

DECENTRALIZED ROBUST H_∞ CONTROLLER DESIGN FOR A HALF-CAR ACTIVE SUSPENSION SYSTEM

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ABSTRACT

In this paper an H_∞ controller is designed for a hydraulically actuated active suspension system of a half-modeled vehicle in a cascade feedback structure. Using the proposed structure the nonlinear behavior of actuator is reduced significantly. In the controller synthesis, a proportional controller is used in the inner loop, and a robust H_∞ controller forms the outer loop. Two H_∞ controllers are designed for this system. First unstructured uncertainty is not considered in the design procedure and secondly, the controller is designed considering uncertainty. Each of these controllers is designed in a decentralized fashion and the vehicle oscillation in the human sensitivity frequency range is reduced to a minimum. Statistical analysis of the simulation result using random input as road roughness, illustrates the effectiveness of the proposed control algorithm for both cases.

Keyword: active suspension, cascade feedback, hydraulic actuator, nonlinear model, multiplicative uncertainty, robust H_∞ controller, disturbance input, road random input, statistical analysis.

INTRODUCTION

Demands for ride comfort and controllability of road vehicle are pursued by many automotive industries by using active suspension. These electronically controlled suspension system can improve the ride comfort as well as the road handling of the vehicle. The suspension system can be categorized into three groups: passive suspension systems including conventional springs and dampers. These systems contain no electronic sensor and control, Miller (1). Semi-active suspension systems provide controlled real-time dissipation of energy, (1). Active suspension systems use a hydraulic or pneumatic actuator in parallel with a passive spring and shock absorber, and hence, the measurement of body vibration is used to decide instantaneously the amount of force needed by the actuator. Different characteristics can be considered in a suspension system design namely, ride comfort, body movement, road holding and suspension travel. No suspension system can simultaneously optimize all four mentioned parameters. But a better trade-off among these parameters can be achieved in active suspension system, Taghirad and Esmailzadeh (2). Many researches in recent years have concentrated

on active vehicle suspension system. In these researches a variety of models including 1/4, 1/2 and full-car model have considered. Although experiments show the importance of nonlinear behavior of actuator in determination of a suitable trade-off in a suspension system, in many published researches, no attention is made to the nonlinear behavior of this element, (2), Yamashita et al (3), Wang et al (4). In Some researches use backstepping control is used to investigate nonlinear behavior of actuator in different road condition, Lin and Kanellakopoulos (5), Karlsson et al (6). Moreover, nonlinear optimal control is proposed in Karlsson et al (7, 8), which provides a controller for active suspension system. Nonlinear H_∞ control is another design approach for a quarter-car model, Karlsson et al (9). Also, use of cascade feedback structure is another way for investigation of hydraulic actuator nonlinearity to reduce nonlinear behavior of hydraulic actuator, Fukao et al (10).

In this paper a decentralized robust H_∞ control are designed for a half-modeled vehicle considering nonlinear behavior of hydraulic actuator. Nonlinear system is linearized in different operating conditions and linearization results show the dominance of nonlinear behavior of the hydraulic actuator. To remedy this drawback, cascade feedback structure is used. Using this structure, the behavior of the system is significantly linearized makes it plausible to determine linear model for the system in addition to minimum bounded norm multiplicative uncertainty description. Two H_∞ controllers are designed for the above structure, considering nominal performance for the first case, and robust performance in the second. The solution of mixed sensitivity problem is significantly reducing the vehicle vibration in the human sensitivity frequency range. Statistical analysis of the simulation results using random input as road roughness illustrates that the proposed strategy can provide a suitable trade-off between ride comfort and road holding, despite nonlinear behavior of the actuator.

HALF – CAR MODEL

Figure 1 illustrates the half-car model of a passenger car, in which only four degrees of freedom are considered, Taghirad and Behraves (15). In this model the dynamical motion of the vehicle body and two axles in longitudinal plane is determined. The suspension stiffness and the tires are modeled by linear spring in parallel with viscous dampers. Force-generating

elements in active suspension system are hydraulic actuators, Alley and Hedrick (13). The systems with four degrees of freedom are represented by the following states: body bounce z_s , body pitch θ , front and rear tire deflection, z_{u1} , z_{u2} .

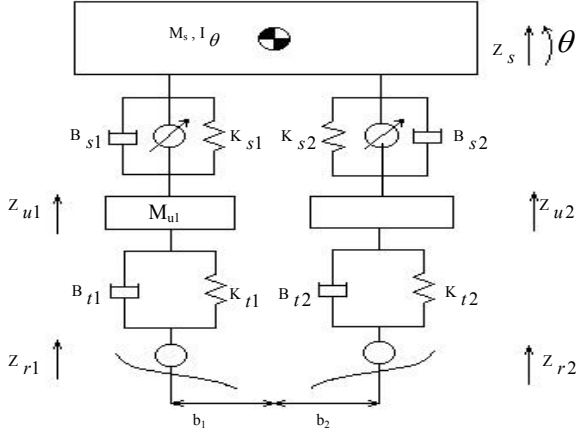


Figure 1 : Half-car Suspension System

The following equations of motion are derived for the model using Newton – Euler method:

$$\begin{aligned}\ddot{z}_s &= \frac{1}{M_s} [-(K_{s1} + K_{s2})z_s - (K_{s1}b_1 - K_{s2}b_2)\theta + K_{s1}z_{u1} + K_{s2}z_{u2} + B_{s2}\dot{z}_{u2} \\ &\quad - (B_{s1} + B_{s2})\dot{z}_s - (B_{s1} - B_{s2}b_2)\dot{\theta} + F_{A1} + F_{A2} - F_{f1} - f_{f2}] \\ \ddot{\theta} &= \frac{1}{I_\theta} [-(K_{s1}b_1 - K_{s2}b_2)z_s - (K_{s1}b_1^2 + K_{s2}b_2^2)\theta - K_{s1}b_1z_{u1} + K_{s2}b_2z_{u2} \\ &\quad - (B_{s1}b_1 - B_{s2}b_2)\dot{z}_s - (B_{s1}b_1^2 + B_{s2}b_2^2)\dot{\theta} - B_{s1}b_1\dot{z}_{u1} + B_{s2}b_2\dot{z}_{u2} \\ &\quad + (F_{A1}b_1 - F_{A2}b_2)] \\ \ddot{z}_{u1} &= \frac{1}{M_{u1}} [K_{s1}z_s + K_{s1}b_1\theta - (K_{s1} + K_{u1})z_{u1} + K_{u1}z_{r1} - F_{A1} + B_{s1}\dot{z}_s + B_{u1}\dot{\theta} \\ &\quad - (B_{s1} + B_{u1})\dot{z}_{u1} + B_{u2}\dot{z}_{r2}] \\ \ddot{z}_{u2} &= \frac{1}{M_{u2}} [K_{s2}z_s - K_{s2}b_2\theta - (K_{s2} + K_{u2})z_{u2} + K_{u2}z_{r2} - F_{A2} + B_{s2}\dot{z}_s - B_{s2}b_2\dot{\theta} \\ &\quad - (B_{s2} + B_{u2})\dot{z}_{u2} + B_{u2}\dot{z}_{r2}]\end{aligned}\quad (1)$$

In which, F_A is the hydraulic force provided by the actuator and F_f is the frictional force due to rubbing of piston seals with the cylinders wall inside the actuator and z_r is the road disturbance input. The relation between the spool valve movement, u , and the output force of this actuator, $F_A = P_L \times A$, possess a nonlinear dynamics, Merritt (14).

$$\dot{F}_A = A\alpha \left[C_d \alpha x_v \sqrt{\frac{P_s - \text{sgn}(x_v)P_L}{\rho}} - C_{m1}P_L - A(\dot{z}_s - \dot{z}_u) \right] \quad (2)$$

Frictional force is modeled with a smooth approximation of Signum function.

$$F_f = \begin{cases} \mu \text{sgn}(\dot{z}_s - \dot{z}_u), & \text{if } |\dot{z}_s - \dot{z}_u| \geq 0.01 \text{ m/s} \\ \mu \sin\left(\frac{\dot{z}_s - \dot{z}_u \pi}{0.01 \cdot 2}\right), & \text{if } |\dot{z}_s - \dot{z}_u| < 0.01 \text{ m/s} \end{cases} \quad (3)$$

This model is also found to match suitably with experiments, Rajamani, and Hedrick (12). The value of μ is estimated about 240N. Also experimental values of parameter are shown in Table 1.

TABLE 1 : The values of Parameters in Half-car Suspension System

Description	Units	Values
Body Mass	$M_s(\text{kg})$	897.35
Front Axle Mass	$M_{u1}(\text{kg})$	43.76
Front Suspension Stiffness	$K_{s1}(\text{N/m})$	66793
Front Suspension Damping	$B_{s1}(\text{Ns/m})$	1189
Front Tire Stiffness	$K_{t1}(\text{N/m})$	201021
Front Tire Damping	$B_{t1}(\text{Ns/m})$	14.6
Rear Axle Mass	$M_{u2}(\text{kg})$	70.53
Rear Suspension Stiffness	$K_{s2}(\text{N/m})$	18606
Rear Suspension Damping	$B_{s2}(\text{Ns/m})$	998
Rear Tire Stiffness	$K_{t2}(\text{N/m})$	201021
Rear Tire Damping	$B_{t2}(\text{Ns/m})$	14.6
Body Inertia	$I_\theta(\text{N.m.s}^2)$	1283.83

DECETERALIZED ROBUST H_∞ CONTROLLER DESIGN

Half-car suspension system is a 2×2 -system. Considering the ability of decentralized control in multi-input, multi output systems, we use this method in control structure. For this purpose each suspension system in front and rear of the model, are considered as an independent suspension system. Hence, nonlinear model of the system can be approximated with a linear model in addition to a multiplicative uncertainty. The linear model is called nominal model of the system, and the variations of each frequency response estimate from the nominal model can be shown by a multiplicative uncertainty. Assuming that the nominal plant transfer function is $P(s)$. Define p as the family of possible model of the system, which includes all the systems, by multiplicative uncertainty we consider:

$$\forall P(s) \in p, \quad P(s) = (1 + \Delta(s)W(s))P_0(s) \quad (4)$$

Where, $W(s)$ is the uncertainty weighting function and Δ is a memoryless operator of induced norm less than unity. In this representation $\Delta(s)w(s)$ gives the percentage of the normalized system variation at each frequency:

$$\frac{P(j\omega)}{P_0(j\omega)} - 1 = \Delta(j\omega)W(j\omega) \quad (5)$$

Hence, since $|\Delta| \leq 1$, then

$$\left| \frac{P(j\omega)}{P_0(j\omega)} - 1 \right| \leq |W(j\omega)|, \quad \forall \omega \quad (6)$$

By plotting the system variations $|P(j\omega)/P_0(j\omega) - 1|$, for all experimental frequency response estimates of the system $P(j\omega)$, and estimating an upper bound to those variations as a transfer function, the multiplicative uncertainty weighting function $W(s)$ can be easily obtained. The nonlinear behavior of the hydraulic actuator causes high uncertainty in suspension system. On the other hand, one important results of using feedback is the reduction of the nonlinear behavior of the system. Therefore, by using cascade feedback structure the nonlinear behavior of the actuator will be

reduced significantly, Taghirad, and Shariati (16), (10). This structure consists of two parallel feedbacks as shown in figure (2).

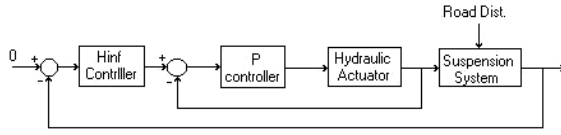


Figure 2 : Control system with cascade feedback structure

The inner loop is designed for linearization of hydraulic actuator with a greater bandwidth compared to the outer loop. The hydraulic actuator is essentially a non-self-regulating system, that is, its output is constant just for zero input in the operating band. Therefore it can be linearized just by a proportional controller without need to an integrator. So for nonlinearity reduction of the hydraulic actuators in half-car suspension system, a proportional controller is proposed in the inner loop. The high gain controller is used 10^{-4} . This value of gain provides a good force tracking in hydraulic actuator as shown in figure (3).

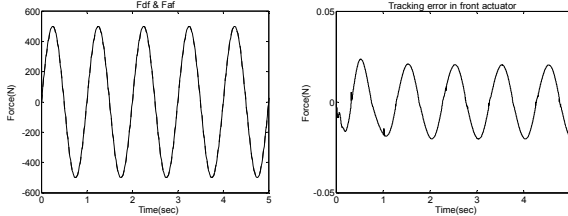


Figure 3 : Force tracking in the front actuator and tracking error

Half-car suspension system identification with inner loops

Using cascade feedback structure, we expect a linear behavior of the system. The actuator forces are considered as two inputs and the rear and the front vertical body motion are considered as two outputs. Therefore, the half-car suspension system is a two-input, two output system in identification process. In this stage the range of the system transfer function variations is specified with a series of frequency response estimates. Figure (4) shows the frequency response of the identified models for the G_{11} of the system transfer function for many different input-output patterns. As expected the nonlinear behavior of the system has reduced significantly using cascade feedback structure, and the identified models are close to each other. The nominal model of the system is a 2×2 transfer matrix consisting of the average of the identified transfer function in each array. The numerical results obtained through comprehensive identification procedure for the system is given in Equation (7).

Input-output pairing is very important to design decentralized controller, and Gershgorian bounds can be used as a measure in this case, Sedigh (18). As shown in Figure (4) the half-car suspension system has a linear behavior in presence of cascade feedback. Therefore, we can use the nominal model to investigate the interaction of the system. Figure (5) illustrates the Gershgorian bounds on the system, by which it can be

concluded that the identified system as $G(s)$ is diagonally dominant (18).

For decentralized controller design, the nominal model of the system is considered diagonal and the effectiveness of the off-diagonal elements are considered in the uncertainty of the system. In a multi-input, multi-output system, the maximum singular value of the matrix $(P - P_o)P_o^{-1}$ is considered as the uncertainty imposed on the system by this means, where, P_o is the nominal model and P is the family of the transfer functions provided in a series of frequency response estimates.

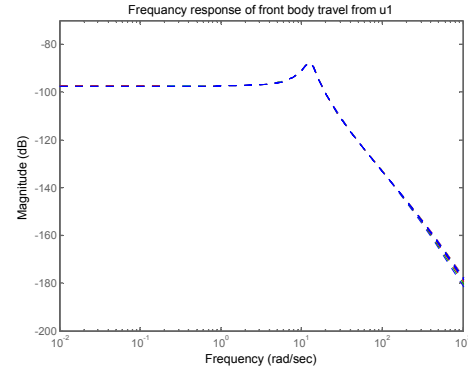


Figure 4 : Frequency Response of $G(1,1)$

$$G(s) = \begin{bmatrix} \frac{F. \text{ Body travel}}{F_{f. act}} & \frac{F. \text{ Body travel}}{F_{r. act}} \\ \frac{R. \text{ Body travel}}{F_{f. act}} & \frac{R. \text{ Body travel}}{F_{r. act}} \end{bmatrix} = \begin{bmatrix} \frac{7.5 \times 10^{-7} s^2 + 2.8 \times 10^{-3} s + 3.16}{s^3 + 1288s^2 + 5785s + 2.1 \times 10^5} & \frac{2.1 \times 10^{-8} s^2 + 6.24 \times 10^{-5} s + 0.0625}{s^3 + 29.09s^2 + 723.2s + 4863} \\ \frac{4.5 \times 10^{-7} s^2 + 3.3 \times 10^{-3} s + 3 \times 10^{-3}}{s^3 + 18.36s^2 + 224.8s + 1727} & \frac{1.02 \times 10^{-6} s^2 + 3.6 \times 10^{-3} s + 0.07}{s^3 + 23.19s^2 + 170s + 1033} \end{bmatrix} \quad (7)$$

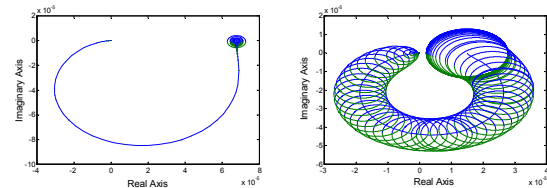


Figure 5 : Nyquist arrays and gershgorian bounds

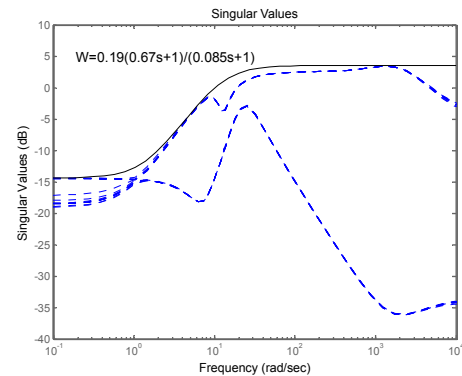


Figure 6 : Singular value variation of uncertainty matrix

The singular values of the system are shown in Figure 6, where its upper bound is considered as the multiplicative uncertainty weighting function for the system. From this figure, it is observed that the interaction of the system is more dominant in the frequencies more than 20 rad/sec, which is usually more than the required bandwidth for the active suspension system. In order to design the decentralized controller, the diagonal matrix of the uncertainty and the nominal model is used as following, and H_∞ controllers are designed separately for each of them.

$$W(s) = \begin{bmatrix} \frac{0.19(0.67s+1)}{(0.085s+1)} & 0 \\ 0 & \frac{0.19(0.67s+1)}{(0.085s+1)} \end{bmatrix} \quad (8)$$

$$G(s) = \begin{bmatrix} \frac{7.5 \times 10^{-7}(s+18.84 \pm 811j)}{(s+1284)(s+2.2 \pm 12.61j)} & 0 \\ 0 & \frac{1.02 \times 10^{-6}(s+3491)(s+19.77)}{(s+16.72)(s+3.24 \pm 7.16j)} \end{bmatrix} \quad (9)$$

Decentralized H_∞ controller design for nominal performance

To reach to a high quality in the closed loop performance, first the nominal performance index, $\|W_s S\|_{\infty} < 1$, is solved, and the robust stability of the system will be investigated next by μ -analysis. Considering the bandwidth requirement for disturbance rejection in human sensitivity range, Thomson (17), the sensitivity weighting function is selected as $W_s(s) = 155/(s+51)$. Using μ -analysis toolbox decentralized H_∞ controller is designed to solve the above performance as follows:

$$C(s) = \begin{bmatrix} \frac{4.61 \times 10^{12}(s+1284)(s+2.2 \pm 12.61j)}{(s+7707)(s+51)(s+1902.5 \pm 838.7j)} & 0 \\ 0 & \frac{4.54 \times 10^{12}(s+16.72)(s+3.24 \pm 7.16j)}{(s+9948)(s+3704)(s+51)(s+19.77)} \end{bmatrix} \quad (10)$$

The performance of the closed loop system has been investigated and its performance is compared to of the passive suspension as illustrated in Figure (7). Using random input as road roughness, statistical analysis of the simulation results provides quantitative measures as given TABLE 2. As it is clearly seen in the results, all the variables in active suspension system has been significantly reduced in the active suspension except for the front suspension travel. The front body acceleration has reduced about 4.5 times and a 9 times reduction is seen in the rear body acceleration compare to the passive suspension system. Since, ride comfort is related to the acceleration, the proposed control strategy for the active suspension is significantly contributing in the passenger ride comfort. Also, the road handling has been improved as the reduction of tire deflection. Also, suspension travel constraints are satisfied in front and rear axles, as well as the constraints on the control valve movements.

ROBUST STABILITY ANALYSIS

Although very suitable performance is obtained through the solution of the nominal performance for the system, no guarantee is given for the robust stability of the system, especially for the interaction of the system off-diagonal elements. To investigate the system robust stability, μ -analysis is performed numerical evaluation of μ through μ -synthesis toolbox of Matlab software. Figure (8) illustrates the numerical values of μ for the perturbed uncertain system. As it is shown, the robust stability of the system is not guaranteed in the (10^{-3}) rad/sec frequency range.

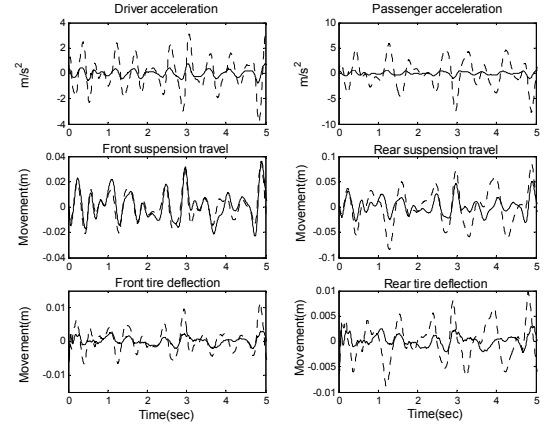


Figure 7 : Comparison between passive and active suspension Systems. Dotted: Passive suspension, Solid: Active suspension

TABLE 2 : Comparison between the parameters of the passive and active suspension system

90% Probability Bound					
	Passive	Active		Passive	Active
F. body acc.	2.31	0.53	R. body acc.	5.27	0.6
F. tire deflection	6.1	2.2	R. tire deflection	7.5	5.7
F. body travel	16.2	2.8	R. body travel	73.7	6.3
F. sus. travel	20	20	R. sus. travel	72.5	28.6
F. sevovalve input	1.1	-	R. sevovalve in.	1.5	-
F. actuator force	892	-	R. actuator force	1094	-
Body pitch	0.019	0.002			

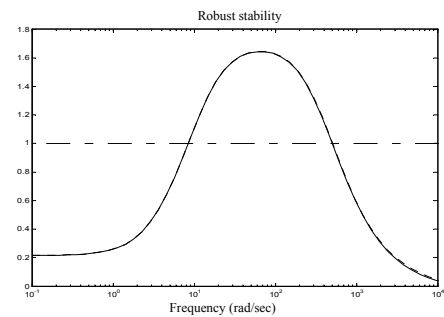


Figure 8 : Robust Stability of the Closed-loop System

Although, no instability indications are seen in many simulations made for the closed loop system, in order to be able to claim robust stability for the system, robust performance of the system is also considered as the design objective and the mixed-sensitivity solution is elaborated in the next section.

Decentralized H_∞ controller design for robust performance

The second H_∞ controller is designed with the above-mentioned nominal models and uncertainty weighting functions for the mixed-sensitivity problem including the previous objective in addition to robust stability. Adding this requirement into the controller objectives causes a reduction in system bandwidth, and sensitivity weighting function is selected as $W_S(s) = 43/(s+19)$ for the system controller to be achievable. The designed H_∞ controller is as following:

$$C(s) = \begin{bmatrix} \frac{2.24 \times 10^{12}(s+1284)(s+11.8)(s+2.2 \pm 12.61j)}{(s+1.12 \times 10^4)(s+71.04)(s+19)(s+1895 \pm 824.6j)} & 0 \\ 0 & \frac{2.23 \times 10^{12}(s+16.72)(s+11.8)(s+3.24 \pm 7.16j)}{(s+3587)(s+1.5 \times 10^4)(s+7092)(s+19)(s+19.77)} \end{bmatrix} \quad (11)$$

The performance of this controller is investigated in cascade feedback structure. The comparison between the variables of the passive and active suspension system using random input as road roughness is shown in Figure (9) and TABLE 3. Although the performance of the closed loop system with the first controller is better compared to that of the second controller, this controller possesses robust stability and still favorable performance compared to that of passive suspension system. As seen in Table 3, the front and rear body acceleration are reduced about 1.67 and 2.8 times, respectively.

CONCLUSIONS

In this paper, decentralized robust H_∞ control for a half-car active suspension system has been investigated. Cascade feedback structure has been used to reduce the nonlinear behavior of the hydraulic actuator. Two H_∞ controllers have been designed for this system, considering nominal and robust performance. Both controllers give favorable performance compare to passive suspension system. Although, the controller designed based on nominal performance can significantly improve ride comfort and road handling. Because of the conservatism structure of H_∞ control synthesis, we propose using this controller for further experiments, subject to a thorough experimental assessment of the closed loop stability in practice.

TABLE 3 : Comparison between the Parameters of the Passive and Active Suspension System

90% Probability Bound					
	Passive	Active		Passive	Active
F. body acc.	2.31	1.39	R. body acc.	5.27	1.89
F. tire deflection	6.1	3.4	R. tire deflection	7.5	5
F. body travel	16.2	8	R. body travel	73.7	24.6
F. sus. travel	20	18	R. sus. travel	72.5	33.2
F. sevovalve input	1	-	R. sevovalve in.	1.2	-
F. actuator force	493.8	-	R. actuator force	677.3	-
Body pitch	0.019	0.008			

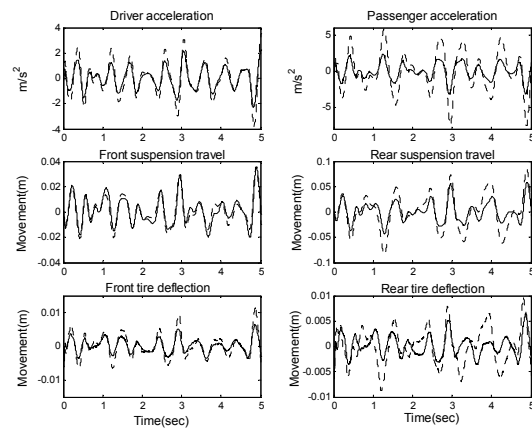


Figure 9 : Comparison between Passive and Active Suspension Systems. Dotted: Passive Suspension, Solid: Active Suspension

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