

THE HANDBOOK
OF
**Formulas and
Tables for
Signal Processing**

The Electrical Engineering Handbook Series

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THE HANDBOOK OF Formulas and Tables for Signal Processing

Alexander D. Poularikas

Department of Electrical and Computer Engineering
The University of Alabama in Huntsville



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About The Author

Alexander D. Pouliarikas is a professor in the Department of Electrical and Computer Engineering at the University of Alabama in Huntsville. He received a B.S. degree in Electrical Engineering in 1960, an M.S. degree in Physics in 1963, and a Ph.D. in 1965, all at the University of Arkansas, Fayetteville.

He has held positions as assistant, associate, and professor at the University of Rhode Island (1965–1983), professor and Chairman of the Engineering Department at the University of Denver (1983–1985), and professor (1985–) and Chairman (1985–1989) at the University of Alabama in Huntsville. Dr. Pouliarikas was a visiting scientist at MIT (1971–1972), and summer faculty fellow at NASA (1968, 1972), at Stanford University (1966), and at Underwater Systems Center (1971, 1973, 1974).

He has coauthored the books *Electromagnetics* (Marcel Dekker, 1997), *Electrical Engineering: Introduction and Concepts* (Matrix Publishers, 1982), *Workbook for Electrical Engineers* (Matrix Publishers, 1983), *Signals and Systems* (Brooks/Cole, 1985), *Elements of Signals and Systems* (PWS-KENT, 1987), and *Signals and Systems* (2nd edition) (PWS-KENT, 1992). He is Editor-in-Chief for the books *Transforms and Applications Handbook* (CRC Press, 1995) and *Handbook of Formulas and Tables for Signal Processing* (CRC Press, 1999).

Dr. Pouliarikas is a senior member of the IEEE, was a Fulbright scholar and was awarded the Outstanding Educator's Award by the IEEE Huntsville Section in 1990 and 1996. His main interest is in the area of signal processing.

PREFACE

The purpose of *The Handbook of Formulas and Tables for Signal Processing* is to include in a single volume the most important and most useful tables and formulas that are used by engineers and students involved in signal processing. This includes deterministic as well as statistical signal processing applications. The handbook contains a large number of standard mathematical tables, so it can also be used as a mathematical formulas handbook.

The handbook is organized into 45 chapters. Each contains tables, formulas, definitions, and other information needed for the topic at hand. Each chapter also contains numerous examples to explain how to use the tables and formulas. Some of the figures were created using MATLAB and MATHEMATICA.

The editor and CRC Press would be grateful if readers would send their opinions about the handbook, any error they may detect, suggestions for additional material for future editions, and suggestions for deleting material.

The handbook is testimony to the efforts of colleagues whose contributions were invaluable, Nora Konopka, Associate Editor at CRC Press, the commitment of the Editor-in-Chief of the series, Dr. Richard Dorf, and others. Special thanks go to Dr. Yunlong Sheng for contributing Chapter 42.

Alexander D. Poularikas
Huntsville, Alabama
July 1998

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Fourier Series

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1.1 Definitions and Series Formulas

- 1.1.1 A function is **periodic** if $f(t) = f(t + nT)$, where n is an integer and T is the period of the function.
- 1.1.2 The function $f(t)$ is **absolutely integrable** if $\int_a^b |f(t)| dt < \infty$.
- 1.1.3 An **infinite series** of function

$$f_1(t) + f_2(t) + \dots + f_k(t) + \dots = \sum_{k=1}^{\infty} f_k(t)$$

converges at a given value of t if its **partial sums**

$$s_n(t) = \sum_{k=1}^{\infty} f_k(t), \quad (n = 1, 2, 3, \dots)$$

have a finite limit $s(t) = \lim_{n \rightarrow \infty} s_n(t)$.

- 1.1.4 The series in 1.1.3 is **uniformly convergent** in $[a, b]$ if, for any positive number ε , there exists a number N such that the inequality $|s(t) - s_n(t)| \leq \varepsilon$ holds for all $n \geq N$ and for all t in the interval $[a, b]$.
- 1.1.5 **Complex form** of the series:

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_o t} = \sum_{n=-\infty}^{\infty} |\alpha_n| e^{j(n\omega_o t + \varphi_n)}, \quad t_o \leq t \leq t_o + T$$

$$\alpha_n = \frac{1}{T} \int_{t_o}^{t_o+T} f(t) e^{-j n \omega_o t} dt, \quad \omega_o = \frac{2\pi}{T}, \quad T = \text{period}$$

$$\alpha_n = |\alpha_n| e^{j \varphi_n} = |\alpha_n| \cos \varphi_n + j |\alpha_n| \sin \varphi_n, \quad \alpha_{-n} = \alpha_n^*, \quad t_o = \text{any real value.}$$

1.1.6 Trigonometric form of the series

$$\begin{aligned} f(t) &= \frac{A_o}{2} + \sum_{n=1}^{\infty} (A_n \cos n \omega_o t + B_n \sin n \omega_o t), \quad A_o = 2 \alpha_o = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) dt \\ A_n &= (\alpha_n + \alpha_n^*) = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \cos n \omega_o t dt, \quad B_n = j(\alpha_n - \alpha_n^*) = \frac{2}{T} \int_{t_o}^{t_o+T} f(t) \sin n \omega_o t dt \\ f(t) &= \frac{A_o}{2} + \sum_{n=1}^{\infty} C_n \cos(n \omega_o t + \varphi_n), \quad C_n = (A_n^2 + B_n^2)^{1/2}, \quad \varphi_n = -\tan^{-1}(B_n / A_n) \end{aligned}$$

1.1.7 Parseval's formula

$$\frac{1}{T} \int_{t_o}^{t_o+T} |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |\alpha_n|^2 = \frac{A_o^2}{4} + \sum_{n=1}^{\infty} \left(\frac{A_n^2}{2} + \frac{B_n^2}{2} \right) = \frac{A_o^2}{4} + \sum_{n=1}^{\infty} \frac{C_n^2}{2}$$

1.1.8 Sum of cosines

$$\frac{1}{2} + \cos t + \cos 2t + \dots + \cos nt = \frac{\sin(n + \frac{1}{2})t}{2 \sin \frac{t}{2}}$$

1.1.9 Truncated Fourier series

$$\begin{aligned} f_N(t) &= \frac{A_o}{2} + \sum_{n=1}^N (A_n \cos n \omega_o t + B_n \sin n \omega_o t) \\ &= \frac{1}{T} \int_{-T/2}^{T/2} f(v) \frac{\sin \left[(2N+1) \omega_o \frac{t-v}{2} \right]}{\sin \left[\omega_o \frac{t-v}{2} \right]} dv \end{aligned}$$

1.1.10 Sum and difference functions

$$p(t) = C_1 f(t) \pm C_2 h(t) = \sum_{n=-\infty}^{\infty} [C_1 \beta_n \pm C_2 \gamma_n] e^{jn \omega_o t} = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn \omega_o t},$$

C_1 = constant, C_2 = constant, β_n = Fourier expansion coefficients of $f(t)$, γ_n = Fourier expansion coefficients of $h(t)$, $\alpha_n = C_1 \beta_n \pm C_2 \gamma_n$, $f(t)$ and $h(t)$ are periodic with same period.

1.1.11 Product of two functions

$$p(t) = f(t)h(t) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} (\beta_{n-m} \gamma_m) e^{j n \omega_o t} = \sum_{n=-\infty}^{\infty} \alpha_n e^{j n \omega_o t}$$

$$\alpha_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t)h(t) e^{-j n \omega_o t} dt = \sum_{m=-\infty}^{\infty} (\beta_{n-m} \gamma_m)$$

β_n = Fourier expansion coefficients of $f(t)$, γ_n = Fourier expansion coefficients of $h(t)$, $f(t)$ and $h(t)$ are periodic with same period.

1.1.12 Convolution of two functions

$$g(t) = \frac{1}{T} \int_{-T/2}^{T/2} f(\tau)h(t-\tau) d\tau = \sum_{n=-\infty}^{\infty} \alpha_n e^{j n \omega_o t} = \sum_{n=-\infty}^{\infty} \beta_n \gamma_n e^{j n \omega_o t}$$

$\alpha_n = \beta_n \gamma_n$. β_n = Fourier expansion coefficients of $f(t)$, γ_n = Fourier expansion coefficients of $h(t)$, $f(t)$, and $h(t)$ are periodic with same period.

1.1.13 If $H(\omega)$ (*transfer function*) is the Fourier transform of the *impulse response* $h(t)$ of a linear time invariant system (LTI), then its output due to a periodic input function $f(t)$ is

$$y(t) = \frac{A_o}{2} H(0) + \sum_{n=1}^{\infty} |H(n\omega_o)| [A_n \cos[n\omega_o t + \phi(n\omega_o)] + B_n \sin[n\omega_o t + \phi(n\omega_o)]]$$

$$H(n\omega_o) = H_r(n\omega_o) + jH_i(n\omega_o) = [H_r^2(n\omega_o) + H_i^2(n\omega_o)]^{1/2} e^{j\phi(n\omega_o)}$$

$$\phi(n\omega_o) = \tan^{-1}[H_i(n\omega_o)/H_r(n\omega_o)]$$

$H_r(\cdot)$ and $H_i(\cdot)$ are real functions.

1.1.14 Lanczos *smoothing* factor

$$f_N(t) = \frac{A_o}{2} + \sum_{n=1}^N \frac{\sin(n\pi/N)}{n\pi/N} [A_n \cos n\omega_o t + B_n \sin n\omega_o t]$$

where A_0 , A_n , and B_n are the trigonometric expansion Fourier series coefficients (see 1.1.6).

1.1.15 Fej  smoothing series

$$f_N(t) = \frac{A_o}{2} + \sum_{n=1}^N \frac{N-n}{N} [A_n \cos n\omega_o t + B_n \sin n\omega_o t]$$

where A_0 , A_n , and B_n are the trigonometric expansion Fourier series coefficients (see 1.1.6).

1.1.16 Transformation from 2ℓ to 2π

If the period is 2ℓ , then the Fourier series of $f(t)$ is

$$f(t) = \frac{A_o}{2} + \sum_{k=1}^N \left[A_k \cos \frac{\pi k t}{\ell} + B_k \sin \frac{\pi k t}{\ell} \right]$$

If we set $\pi t/\ell = x$ or $t = x\ell/\pi$, we obtain the equivalent series

$$\varphi(x) = f\left(\frac{x\ell}{\pi}\right) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos kx + b_k \sin kx]$$

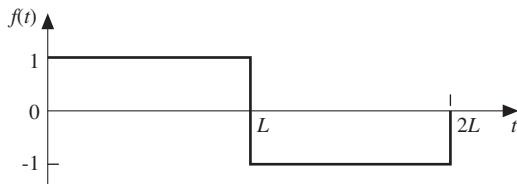
The above means: If $f(t)$ has period 2ℓ , then $\varphi(x) = f(x\ell/\pi)$ has a period 2π .

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos kx dx \quad k = 0, 1, 2, \dots$$

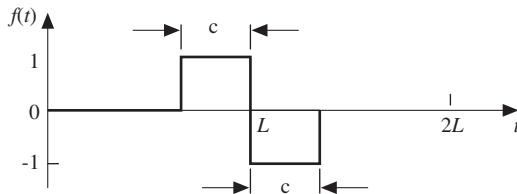
$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin kx dx \quad k = 1, 2, \dots$$

1.1.17 Table of Fourier Series Expansions

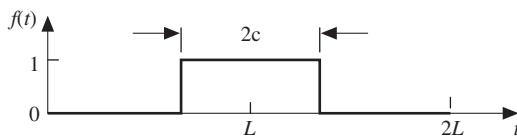
$$1. \quad f(t) = \frac{1}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \sin \frac{n\pi t}{L}$$



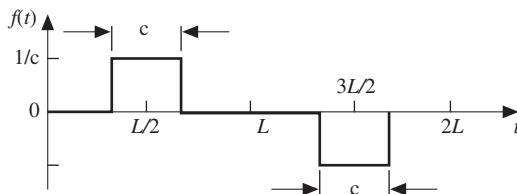
$$2. \quad f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\cos \frac{n\pi c}{L} - 1 \right) \sin \frac{n\pi t}{L}$$



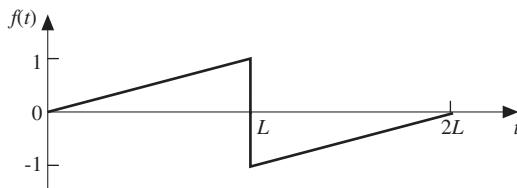
$$3. \quad f(t) = \frac{c}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi c}{L} \cos \frac{n\pi t}{L}$$



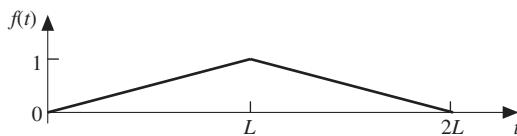
$$4. \quad f(t) = \frac{2}{L} \sum_{n=1}^{\infty} \sin \frac{n\pi}{2} \frac{\sin \frac{1}{2}n\pi c/L}{\frac{1}{2}n\pi c/L} \sin \frac{n\pi t}{L}$$



5.
$$f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi t}{L}$$

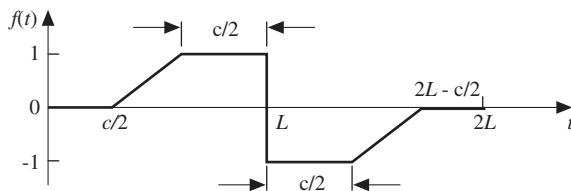


6.
$$f(t) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{n=1,3,5,\dots} \frac{1}{n^2} \cos \frac{n\pi t}{L}$$

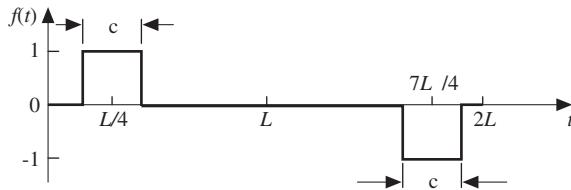


7.
$$f(t) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left[1 + \frac{1+(-1)^n}{n\pi(1-2a)} \sin n\pi a \right] \sin \frac{n\pi t}{L}; \quad a = \frac{c}{2L}$$

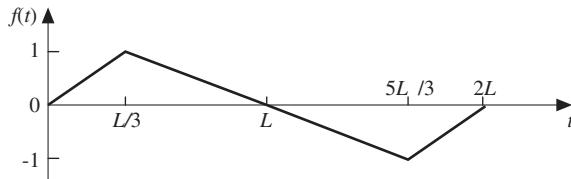
$$\left[1 + \frac{1+(-1)^n}{n\pi(1-2a)} \sin n\pi a \right] \sin \frac{n\pi t}{L}; \quad a = \frac{c}{2L}$$



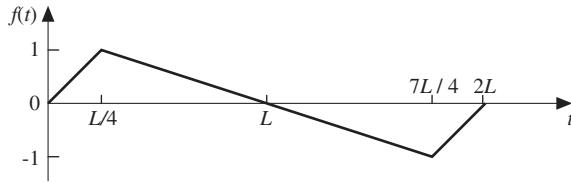
8.
$$f(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{4} \sin n\pi a \sin \frac{n\pi t}{L}; \quad a = \frac{c}{2L}$$



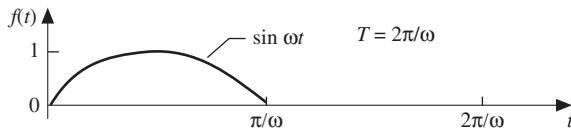
9.
$$f(t) = \frac{9}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{3} \sin \frac{n\pi t}{L}$$



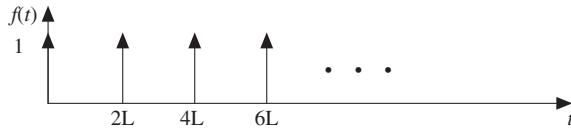
10.
$$f(t) = \frac{32}{3\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{4} \sin \frac{n\pi t}{L}$$



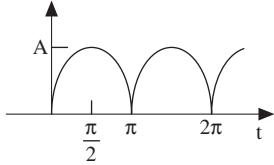
11.
$$f(t) = \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \sum_{n=2,4,6,\dots} \frac{1}{n^2 - 1} \cos n\omega t$$



12.
$$f(t) = \frac{1}{2L} + \frac{1}{L} \sum_{n=1}^{\infty} \cos \frac{n\pi t}{L}$$



13.
$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos 2nt$$



1.2 Orthogonal Systems and Fourier Series

1.2.1 An infinite system of real functions $\varphi_0(t), \varphi_1(t), \varphi_2(t), \dots, \varphi_n(t), \dots$ is said to be **orthogonal** on an

interval $[a,b]$ if $\int_a^b \varphi_n(t)\varphi_m(t)dt = 0$ for $n \neq m$ and $n,m = 0,1,2,\dots$. It is assumed that

$$\int_a^b \varphi_n^2(t)dt \neq 0 \text{ for } n = 0,1,2,\dots$$

1.2.2 The expansion of a function $f(t)$ in $[a,b]$ is given by

$$f(t) = c_0 \varphi_0(t) + c_1 \varphi_1(t) + \dots + c_n \varphi_n(t) + \dots$$

$$c_n = \frac{\int_a^b f(t) \varphi_n(t) dt}{\int_a^b \varphi_n^2(t) dt} = \frac{\int_a^b f(t) \varphi_n(t) dt}{\|\varphi_n\|^2} \quad n = 0, 1, 2, \dots$$

1.2.3 **Bessel's** inequality

$$\int_a^b f^2(t) dt \geq \sum_{k=0}^n c_k^2 \|\varphi_k\|^2 \quad n = \text{arbitrary}$$

- 1.2.4 **Completeness** of the system (1.2.1): A necessary and sufficient condition for the system (1.2.1) to be complete is that the Fourier series of any square integrable function $f(t)$ converges to $f(t)$ in the mean.

If the system (1.2.1) is complete, then every square integrable function $f(t)$ is completely determined (except for its values at a finite number of points) by its Fourier series.

- 1.2.5 The **limits** as $n \rightarrow \infty$ of the **trigonometric** integrals

$$\lim_{n \rightarrow \infty} \int_{-T/2}^{T/2} f(t) \cos \frac{2\pi nt}{T} dt = \lim_{n \rightarrow \infty} \int_{-T/2}^{T/2} f(t) \sin \frac{2\pi nt}{T} dt$$

- 1.2.6 **Convergence** in discontinuity: If $f(t)$ is the absolutely integrable function of period T , then at every point of discontinuity where $f(t)$ has a right-hand and left-hand derivative, the Fourier series of $f(t)$ converges to the value $[f(t+0) + f(t-0)]/2$.

1.3 Decreasing Coefficients of Trigonometric Series

- 1.3.1 **Abel** lemma: Let $u_0 + u_1 + u_2 + \dots + u_n + \dots$ be a numerical series whose partial sums σ_n satisfy the condition $|\sigma_n| \leq M$, where M is a constant. Then, if the positive numbers $\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_n, \dots$ approach zero monotonically, the series $\alpha_0 u_0 + \alpha_1 u_1 + \dots + \alpha_n u_n + \dots$ converges, and the sum S satisfies the inequality $|S| \leq M \alpha_0$.

- 1.3.2 The **sum** of sines

$$\sin t + \sin 2t + \sin 3t + \dots + \sin nt = \frac{\cos \frac{t}{2} - \cos \left[\left(n + \frac{1}{2} \right) t \right]}{2 \cos \frac{t}{2}}$$

$$1 + \frac{\cos t}{p} + \frac{\cos 2t}{p^2} + \dots + \frac{\cos nt}{p^n} + \dots = \frac{p(p - \cos t)}{p^2 - 2p \cos t + 1}$$

$$\frac{\sin t}{p} + \frac{\sin 2t}{p^2} + \dots + \frac{\sin nt}{p^n} + \dots = \frac{p \sin t}{p^2 - 2p \cos t + 1}$$

1.4 Operations on Fourier Series

- 1.4.1 **Integration** of Fourier series: If the absolutely integrable function $f(t)$ of period T is specified by its Fourier series (1.1.6) then

$$\int_a^b f(t) dt$$

can be found by term-by-term integration of the series.

- 1.4.2 **Differentiation** of Fourier series: If $f(t)$ is a continuous function of period T with absolutely integrable derivative, which may not exist at certain points, then the Fourier series of $df(t)/dt$ can be obtained from the Fourier series of $f(t)$ by term-by-term differentiation.

1.5 Two-Dimensional Fourier Series

- 1.5.1 Complex form

$$f(x, y) = \sum_{m,n=-\infty}^{\infty} c_{mn} e^{j\pi(\frac{mx}{l} + \frac{ny}{h})} \quad R\{-l \leq x \leq l, -h \leq y \leq h\}$$

$$c_{mn} = \frac{1}{2lh} \iint_R f(x, y) e^{-j\pi(\frac{mx}{l} + \frac{ny}{h})} dx dy \quad m, n = 0, \pm 1, \pm 2, \dots$$

- 1.5.2 Trigonometric form

$$f(x, y) = \sum_{m,n=0}^{\infty} \left[A_{mn} \cos \frac{\pi mx}{l} \cos \frac{\pi ny}{h} + B_{mn} \sin \frac{\pi mx}{l} \cos \frac{\pi ny}{h} \right. \\ \left. + C_{mn} \cos \frac{\pi mx}{l} \sin \frac{\pi ny}{h} + D_{mn} \sin \frac{\pi mx}{l} \sin \frac{\pi ny}{h} \right] \quad R\{-l \leq x \leq l, -h \leq y \leq h\}$$

$$A_{mn} = \frac{1}{lh} \iint_{-l-h}^{l-h} f(x, y) \cos \frac{\pi mx}{l} \cos \frac{\pi ny}{h} dx dy$$

$$B_{mn} = \frac{1}{lh} \iint_{-l-h}^{l-h} f(x, y) \sin \frac{\pi mx}{l} \cos \frac{\pi ny}{h} dx dy$$

$$C_{mn} = \frac{1}{lh} \iint_{-l-h}^{l-h} f(x, y) \cos \frac{\pi mx}{l} \sin \frac{\pi ny}{h} dx dy$$

$$D_{mn} = \frac{1}{lh} \iint_{-l-h}^{l-h} f(x, y) \sin \frac{\pi mx}{l} \sin \frac{\pi ny}{h} dx dy$$

1.5.3 Trigonometric form with limits $-\pi \leq x \leq \pi, -\pi \leq y \leq \pi$

$$f(x,y) = \sum_{m,n=0}^{\infty} \lambda_{mn} [a_{mn} \cos mx \cos ny + b_{mn} \sin mx \cos ny \\ + c_{mn} \cos mx \sin ny + d_{mn} \sin mx \sin ny]$$

$R\{-\pi \leq x \leq \pi, -\pi \leq y \leq \pi\}$

$$a_{mn} = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x,y) \cos mx \cos ny \, dx dy$$

$$b_{mn} = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x,y) \sin mx \cos ny \, dx dy$$

$$c_{mn} = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x,y) \cos mx \sin ny \, dx dy$$

$$d_{mn} = \frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x,y) \sin mx \sin ny \, dx dy$$

$$\lambda_{mn} = \begin{cases} \frac{1}{4} & m = n = 0 \\ \frac{1}{2} & m > 0, n = 0, \text{ or } m = 0, n > 0 \\ 1 & m > 0, n > 0 \end{cases}$$

$$m, n = 0, 1, 2, 3, 4 \dots$$

1.5.4 Parseval's formula

$$\frac{1}{\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f^2(x,y) \, dx dy = \sum_{m,n=0}^{\infty} \lambda_{mn} (a_{mn}^2 + b_{mn}^2 + c_{mn}^2 + d_{mn}^2)$$

Appendix 1

Examples

Example 1

Expand the function shown in [Figure 1.1](#) in Fourier series and plot the results.

$$\alpha_n = \frac{1}{3.5} \int_{-0.5}^3 f(t) e^{-j n \omega_o t} \, dt = \frac{1}{3.5} \left[\int_{-0.5}^1 1 \cdot e^{-j n \omega_o t} \, dt + \int_1^3 0 \cdot e^{-j n \omega_o t} \, dt \right] \\ = \frac{1}{3.5(-j n \omega_o)} e^{-j n \omega_o t} \Big|_{-0.5}^1 = \frac{1}{-j 3.5 n \omega_o} (e^{-j n \omega_o} - e^{j 0.5 n \omega_o})$$

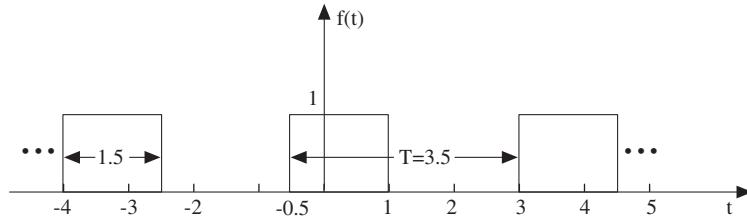


FIGURE 1.1

$$\alpha_o = \frac{1}{3.5} \int_{-0.5}^1 dt = \frac{3}{7}, \quad \omega_o = \frac{2\pi}{3.5}$$

$$\begin{aligned}
 f(t) &= \alpha_o + \sum_{n=-\infty}^{\infty} \alpha_n e^{j n \omega_o t} = \frac{3}{7} + \sum_{n=1}^{\infty} \left\{ \left[\frac{1}{-j 3.5 n \omega_o} (e^{-j n \omega_o} - e^{j 0.5 n \omega_o}) + \frac{1}{j 3.5 n \omega_o} (e^{j n \omega_o} - e^{-j 0.5 n \omega_o}) \right] \cos n \omega_o t \right. \\
 &\quad \left. + j \left[\frac{1}{-j 3.5 n \omega_o} (e^{-j n \omega_o} - e^{j 0.5 n \omega_o}) - \frac{1}{j 3.5 n \omega_o} (e^{j n \omega_o} - e^{-j 0.5 n \omega_o}) \right] \sin n \omega_o t \right\} \\
 &= \frac{3}{7} + \sum_{n=1}^{\infty} \left[\frac{4}{3.5 n \omega_o} [(\sin 0.75 n \omega_o \cos 0.25 n \omega_o) \cos n \omega_o t + (\sin 0.75 n \omega_o \sin 0.25 n \omega_o) \sin n \omega_o t] \right]
 \end{aligned}$$

Figure 1.2 shows $f(t)$ for the cases $1 \leq n \leq 3$ (curve 1) and $1 \leq n \leq 10$ (curve 2). Figure 1.3 shows $f(t)$ for $10 \leq n \leq 50$, and Figure 1.4 shows $f(t)$ for $1 \leq n \leq 60$. Observe the Gibbs phenomenon in Figures 1.2 and 1.4.

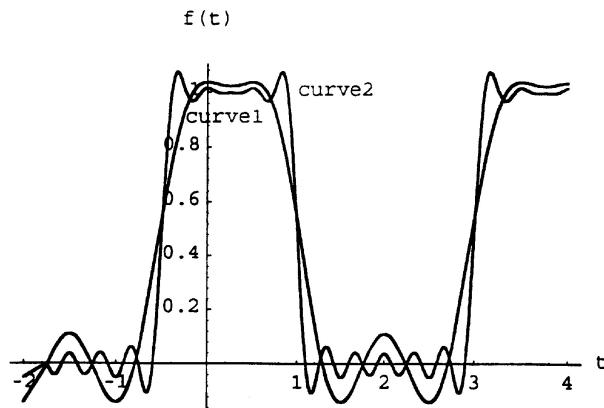


FIGURE 1.2

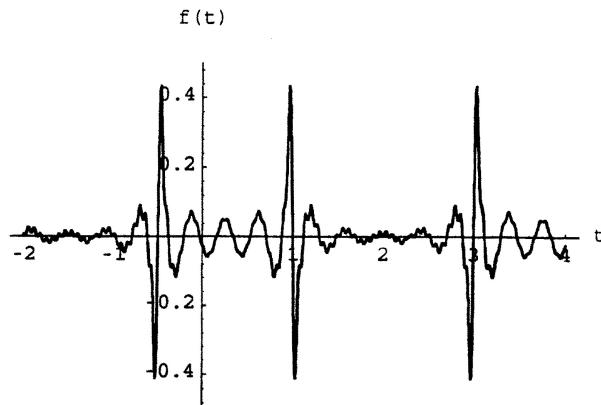


FIGURE 1.3

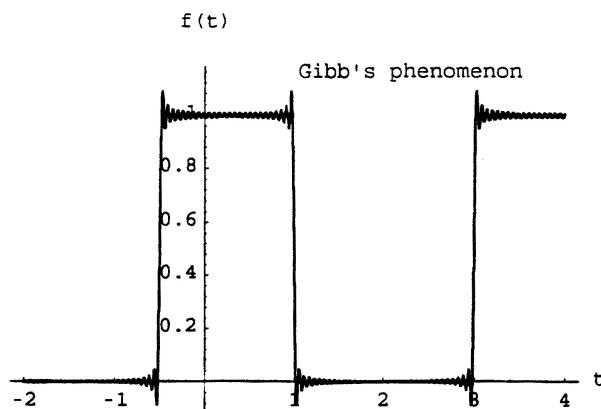


FIGURE 1.4

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