cardinality of a Set

Cardinality of a set S, denoted by |S||S|, is the number of elements of the set. The number is also referred as the cardinal number. If a set has an infinite number of elements, its cardinality is ∞∞.

Example – $|\{1,4,3,5\}| = 4, |\{1,2,3,4,5,\ldots\}| = \infty$

If there are two sets X and Y, $|X| = |Y|$ denotes two sets X and Y having same cardinality. It occurs when the number of elements in X is exactly equal to the number of elements in Y. In this case, there exists a bijective function 'f' from X to Y.

$|X| \leq |Y|$ denotes that set X's cardinality is less than or equal to set Y's cardinality. It occurs when the number of elements in X is less than or equal to that of Y. Here, there exists an injective function 'f' from X to Y.

$|X| < |Y|$ denotes that set X's cardinality is less than set Y's cardinality. It occurs when the number of elements in X is less than that of Y. Here, the function 'f' from X to Y is injective function but not bijective.

If $|X| \leq |Y|$ and $|X| \leq |Y|$ then $|X| = |Y|$. The sets X and Y are commonly referred as **equivalent sets**.

Types of Sets

Sets can be classified into many types; some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Finite Set

A set which contains a definite number of elements is called a finite set.

Example – $S = \{x | x \in N \; and \; 70 > x > 50\}$

Infinite Set

A set which contains infinite number of elements is called an infinite set.

Example – $S = \{x | x \in N \; and \; x > 10\}$

Subset

A set X is a subset of set Y (Written as $X \subseteq Y$) if every element of X is an element of set Y.

Example 1 – Let, $X = \{1,2,3,4,5,6\}$ and $Y = \{1,2\}$. Here set Y is a subset of set X as all the elements of set Y is in set X. Hence, we can write Y⊆X.

Example 2 – Let, $X = \{1,2,3\}$ and $Y = \{1,2,3\}$. Here set Y is a subset (not a proper subset) of set X as all the elements of set Y is in set X. Hence, we can write Y⊆X.

Proper Subset

tutorialspoint
SIMPLYEASYLEARNING

The term "proper subset" can be defined as "subset of but not equal to". A Set X is a proper subset of set Y (Written as $X \subset Y$) if every element of X is an element of set Y and $|X| < |Y|$.

Example – Let, $X = \{1,2,3,4,5,6\}$ and $Y = \{1,2\}$. Here set $Y \subset X$, since all elements in Y are contained in X too and X has at least one element which is more than set Y.

Universal Set

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as U.

Example – We may define U as the set of all animals on earth. In this case, a set of all mammals is a subset of U, a set of all fishes is a subset of U, a set of all insects is a subset of U, and so on.

Empty Set or Null Set

An empty set contains no elements. It is denoted by ∅. As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empty set or null set is zero.

Example – $S = \{x | x \in N \; and \; 7 < x < 8\} = \emptyset$

Singleton Set or Unit Set

A Singleton set or Unit set contains only one element. A singleton set is denoted by {s}.

Example – $S = \{x | x \in N, \; 7 < x < 9\} = \{8\}$

Equal Set

If two sets contain the same elements, they are said to be equal.

Example – If $A = \{1,2,6\}$ and $B = \{6,1,2\}$, they are equal as every element of set A is an element of set B and every element of set B is an element of set A.

Equivalent Set

If the cardinalities of two sets are same, they are called equivalent sets.

Example – If $A = \{1,2,6\}$ and $B = \{16,17,22\}$, they are equivalent as cardinality of A is equal to the cardinality of B. i.e. $|A| = |B| = 3$

Overlapping Set

Two sets that have at least one common element are called overlapping sets. In case of overlapping sets –

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

$$n(A) = n(A - B) + n(A \cap B)$$

$$n(B) = n(B - A) + n(A \cap B)$$

tutorialspoint
SIMPLYEASYLEARNING

Example – Let, $A = \{1,2,6\}$ and $B = \{6,12,42\}$. There is a common element '6', hence these sets are overlapping sets.

Disjoint Set

Two sets A and B are called disjoint sets if they do not have even one element in common. Therefore, disjoint sets have the following properties –

$$n(A \cap B) = \emptyset$$

$$n(A \cup B) = n(A) + n(B)$$

Example – Let, $A = \{1,2,6\}$ and $B = \{7,9,14\}$, there is not a single common element, hence these sets are overlapping sets.

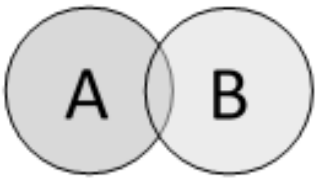
Operations on Classical Sets

Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian Product.

Union

The union of sets A and B (denoted by $A \cup BA \cup B$) is the set of elements which are in A, in B, or in both A and B. Hence, $A \cup B = \{x|x \in A\ OR\ x \in B\}$.

Example – If $A = \{10,11,12,13\}$ and $B = \{13,14,15\}$, then $A \cup B = \{10,11,12,13,14,15\}$ – The common element occurs only once.



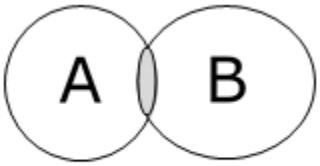
A ∪ B

Intersection

The intersection of sets A and B (denoted by $A \cap B$) is the set of elements which are in both A and B. Hence, $A \cap B = \{x|x \in A\ AND\ x \in B\}$.

Example – If $A = \{11,12,13\}$ and B=$\{13,14,15\}$, then $A \cap B = \{13\}$.

tutorialspoint
SIMPLYEASYLEARNING

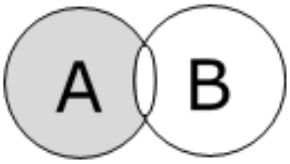


A ∩ B

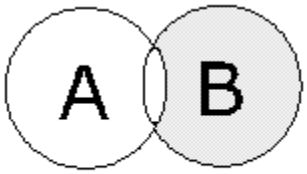
Difference/ Relative Complement

The set difference of sets A and B (denoted by A–B) is the set of elements which are only in A but not in B. Hence, $A - B = \{x | x \in A\ AND\ x \notin B\}$.

Example – If $A = \{10,11,12,13\}\ and\ B = \{13,14,15\}$, then $(A - B) = \{10,11,12\} and\ (B - A) = \{14,15\}$. Here, we can see $(A - B)\ \neq\ (B - A)$



A - B



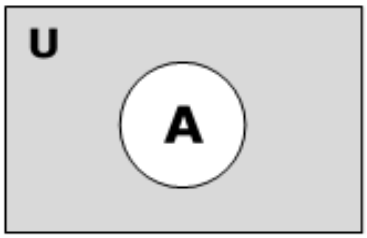
B - A

Complement of a Set

The complement of a set A (denoted by A′) is the set of elements which are not in set A. Hence, $A' = \{x | x \notin A\}$.

More specifically, A′=(U–A) where U is a universal set which contains all objects.

Example – If $A = \{x | x\ belongs\ to\ set\ of\ odd\ integers\}$ then $A' =$
$\{y | y\ does\ not\ belong\ to\ set\ of\ odd\ integers\}$



Complement of set A

Cartesian Product / Cross Product

The Cartesian product of n number of sets $A1, A2, \ldots An$ denoted as $A1 \times A2 \cdots \times An$ can be defined as all possible ordered pairs $(x1, x2, \ldots xn)\ where\ x1 \in A1, x2 \in A2, \ldots xn \in An$

tutorialspoint
SIMPLYEASYLEARNING

Example − If we take two sets $A = \{a, b\}$ and $B = \{1,2\}$,

The Cartesian product of A and B is written as − $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

And, the Cartesian product of B and A is written as − $B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$

Properties of Classical Sets

Properties on sets play an important role for obtaining the solution. Following are the different properties of classical sets:

Commutative Property

Having two sets **A** and **B**, this property states:

$$\boldsymbol{A \cup B = B \cup A}$$

$$\boldsymbol{A \cap B = B \cap A}$$

Associative Property

Having three sets **A, B** and **C**, this property states:

$$\boldsymbol{A \cup (B \cup C) = (A \cup B) \cup C}$$

$$\boldsymbol{A \cap (B \cap C) = (A \cap B) \cap C}$$

Distributive Property

Having three sets **A, B** and **C**, this property states:

$$\boldsymbol{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$$

$$\boldsymbol{A \cap (B \cup C) = A \cap B) \cup (A \cap C)}$$

Idempotency Property

For any set **A**, this property states:

$$\boldsymbol{A \cup A = A}$$

$$\boldsymbol{A \cap A = A}$$

Identity Property

For set **A** and universal set **X**, this property states:

$$\boldsymbol{A \cup \varphi = A}$$

$$\boldsymbol{A \cap X = A}$$

$$\boldsymbol{A \cap \varphi = \varphi}$$

$$\boldsymbol{A \cup X = X}$$

tutorialspoint
SIMPLYEASYLEARNING

Transitive Property

Having three sets **A, B** and **C**, the property states:

$$If\ A \subseteq B \subseteq C, then\ A \subseteq C$$

Involution Property

For any set **A**, this property states:

$$\overline{\overline{A}} = A$$

De Morgan's Law

It is a very important law and supports in proving tautologies and contradiction. This law states:

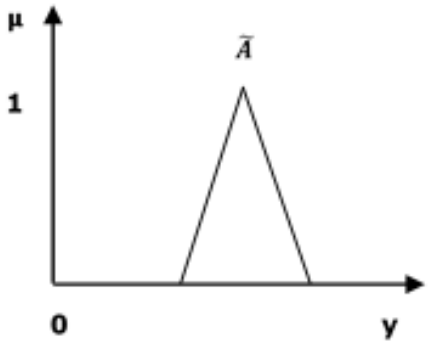
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

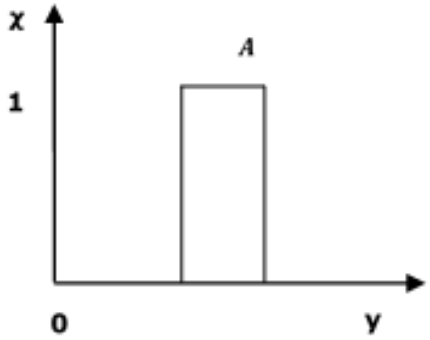
tutorialspoint
SIMPLYEASYLEARNING

3. Fuzzy Logic – Fuzzy Set Theory

Fuzzy sets can be considered as an extension and gross oversimplification of classical sets. It can be best understood in the context of set membership. Basically it allows partial membership which means that it contain elements that have varying degrees of membership in the set. From this, we can understand the difference between classical set and fuzzy set. Classical set contains elements that satisfy precise properties of membership while fuzzy set contains elements that satisfy imprecise properties of membership.



Membership Function of fuzzy set $\tilde{A}$



Membership Function of classical set A

Mathematical Concept

A fuzzy set $\tilde{A}$ in the universe of information U can be defined as a set of ordered pairs and it can be represented mathematically as:

$$\tilde{A} = \{(y, \mu_{\tilde{A}}(y)) | \, y \in U\}$$

Here $\mu_{\tilde{A}}(y)$ = degree of membership of y in $\tilde{A}$, assumes values in the range from 0 to 1, i.e., $\mu_{\tilde{A}}(y) \in [0,1]$.

Representation of fuzzy set

Let us now consider two cases of universe of information and understand how a fuzzy set can be represented.

Case 1

When universe of information U is discrete and finite -

tutorialspoint
SIMPLYEASYLEARNING

$$\tilde{A} = \left\{\frac{\mu_{\tilde{A}}(y_1)}{y_1} + \frac{\mu_{\tilde{A}}(y_2)}{y_2} + \frac{\mu_{\tilde{A}}(y_3)}{y_3} + \cdots\right\}$$

$$= \left\{\sum_{i=1}^{n} \frac{\mu_{\tilde{A}}(y_i)}{y_i}\right\}$$

Case 2

When universe of information U is continuous and infinite -

$$\tilde{A} = \left\{\int \frac{\mu_{\tilde{A}}(y)}{y}\right\}$$

In the above representation, the summation symbol represents the collection of each element.

Operations on Fuzzy Sets

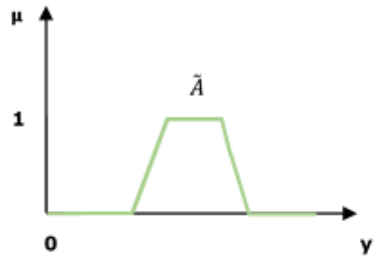
Having two fuzzy sets $\tilde{A}$ and $\tilde{B}$, the universe of information U and an element y of the universe, the following relations express the union, intersection and complement operation on fuzzy sets.

Union/Fuzzy 'OR'

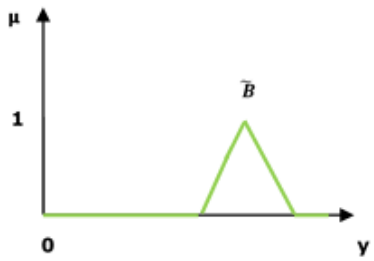
Let us consider the following representation to understand how the **Union/Fuzzy 'OR'** relation works -

$$\mu_{\tilde{A} \cup \tilde{B}}(y) = \mu_{\tilde{A}} \vee \mu_{\tilde{B}} \quad \forall\, y \in U$$

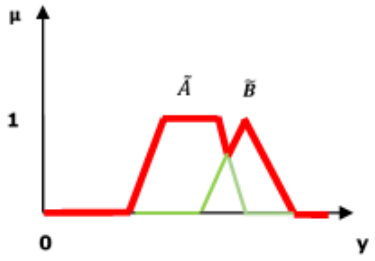
Here ∨ represents the 'max' operation.



Fuzzy set $\tilde{A}$



Fuzzy set $\tilde{B}$



Union of two Fuzzy sets

Intersection/Fuzzy 'AND'

Let us consider the following representation to understand how the **Intersection/Fuzzy 'AND'** relation works -

$$\mu_{\tilde{A} \cap \tilde{B}}(y) = \mu_{\tilde{A}} \wedge \mu_{\tilde{B}} \quad \forall\, y \in U$$

tutorialspoint
SIMPLYEASYLEARNING

End of ebook preview
If you liked what you saw…
Buy it from our store @ **https://store.tutorialspoint.com**

tutorialspoint
SIMPLYEASYLEARNING