ENGINEERING PROBABILITY

HOMEWORK # 12: Posted on 04/18/2018

Please work out the **ten** (10) problems stated below – BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem **1.55** (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). Show work and explain reasoning.

1. _

The rvs X and Y are known to be jointly continuous with a probability density function $f_{X,Y}: \mathbb{R}^2 \to \mathbb{R}_+$ given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{e^{-y}}{y} & \text{if } 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

1.a. Show that the rvs X and Y are each of continuous type, and identify their probability distribution functions $f_X, f_Y : \mathbb{R} \to \mathbb{R}$.

1.b. Are the rvs X and Y independent? Explain.

1.c. What is the conditional distribution of the rv X given Y = y for y > 0?

1.d. Is it easy to identify the conditional distribution of the rv Y given X = x for x > 0? Comments welcome!

1.e. Compute

$$\mathbb{E}\left[X^3|Y=y\right], \quad y>0.$$

2. _____

The rvs X and Y are known to be jointly continuous with a probability density function $f_{X,Y}: \mathbb{R}^2 \to \mathbb{R}_+$ given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{e^{-\frac{x}{y}}e^{-y}}{y} & \text{if } 0 < x, y < \infty \\ 0 & \text{otherwise} \end{cases}$$

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2.a. Show that the rvs X and Y are each of continuous type, and identify their probability distribution functions $f_X, f_Y : \mathbb{R} \to \mathbb{R}$.

2.b. Are the rvs X and Y independent? Explain.

2.c. Compute

$$\mathbb{E}\left[X^2|Y=y\right], \quad y>0.$$

3. ____

The rvs X and Y are jointly continuous with a probability density function $f_{X,Y} : \mathbb{R}^2 \to \mathbb{R}_+$ given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{5} + cy & \text{if } 0 < x < 1, 1 < y < 5 \\ 0 & \text{otherwise} \end{cases}$$

for some c > 0.

- **3.a.** What is the value of c > 0?
- **3.b.** Are the rvs X and Y independent?

3.c. Evaluate $\mathbb{P}[X + Y > 3]$. Recall that

$$[X + Y > 3] = [(X, Y) \in B]$$

where

$$B = \{(x, y) \in \mathbb{R}^2 : x + y > 3\}.$$

4. _

The rvs X and Y are independent rvs with X (resp. Y) exponentially distributed with parameter 2 (resp. 3), respectively. With event $A = [X + Y \leq 1]$, show that the joint conditional distribution of the pair (X, Y) given A is of continuous type. Identify the joint conditional density function of the pair (X, Y) given A.

5. ____

Problem 4.1 (BT)

6. _____

Problem 4.2 (BT)

7. ____

Problem **4.3** (BT)

8. _____

Problem 4.4 (BT)

9. _____

Problem 4.5 (BT)

10. _____

Problem 4.6 (BT)