# ENGINEERING PROBABILITY <br> HOMEWORK \# 12: <br> Posted on 04/18/2018 

Please work out the ten (10) problems stated below - BT refers to the text: D.P. Bertsekas and J.N. Tsitsiklis, Introduction to Probability (Second Edition), Athena Scientific (2008). Problem 1.55 (BT) refers to Problem 55 for Chapter 1 of BT (to be found at the end of Chapter 1). Show work and explain reasoning.
1.

The rvs $X$ and $Y$ are known to be jointly continuous with a probability density function $f_{X, Y}: \mathbb{R}^{2} \rightarrow \mathbb{R}_{+}$given by

$$
f_{X, Y}(x, y)= \begin{cases}\frac{e^{-y}}{y} & \text { if } 0<x<y<\infty \\ 0 & \text { otherwise }\end{cases}
$$

1.a. Show that the rvs $X$ and $Y$ are each of continuous type, and identify their probability distribution functions $f_{X}, f_{Y}: \mathbb{R} \rightarrow \mathbb{R}$.
1.b. Are the rvs $X$ and $Y$ independent? Explain.
1.c. What is the conditional distribution of the rv $X$ given $Y=y$ for $y>0$ ?
1.d. Is it easy to identify the conditional distribution of the rv $Y$ given $X=x$ for $x>0$ ? Comments welcome!
1.e. Compute

$$
\mathbb{E}\left[X^{3} \mid Y=y\right], \quad y>0
$$

2. 

The rvs $X$ and $Y$ are known to be jointly continuous with a probability density function $f_{X, Y}: \mathbb{R}^{2} \rightarrow \mathbb{R}_{+}$given by

$$
f_{X, Y}(x, y)= \begin{cases}\frac{e^{-\frac{x}{y}} e^{-y}}{y} & \text { if } 0<x, y<\infty \\ 0 & \text { otherwise }\end{cases}
$$

2.a. Show that the rvs $X$ and $Y$ are each of continuous type, and identify their probability distribution functions $f_{X}, f_{Y}: \mathbb{R} \rightarrow \mathbb{R}$.
2.b. Are the rvs $X$ and $Y$ independent? Explain.
2.c. Compute

$$
\mathbb{E}\left[X^{2} \mid Y=y\right], \quad y>0
$$

3. 

The rvs $X$ and $Y$ are jointly continuous with a probability density function $f_{X, Y}: \mathbb{R}^{2} \rightarrow$ $\mathbb{R}_{+}$given by

$$
f_{X, Y}(x, y)= \begin{cases}\frac{x}{5}+c y & \text { if } 0<x<1,1<y<5 \\ 0 & \text { otherwise }\end{cases}
$$

for some $c>0$.
3.a. What is the value of $c>0$ ?
3.b. Are the rvs $X$ and $Y$ independent?
3.c. Evaluate $\mathbb{P}[X+Y>3]$. Recall that

$$
[X+Y>3]=[(X, Y) \in B]
$$

where

$$
B=\left\{(x, y) \in \mathbb{R}^{2}: x+y>3\right\} .
$$

4. 

The rvs $X$ and $Y$ are independent rvs with $X$ (resp. $Y$ ) exponentially distributed with parameter 2 (resp. 3), respectively. With event $A=[X+Y \leq 1]$, show that the joint conditional distribution of the pair $(X, Y)$ given $A$ is of continuous type. Identify the joint conditional density function of the pair $(X, Y)$ given $A$.
5.

Problem 4.1 (BT)
6.

Problem 4.2 (BT)
7.

Problem 4.3 (BT)
8.

Problem 4.4 (BT)
9.

Problem 4.5 (BT)
10.

Problem 4.6 (BT)

