Reading, Writing, and Proving (Second Edition)

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Solutions to Chapter 3: Introducing the Contrapositive and Converse

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If you discover errors in these solutions or feel you have a better solution, please write to us at udaepp@bucknell.edu or pgorkin@bucknell.edu. We hope that you have fun with these problems.

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Solution to Problem 3.3. *(a) Contrapositive: If you don't live in a white house, then you are not the President of the United States.*

Converse: If you live in a white house, then you are the President of the United States.

(b) Contrapositive: If you do not need eggs, then you are not going to bake a soufflé.

Converse: If you need eggs, then you are going to bake a soufflé.

(c) Contrapositive: If x is not an integer, then x is not a real number.

Converse: If x is an integer, then x is a real number.

(d) Contrapositive: If $x^2 \not< 0$, then x is not a real number.

Converse: If $x^2 < 0$, then x is a real number.

Solution to Problem 3.6. *(a) If it does not rain, then it does not pour.*

(b) If I am not living abroad, then I do not need brownies.

(c) We first realize that the given statement may be rewritten as "If one has long legs, then one runs quickly." Thus the inverse is: If one does not have long legs, then one cannot run quickly.

(d) Again, we first turn the given statement into standard form: "If one makes good chocolate chip cookies, then one has baking soda." Thus the inverse statement is: If one does not make good chocolate chip cookies, then one does not have baking soda.

Solution to Problem 3.9. *We will show the contrapositive. Let x and y be real numbers. If $2x + 4 = 2y + 4$, then $x = y$.*

Given is $2x + 4 = 2y + 4$. Subtracting 4 from both sides leads to $2x = 2y$. Finally we divide both sides by 2 to get $x = y$.

Since a statement and its contrapositive are equivalent, the original statement is also proven.

Solution to Problem 3.12. *(a)*

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{P} \to \mathbf{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

$\mathbf{P}$	$\mathbf{Q}$	$\mathbf{P} \to (\mathbf{Q} \vee \neg\mathbf{P})$
T	T	T
T	F	F
F	T	T
F	F	T

(b) *Since both statement forms have the same truth tables, Theorem 2.7 implies that the two forms are equivalent. That is, we have shown that*

$$(P \to Q) \leftrightarrow (P \to (Q \vee \neg P)).$$

Solution to Problem 3.15. *We will prove the contrapositive which will imply the truth of the original statement. For a natural number x, if $\sqrt{2x}$ is an integer, then x is even.*

Proof. We assume that $\sqrt{2x} = n$ for some integer n. Then $n^2 = 2x$ and thus n^2 is even. From Problem 3.2 we conclude that n is even. Hence $n = 2m$ for some integer m. Thus $2x = (2m)^2 = 4m^2$. Hence $x = 2(m^2)$ and m^2 is an integer. This shows that x is even. □

Solution to Problem 3.18. *We will prove the contrapositive statement which is: If at least one of two integers, x and y, is even, then the product is even.*

Without loss of generality we may assume that x is even; that is, $x = 2n$ for some integer n. Then $xy = 2ny = 2(ny)$ with ny an integer; that is, xy is even. The contrapositive statement is proven and thus the equivalent original statement also holds.