Answer Key: The Math That Makes Videos Go Viral: Exponential Growth & Decay
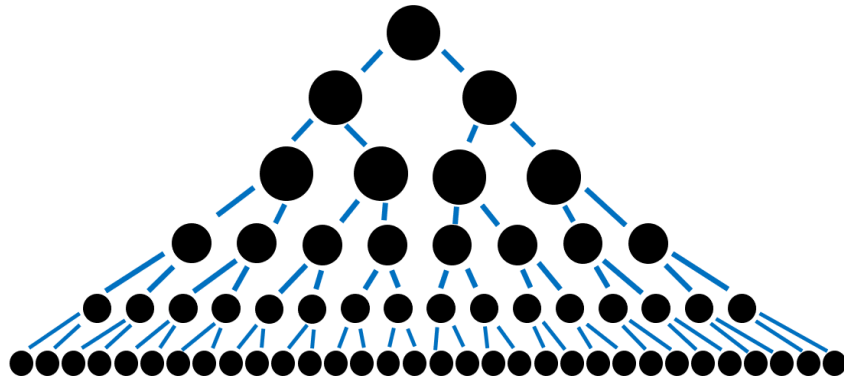*Lesson Plan by Dashiell Young-Saver*

Math Activity #1: High Sharing Rates

Fill in the rest of the tree diagram and table, assuming that the sharing rate remains at 2 per day for each new viewer.

Answer:

Days After Posting	New Viewers
0	1
1	2
2	4
3	8
4	16
5	32

Look at the table: What mathematical pattern do you notice? Why does this pattern occur?
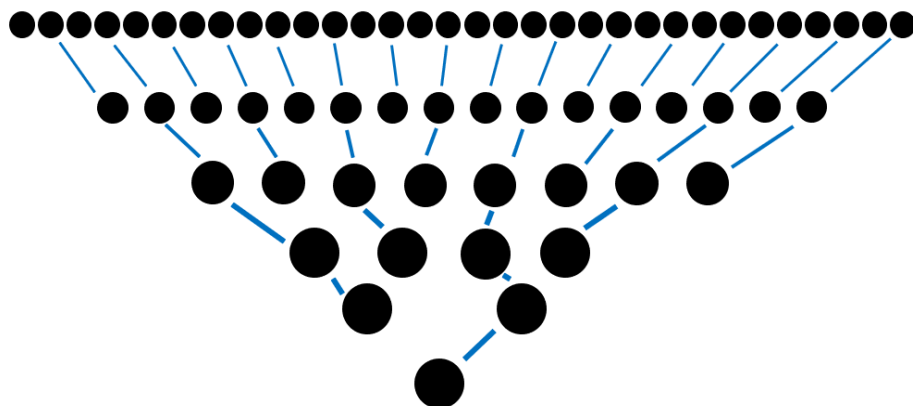
Answer: The number of new viewers is double the number from the previous day. So, to get tomorrow's new view count, we can just multiply today's number by 2. This makes sense: Each person shares with two new people, and then those people share with two more each. This is the same as repeatedly multiplying by 2.

Math Activity #2: Low Sharing Rates

Fill in the rest of the tree diagram and table, assuming that the sharing rate remains the same.

Answer:

Days After Peak	New Viewers
0	32
1	16
2	8
3	4
4	2
5	1

Answer Key: The Math That Makes Videos Go Viral: Exponential Growth & Decay
Lesson Plan by Dashiell Young-Saver

Look at the table: What mathematical pattern do you notice? Why does this pattern occur?

Answer: The number of new viewers is half the number from the previous day. So, to get tomorrow's new view count, we can just multiply today's number by 0.5. This makes sense: Only half of the people share the video. This is the same as repeatedly multiplying by 0.5.

Math Activity #3: Exponential Growth & Decay

Let's analyze both of the video sharing patterns - high sharing and low sharing. Using these tables from both videos, fill in the following graph:
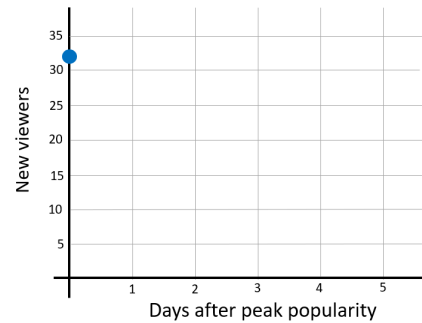
High Share Rate (GenZ Video)

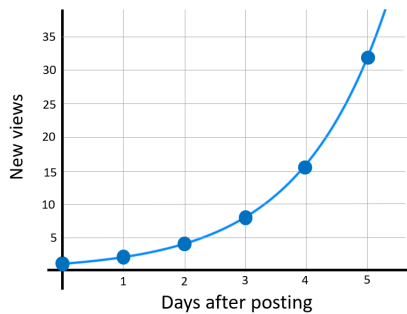| Days After Posting (x)	New Viewers (y)
0	1
1	2
2	4
3	8
4	16
5	32
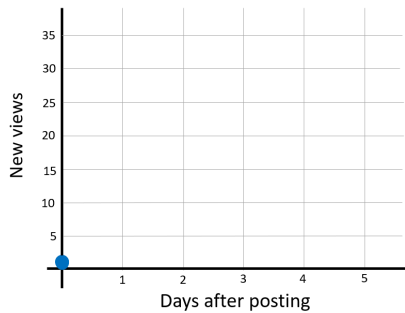
Low Share Rate (Professional Video)

Days After Peak (x)	New Viewers (y)
0	32
1	16
2	8
3	4
4	2
5	1

Answer:

Days After Posting (x)	New Viewers (y)
0	1
1	2
2	4
3	8
4	16
5	32

Days After Peak (x)	New Viewers (y)
0	32
1	16
2	8
3	4
4	2
5	1
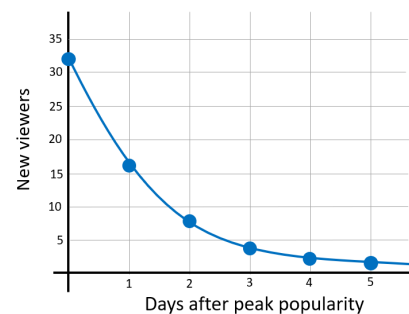
For each graph, describe what happens to the <u>number</u> of new viewers over time.

Answer: For the video with a higher rate of sharing, the number of new views increases over time. The number of new viewers decreases over time for the other video.

Answer Key: The Math That Makes Videos Go Viral: Exponential Growth & Decay
Lesson Plan by Dashiell Young-Saver

For each graph, describe what happens to the <u>rate of change</u> of new viewers (i.e. the change in new viewers from day-to-day) over time.

Answer: For the video with a higher rate of sharing, the number of new views increases slowly at first, then accelerates. For the other video, the number of new viewers declines rapidly at first, then decelerates over time.

Repeated multiplication creates what we call *exponential* patterns. If you've learned about exponents before, this name should make some sense. Exponents represent repeated multiplication. For example, the exponential pattern for the high-sharing video can be written like this:

<u>Day 0:</u> 1 view

<u>Day 1:</u> $1 * 2 = 1 * (2)^1 = 2$ new views

<u>Day 2:</u> $1 * 2 * 2 = 1 * (2)^2 = 4$ new views

<u>Day 3:</u> $1 * 2 * 2 * 2 = 1 * (2)^3 = 8$ new views

<u>Day 4:</u> $1 * 2 * 2 * 2 * 2 = 1 * (2)^4 = 16$ new views

<u>Day 5:</u> $1 * 2 * 2 * 2 * 2 * 2 = 1 * (2)^5 = 32$ new views

In the pattern above, what does the exponent represent? What does the (1) represent? What does the (2) represent?

Answer: The exponent represents the Day. The 1 represents the initial number of views. The 2 represents the rate of sharing.

Let's call y the number of views. Let's call x the day. Write the formula for an exponential model, using the pattern above, that gives the number of views (y) depending on the day (x).

Answer: $y = 1 * (2)^x$

Now, write the same formula, but for the low-sharing rate video:

Answer: $y = 32 * (0.5)^x$

Looking at the two formulas, what is similar? What is different?

Answer: The structure of the formulas (exponential) is similar. The initial number of views (1 vs 32) and the rate of sharing (2 vs 0.5) are different.

Using both models, find the number of new views you'd expect after 20 days.

Answer Key: The Math That Makes Videos Go Viral: Exponential Growth & Decay
Lesson Plan by Dashiell Young-Saver

Answer: High sharing video: $y = 1 * (2)^{20} = 1,048,576$ *new views (≈1 million)*
Low sharing video: $y = 32 * (0.5)^{20} ≈ 0$ *new views*

Clearly, one video went viral. The other died out. What was more important to the video's popularity: the initial number of views or the rate of sharing? Explain.

Answer: The rate of sharing is more important. Even though it only started with 1 view, the high sharing video (share rate of 2) was able to grow and accelerate its viewers. The other video started higher but quickly declined, since its rate of sharing was below 1 (0.5).

In math, we call the rate of sharing the **growth rate**. Check out this Desmos screen. It has the exponential formula, with a slider for different values of the growth rate (b). Play around with slider for the growth rate, then answer the following question:

What growth rate values lead to **exponential growth** (increasing views and accelerating rate of change) and what values lead to **exponential decay** (decreasing views and decelerating rate of change)?

Answer: Growth rate values greater than 1 lead to exponential growth. Growth rate values less than 1 (technically, between 0-1) lead to exponential decay.