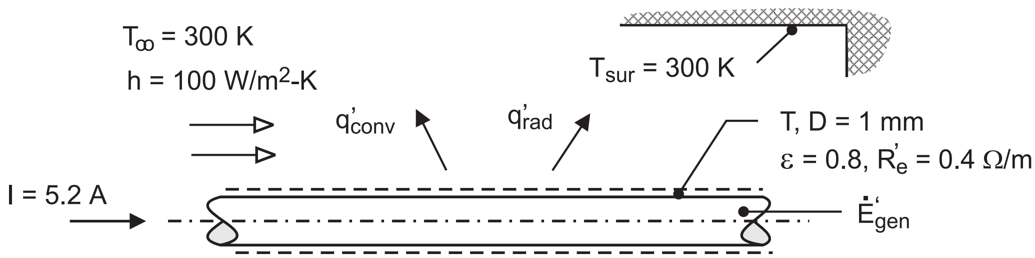
PROBLEM 1.45

KNOWN: Rod of prescribed diameter experiencing electrical dissipation from passage of electrical current and convection under different air velocity conditions. See Example 1.3.

FIND: Rod temperature as a function of the electrical current for $0 \le I \le 10$ A with convection coefficients of 50, 100 and 250 $W/m^2 \cdot K$. Will variations in the surface emissivity have a significant effect on the rod temperature?

SCHEMATIC:



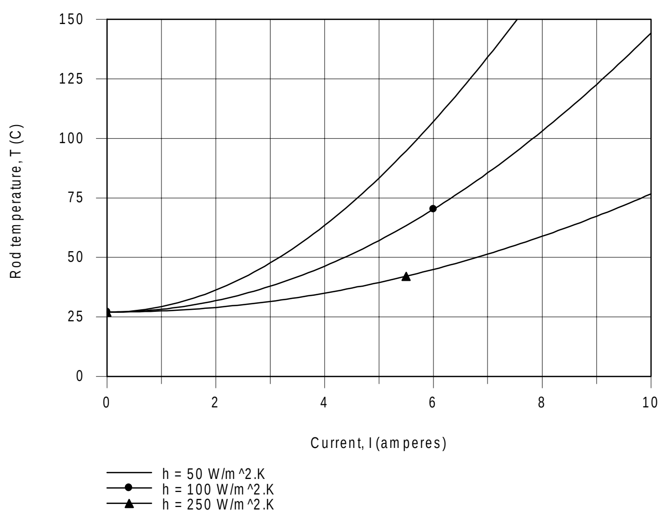
ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform rod temperature, (3) Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and large enclosure.

ANALYSIS: The energy balance on the rod for steady-state conditions has the form,

$$q'_{conv} + q'_{rad} = \dot{E}'_{gen}$$

$$\pi D h \left(T - T_\infty\right) + \pi D \varepsilon \sigma \left(T^4 - T_{sur}^4\right) = I^2 R'_e$$

Using this equation in the Workspace of IHT, the rod temperature is calculated and plotted as a function of current for selected convection coefficients.



COMMENTS: (1) For forced convection over the cylinder, the convection heat transfer coefficient is dependent upon air velocity approximately as $h \sim V^{0.6}$. Hence, to achieve a 5-fold change in the convection coefficient (from 50 to 250 $W/m^2 \cdot K$), the air velocity must be changed by a factor of nearly 15.

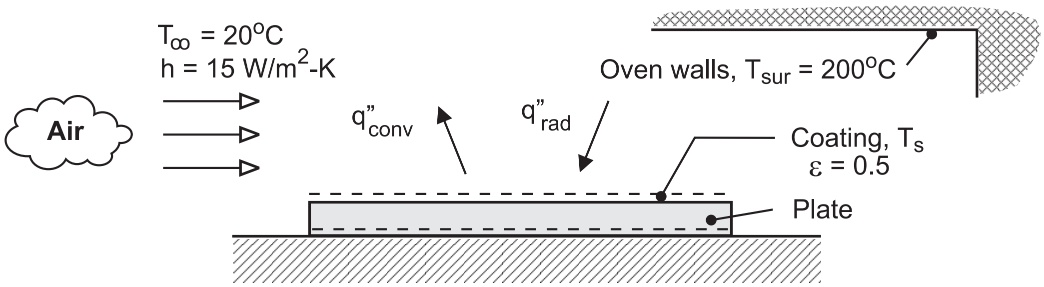
Continued …..

PROBLEM 1.66

KNOWN: Hot-wall oven, in lieu of infrared lamps, with temperature T_{sur} = 200°C for heating a coated plate to the cure temperature. See Example 1.6.

FIND: (a) The plate temperature T_s for prescribed convection conditions and coating emissivity, and (b) Calculate and plot T_s as a function of T_{sur} for the range $150 \le T_{sur} \le 250$°C for ambient air temperatures of 20, 40 and 60°C; identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat loss from back surface of plate, (3) Plate is small object in large isothermal surroundings (hot oven walls).

ANALYSIS: (a) The temperature of the plate can be determined from an energy balance on the plate, considering radiation exchange with the hot oven walls and convection with the ambient air.

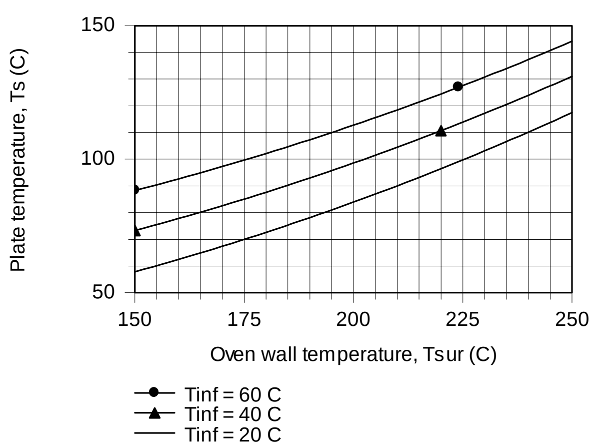
$$\dot{E}''_{in} - \dot{E}''_{out} = 0 \qquad \text{or} \qquad q''_{rad} - q''_{conv} = 0$$

$$\varepsilon\sigma\left(T_{sur}^4 - T_s^4\right) - h\left(T_s - T_\infty\right) = 0$$

$$0.5\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left([200+273]^4 - T_s^4\right)\mathrm{K}^4 - 15\,\mathrm{W/m^2\cdot K}\left(T_s - [20+273]\right)\mathrm{K} = 0$$

$$T_s = 357\ \mathrm{K} = 84°\mathrm{C} \qquad <$$

(b) Using the energy balance relation in the Workspace of IHT, the plate temperature can be calculated and plotted as a function of oven wall temperature for selected ambient air temperatures.



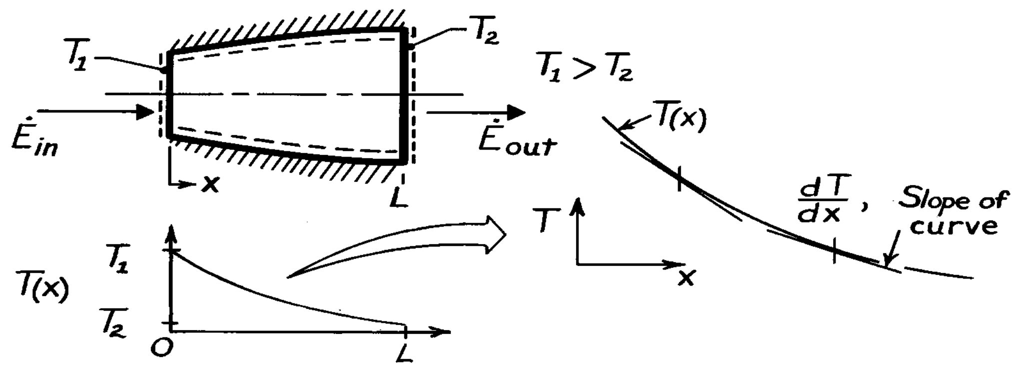
COMMENTS: From the graph, acceptable cure temperatures between 100 and 110°C can be maintained for these conditions: with T_∞ = 20°C when $225 \le T_{sur} \le 240$°C; with T_∞ = 40°C when $205 \le T_{sur} \le 220$°C; and with T_∞ = 60°C when $175 \le T_{sur} \le 195$°C.

PROBLEM 2.1

KNOWN: Steady-state, one-dimensional heat conduction through an axisymmetric shape.

FIND: Sketch temperature distribution and explain shape of curve.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

ANALYSIS: Performing an energy balance on the object according to Eq. 1.11a, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

and that $q_x \neq q_x(x)$. That is, the heat rate within the object is everywhere constant. From Fourier's law,

$$q_x = -kA_x \frac{dT}{dx},$$

and since q_x and k are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{Constant.}$$

That is, the product of the cross-sectional area normal to the heat rate and temperature gradient remains a constant and independent of distance x. It follows that since A_x increases with x, then dT/dx must decrease with increasing x. Hence, the temperature distribution appears as shown above.

COMMENTS: (1) Be sure to recognize that dT/dx is the slope of the temperature distribution. (2) What would the distribution be when $T_2 > T_1$? (3) How does the heat flux, q''_x, vary with distance?

$$q_{iron} = q_{heater} - q_{ss} = 100\text{V} \times 0.601\text{A} - 15.0\ \text{W/m}\cdot\text{K} \times \frac{\pi(0.030\ \text{m})^2}{4} \times \frac{15.0^\circ\text{C}}{0.015\ \text{m}}$$

$$q_{iron} = (60.1 - 10.6)\text{W} = 49.5\ \text{W}$$

where

$$q_{ss} = k_{ss} A_c \Delta T_2 / \Delta x_2 .$$

Applying Fourier's law to the iron sample,

$$k_{iron} = \frac{q_{iron}\Delta x_2}{A_c \Delta T_2} = \frac{49.5\ \text{W} \times 0.015\ \text{m}}{\pi(0.030\ \text{m})^2 / 4 \times 15.0^\circ\text{C}} = 70.0\ \text{W/m}\cdot\text{K}. \qquad <$$

The total drop across the iron sample is 15°C(60/15) = 60°C; the heater temperature is (77 + 60)°C = 137°C. Hence the average temperature of the iron sample is

$$\overline{T} = (137 + 77)^\circ\text{C}/2 = 107^\circ\text{C} = 380\ \text{K}. \qquad <$$

We compare the computed value of k with the tabulated value (see above) at 380 K and note the good agreement.

(c) The principal advantage of having two identical samples is the assurance that all the electrical power dissipated in the heater will appear as equivalent heat flows through the samples. With only one sample, heat can flow from the backside of the heater even though insulated.

Heat leakage out the lateral surfaces of the cylindrically shaped samples will become significant when the sample thermal conductivity is comparable to that of the insulating material. Hence, the method is suitable for metallics, but must be used with caution on nonmetallic materials.

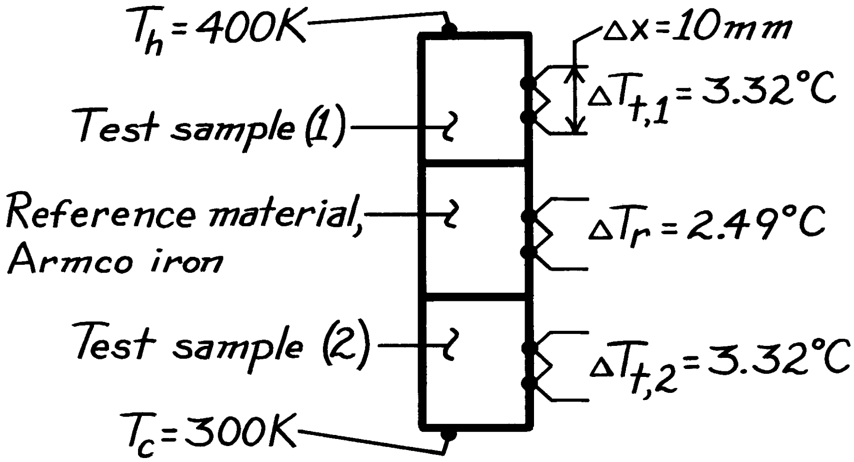
For any combination of materials in the upper and lower position, we expect $\Delta T_1 = \Delta T_2$. However, if the insulation were improperly applied along the lateral surfaces, it is possible that heat leakage will occur, causing $\Delta T_1 \neq \Delta T_2$.

PROBLEM 2.18

KNOWN: Comparative method for measuring thermal conductivity involving two identical samples stacked with a reference material.

FIND: (a) Thermal conductivity of test material and associated temperature, (b) Conditions for which $\Delta T_{t,1} \neq \Delta T_{t,2}$

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through samples and reference material, (3) Negligible thermal contact resistance between materials.

PROPERTIES: *Table A.2*, Armco iron $\left(\overline{T} = 350\ K\right)$: $k_r = 69.2\ W/m\cdot K$.

ANALYSIS: (a) Recognizing that the heat rate through the samples and reference material, all of the same diameter, is the same, it follows from Fourier's law that

$$k_t \frac{\Delta T_{t,1}}{\Delta x} = k_r \frac{\Delta T_r}{\Delta x} = k_t \frac{\Delta T_{t,2}}{\Delta x}$$

$$k_t = k_r \frac{\Delta T_r}{\Delta T_t} = 69.2\ W/m\cdot K \frac{2.49°C}{3.32°C} = 51.9\ W/m\cdot K. \qquad <$$

We should assign this value a temperature of 350 K. <

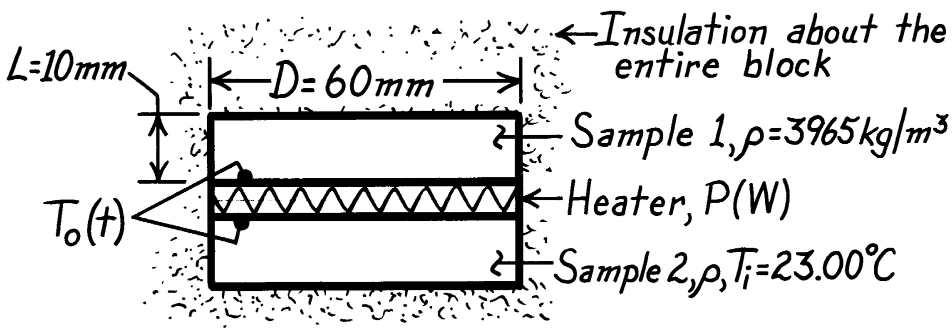
(b) If the test samples are identical in every respect, $\Delta T_{t,1} \neq \Delta T_{t,2}$ if the thermal conductivity is highly dependent upon temperature. Also, if there is heat leakage out the lateral surface, we can expect $\Delta T_{t,2} < \Delta T_{t,1}$. Leakage could be influential, if the thermal conductivity of the test material were less than an order of magnitude larger than that of the insulating material.

PROBLEM 2.19

KNOWN: Identical samples of prescribed diameter, length and density initially at a uniform temperature T_i, sandwich an electric heater which provides a uniform heat flux q''_o for a period of time Δt_o. Conditions shortly after energizing and a long time after de-energizing heater are prescribed.

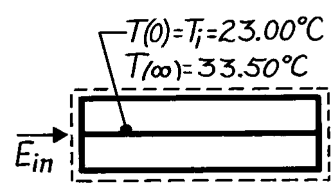
FIND: Specific heat and thermal conductivity of the test sample material. From these properties, identify type of material using Table A.1 or A.2.

SCHEMATIC:



ASSUMPTIONS: (1) One dimensional heat transfer in samples, (2) Constant properties, (3) Negligible heat loss through insulation, (4) Negligible heater mass.

ANALYSIS: Consider a control volume about the samples and heater, and apply conservation of energy over the time interval from t = 0 to ∞



$$E_{in} - E_{out} = \Delta E = E_f - E_i$$

$$P\Delta t_o - 0 = Mc_p\left[T(\infty) - T_i\right]$$

where energy inflow is prescribed by the Case A power condition and the final temperature T_f by Case B. Solving for c_p,

$$c_p = \frac{P\Delta t_o}{M\left[T(\infty) - T_i\right]} = \frac{15\ \text{W} \times 120\ \text{s}}{2 \times 3965\ \text{kg/m}^3\left(\pi \times 0.060^2/4\right)\text{m}^2 \times 0.010\ \text{m}\left[33.50 - 23.00\right]^\circ\text{C}}$$

$$c_p = 765\ \text{J/kg}\cdot\text{K}$$ <

where $M = \rho V = 2\rho(\pi D^2/4)L$ is the mass of both samples. For Case A, the transient thermal response of the heater is given by

Continued …..

$$T_o(t) - T_i = 2q_o''\left[\frac{t}{\pi\rho c_p k}\right]^{1/2}$$

$$k = \frac{t}{\pi\rho c_p}\left[\frac{2q_o''}{T_o(t) - T_i}\right]^2$$

$$k = \frac{30\text{ s}}{\pi \times 3965\text{ kg/m}^3 \times 765\text{ J/kg}\cdot\text{K}}\left[\frac{2 \times 2653\text{ W/m}^2}{(24.57\ -\ 23.00)^\circ\text{C}}\right]^2 = 36.0\text{ W/m}\cdot\text{K}$$ <

where

$$q_o'' = \frac{P}{2A_s} = \frac{P}{2(\pi D^2/4)} = \frac{15\text{ W}}{2(\pi \times 0.060^2/4)\text{m}^2} = 2653\text{ W/m}^2.$$

With the following properties now known,

ρ = 3965 kg/m^3 c_p = 765 J/kg·K k = 36 W/m·K

entries in Table A.1 are scanned to determine whether these values are typical of a metallic material. Consider the following,

- metallics with low ρ generally have higher thermal conductivities,
- specific heats of both types of materials are of similar magnitude,
- the low k value of the sample is typical of poor metallic conductors which generally have much higher specific heats,
- more than likely, the material is nonmetallic.

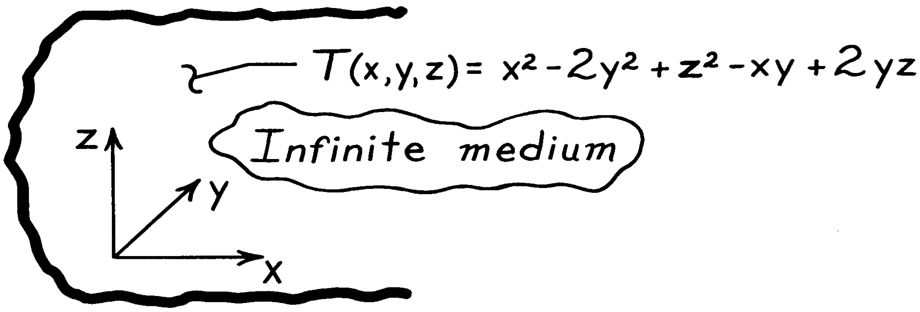
From Table A.2, the second entry, polycrystalline aluminum oxide, has properties at 300 K corresponding to those found for the samples. <

PROBLEM 2.20

KNOWN: Temperature distribution, T(x,y,z), within an infinite, homogeneous body at a given instant of time.

FIND: Regions where the temperature changes with time.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties of infinite medium and (2) No internal heat generation.

ANALYSIS: The temperature distribution throughout the medium, at any instant of time, must satisfy the heat equation. For the three-dimensional cartesian coordinate system, with constant properties and no internal heat generation, the heat equation, Eq. 2.15, has the form

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}. \qquad (1)$$

If T(x,y,z) satisfies this relation, conservation of energy is satisfied at every point in the medium. Substituting T(x,y,z) into the Eq. (1), first find the gradients, $\partial T/\partial x$, $\partial T/\partial y$, and $\partial T/\partial z$.

$$\frac{\partial}{\partial x}(2x - y)+\frac{\partial}{\partial y}(-4y - x + 2z)+\frac{\partial}{\partial z}(2z + 2y)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Performing the differentiations,

$$2-4+2=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Hence,

$$\frac{\partial T}{\partial t}=0$$

which implies that, at the prescribed instant, the temperature is everywhere independent of time.

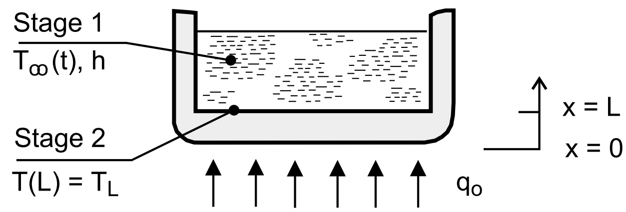
COMMENTS: Since we do not know the initial and boundary conditions, we cannot determine the temperature distribution, T(x,y,z), at any future time. We can only determine that, for this special instant of time, the temperature will not change.

PROBLEM 2.21

KNOWN: Diameter D, thickness L and initial temperature T_i of pan. Heat rate from stove to bottom of pan. Convection coefficient h and variation of water temperature $T_\infty(t)$ during Stage 1. Temperature T_L of pan surface in contact with water during Stage 2.

FIND: Form of heat equation and boundary conditions associated with the two stages.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in pan bottom, (2) Heat transfer from stove is uniformly distributed over surface of pan in contact with the stove, (3) Constant properties.

ANALYSIS:

Stage 1

Heat Equation: $$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Boundary Conditions: $$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_o'' = \frac{q_o}{\left(\pi D^2/4\right)}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty(t)\right]$$

Initial Condition: $$T(x,0) = T_i$$

Stage 2

Heat Equation: $$\frac{d^2 T}{dx^2} = 0$$

Boundary Conditions: $$-k\left.\frac{dT}{dx}\right|_{x=0} = q_o''$$

$$T(L) = T_L$$

COMMENTS: Stage 1 is a transient process for which $T_\infty(t)$ must be determined separately. As a first approximation, it could be estimated by neglecting changes in thermal energy storage by the pan bottom and assuming that all of the heat transferred from the stove acted to increase thermal energy storage within the water. Hence, with $q \approx Mc_p\, d\,T_\infty/dt$, where M and c_p are the mass and specific heat of the water in the pan, $T_\infty(t) \approx (q/Mc_p)\, t$.

PROBLEM 2.36

KNOWN: Three-dimensional system – described by cylindrical coordinates (r,ϕ,θ) – experiences transient conduction and internal heat generation.

FIND: Heat diffusion equation.

SCHEMATIC: See Figure 2.10.

ASSUMPTIONS: (1) Homogeneous medium.

ANALYSIS: The differential control volume is V = dr·rsinθdϕ·rdθ, and the conduction terms are identified in Figure 2.10. Conservation of energy requires

$$q_r - q_{r+dr} + q_\phi - q_{\phi+d\phi} + q_\theta - q_{\theta+d\theta} + \dot{E}_g = \dot{E}_{st}. \qquad (1)$$

The generation and storage terms, both representing volumetric phenomena, are

$$\dot{E}_g = \dot{q}V = \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] \qquad \dot{E}_{st} = \rho Vc\frac{\partial T}{\partial t} = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t}. \qquad (2,3)$$

Using a Taylor series expansion, we can write

$$q_{r+dr} = q_r + \frac{\partial}{\partial r}(q_r)dr, \quad q_{\phi+d\phi} = q_\phi + \frac{\partial}{\partial\phi}(q_\phi)d\phi, \quad q_{\theta+d\theta} = q_\theta + \frac{\partial}{\partial\theta}(q_\theta)d\theta. \qquad (4,5,6)$$

From Fourier's law, the conduction heat rates have the following forms.

$$q_r = -kA_r\partial T/\partial r = -k[r\sin\theta d\phi \cdot rd\theta]\partial T/\partial r \qquad (7)$$

$$q_\phi = -kA_\phi\partial T/r\sin\theta\partial\phi = -k[dr \cdot rd\theta]\partial T/r\sin\theta\partial\phi \qquad (8)$$

$$q_\theta = -kA_\theta\partial T/r\partial\theta = -k[dr \cdot r\sin\theta d\phi]\partial T/r\partial\theta. \qquad (9)$$

Substituting Eqs. (2), (3) and (4), (5), (6) into Eq. (1), the energy balance becomes

$$-\frac{\partial}{\partial r}(q_r)dr - \frac{\partial}{\partial_\phi}(q_\phi)d\phi - \frac{\partial}{\partial\theta}(q_\theta)d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (10)$$

Substituting Eqs. (7), (8) and (9) for the conduction rates, find

$$-\frac{\partial}{\partial\theta}\left[-k[r\sin\theta d\phi \cdot rd\theta]\frac{\partial T}{\partial r}\right]dr - \frac{\partial}{\partial\phi}\left[-k[dr \cdot rd\theta]\frac{\partial T}{r\sin\theta\partial\phi}\right]d\phi$$

$$-\frac{\partial}{\partial\theta}\left[-k[dr \cdot r\sin\theta d\phi]\frac{\partial T}{r\partial\theta}\right]d\theta + \dot{q}[dr \cdot r\sin\theta d\phi \cdot rd\theta] = \rho[dr \cdot r\sin\theta d\phi \cdot rd\theta]c\frac{\partial T}{\partial t} \qquad (11)$$

Dividing Eq. (11) by the volume of the control volume, V, Eq. 2.23 is obtained.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[kr^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left[k\frac{\partial T}{\partial\phi}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[k\sin\theta\frac{\partial T}{\partial\theta}\right] + \dot{q} = \rho c\frac{\partial T}{\partial t}. \qquad <$$

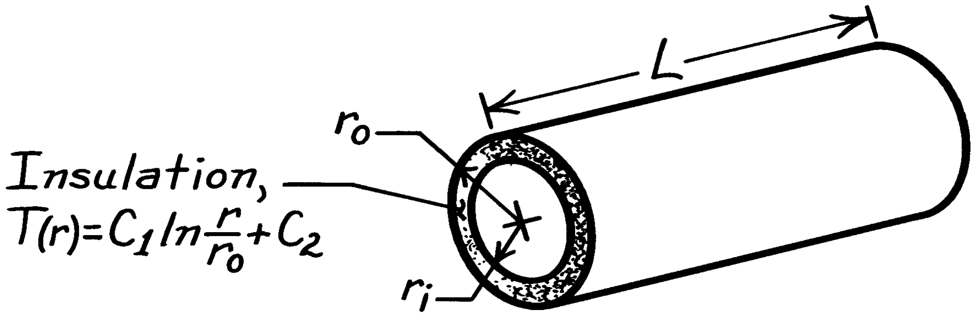
COMMENTS: Note how the temperature gradients in Eqs. (7) - (9) are formulated. The numerator is always ∂T while the denominator is the dimension of the control volume in the specified coordinate direction.

PROBLEM 2.37

KNOWN: Temperature distribution in steam pipe insulation.

FIND: Whether conditions are steady-state or transient. Manner in which heat flux and heat rate vary with radius.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r, (2) Constant properties.

ANALYSIS: From Equation 2.20, the heat equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}.$$

Substituting for T(r),

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{C_1}{r}\right)=0.$$

Hence, steady-state conditions exist. <

From Equation 2.19, the radial component of the heat flux is

$$q_r''=-k\frac{\partial T}{\partial r}=-k\frac{C_1}{r}.$$

Hence, q_r'' decreases with increasing $r\left(q_r''\alpha 1/r\right)$. <

At any radial location, the heat rate is

$$q_r=2\pi r L q_r''=-2\pi k C_1 L$$

Hence, q_r is independent of r. <

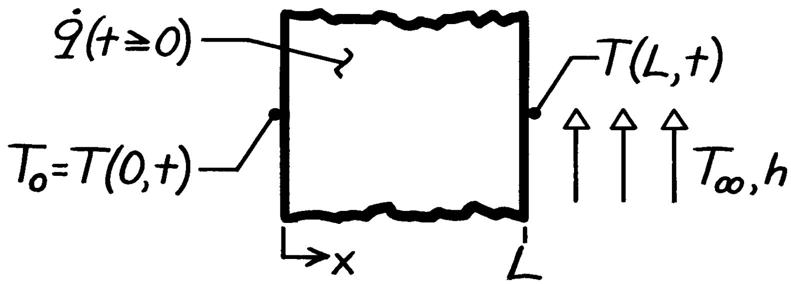
COMMENTS: The requirement that q_r is invariant with r is consistent with the energy conservation requirement. If q_r is constant, the flux must vary inversely with the area perpendicular to the direction of heat flow. Hence, q_r'' varies inversely with r.

PROBLEM 2.48

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty > T_o$,h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. X) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; also show distribution when there is no heat flow at the x = L boundary, (b) Sketch the heat flux $\left(q_x'' \text{ vs. } t\right)$ at the boundaries x = 0 and L.

SCHEMATIC:



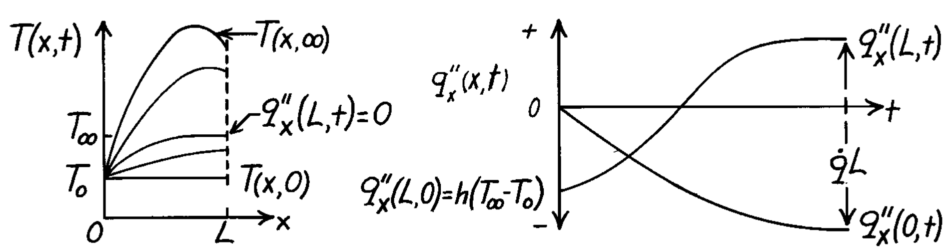
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_o < T_\infty$ and $\dot{q}$ large enough that $T(x,\infty) > T_\infty$.

ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	$x = 0 \quad T(0,t) = T_o$	Constant temperature
	$x = L \quad -k\left.\dfrac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note the special condition when the heat flux at (x = L) is zero.

(b) The heat flux as a function of time at the boundaries, $q_x''(0,t)$ and $q_x''(L,t)$, can be inferred from the temperature distributions using Fourier's law.



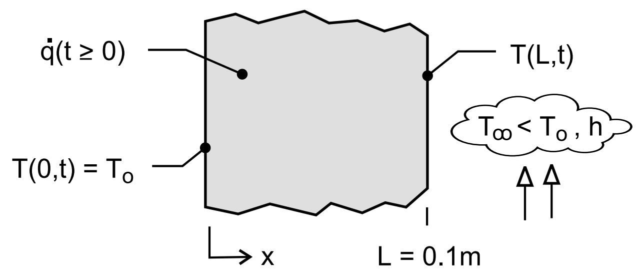
COMMENTS: Since $T(x,\infty) > T_\infty$ and $T_\infty > T_o$, heat transfer at both boundaries must be out of the wall. Hence, it follows from an overall energy balance on the wall that $+q_x''(0,\infty) - q_x''(L,\infty) + \dot{q}L = 0$.

PROBLEM 2.49

KNOWN: Plane wall, initially at a uniform temperature T_o, has one surface (x = L) suddenly exposed to a convection process ($T_\infty < T_o$, h), while the other surface (x = 0) is maintained at T_o. Also, wall experiences uniform volumetric heating $\dot{q}$ such that the maximum steady-state temperature will exceed T_∞.

FIND: (a) Sketch temperature distribution (T vs. x) for following conditions: initial ($t \le 0$), steady-state ($t \to \infty$), and two intermediate times; identify key features of the distributions, (b) Sketch the heat flux $\left(q''_x \text{ vs. } t\right)$ at the boundaries x = 0 and L; identify key features of the distributions.

SCHEMATIC:



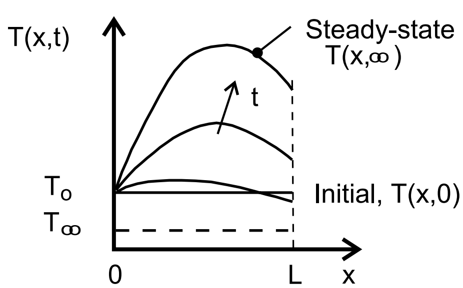
ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) Uniform volumetric generation, (4) $T_\infty < T_o$ and $\dot{q}$ large enough that $T(x,\infty) > T_o$.

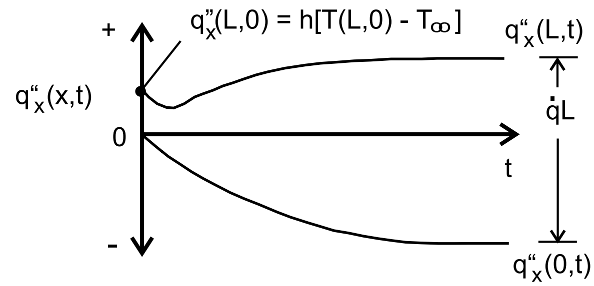
ANALYSIS: (a) The initial and boundary conditions for the wall can be written as

Initial ($t \le 0$):	$T(x,0) = T_o$	Uniform temperature
Boundary:	$x = 0 \quad T(0,t) = T_o$	Constant temperature
	$x = L \quad -k\left.\dfrac{\partial T}{\partial x}\right)_{x=L} = h\left[T(L,t) - T_\infty\right]$	Convection process.

The temperature distributions are shown on the T-x coordinates below. Note that the maximum temperature occurs under steady-state conditions not at the midplane, but to the right toward the surface experiencing convection. The temperature gradients at x = L increase for t > 0 since the convection heat rate from the surface increases as the surface temperature increases.

(b) The heat flux as a function of time at the boundaries, $q''_x(0,t)$ and $q''_x(L,t)$, can be inferred from the temperature distributions using Fourier's law. At the surface x = L, the convection heat flux at t = 0 is $q''_x(L,0) = h\left(T_o - T_\infty\right)$. Because the surface temperature dips slightly at early times, the convection heat flux decreases slightly, and then increases until the steady-state condition is reached. For the steady-state condition, heat transfer at both boundaries must be out of the wall. It follows from an overall energy balance on the wall that $+q''_x(0,\infty) - q''_x(L,\infty) + \dot{q}L = 0$.



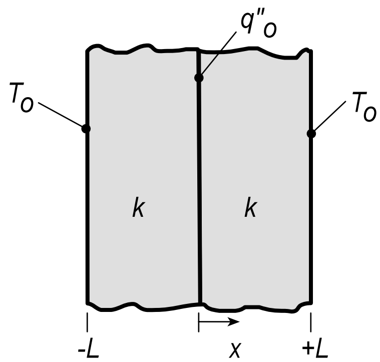


PROBLEM 2.50

KNOWN: Interfacial heat flux and outer surface temperature of adjoining, equivalent plane walls.

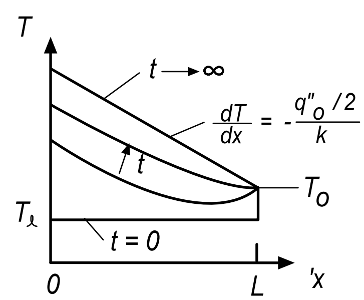
FIND: (a) Form of temperature distribution at representative times during the heating process, (b) Variation of heat flux with time at the interface and outer surface.

SCHEMATIC:

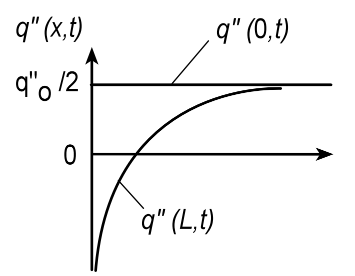


ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: (a) With symmetry about the interface, consideration of the temperature distribution may be restricted to $0 \le x \le L$. During early stages of the process, heat transfer is *into* the material from the outer surface, as well as from the interface. During later stages and the eventual steady state, heat is transferred *from* the material at the outer surface. At steady-state, $dT/dx = -\left(q_o''/2\right)/k = \text{const}$. and $T(0,t) = T_o + \left(q_o''/2\right)L/k$.



(b) At the outer surface, the heat flux is initially negative, but increases with time, approaching $q_o''/2$. It is zero when $dT/dx|_{x=L} = 0$.

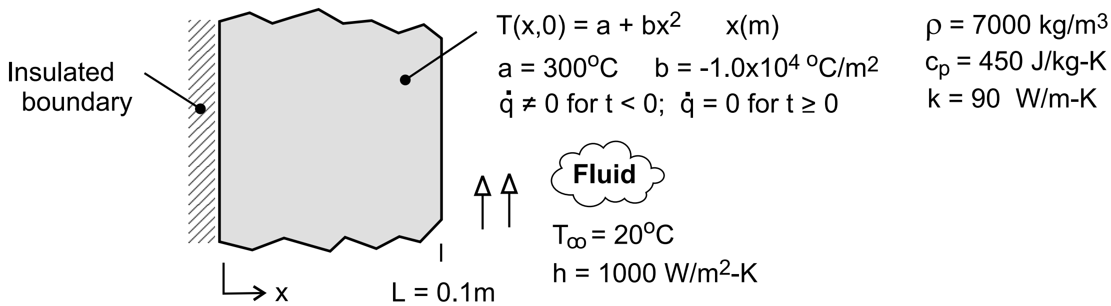


PROBLEM 2.51

KNOWN: Temperature distribution in a plane wall of thickness L experiencing uniform volumetric heating $\dot{q}$ having one surface (x = 0) insulated and the other exposed to a convection process characterized by T_∞ and h. Suddenly the volumetric heat generation is deactivated while convection continues to occur.

FIND: (a) Determine the magnitude of the volumetric energy generation rate associated with the initial condition, (b) On T-x coordinates, sketch the temperature distributions for the initial condition ($T \le 0$), the steady-state condition ($t \to \infty$), and two intermediate times; (c) On q''_x - t coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L,t)$; calculate the corresponding value of the heat flux at t = 0; and (d) Determine the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, and (3) Uniform internal volumetric heat generation for t < 0.

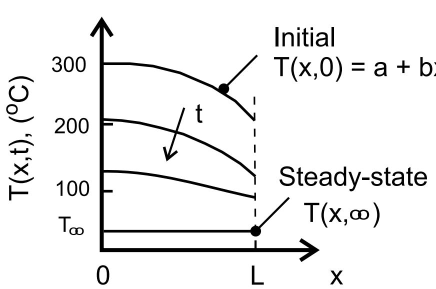
ANALYSIS: (a) The volumetric heating rate can be determined by substituting the temperature distribution for the initial condition into the appropriate form of the heat diffusion equation.

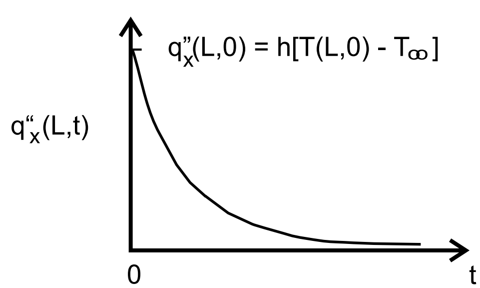
$$\frac{d}{dx}\left(\frac{dT}{dx}\right)+\frac{\dot{q}}{k}=0 \qquad \text{where} \qquad T(x,0)=a+bx^2$$

$$\frac{d}{dx}(0+2bx)+\frac{\dot{q}}{k}=0+2b+\frac{\dot{q}}{k}=0$$

$$\dot{q}=-2kb=-2\times 90\,W/m\cdot K\left(-1.0\times 10^{4}\,{}^\circ C/m^2\right)=1.8\times 10^{6}\,W/m^3 \qquad <$$

(b) The temperature distributions are shown in the sketch below.





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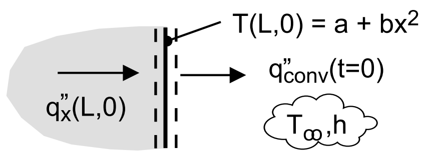
(c) The heat flux at the exposed surface x = L, $q''_x(L,0)$, is initially a maximum value and decreases with increasing time as shown in the sketch above. The heat flux at t = 0 is equal to the convection heat flux with the surface temperature T(L,0). See the surface energy balance represented in the schematic.

$$q''_x(L,0) = q''_{conv}(t=0) = h\left(T(L,0) - T_\infty\right) = 1000\,W/m^2 \cdot K\,(200-20)°C = 1.80\times10^5\,W/m^2 \quad <$$

where $T(L,0) = a + bL^2 = 300°C - 1.0\times10^4 °C/m^2\,(0.1m)^2 = 200°C.$



(d) The energy removed from the wall to the fluid as it cools from its initial to steady-state condition can be determined from an energy balance on a time interval basis, Eq. 1.11b. For the initial state, the wall has the temperature distribution $T(x,0) = a + bx^2$; for the final state, the wall is at the temperature of the fluid, $T_f = T_\infty$. We have used T_∞ as the reference condition for the energy terms.

$$E''_{in} - E''_{out} = \Delta E''_{st} = E''_f - E''_i \qquad \text{with} \qquad E''_{in} = 0$$

$$-E''_{out} = \rho c_p L\left[T_f - T_\infty\right] - \rho c_p \int_{x=0}^{x=L} \left[T(x,0) - T_\infty\right] dx$$

$$E''_{out} = \rho c_p \int_{x=0}^{x=L} \left[a + bx^2 - T_\infty\right] dx = \rho c_p \left[ax + bx^3/3 - T_\infty x\right]_0^L$$

$$E''_{out} = 7000\,kg/m^3 \times 450\,J/kg\cdot K\left[300\times0.1 - 1.0\times10^4\,(0.1)^3/3 - 20\times0.1\right] K\cdot m$$

$$E''_{out} = 7.77\times10^7\,J/m^2 \quad <$$

COMMENTS: (1) In the temperature distributions of part (a), note these features: initial condition has quadratic form with zero gradient at the adiabatic boundary; for the steady-state condition, the wall has reached the temperature of the fluid; for all distributions, the gradient at the adiabatic boundary is zero; and, the gradient at the exposed boundary decreases with increasing time.

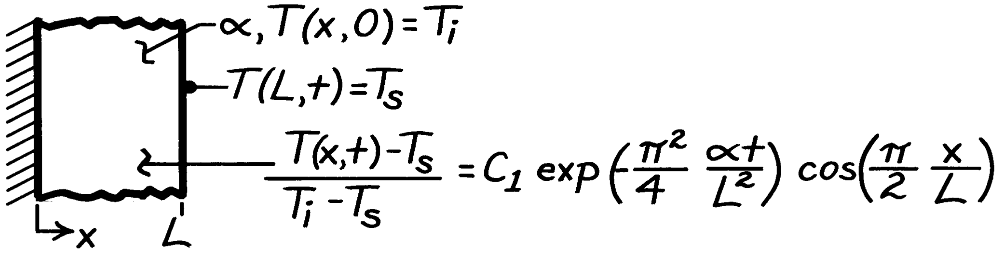
(2) In this thermodynamic analysis, we were able to determine the energy transferred during the cooling process. However, we cannot determine the rate at which cooling of the wall occurs without solving the heat diffusion equation.

PROBLEM 2.52

KNOWN: Temperature as a function of position and time in a plane wall suddenly subjected to a change in surface temperature, while the other surface is insulated.

FIND: (a) Validate the temperature distribution, (b) Heat fluxes at x = 0 and x = L, (c) Sketch of temperature distribution at selected times and surface heat flux variation with time, (d) Effect of thermal diffusivity on system response.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in x, (2) Constant properties.

ANALYSIS: (a) To be valid, the temperature distribution must satisfy the appropriate forms of the heat equation and boundary conditions. Substituting the distribution into Equation 2.15, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$-C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\left(\frac{\pi}{2L}\right)^2 \cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

$$= -\frac{C_1}{\alpha}(T_i - T_s)\left(\frac{\pi^2}{4}\frac{\alpha}{L^2}\right)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right).$$ <

Hence, the heat equation is satisfied. Applying boundary conditions at x = 0 and x = L, it follows that

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = -\frac{C_1 \pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right)\bigg|_{x=0} = 0$$ <

and

$$T(L,t) = T_s + C_1(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)\bigg|_{x=L} = T_s.$$ <

Hence, the boundary conditions are also satisfied.

(b) The heat flux has the form

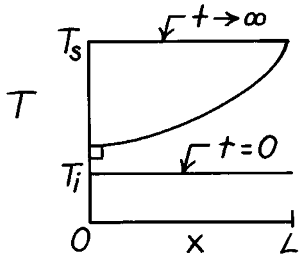
$$q''_x = -k\frac{\partial T}{\partial x} = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\sin\left(\frac{\pi}{2}\frac{x}{L}\right).$$

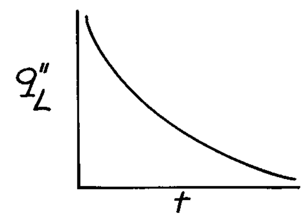
Continued …..

Hence, $q''_x(0) = 0,$ <

$$q''_x(L) = +\frac{kC_1\pi}{2L}(T_i - T_s)\exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right).$$ <

(c) The temperature distribution and surface heat flux variations are:





(d) For materials A and B of different α,

$$\frac{\left[T(x,t) - T_s\right]_A}{\left[T(x,t) - T_s\right]_B} = \exp\left[-\frac{\pi^2}{4L^2}(\alpha_A - \alpha_B)t\right]$$

Hence, if $\alpha_A > \alpha_B$, $T(x,t) \rightarrow T_s$ more rapidly for Material A. If $\alpha_A < \alpha_B$, $T(x,t) \rightarrow T_s$ more rapidly for Material B. <

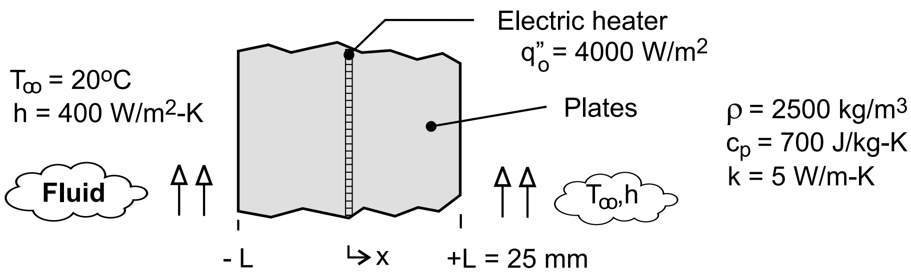
COMMENTS: Note that the prescribed function for T(x,t) does not reduce to T_i for $t \rightarrow 0$. For times at or close to zero, the function is not a valid solution of the problem. At such times, the solution for T(x,t) must include additional terms. The solution is consideed in Section 5.5.1 of the text.

PROBLEM 2.53

KNOWN: Thin electrical heater dissipating 4000 W/m^2 sandwiched between two 25-mm thick plates whose surfaces experience convection.

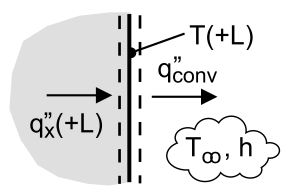
FIND: (a) On T-x coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$; calculate values for the surfaces x = L and the mid-point, x = 0; label this distribution as Case 1 and explain key features; (b) Case 2: sudden loss of coolant causing existence of adiabatic condition on the x = +L surface; sketch temperature distribution on same T-x coordinates as part (a) and calculate values for x = 0, ± L; explain key features; (c) Case 3: further loss of coolant and existence of adiabatic condition on the x = - L surface; situation goes undetected for 15 minutes at which time power to the heater is deactivated; determine the eventual ($t \to \infty$) uniform, steady-state temperature distribution; sketch temperature distribution on same T-x coordinates as parts (a,b); and (d) On T-t coordinates, sketch the temperature-time history at the plate locations x = 0, ± L during the transient period between the steady-state distributions for Case 2 and Case 3; at what location and when will the temperature in the system achieve a maximum value?

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal volumetric generation in plates, and (3) Negligible thermal resistance between the heater surfaces and the plates.

ANALYSIS: (a) Since the system is symmetrical, the heater power results in equal conduction fluxes through the plates. By applying a surface energy balance on the surface x = +L as shown in the schematic, determine the temperatures at the mid-point, x = 0, and the exposed surface, x + L.



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_x(+L) - q''_{conv} = 0 \qquad \text{where} \qquad q''_x(+L) = q''_o/2$$

$$q''_o/2 - h\left[T(+L) - T_\infty\right] = 0$$

$$T_1(+L) = q''_o/2h + T_\infty = 4000\,\text{W/m}^2/\left(2\times 400\,\text{W/m}^2\cdot\text{K}\right) + 20°\text{C} = 25°\text{C} \qquad <$$

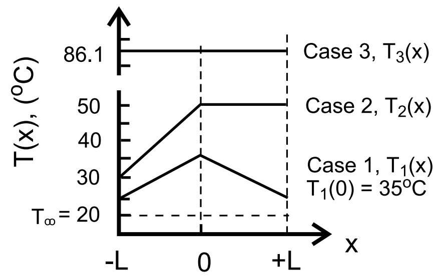
From Fourier's law for the conduction flux through the plate, find T(0).

$$q''_x = q''_o/2 = k\left[T(0) - T(+L)\right]/L$$

$$T_1(0) = T_1(+L) + q''_o L/2k = 25°\text{C} + 4000\,\text{W/m}^2\cdot\text{K}\times 0.025\text{m}/\left(2\times 5\,\text{W/m}\cdot\text{K}\right) = 35°\text{C} \qquad <$$

The temperature distribution is shown on the T-x coordinates below and labeled Case 1. The key features of the distribution are its symmetry about the heater plane and its linear dependence with distance.

Continued …..



(b) Case 2: sudden loss of coolant with the existence of an adiabatic condition on surface x = +L. For this situation, all the heater power will be conducted to the coolant through the left-hand plate. From a surface energy balance and application of Fourier's law as done for part (a), find

$$T_2(-L) = q_o''/h + T_\infty = 4000\,W/m^2 / 400\,W/m^2 \cdot K + 20°C = 30°C \qquad <$$

$$T_2(0) = T_2(-L) + q_o'' L/k = 30°C + 4000\,W/m^2 \times 0.025\,m / 5\,W/m\cdot K = 50°C \qquad <$$

The temperature distribution is shown on the T-x coordinates above and labeled Case 2. The distribution is linear in the left-hand plate, with the maximum value at the mid-point. Since no heat flows through the right-hand plate, the gradient must zero and this plate is at the maximum temperature as well. The maximum temperature is higher than for Case 1 because the heat flux through the left-hand plate has increased two-fold.

(c) Case 3: sudden loss of coolant occurs at the x = -L surface also. For this situation, there is no heat transfer out of either plate, so that for a 15-minute period, Δt_o, the heater dissipates 4000 W/m^2 and then is deactivated. To determine the eventual, uniform steady-state temperature distribution, apply the conservation of energy requirement on a time-interval basis, Eq. 1.11b. The initial condition corresponds to the temperature distribution of Case 2, and the final condition will be a uniform, elevated temperature $T_f = T_3$ representing Case 3. We have used T_∞ as the reference condition for the energy terms.

$$E_{in}'' - E_{out}'' + E_{gen}'' = \Delta E_{st}'' = E_f'' - E_i'' \qquad (1)$$

Note that $E_{in}'' - E_{out}'' = 0$, and the dissipated electrical energy is

$$E_{gen}'' = q_o'' \Delta t_o = 4000\,W/m^2 (15 \times 60)s = 3.600 \times 10^6\,J/m^2 \qquad (2)$$

For the final condition,

$$E_f'' = \rho c(2L)[T_f - T_\infty] = 2500\,kg/m^3 \times 700\,J/kg\cdot K(2 \times 0.025m)[T_f - 20]°C$$
$$E_f'' = 8.75 \times 10^4 [T_f - 20]\,J/m^2 \qquad (3)$$

where $T_f = T_3$, the final uniform temperature, Case 3. For the initial condition,

$$E_i'' = \rho c \int_{-L}^{+L} [T_2(x) - T_\infty]dx = \rho c \left\{ \int_{-L}^{0} [T_2(x) - T_\infty]dx + \int_{0}^{+L} [T_2(0) - T_\infty]dx \right\} \qquad (4)$$

where $T_2(x)$ is linear for $-L \le x \le 0$ and constant at $T_2(0)$ for $0 \le x \le +L$.

$$T_2(x) = T_2(0) + [T_2(0) - T_2(L)]x/L \qquad -L \le x \le 0$$

$$T_2(x) = 50°C + [50 - 30]°C x / 0.025m$$

$$T_2(x) = 50°C + 800x \qquad (5)$$

Substituting for $T_2(x)$, Eq. (5), into Eq. (4)

Continued …..

PROBLEM 2.53 (Cont.)

$$E_i'' = \rho\, c \left\{ \int_{-L}^{0} \left[50 + 800x - T_\infty \right] dx + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho\, c \left\{ \left[50x + 400x^2 - T_\infty x \right]_{-L}^{0} + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho\, c \left\{ -\left[-50L + 400L^2 + T_\infty L \right] + \left[T_2(0) - T_\infty \right] L \right\}$$

$$E_i'' = \rho\, cL \left\{ +50 - 400L - T_\infty + T_2(0) - T_\infty \right\}$$

$$E_i'' = 2500\,\mathrm{kg/m^3} \times 700\,\mathrm{J/kg\cdot K} \times 0.025\,\mathrm{m} \left\{ +50 - 400 \times 0.025 - 20 + 50 - 20 \right\} \mathrm{K}$$

$$E_i'' = 2.188 \times 10^6\,\mathrm{J/m^2} \tag{6}$$

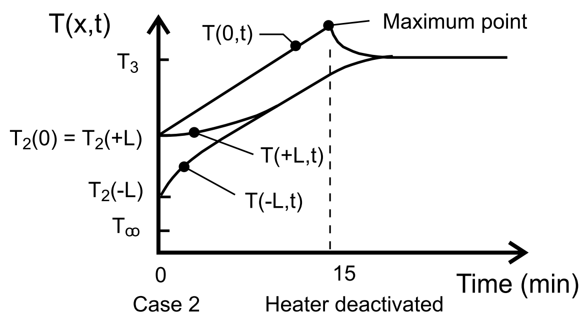
Returning to the energy balance, Eq. (1), and substituting Eqs. (2), (3) and (6), find $T_f = T_3$.

$$3.600 \times 10^6\,\mathrm{J/m^2} = 8.75 \times 10^4 \left[T_3 - 20 \right] - 2.188 \times 10^6\,\mathrm{J/m^2}$$

$$T_3 = (66.1 + 20)\,^\circ\mathrm{C} = 86.1\,^\circ\mathrm{C}$$ <

The temperature distribution is shown on the T-x coordinates above and labeled Case 3. The distribution is uniform, and considerably higher than the maximum value for Case 2.

(d) The temperature-time history at the plate locations x = 0, ± L during the transient period between the distributions for Case 2 and Case 3 are shown on the T-t coordinates below.



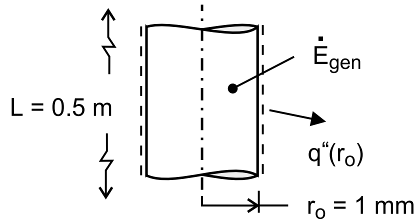
Note the temperatures for the locations at time t = 0 corresponding to the instant when the surface x = - L becomes adiabatic. These temperatures correspond to the distribution for Case 2. The heater remains energized for yet another 15 minutes and then is deactivated. The midpoint temperature, T(0,t), is always the hottest location and the maximum value slightly exceeds the final temperature T_3.

PROBLEM 2.54

KNOWN: Radius and length of coiled wire in hair dryer. Electric power dissipation in the wire, and temperature and convection coefficient associated with air flow over the wire.

FIND: (a) Form of heat equation and conditions governing transient, thermal behavior of wire during start-up, (b) Volumetric rate of thermal energy generation in the wire, (c) Sketch of temperature distribution at selected times during start-up, (d) Variation with time of heat flux at r = 0 and r = r_o.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties, (3) Uniform volumetric heating, (4) Negligible radiation from surface of wire.

ANALYSIS: (a) The general form of the heat equation for cylindrical coordinates is given by Eq. 2.20. For one-dimensional, radial conduction and constant properties, the equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\dot{q}=\frac{\rho c_p}{k}\frac{\partial T}{\partial t}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ <

The initial condition is $\qquad T(r,0)=T_i$ <

The boundary conditions are: $\qquad \partial T/\partial r|_{r=0}=0$ <

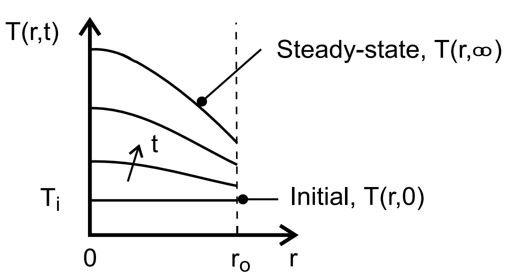
$$-k\left.\frac{\partial T}{\partial r}\right|_{r=r_o}=h\left[T(r_o,t)-T_\infty\right]$$ <

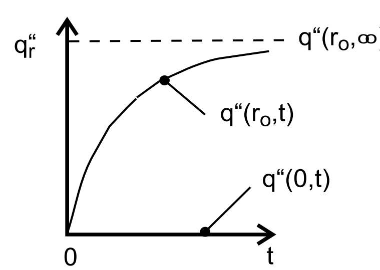
(b) The volumetric rate of thermal energy generation is

$$\dot{q}=\frac{\dot{E}_g}{\forall}=\frac{P_{elec}}{\pi r_o^2 L}=\frac{500\ \text{W}}{\pi(0.001\text{m})^2(0.5\text{m})}=3.18\times10^8\ \text{W/m}^3$$ <

Under steady-state conditions, all of the thermal energy generated within the wire is transferred to the air by convection. Performing an energy balance for a control surface about the wire, $-\dot{E}_{out}+\dot{E}_g=0$, it follows that $-2\pi r_o L\ q''(r_o, t\to\infty)+P_{elec}=0$. Hence,

$$q''(r_o,t\to\infty)=\frac{P_{elec}}{2\pi r_o L}=\frac{500\ \text{W}}{2\pi(0.001\text{m})0.5\text{m}}=1.59\times10^5\ \text{W/m}^2$$ <





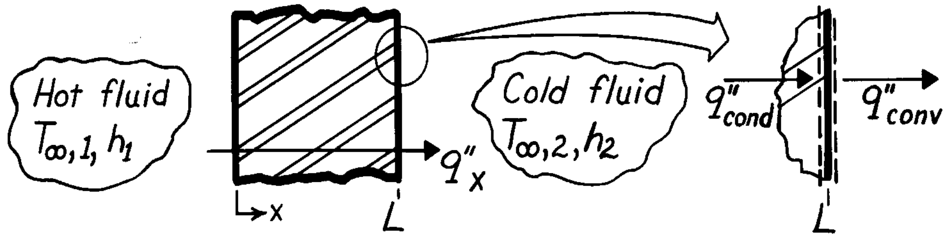
COMMENTS: The symmetry condition at r = 0 imposes the requirement that $\partial T/\partial r|_{r=0}=0$, and hence $q''(0,t)=0$ throughout the process. The temperature at r_o, and hence the convection heat flux, increases steadily during the start-up, and since conduction to the surface must be balanced by convection from the surface at all times, $\left|\partial T/\partial r\right|_{r=r_o}$ also increases during the start-up.

PROBLEM 3.1

KNOWN: One-dimensional, plane wall separating hot and cold fluids at $T_{\infty,1}$ and $T_{\infty,2}$, respectively.

FIND: Temperature distribution, T(x), and heat flux, q''_x, in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k and L.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation, (5) No generation.

ANALYSIS: For the foregoing conditions, the general solution to the heat diffusion equation is of the form, Equation 3.2,

$$T(x) = C_1 x + C_2. \tag{1}$$

The constants of integration, C_1 and C_2, are determined by using surface energy balance conditions at x = 0 and x = L, Equation 2.23, and as illustrated above,

$$-k\left.\frac{dT}{dt}\right]_{x=0} = h_1\left[T_{\infty,1} - T(0)\right] \qquad -k\left.\frac{dT}{dx}\right]_{x=L} = h_2\left[T(L) - T_{\infty,2}\right]. \tag{2,3}$$

For the BC at x = 0, Equation (2), use Equation (1) to find

$$-k(C_1 + 0) = h_1\left[T_{\infty,1} - (C_1 \cdot 0 + C_2)\right] \tag{4}$$

and for the BC at x = L to find

$$-k(C_1 + 0) = h_2\left[(C_1 L + C_2) - T_{\infty,2}\right]. \tag{5}$$

Multiply Eq. (4) by h_2 and Eq. (5) by h_1, and add the equations to obtain C_1. Then substitute C_1 into Eq. (4) to obtain C_2. The results are

$$C_1 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{k\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} \qquad C_2 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{h_1\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} + T_{\infty,1}$$

$$T(x) = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}\left[\frac{x}{k} + \frac{1}{h_1}\right] + T_{\infty,1}.$$ <

From Fourier's law, the heat flux is a constant and of the form

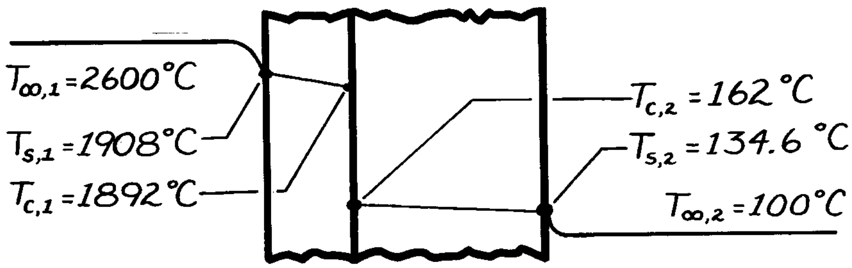
$$q''_x = -k\frac{dT}{dx} = -k\,C_1 = +\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}.$$ <

PROBLEM 3.20 (Cont.)

and with $q'' = (k_B / L_B)(T_{c,2} - T_{s,2})$,

$$T_{s,2} = T_{c,2} - \frac{L_B q''}{k_B} = 162^\circ C - \frac{0.02\text{m} \times 34{,}600 \text{ W/m}^2}{25.4 \text{ W/m} \cdot \text{K}} = 134.6^\circ C.$$

The temperature distribution is therefore of the following form:



<

COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q'' = h_2 (T_{s,2} - T_{\infty,2}) = 1000 \text{W/m}^2 \cdot \text{K} (134.6\text{-}100)^\circ \text{C} = 34{,}600 \text{W/m}^2$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to T ≈ 1900°C for the oxide and T ≈ 115°C for the steel.

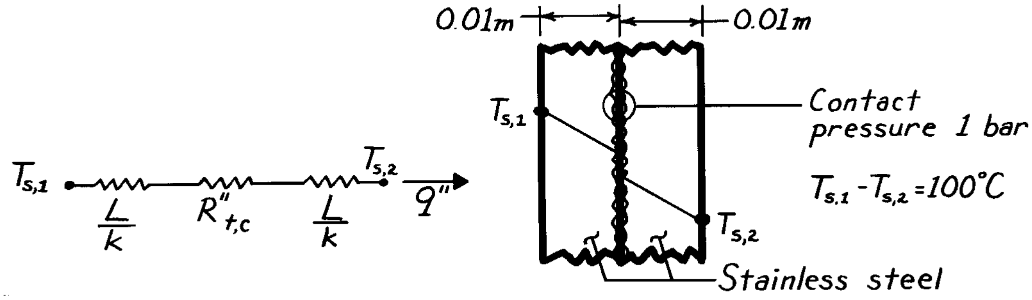
(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.

PROBLEM 3.21

KNOWN: Thickness, overall temperature difference, and pressure for two stainless steel plates.

FIND: (a) Heat flux and (b) Contact plane temperature drop.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: *Table A-1*, Stainless Steel (T ≈ 400K): k = 16.6 W/m·K.

ANALYSIS: (a) With $R''_{t,c} \approx 15\times10^{-4}\ m^2\cdot K/W$ from Table 3.1 and

$$\frac{L}{k} = \frac{0.01m}{16.6\ W/m\cdot K} = 6.02\times10^{-4} m^2\cdot K/W,$$

it follows that

$$R''_{tot} = 2(L/k) + R''_{t,c} \approx 27\times10^{-4} m^2\cdot K/W;$$

hence

$$q'' = \frac{\Delta T}{R''_{tot}} = \frac{100^\circ C}{27\times10^{-4} m^2\cdot K/W} = 3.70\times10^4 W/m^2. \qquad <$$

(b) From the thermal circuit,

$$\frac{\Delta T_c}{T_{s,1} - T_{s,2}} = \frac{R''_{t,c}}{R''_{tot}} = \frac{15\times10^{-4} m^2\cdot K/W}{27\times10^{-4} m^2\cdot K/W} = 0.556.$$

Hence,

$$\Delta T_c = 0.556\left(T_{s,1} - T_{s,2}\right) = 0.556\left(100^\circ C\right) = 55.6^\circ C. \qquad <$$

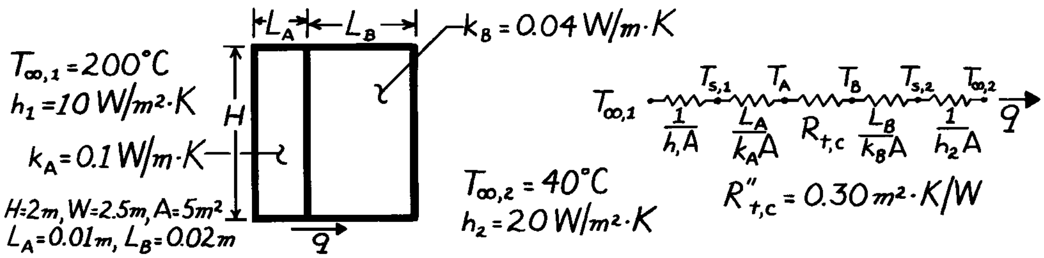
COMMENTS: The contact resistance is significant relative to the conduction resistances. The value of $R''_{t,c}$ would diminish, however, with increasing pressure.

PROBLEM 3.22

KNOWN: Temperatures and convection coefficients associated with fluids at inner and outer surfaces of a composite wall. Contact resistance, dimensions, and thermal conductivities associated with wall materials.

FIND: (a) Rate of heat transfer through the wall, (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible radiation, (4) Constant properties.

ANALYSIS: (a) Calculate the total resistance to find the heat rate,

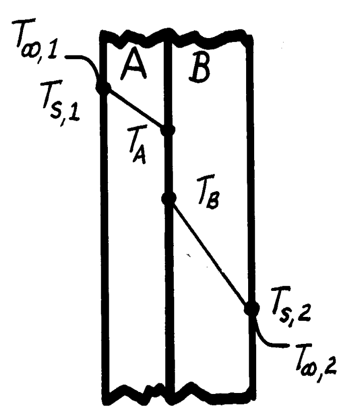
$$R_{tot} = \frac{1}{h_1 A} + \frac{L_A}{k_A A} + R_{t,c} + \frac{L_B}{k_B A} + \frac{1}{h_2 A}$$

$$R_{tot} = \left[\frac{1}{10 \times 5} + \frac{0.01}{0.1 \times 5} + \frac{0.3}{5} + \frac{0.02}{0.04 \times 5} + \frac{1}{20 \times 5}\right]\frac{K}{W}$$

$$R_{tot} = [0.02 + 0.02 + 0.06 + 0.10 + 0.01]\frac{K}{W} = 0.21\frac{K}{W}$$

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{(200-40)^\circ C}{0.21\ K/W} = 762\ W.$$ <

(b) It follows that



$$T_{s,1} = T_{\infty,1} - \frac{q}{h_1 A} = 200^\circ C - \frac{762\ W}{50\ W/K} = 184.8^\circ C$$

$$T_A = T_{s,1} - \frac{qL_A}{k_A A} = 184.8^\circ C - \frac{762 W \times 0.01 m}{0.1 \frac{W}{m \cdot K} \times 5 m^2} = 169.6^\circ C$$

$$T_B = T_A - qR_{t,c} = 169.6^\circ C - 762 W \times 0.06 \frac{K}{W} = 123.8^\circ C$$

$$T_{s,2} = T_B - \frac{qL_B}{k_B A} = 123.8^\circ C - \frac{762 W \times 0.02 m}{0.04 \frac{W}{m \cdot K} \times 5 m^2} = 47.6^\circ C$$

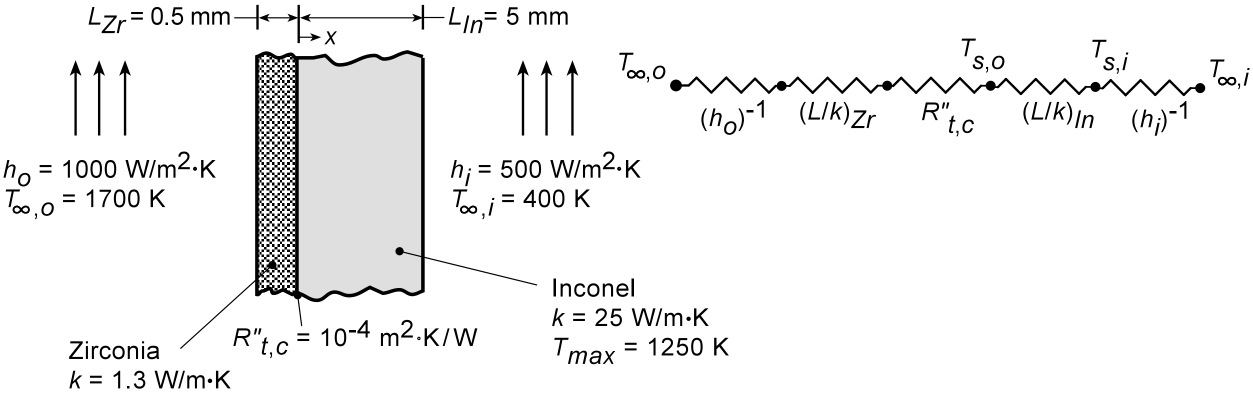
$$T_{\infty,2} = T_{s,2} - \frac{q}{h_2 A} = 47.6^\circ C - \frac{762 W}{100 W/K} = 40^\circ C$$

PROBLEM 3.23

KNOWN: Outer and inner surface convection conditions associated with zirconia-coated, Inconel turbine blade. Thicknesses, thermal conductivities, and interfacial resistance of the blade materials. Maximum allowable temperature of Inconel.

FIND: Whether blade operates below maximum temperature. Temperature distribution in blade, with and without the TBC.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right)m^2\cdot K/W = 3.69\times10^{-3}\,m^2\cdot K/W$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,K}{3.69\times10^{-3}\,m^2\cdot K/W} = 3.52\times10^5\,W/m^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + (q''_w/h_i) = 400\,K + \left(3.52\times10^5\,W/m^2 / 500\,W/m^2\cdot K\right) = 1104\,K$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_w = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right)m^2\cdot K/W\left(3.52\times10^5\,W/m^2\right) = 1174\,K$$

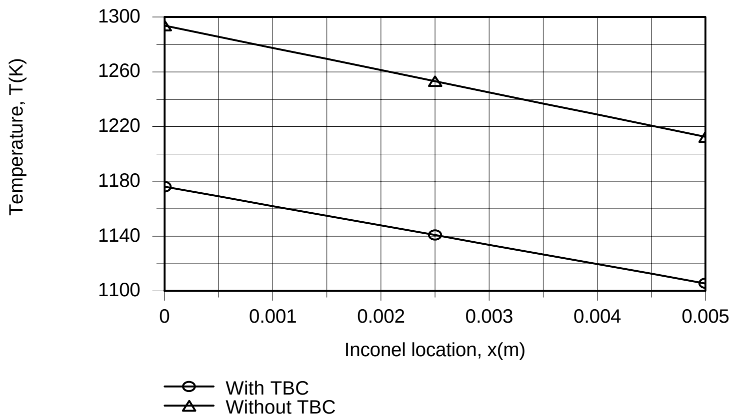
Without the TBC, $R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3}\,m^2\cdot K/W$, and $q''_{wo} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{tot,wo}$ = (1300 K)/3.20×10^{-3} m²·K/W = 4.06×10^5 W/m². The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(wo)} = T_{\infty,i} + (q''_{wo}/h_i) = 400\,K + \left(4.06\times10^5\,W/m^2 / 500\,W/m^2\cdot K\right) = 1212\,K$$

$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_{wo} = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right)m^2\cdot K/W\left(4.06\times10^5\,W/m^2\right) = 1293\,K$$

Continued...

PROBLEM 3.23 (Cont.)



Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

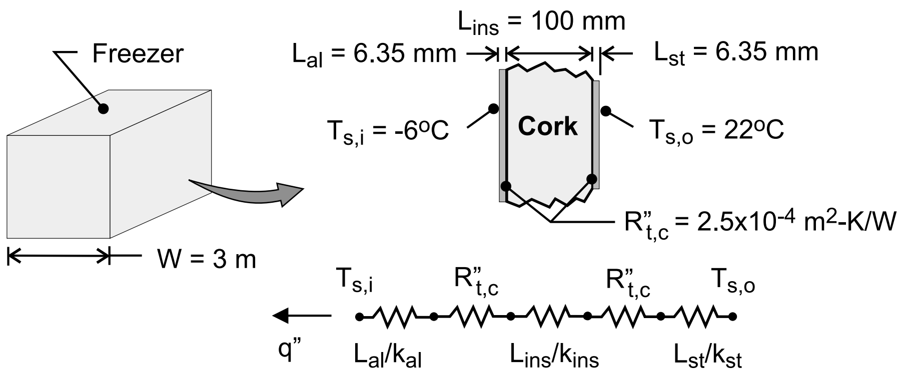
COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to the thickness are associated with reliability considerations.

PROBLEM 3.24

KNOWN: Size and surface temperatures of a cubical freezer. Materials, thicknesses and interface resistances of freezer wall.

FIND: Cooling load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum 2024 (~267K): k_{al} = 173 W/m·K. *Table A-1*, Carbon steel AISI 1010 (~295K): k_{st} = 64 W/m·K. *Table A-3* (~300K): k_{ins} = 0.039 W/m·K.

ANALYSIS: For a unit wall surface area, the total thermal resistance of the composite wall is

$$R''_{tot} = \frac{L_{al}}{k_{al}} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + R''_{t,c} + \frac{L_{st}}{k_{st}}$$

$$R''_{tot} = \frac{0.00635\text{m}}{173\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.100\text{m}}{0.039\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.00635\text{m}}{64\ \text{W/m}\cdot\text{K}}$$

$$R''_{tot} = \left(3.7\times10^{-5} + 2.5\times10^{-4} + 2.56 + 2.5\times10^{-4} + 9.9\times10^{-5}\right)\text{m}^2\cdot\text{K/W}$$

Hence, the heat flux is

$$q'' = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} = \frac{\left[22 - (-6)\right]°\text{C}}{2.56\ \text{m}^2\cdot\text{K/W}} = 10.9\ \frac{\text{W}}{\text{m}^2}$$

and the cooling load is

$$q = A_s\, q'' = 6\,W^2\, q'' = 54\text{m}^2 \times 10.9\ \text{W/m}^2 = 590\,\text{W}$$ <

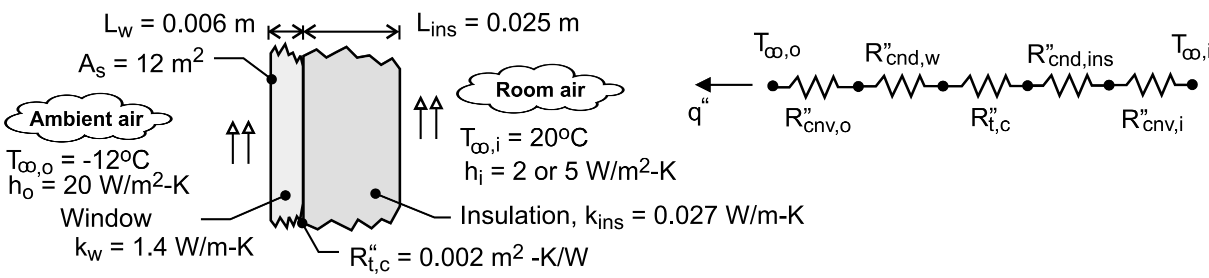
COMMENTS: Thermal resistances associated with the cladding and the adhesive joints are negligible compared to that of the insulation.

PROBLEM 3.25

KNOWN: Thicknesses and thermal conductivity of window glass and insulation. Contact resistance. Environmental temperatures and convection coefficients. Furnace efficiency and fuel cost.

FIND: (a) Reduction in heat loss associated with the insulation, (b) Heat losses for prescribed conditions, (c) Savings in fuel costs for 12 hour period.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

ANALYSIS: (a) The percentage reduction in heat loss is

$$R_q = \frac{q''_{wo} - q''_{with}}{q''_{wo}} \times 100\% = \left(1 - \frac{q''_{with}}{q''_{wo}}\right) \times 100\% = \left(1 - \frac{R''_{tot,wo}}{R''_{tot,with}}\right) \times 100\%$$

where the total thermal resistances without and with the insulation, respectively, are

$$R''_{tot,wo} = R''_{cnv,o} + R''_{cnd,w} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + \frac{1}{h_i}$$

$$R''_{tot,wo} = (0.050 + 0.004 + 0.200)\,m^2 \cdot K/W = 0.254\ m^2 \cdot K/W$$

$$R''_{tot,with} = R''_{cnv,o} + R''_{cnd,w} + R''_{t,c} + R''_{cnd,ins} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h_i}$$

$$R''_{tot,with} = (0.050 + 0.004 + 0.002 + 0.926 + 0.500)\,m^2 \cdot K/W = 1.482\ m^2 \cdot K/W$$

$$R_q = (1 - 0.254/1.482) \times 100\% = 82.9\% \qquad <$$

(b) With $A_s = 12\ m^2$, the heat losses without and with the insulation are

$$q_{wo} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,wo} = 12\,m^2 \times 32°C / 0.254\,m^2 \cdot K/W = 1512\,W \qquad <$$

$$q_{with} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,with} = 12\,m^2 \times 32°C / 1.482\,m^2 \cdot K/W = 259\ W \qquad <$$

(c) With the windows covered for 12 hours per day, the daily savings are

$$S = \frac{(q_{wo} - q_{with})}{\eta_f}\Delta t\ C_g \times 10^{-6}\ MJ/J = \frac{(1512 - 259)\,W}{0.8} 12h \times 3600\,s/h \times \$0.01/MJ \times 10^{-6}\,MJ/J = \$0.677$$

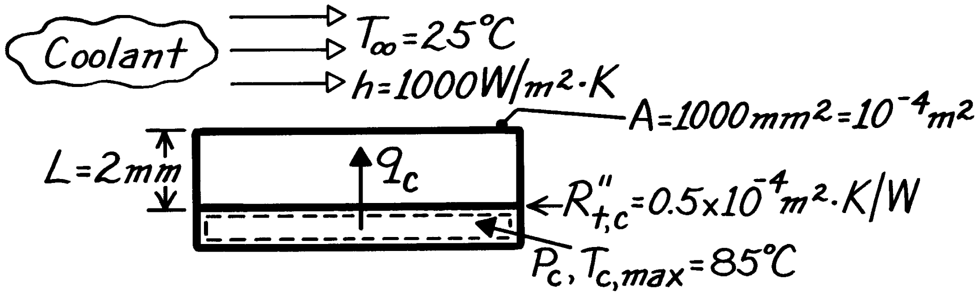
COMMENTS: (1) The savings may be insufficient to justify the cost of the insulation, as well as the daily tedium of applying and removing the insulation. However, the losses are significant and unacceptable. The owner of the building should install double pane windows. (2) The dominant contributions to the total thermal resistance are made by the insulation and convection at the inner surface.

PROBLEM 3.26

KNOWN: Surface area and maximum temperature of a chip. Thickness of aluminum cover and chip/cover contact resistance. Fluid convection conditions.

FIND: Maximum chip power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible heat loss from sides and bottom, (4) Chip is isothermal.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 325 K): k = 238 W/m·K.

ANALYSIS: For a control surface about the chip, conservation of energy yields

$$\dot{E}_g - \dot{E}_{out} = 0$$

or

$$P_c - \frac{(T_c - T_\infty)A}{\left[(L/k) + R''_{t,c} + (1/h)\right]} = 0$$

$$P_{c,max} = \frac{(85-25)^\circ C\left(10^{-4}m^2\right)}{\left[(0.002/238) + 0.5\times10^{-4} + (1/1000)\right]m^2\cdot K/W}$$

$$P_{c,max} = \frac{60\times10^{-4}\ ^\circ C\cdot m^2}{\left(8.4\times10^{-6} + 0.5\times10^{-4} + 10^{-3}\right)m^2\cdot K/W}$$

$$P_{c,max} = 5.7\ W.$$ <

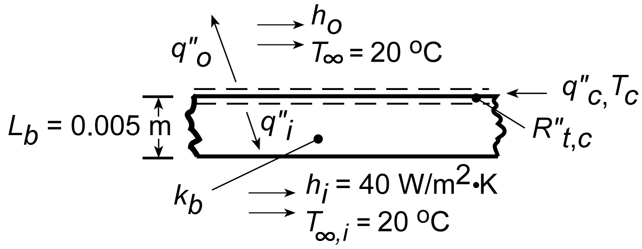
COMMENTS: The dominant resistance is that due to convection $\left(R_{conv} > R_{t,c} >> R_{cond}\right)$.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid ($h_o = 1000\ W/m^2{\cdot}K$) and air ($h_o = 100\ W/m^2{\cdot}K$). Effect of changes in circuit board temperature and contact resistance.

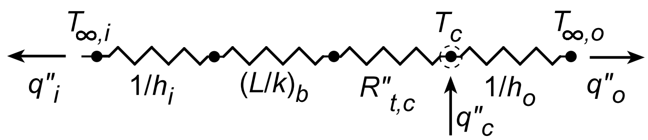
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): $k_b = 32.4\ W/m{\cdot}K$.

ANALYSIS: (a)



(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ W/m^2$, $h_o = 1000\ W/m^2{\cdot}K$, $k_b = 1\ W/m{\cdot}K$ and $R''_{t,c} = 10^{-4}\ m^2\cdot K/W$,

$$3\times10^4\ W/m^2 = \frac{T_c - 20^\circ C}{\left(1/40 + 0.005/1 + 10^{-4}\right)m^2\cdot K/W} + \frac{T_c - 20^\circ C}{(1/1000)\,m^2\cdot K/W}$$

$$3\times10^4\ W/m^2 = (33.2T_c - 664 + 1000T_c - 20{,}000)\,W/m^2\cdot K$$

$$1003T_c = 50{,}664$$

$$T_c = 49^\circ C.$$ <

(c) For $T_c = 85^\circ C$ and $h_o = 1000\ W/m^2{\cdot}K$, the foregoing energy balance yields

$$q''_c = 67{,}160\ W/m^2$$ <

with $q''_o = 65{,}000\ W/m^2$ and $q''_i = 2160\ W/m^2$. Replacing the dielectric with air ($h_o = 100\ W/m^2{\cdot}K$), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

k_b (W/m·K)	$R''_{t,c}$ (m^2·K/W)	q''_i (W/m^2)	q''_o (W/m^2)	q''_c (W/m^2)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

<

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m^2·K/W, while the outer resistance is 0.001 m^2·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30 .$$

and only approximately 3% of the heat is dissipated through the board.

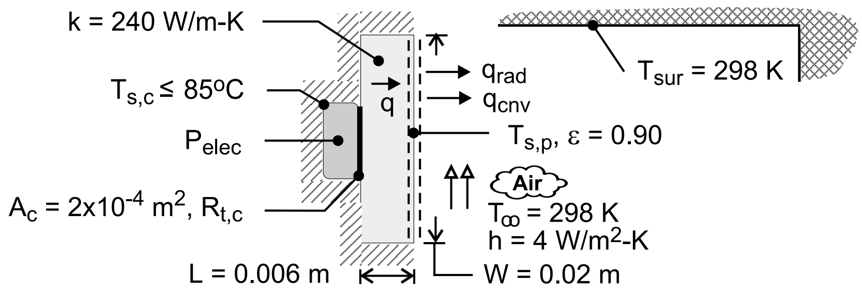
2. With h_o = 100 W/m^2·K, the outer resistance increases to 0.01 m^2·K/W, in which case $q''_o/q''_i = R''_i/R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m^2) by reducing R''_i from 0.0301 to 0.0253 m^2·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4} m^2 \cdot K/W$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.28

KNOWN: Dimensions, thermal conductivity and emissivity of base plate. Temperature and convection coefficient of adjoining air. Temperature of surroundings. Maximum allowable temperature of transistor case. Case-plate interface conditions.

FIND: (a) Maximum allowable power dissipation for an air-filled interface, (b) Effect of convection coefficient on maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from the enclosure, to the surroundings. (3) One-dimensional conduction in the base plate, (4) Radiation exchange at surface of base plate is with large surroundings, (5) Constant thermal conductivity.

PROPERTIES: Aluminum-aluminum interface, air-filled, 10 μm roughness, 10^5 N/m^2 contact pressure (Table 3.1): $R''_{t,c} = 2.75\times10^{-4}\ m^2\cdot K/W$.

ANALYSIS: (a) With all of the heat dissipation transferred through the base plate,

$$P_{elec} = q = \frac{T_{s,c} - T_\infty}{R_{tot}} \qquad (1)$$

where $R_{tot} = R_{t,c} + R_{cnd} + \left[(1/R_{cnv}) + (1/R_{rad})\right]^{-1}$

$$R_{tot} = \frac{R''_{t,c}}{A_c} + \frac{L}{kW^2} + \frac{1}{W^2}\left(\frac{1}{h + h_r}\right) \qquad (2)$$

and $$h_r = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) \qquad (3)$$

To obtain $T_{s,p}$, the following energy balance must be performed on the plate surface,

$$q = \frac{T_{s,c} - T_{s,p}}{R_{t,c} + R_{cnd}} = q_{cnv} + q_{rad} = hW^2\left(T_{s,p} - T_\infty\right) + h_r W^2\left(T_{s,p} - T_{sur}\right) \qquad (4)$$

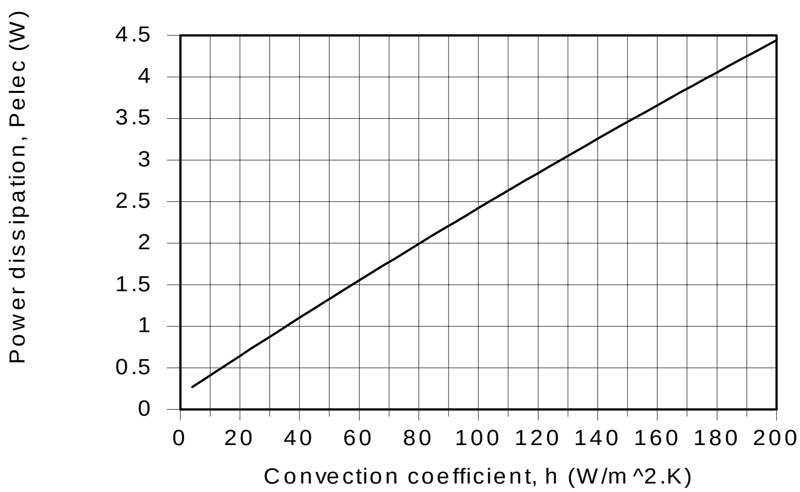
With $R_{t,c} = 2.75 \times 10^{-4}\ m^2\cdot K/W/2\times10^{-4}\ m^2 = 1.375$ K/W, $R_{cnd} = 0.006\ m/(240\ W/m\cdot K \times 4 \times 10^{-4}\ m^2)$ = 0.0625 K/W, and the prescribed values of h, W, $T_\infty = T_{sur}$ and ε, Eq. (4) yields a surface temperature of $T_{s,p}$ = 357.6 K = 84.6°C and a power dissipation of

Continued …..

$$P_{elec} = q = 0.268 \text{ W}$$ <

The convection and radiation resistances are R_{cnv} = 625 m·K/W and R_{rad} = 345 m·K/W, where h_r = 7.25 W/m^2·K.

(b) With the major contribution to the total resistance made by convection, significant benefit may be derived by increasing the value of h.



For h = 200 W/m^2·K, R_{cnv} = 12.5 m·K/W and $T_{s,p}$ = 351.6 K, yielding R_{rad} = 355 m·K/W. The effect of radiation is then negligible.

COMMENTS: (1) The plate conduction resistance is negligible, and even for h = 200 W/m^2·K, the contact resistance is small relative to the convection resistance. However, $R_{t,c}$ could be rendered negligible by using indium foil, instead of an air gap, at the interface. From Table 3.1, $R''_{t,c} = 0.07 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$, in which case $R_{t,c}$ = 0.035 m·K/W.

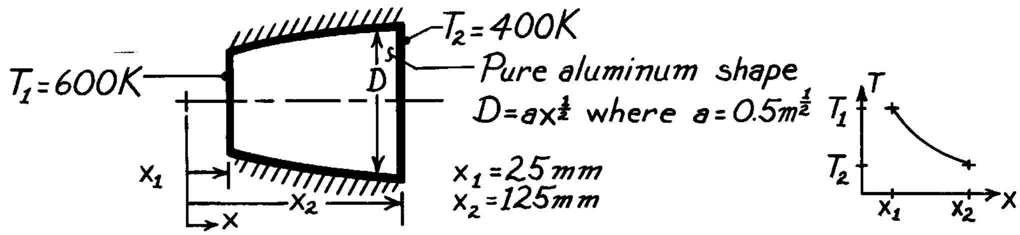
(2) Because $A_c < W^2$, heat transfer by conduction in the plate is actually two-dimensional, rendering the conduction resistance even smaller.

PROBLEM 3.29

KNOWN: Conduction in a conical section with prescribed diameter, D, as a function of x in the form $D = ax^{1/2}$.

FIND: (a) Temperature distribution, T(x), (b) Heat transfer rate, q_x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation, (4) Constant properties.

PROPERTIES: *Table A-2*, Pure Aluminum (500K): k= 236 W/m·K.

ANALYSIS: (a) Based upon the assumptions, and following the same methodology of Example 3.3, q_x is a constant independent of x. Accordingly,

$$q_x = -kA\frac{dT}{dx} = -k\left[\pi\left(ax^{1/2}\right)^2/4\right]\frac{dT}{dx} \qquad (1)$$

using $A = \pi D^2/4$ where $D = ax^{1/2}$. Separating variables and identifying limits,

$$\frac{4q_x}{\pi a^2 k}\int_{x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T} dT. \qquad (2)$$

Integrating and solving for T(x) and then for T_2,

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x}{x_1} \qquad T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x_2}{x_1}. \qquad (3,4)$$

Solving Eq. (4) for q_x and then substituting into Eq. (3) gives the results,

$$q_x = -\frac{\pi}{4}a^2 k\left(T_1 - T_2\right)/1n\left(x_1/x_2\right) \qquad (5)$$

$$T(x) = T_1 + \left(T_1 - T_2\right)\frac{\ln\left(x/x_1\right)}{\ln\left(x_1/x_2\right)}. \qquad <$$

From Eq. (1) note that (dT/dx)·x = Constant. It follows that T(x) has the distribution shown above.

(b) The heat rate follows from Eq. (5),

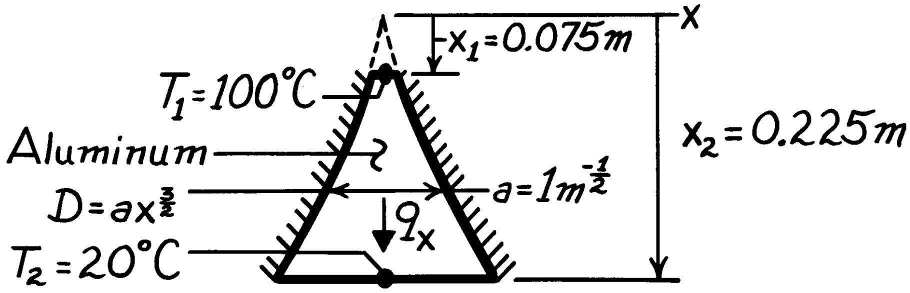
$$q_x = \frac{\pi}{4}\times 0.5^2 m\times 236\frac{W}{m\cdot K}\left(600-400\right)K/\ln\frac{25}{125} = 5.76kW. \qquad <$$

PROBLEM 3.30

KNOWN: Geometry and surface conditions of a truncated solid cone.

FIND: (a) Temperature distribution, (b) Rate of heat transfer across the cone.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum (333K): k = 238 W/m·K.

ANALYSIS: (a) From Fourier's law, Eq. (2.1), with $A = \pi D^2/4 = \left(\pi a^2/4\right)x^3$, it follows that

$$\frac{4q_x dx}{\pi a^2 x^3} = -k dT.$$

Hence, since q_x is independent of x,

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^3} = -k\int_{T_1}^{T} dT$$

or

$$\frac{4q_x}{\pi a^2}\left[-\frac{1}{2x^2}\right]\Bigg|_{x_1}^{x} = -k\left(T - T_1\right).$$

Hence

$$T = T_1 + \frac{2q_x}{\pi a^2 k}\left[\frac{1}{x^2} - \frac{1}{x_1^2}\right]. \qquad <$$

(b) From the foregoing expression, it also follows that

$$q_x = \frac{\pi a^2 k}{2}\frac{T_2 - T_1}{\left[1/x_2^2 - 1/x_1^2\right]}$$

$$q_x = \frac{\pi\left(1m^{-1}\right)238 \text{ W/m}\cdot\text{K}}{2}\times\frac{(20-100)^\circ\text{C}}{\left[(0.225)^{-2} - (0.075)^{-2}\right]m^{-2}}$$

$$q_x = 189 \text{ W}. \qquad <$$

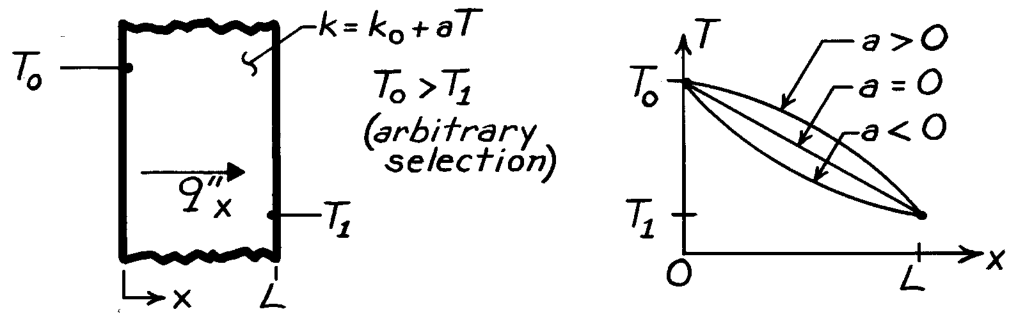
COMMENTS: The foregoing results are approximate due to use of a one-dimensional model in treating what is inherently a two-dimensional problem.

PROBLEM 3.31

KNOWN: Temperature dependence of the thermal conductivity, k.

FIND: Heat flux and form of temperature distribution for a plane wall.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: For the assumed conditions, q_x and A(x) are constant and Eq. 3.21 gives

$$q''_x \int_0^L dx = -\int_{T_o}^{T_1} (k_o + aT)dT$$

$$q''_x = \frac{1}{L}\left[k_o (T_o - T_1) + \frac{a}{2}\left(T_o^2 - T_1^2\right)\right].$$

From Fourier's law,

$$q''_x = -(k_o + aT)\ dT/dx.$$

Hence, since the product of (k_o+aT) and dT/dx) is constant, decreasing T with increasing x implies,

a > 0: decreasing (k_o+aT) and increasing |dT/dx| with increasing x

a = 0: $k = k_o$ => constant (dT/dx)

a < 0: increasing (k_o+aT) and decreasing |dT/dx| with increasing x.

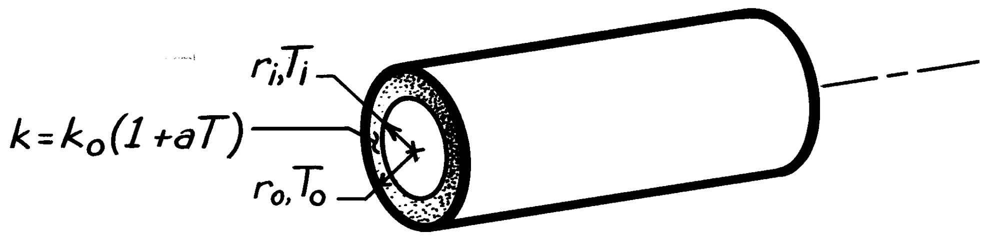
The temperature distributions appear as shown in the above sketch.

PROBLEM 3.32

KNOWN: Temperature dependence of tube wall thermal conductivity.

FIND: Expressions for heat transfer per unit length and tube wall thermal (conduction) resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal heat generation.

ANALYSIS: From Eq. 3.24, the appropriate form of Fourier's law is

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr}$$

$$q_r' = -2\pi\ kr\frac{dT}{dr}$$

$$q_r' = -2\pi\ rk_o(1 + aT)\frac{dT}{dr}.$$

Separating variables,

$$-\frac{q_r'}{2\pi}\frac{dr}{r} = k_o(1 + aT)dT$$

and integrating across the wall, find

$$-\frac{q_r'}{2\pi}\int_{r_i}^{r_o}\frac{dr}{r} = k_o\int_{T_i}^{T_o}(1+aT)dT$$

$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[T + \frac{aT^2}{2}\right]\Bigg|_{T_i}^{T_o}$$

$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[(T_o - T_i) + \frac{a}{2}\left(T_o^2 - T_i^2\right)\right]$$

$$q_r' = -2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]\frac{(T_o - T_i)}{\ln(r_o/r_i)}.$$ <

It follows that the overall thermal resistance per unit length is

$$R_t' = \frac{\Delta T}{q_r'} = \frac{\ln(r_o/r_i)}{2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]}.$$ <

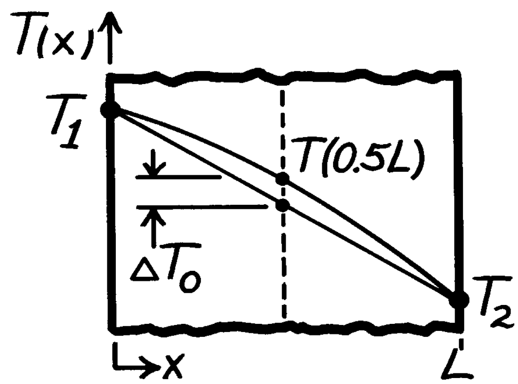
COMMENTS: Note the necessity of the stated assumptions to treating q_r' as independent of r.

PROBLEM 3.33

KNOWN: Steady-state temperature distribution of convex shape for material with $k = k_o(1 + \alpha T)$ where α is a constant and the mid-point temperature is ΔT_o higher than expected for a linear temperature distribution.

FIND: Relationship to evaluate α in terms of ΔT_o and T_1, T_2 (the temperatures at the boundaries).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) α is positive and constant.

ANALYSIS: At any location in the wall, Fourier's law has the form

$$q''_x = -k_o\left(1+\alpha\, T\right)\frac{dT}{dx}. \qquad (1)$$

Since q''_x is a constant, we can separate Eq. (1), identify appropriate integration limits, and integrate to obtain

$$\int_0^L q''_x dx = -\int_{T_1}^{T_2} k_o\left(1+\alpha\, T\right)dT \qquad (2)$$

$$q''_x = -\frac{k_o}{L}\left[\left(T_2 + \frac{\alpha\, T_2^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right]. \qquad (3)$$

We could perform the same integration, but with the upper limits at x = L/2, to obtain

$$q''_x = -\frac{2k_o}{L}\left[\left(T_{L/2} + \frac{\alpha T_{L/2}^2}{2}\right) - \left(T_1 + \frac{\alpha\, T_1^2}{2}\right)\right] \qquad (4)$$

where

$$T_{L/2} = T\left(L/2\right) = \frac{T_1 + T_2}{2} + \Delta T_o. \qquad (5)$$

Setting Eq. (3) equal to Eq. (4), substituting from Eq. (5) for $T_{L/2}$, and solving for α, it follows that

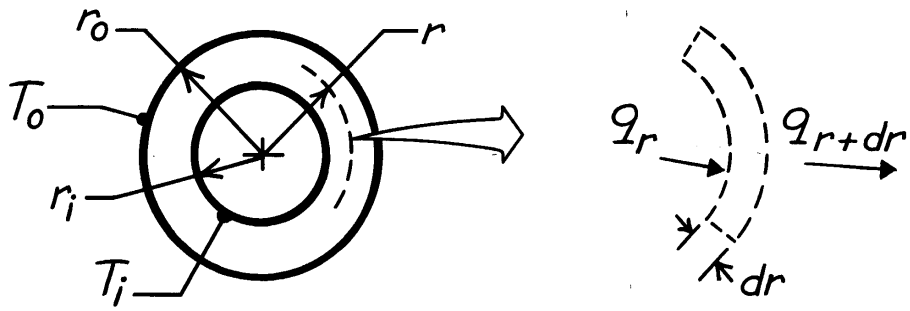
$$\alpha = \frac{2\Delta T_o}{\left(T_2^2 + T_1^2\right)/2 - \left[\left(T_1 + T_2\right)/2 + \Delta T_o\right]^2}.$$ <

PROBLEM 3.34

KNOWN: Hollow cylinder of thermal conductivity k, inner and outer radii, r_i and r_o, respectively, and length L.

FIND: Thermal resistance using the alternative conduction analysis method.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: For the differential control volume, energy conservation requires that $q_r = q_{r+dr}$ for steady-state, one-dimensional conditions with no heat generation. With Fourier's law,

$$q_r = -kA\frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr} \tag{1}$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer. Since q_r is constant, Eq. (1) may be separated and expressed in integral form,

$$\frac{q_r}{2\pi\ L}\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_i}^{T_o} k(T)dT.$$

Assuming k is constant, the heat rate is

$$q_r = \frac{2\pi\ Lk(T_i - T_o)}{\ln(r_o/r_i)}.$$

Remembering that the thermal resistance is defined as

$$R_t \equiv \Delta T/q$$

it follows that for the hollow cylinder,

$$R_t = \frac{\ln(r_o/r_i)}{2\pi\ LK}. \qquad <$$

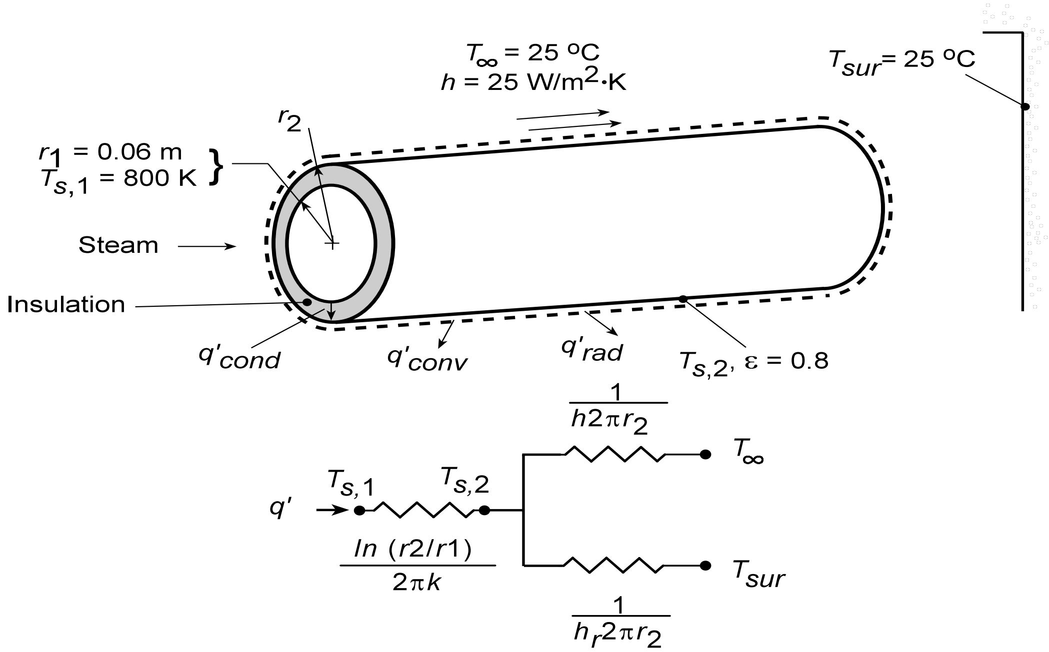
COMMENTS: Compare the *alternative* method used in this analysis with the *standard* method employed in Section 3.3.1 to obtain the same result.

PROBLEM 3.35

KNOWN: Thickness and inner surface temperature of calcium silicate insulation on a steam pipe. Convection and radiation conditions at outer surface.

FIND: (a) Heat loss per unit pipe length for prescribed insulation thickness and outer surface temperature. (b) Heat loss and radial temperature distribution as a function of insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Calcium Silicate (T = 645 K): k = 0.089 W/m·K.

ANALYSIS: (a) From Eq. 3.27 with $T_{s,2}$ = 490 K, the heat rate per unit length is

$$q' = q_r/L = \frac{2\pi k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$q' = \frac{2\pi\left(0.089\,W/m\cdot K\right)\left(800-490\right)K}{\ln\left(0.08\,m/0.06\,m\right)}$$

$$q' = 603\,W/m\,.$$ <

(b) Performing an energy for a control surface around the outer surface of the insulation, it follows that

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,1} - T_{s,2}}{\ln\left(r_2/r_1\right)/2\pi k} = \frac{T_{s,2} - T_\infty}{1/\left(2\pi r_2 h\right)} + \frac{T_{s,2} - T_{sur}}{1/\left(2\pi r_2 h_r\right)}$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Solving this equation for $T_{s,2}$, the heat rate may be determined from

$$q' = 2\pi r_2\left[h\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)\right]$$

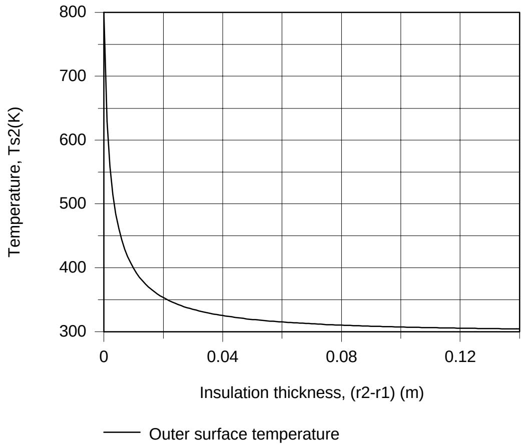
Continued...

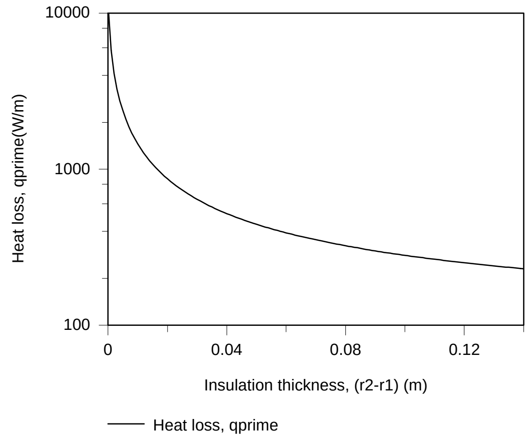
PROBLEM 3.35 (Cont.)

and from Eq. 3.26 the temperature distribution is

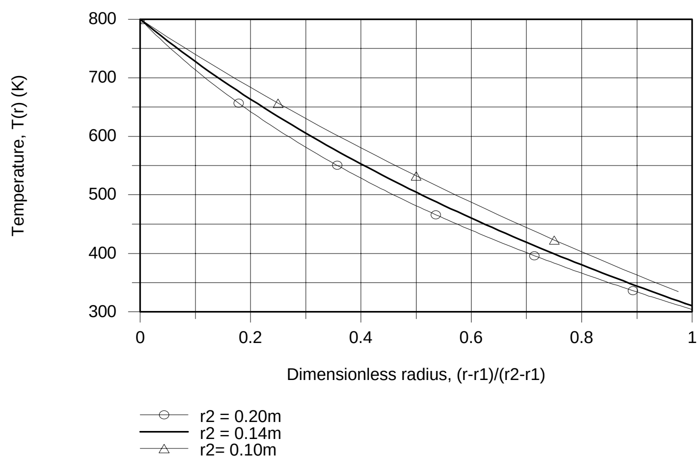
$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

As shown below, the outer surface temperature of the insulation $T_{s,2}$ and the heat loss q' decay precipitously with increasing insulation thickness from values of $T_{s,2} = T_{s,1} = 800$ K and $q' = 11,600$ W/m, respectively, at $r_2 = r_1$ (no insulation).





When plotted as a function of a dimensionless radius, $(r - r_1)/(r_2 - r_1)$, the temperature decay becomes more pronounced with increasing r_2.



Note that $T(r_2) = T_{s,2}$ increases with decreasing r_2 and a linear temperature distribution is approached as r_2 approaches r_1.

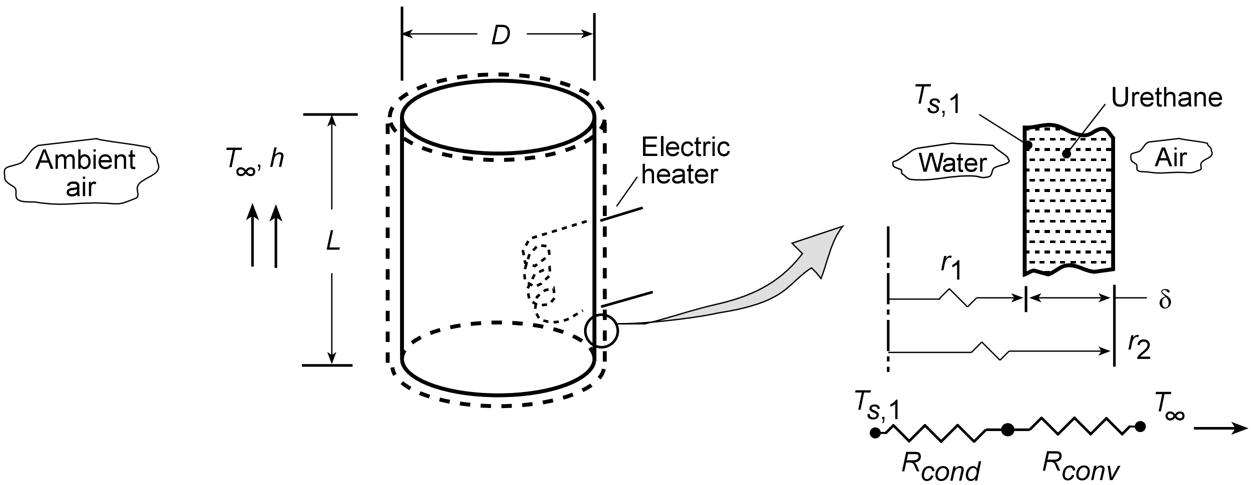
COMMENTS: An insulation layer thickness of 20 mm is sufficient to maintain the outer surface temperature and heat rate below 350 K and 1000 W/m, respectively.

PROBLEM 3.36

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than \$50.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = \left(4\forall/\pi\right)^{1/3} \qquad \text{and} \qquad L = \left(4\forall/\pi\right)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$. Hence, for $\forall$ = 100 gal × 0.00379 m³/gal = 0.379 m³,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ <

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln\left(r_2/r_1\right)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

We begin by estimating the heat loss associated with a 25 mm thick layer of insulation. With $r_1 = D_{op}/2$ = 0.392 m and $r_2 = r_1 + \delta$ = 0.417 m, it follows that

Continued...

$$q = \frac{(55-20)^\circ \text{C}}{\dfrac{\ln(0.417/0.392)}{2\pi(0.026\,\text{W/m}\cdot\text{K})0.784\,\text{m}} + \dfrac{1}{\left(2\,\text{W/m}^2\cdot\text{K}\right)2\pi(0.417\,\text{m})0.784\,\text{m}}}$$

$$+ \frac{2(55-20)^\circ \text{C}}{\dfrac{0.025\,\text{m}}{(0.026\,\text{W/m}\cdot\text{K})\pi/4(0.784\,\text{m})^2} + \dfrac{1}{\left(2\,\text{W/m}^2\cdot\text{K}\right)\pi/4(0.784\,\text{m})^2}}$$

$$q = \frac{35^\circ\text{C}}{(0.483+0.243)\,\text{K/W}} + \frac{2\left(35^\circ\text{C}\right)}{(1.992+1.036)\,\text{K/W}} = (48.2+23.1)\,\text{W} = 71.3\,\text{W}$$

The annual energy loss is therefore

$$Q_{annual} = 71.3\,\text{W}(365\,\text{days})(24\ \text{h/day})\left(10^{-3}\,\text{kW/W}\right) = 625\,\text{kWh}$$

With a unit electric power cost of \$0.08/kWh, the annual cost of the heat loss is

C = (\$0.08/kWh)625 kWh = \$50.00

Hence, an insulation thickness of

δ = 25 mm <

will satisfy the prescribed cost requirement.

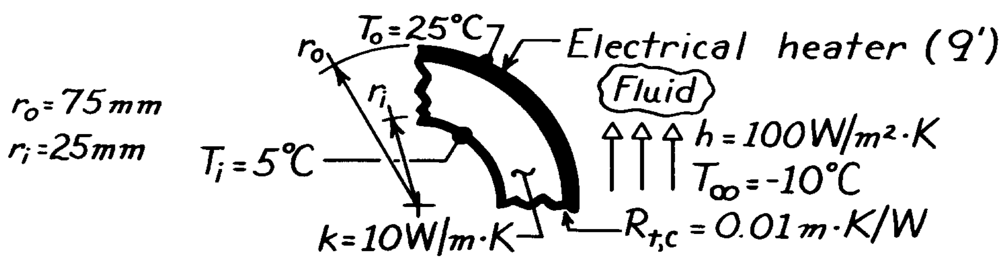
COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3}$ = 0.623 m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 76.1 W and C = \$53.37. The 6.7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

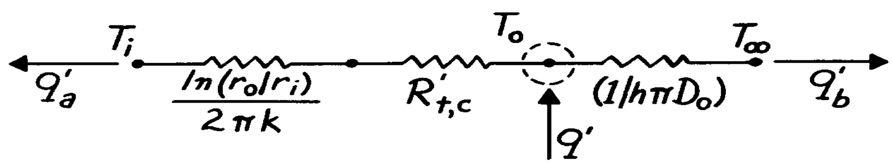
FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form



Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\dfrac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ\text{C}}{\dfrac{\ln(75\text{mm}/25\text{mm})}{2\pi \times 10\ \text{W/m}\cdot\text{K}} + 0.01\dfrac{\text{m}\cdot\text{K}}{\text{W}}} + \frac{\left[25-(-10)\right]^\circ\text{C}}{\left[1/\left(100\ \text{W/m}^2\cdot\text{K}\times\pi\times 0.15\text{m}\right)\right]}$$

$$q' = (728 + 1649)\ \text{W/m}$$

$$q' = 2377\ \text{W/m}. \qquad <$$

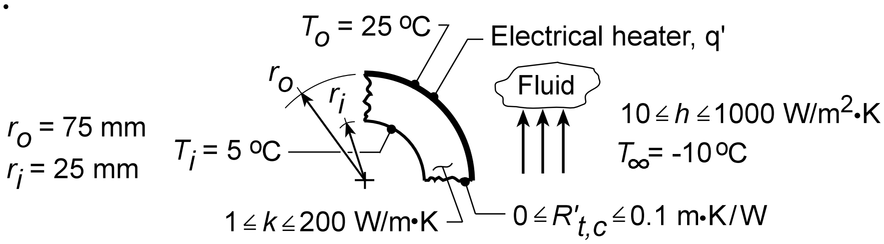
COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

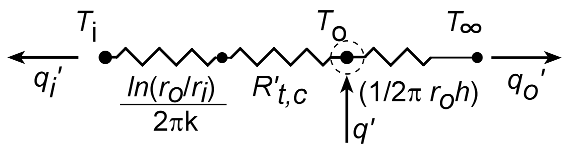
PROBLEM 3.38

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface. Inner and outer wall temperatures. Temperature of fluid adjoining outer wall.

FIND: Effect of wall thermal conductivity, thermal contact resistance, and convection coefficient on total heater power and heat rates to outer fluid and inner surface.

SCHEMATIC:





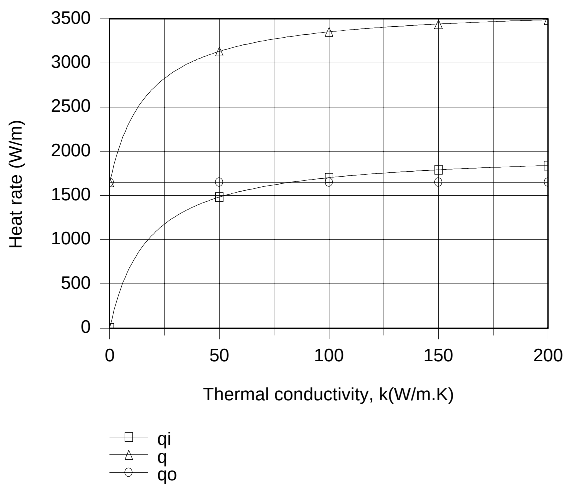
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater, (5) Negligible radiation.

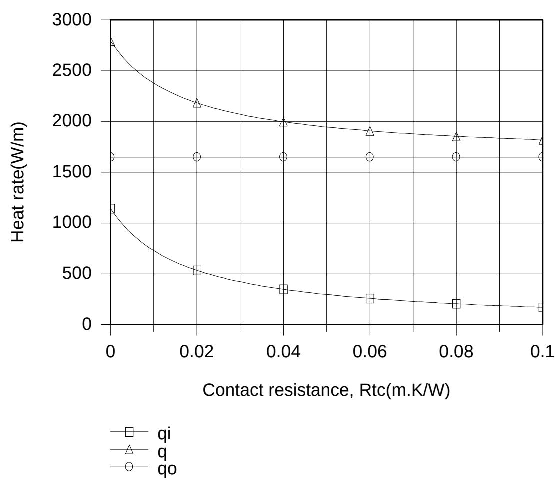
ANALYSIS: Applying an energy balance to a control surface about the heater,

$$q' = q_i' + q_o'$$

$$q' = \frac{T_o - T_i}{\dfrac{\ln(r_o/r_i)}{2\pi k} + R_{t,c}'} + \frac{T_o - T_\infty}{(1/2\pi r_o h)}$$

Selecting nominal values of k = 10 W/m·K, $R_{t,c}'$ = 0.01 m·K/W and h = 100 W/m^2·K, the following parametric variations are obtained





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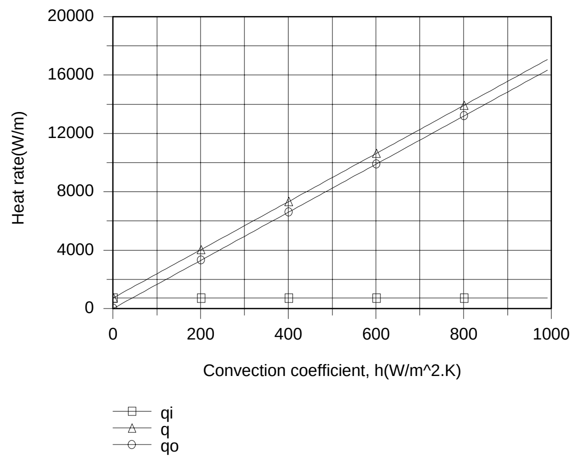


For a prescribed value of h, q'_o is fixed, while q'_i, and hence q', increase and decrease, respectively, with increasing k and $R'_{t,c}$. These trends are attributable to the effects of k and $R'_{t,c}$ on the total (conduction plus contact) resistance separating the heater from the inner surface. For fixed k and $R'_{t,c}$, q'_i is fixed, while q'_o, and hence q', increase with increasing h due to a reduction in the convection resistance.

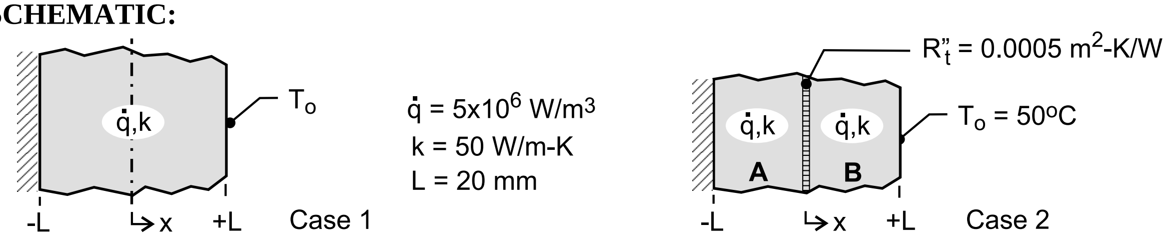
COMMENTS: For the prescribed nominal values of k, $R'_{t,c}$ and h, the electric power requirement is $q' = 2377$ W/m. To maintain the prescribed heater temperature, q' would increase with any changes which reduce the conduction, contact and/or convection resistances.

PROBLEM 3.76

KNOWN: Plane wall of thickness 2L, thermal conductivity k with uniform energy generation $\dot{q}$. For case 1, boundary at x = -L is perfectly insulated, while boundary at x = +L is maintained at T_o = 50°C. For case 2, the boundary conditions are the same, but a thin dielectric strip with thermal resistance $R''_t = 0.0005\ m^2 \cdot K/W$ is inserted at the mid-plane.

FIND: (a) Sketch the temperature distribution for case 1 on T-x coordinates and describe key features; identify and calculate the maximum temperature in the wall, (b) Sketch the temperature distribution for case 2 on the same T-x coordinates and describe the key features; (c) What is the temperature difference between the two walls at x = 0 for case 2? And (d) What is the location of the maximum temperature of the composite wall in case 2; calculate this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the plane and composite walls, and (3) Constant properties.

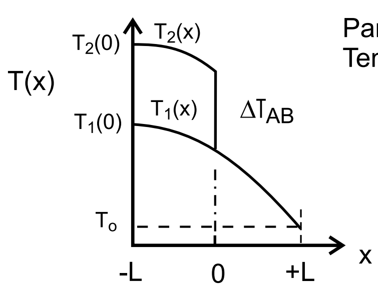
ANALYSIS: (a) For case 1, the temperature distribution, $T_1(x)$ vs. x, is parabolic as shown in the schematic below and the gradient is zero at the insulated boundary, x = -L. From Eq. 3.43,

$$T_1(-L) - T_1(+L) = \frac{\dot{q}(2L)^2}{2k} = \frac{5\times10^6\,W/m^3\,(2\times0.020\ m)^2}{2\times50\ W/m\cdot K} = 80°C$$

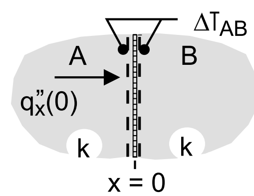
and since $T_1(+L) = T_o$ = 50°C, the maximum temperature occurs at x = -L,

$$T_1(-L) = T_1(+L) + 80°C = 130°C$$

(b) For case 2, the temperature distribution, $T_2(x)$ vs. x, is piece-wise parabolic, with zero gradient at x = -L and a drop across the dielectric strip, ΔT_{AB}. The temperature gradients at either side of the dielectric strip are equal.



Parts (a,b)
Temperature distributions



Part (d) Surface energy balance

(c) For case 2, the temperature drop across the thin dielectric strip follows from the surface energy balance shown above.

$$q''_x(0) = \Delta T_{AB}/R''_t \qquad\qquad q''_x(0) = \dot{q}L$$

$$\Delta T_{AB} = R''_t\,\dot{q}L = 0.0005\ m^2\cdot K/W \times 5\times10^6\,W/m^3 \times 0.020\ m = 50°C.$$

(d) For case 2, the maximum temperature in the composite wall occurs at x = -L, with the value,

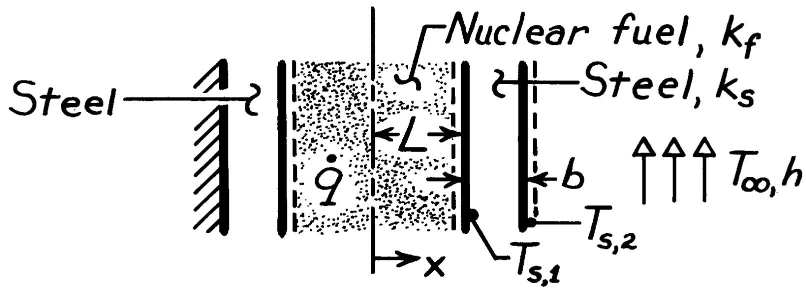
$$T_2(-L) = T_1(-L) + \Delta T_{AB} = 130°C + 50°C = 180°C$$ <

PROBLEM 3.77

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k_f} = 0 \qquad (-L \le x \le +L)$$

is $$T = -\frac{\dot{q}}{2k_f}x^2 + C_1x + C_2.$$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in} = \dot{E}_{out} = 0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L} = -\frac{\dot{q}}{k_f}(-L) + C_1 = 0 \qquad \text{or} \qquad C_1 = -\frac{\dot{q}L}{k_f}$$

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g = q_{cond} = q_{conv}$, or on a unit area basis.

$$\dot{q}(2L) = \frac{k_s}{b}\left(T_{s,1} - T_{s,2}\right) = h\left(T_{s,2} - T_\infty\right).$$

Hence,

$$T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + T_{s,2} \qquad \text{where} \qquad T_{s,2} = \frac{\dot{q}(2L)}{h} + T_\infty$$

$$T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2L)}{h} + T_\infty.$$

Continued …..

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\;\frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

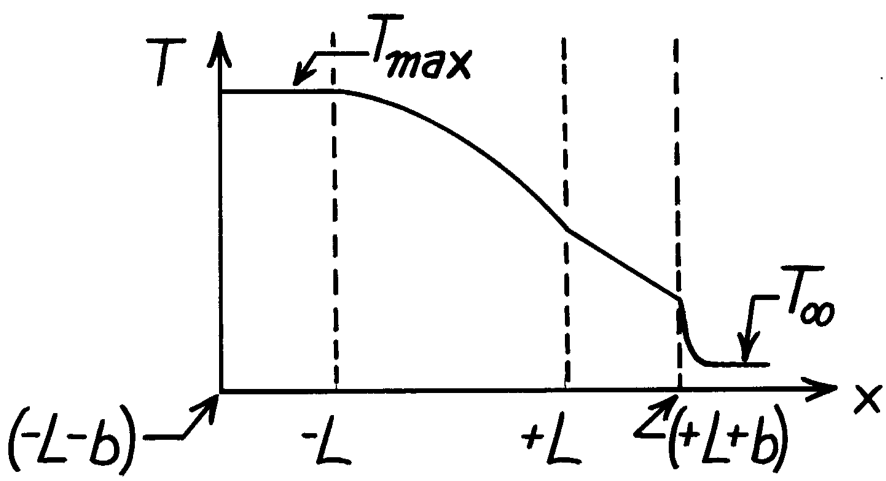
$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right] + T_\infty$$
<

(b) For the temperature distribution shown below,

$(-L-b) \le x \le -L$: $\quad dT/dx=0,\ T=T_{max}$
$-L \le x \le +L$: $\quad |\,dT/dx\,|\ \uparrow$ with $\uparrow$ x
$+L \le x \le L+b$: $\quad (dT/dx)$ is const.

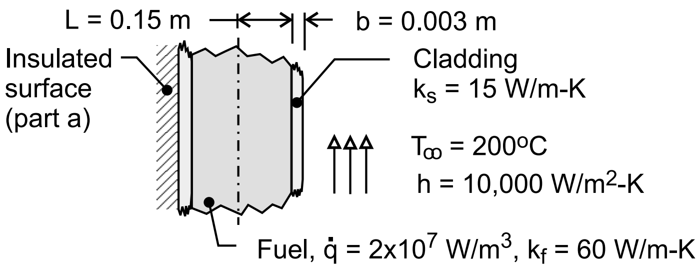


PROBLEM 3.78

KNOWN: Thermal conductivity, heat generation and thickness of fuel element. Thickness and thermal conductivity of cladding. Surface convection conditions.

FIND: (a) Temperature distribution in fuel element with one surface insulated and the other cooled by convection. Largest and smallest temperatures and corresponding locations. (b) Same as part (a) but with equivalent convection conditions at both surfaces, (c) Plot of temperature distributions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance.

ANALYSIS: (a) From Eq. C.1,

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{1}$$

With an insulated surface at x = -L, Eq. C.10 yields

$$T_{s,1} - T_{s,2} = \frac{2\dot{q}L^2}{k_f} \tag{2}$$

and with convection at x = L + b, Eq. C.13 yields

$$U\left(T_{s,2} - T_\infty\right) = \dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{3}$$

Substracting Eq. (2) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{4\dot{q}L^2}{k_f}$$

$$T_{s,2} = T_\infty + \frac{2\dot{q}L}{U} \tag{4}$$

Continued …..

and substituting into Eq. (2)

$$T_{s,1} = T_\infty + 2\dot{q}L\left(\frac{L}{k_f} + \frac{1}{U}\right) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1),

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2}{U} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty$$

or, with $U^{-1} = h^{-1} + b/k_s$,

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty \tag{6}$$ <

The maximum temperature occurs at x = - L and is

$$T(-L) = 2\dot{q}L\left(\frac{b}{k_s} + \frac{1}{h} + \frac{L}{k_f}\right) + T_\infty$$

$$T(-L) = 2\times 2\times 10^7\ \mathrm{W/m^3} \times 0.015\,\mathrm{m}\left(\frac{0.003\mathrm{m}}{15\,\mathrm{W/m\cdot K}} + \frac{1}{10{,}000\,\mathrm{W/m^2\cdot K}} + \frac{0.015\,\mathrm{m}}{60\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 530°\mathrm{C}$$ <

The lowest temperature is at x = + L and is

$$T(+L) = -\frac{3}{2}\frac{\dot{q}L^2}{k_f} + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty = 380°\mathrm{C}$$ <

(b) If a convection condition is maintained at x = - L, Eq. C.12 reduces to

$$U\left(T_\infty - T_{s,1}\right) = -\dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,1} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{7}$$

Subtracting Eq. (7) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty - T_{s,1} + T_\infty\right) \qquad \text{or} \qquad T_{s,1} = T_{s,2}$$

Hence, from Eq. (7)

Continued …..

$$T_{s,1} = T_{s,2} = \frac{\dot{q}L}{U} + T_\infty = \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \tag{8}$$

Substituting into Eq. (1), the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \tag{9}$$ <

The maximum temperature is at x = 0 and is

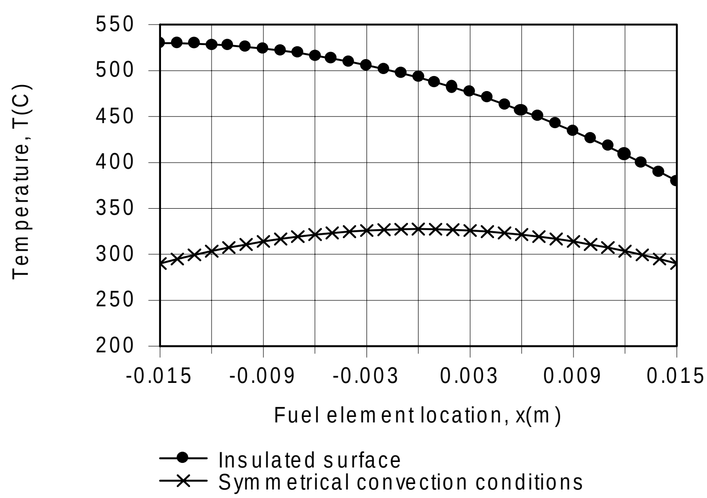
$$T(0) = \frac{2\times10^7\ \text{W/m}^3\,(0.015\,\text{m})^2}{2\times60\ \text{W/m}\cdot\text{K}} + 2\times10^7\ \text{W/m}^3 \times 0.015\,\text{m}\left(\frac{1}{10,000\ \text{W/m}^2\cdot\text{K}} + \frac{0.003\,\text{m}}{15\ \text{W/m}\cdot\text{K}}\right) + 200°\text{C}$$

$$T(0) = 37.5°\text{C} + 90°\text{C} + 200°\text{C} = 327.5°\text{C}$$ <

The minimum temperature at x = ± L is

$$T_{s,1} = T_{s,2} = 2\times10^7\ \text{W/m}^3\,(0.015\,\text{m})\left(\frac{1}{10,000\ \text{W/m}^2\cdot\text{K}} + \frac{0.003\,\text{m}}{15\ \text{W/m}\cdot\text{K}}\right) + 200°\text{C} = 290°\text{C}$$ <

(c) The temperature distributions are as shown.



The amount of heat generation is the same for both cases, but the ability to transfer heat from both surfaces for case (b) results in lower temperatures throughout the fuel element.

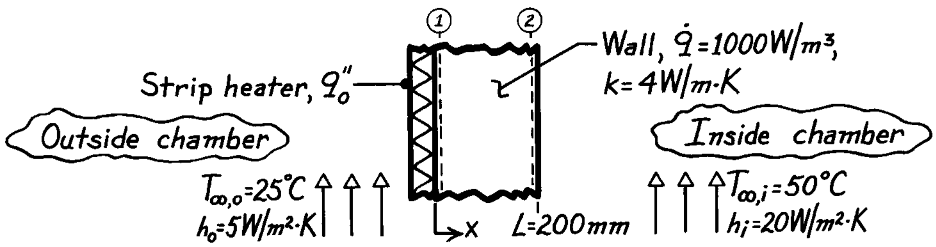
COMMENTS: Note that for case (a), the temperature in the insulated cladding is constant and equivalent to $T_{s,1}$ = 530°C.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

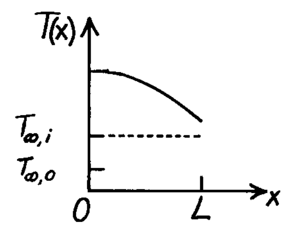
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \qquad (1)$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \qquad (2)$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \qquad (3)$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \qquad (4)$$

Continued …..

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ C + 1000\ W/m^3 \times 0.200\ m / 20\ W/m^2 \cdot K = 50^\circ C + 10^\circ C = 60^\circ C$$ <

$$T_1 = 60^\circ C + 1000\ W/m^3 \times (0.200\ m)^2 / 2 \times 4\ W/m \cdot K = 65^\circ C.$$ <

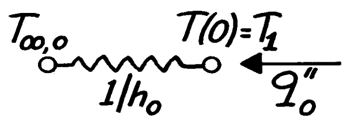
Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

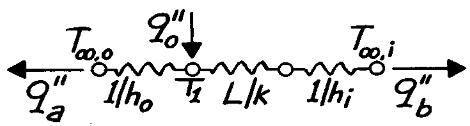
(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ W/m^2 \cdot K\,(65\text{-}25)^\circ C = 200\ W/m^2.$$ <

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

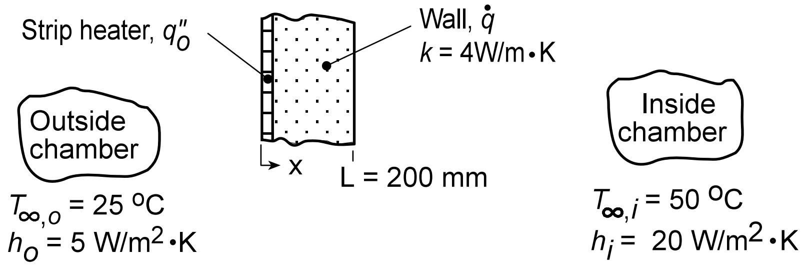
$$T_1 = 55^\circ C.$$ <

PROBLEM 3.80

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation and strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions ($T_{\infty,i}$, h_i, $T_{\infty,o}$, h_o). Strip heater acts to guard against heat losses from the wall to the outside.

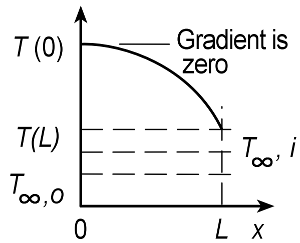
FIND: Compute and plot q''_o and T(0) as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i}$ = 30, 50 and 70°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: If no heat generated within the wall will be lost to the outside of the chamber, the gradient at the position x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution must be parabolic as shown in the sketch.



To determine the required heater flux q''_o as a function of the operation conditions $\dot{q}$ and $T_{\infty,i}$, the analysis begins by considering the temperature distribution in the wall and then surface energy balances at the two wall surfaces. The analysis is organized for easy treatment with equation-solving software.

Temperature distribution in the wall, T(x): The general solution for the temperature distribution in the wall is, Eq. 3.40,

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

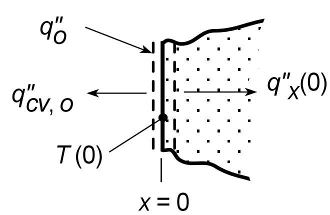
and the guard condition at the outer wall, x = 0, requires that the conduction heat flux be zero. Using Fourier's law,

$$q''_x(0) = -k\left.\frac{dT}{dx}\right)_{x=0} = -kC_1 = 0 \qquad (C_1 = 0) \tag{1}$$

At the outer wall, x = 0,

$$T(0) = C_2 \tag{2}$$

Surface energy balance, x = 0:



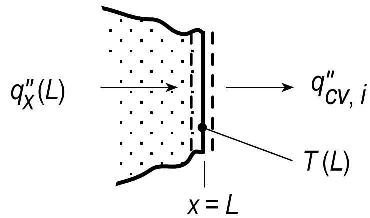
$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_o - q''_{cv,o} - q''_x(0) = 0 \tag{3}$$

$$q''_{cv,o} = h\left(T(0) - T_{\infty,o}\right), q''_x(0) = 0 \tag{4a,b}$$

Continued...

Surface energy balance, x = L:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

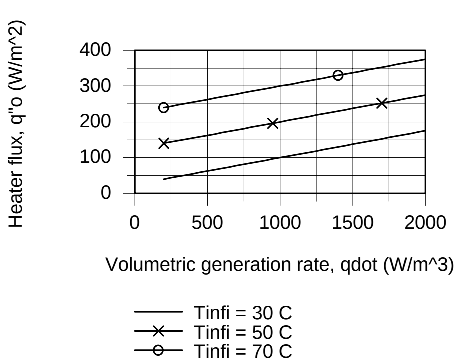
$$q''_x(L) - q''_{cv,i} = 0 \qquad (5)$$

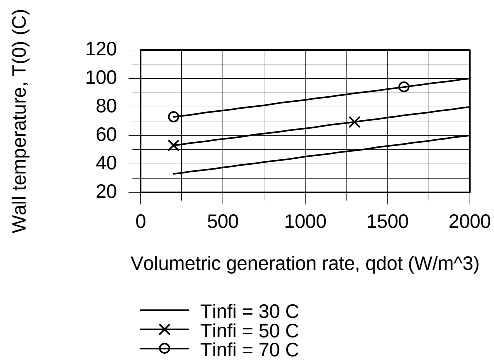
$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = +\dot{q}L \qquad (6)$$

$$q''_{cv,i} = h\left[T(L) - T_{\infty,i}\right]$$

$$q''_{cv,i} = h\left[-\frac{\dot{q}}{2k}L^2 + T(0) - T_{\infty,i}\right] \qquad (7)$$

Solving Eqs. (1) through (7) simultaneously with appropriate numerical values and performing the parametric analysis, the results are plotted below.





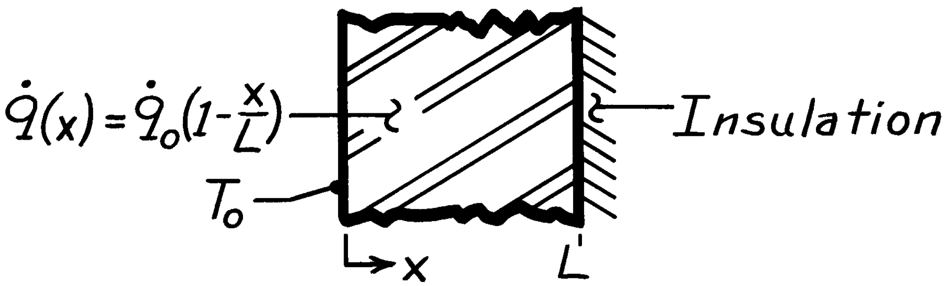
From the first plot, the heater flux q''_o is a linear function of the volumetric generation rate $\dot{q}$. As expected, the higher $\dot{q}$ and $T_{\infty,i}$, the higher the heat flux required to maintain the guard condition ($q''_x(0) = 0$). Notice that for any $\dot{q}$ condition, equal changes in $T_{\infty,i}$ result in equal changes in the required q''_o. The outer wall temperature T(0) is also linearly dependent upon $\dot{q}$. From our knowledge of the temperature distribution, it follows that for any $\dot{q}$ condition, the outer wall temperature T(0) will track changes in $T_{\infty,i}$.

PROBLEM 3.81

KNOWN: Plane wall with prescribed nonuniform volumetric generation having one boundary insulated and the other isothermal.

FIND: Temperature distribution, T(x), in terms of x, L, k, $\dot{q}_o$ and T_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) Constant properties.

ANALYSIS: The appropriate form the heat diffusion equation is

$$\frac{d}{dx}\left[\frac{dT}{dx}\right]+\frac{\dot{q}}{k}=0.$$

Noting that $\dot{q}=\dot{q}(x)=\dot{q}_o\ (1-x/L)$, substitute for $\dot{q}(x)$ into the above equation, separate variables and then integrate,

$$d\left[\frac{dT}{dx}\right]=-\frac{\dot{q}_o}{k}\left[1-\frac{x}{L}\right]dx \qquad \frac{dT}{dx}=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]+C_1.$$

Separate variables and integrate again to obtain the general form of the temperature distribution in the wall,

$$dT=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]dx+C_1dx \qquad T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+C_1x+C_2.$$

Identify the boundary conditions at x = 0 and x = L to evaluate C_1 and C_2. At x = 0,

$$T(0)=T_o=-\frac{\dot{q}_o}{k}(0-0)+C_1\cdot 0+C_2 \qquad \text{hence, } C_2=T_o$$

At x = L,

$$\left.\frac{dT}{dx}\right]_{x=L}=0=-\frac{\dot{q}_o}{k}\left[L-\frac{L^2}{2L}\right]+C_1 \qquad \text{hence, } C_1=\frac{\dot{q}_oL}{2k}$$

The temperature distribution is

$$T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+\frac{\dot{q}_oL}{2k}x+T_o.$$ <

COMMENTS: It is good practice to test the final result for satisfying BCs. The heat flux at x = 0 can be found using Fourier's law or from an overall energy balance

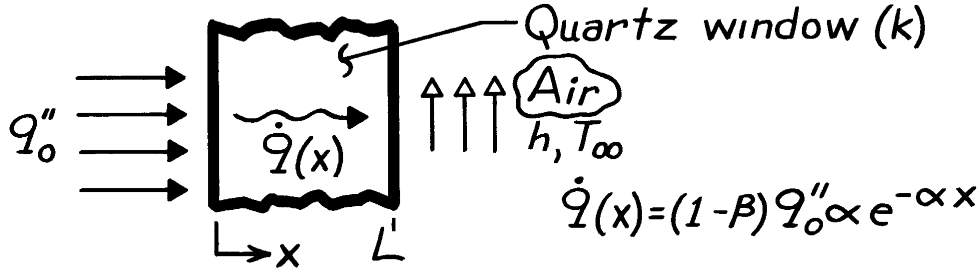
$$\dot{E}_{out}=\dot{E}_g=\int_0^L \dot{q}dV \qquad \text{to obtain} \qquad q''_{out}=\dot{q}_oL/2.$$

PROBLEM 3.82

KNOWN: Distribution of volumetric heating and surface conditions associated with a quartz window.

FIND: Temperature distribution in the quartz.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation emission and convection at inner surface (x = 0) and negligible emission from outer surface, (4) Constant properties.

ANALYSIS: The appropriate form of the heat equation for the quartz is obtained by substituting the prescribed form of $\dot{q}$ into Eq. 3.39.

$$\frac{d^2T}{dx^2}+\frac{\alpha(1-\beta)q''_o}{k}e^{-\alpha x}=0$$

Integrating,

$$\frac{dT}{dx}=+\frac{(1-\beta)q''_o}{k}e^{-\alpha x}+C_1 \qquad T=-\frac{(1-\beta)}{k\alpha}q''_o e^{-\alpha x}+C_1x+C_2$$

Boundary Conditions:

$$-k\,dT/dx)_{x=o}=\beta q''_o$$
$$-k\,dT/dx)_{x=L}=h\left[T(L)-T_\infty\right]$$

Hence, at x = 0:

$$-k\left[\frac{(1\text{-}\beta)}{k}q''_o+C_1\right]=\beta q''_o$$
$$C_1=-q''_o/k$$

At x = L:

$$-k\left[\frac{(1\text{-}\beta)}{k}q''_o e^{-\alpha L}+C_1\right]=h\left[-\frac{(1\text{-}\beta)}{k\alpha}q''_o e^{-\alpha L}+C_1L+C_2-T_\infty\right]$$

Substituting for C_1 and solving for C_2,

$$C_2=\frac{q''_o}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+\frac{q''_o}{k}+\frac{q''_o(1\text{-}\beta)}{k\alpha}e^{-\alpha L}+T_\infty.$$

Hence,
$$T(x)=\frac{(1-\beta)q''_o}{k\alpha}\left[e^{-\alpha L}-e^{-\alpha x}\right]+\frac{q''_o}{k}(L-x)+\frac{q''_o}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+T_\infty. \quad <$$

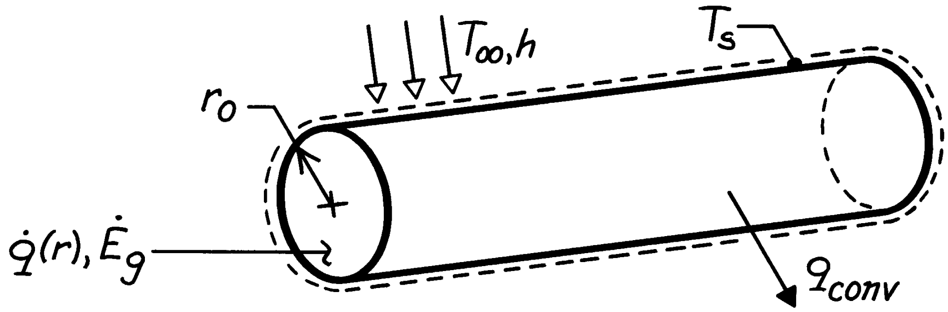
COMMENTS: The temperature distribution depends strongly on the radiative coefficients, α and β. For α → ∞ or β = 1, the heating occurs entirely at x = 0 (no volumetric heating).

PROBLEM 3.83

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left(1-\frac{r^2}{r_o^2}\right)$$

$$r\frac{dT}{dr} = -\frac{\dot{q}_o r^2}{2k} + \frac{\dot{q} r^4}{4kr_o^2} + C_1 \qquad T = -\frac{\dot{q}_o r^2}{4k} + \frac{\dot{q}_o r^4}{16kr_o^2} + C_1 \ln r + C_2.$$

From the boundary conditions,

$$\frac{dT}{dr}\Big|_{r=0} = 0 \rightarrow C_1 = 0 \qquad -k\frac{dT}{dr}\Big|_{r=r_o} = h\left[T(r_o) - T_\infty\right]$$

$$+\frac{\dot{q}_o r_o}{2} - \frac{\dot{q}_o r_o}{4} = h\left[-\frac{\dot{q}_o r_o^2}{4k} + \frac{\dot{q}_o r_o^2}{16k} + C_2 - T_\infty\right]$$

$$C_2 = \frac{\dot{q}_o r_o}{4h} + \frac{3\dot{q}_o r_o^2}{16k} + T_\infty.$$

Hence

$$T(r) = T_\infty + \frac{\dot{q}_o r_o}{4h} + \frac{\dot{q}_o r_o^2}{k}\left[\frac{3}{16} - \frac{1}{4}\left(\frac{r}{r_o}\right)^2 + \frac{1}{16}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (\dot{q}_o r_o)/4h$$

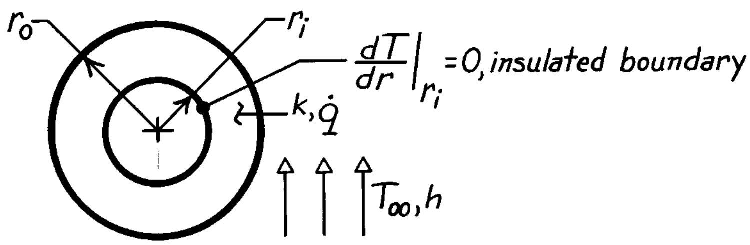
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of r_i, r_o, $\dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $$\left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k}r_i + C_1\frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k}r_i^2$$

at $r = r_o$: $$-k\left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad \text{surface energy balance}$$

$$k\left[-\frac{\dot{q}}{2k}r_o + \left(\frac{\dot{q}}{2k}r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k}r_o^2 + \left(\frac{\dot{q}}{2k}r_i^2\right)\ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q}r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q}r_i^2}{2k}\ln\left(\frac{r}{r_o}\right) - \frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

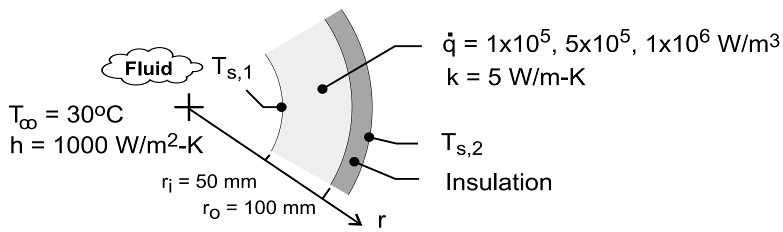
$$q_r'(r) = -k(2\pi\ r_o)\left.\frac{dT}{dr}\right)_{r_o} = -2\pi\ kr_o\left[-\frac{\dot{q}}{2k}r_o + \frac{\dot{q}r_i^2}{2k}\ \frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.85

KNOWN: The solid tube of Example 3.7 with inner and outer radii, 50 and 100 mm, and a thermal conductivity of 5 W/m·K. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 $W/m^2 \cdot K$.

FIND: Calculate and plot the temperature distributions for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. Use Eq. (7) with Eq. (10) of the Example 3.7 in the *IHT Workspace*.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties and (4) Uniform volumetric generation.

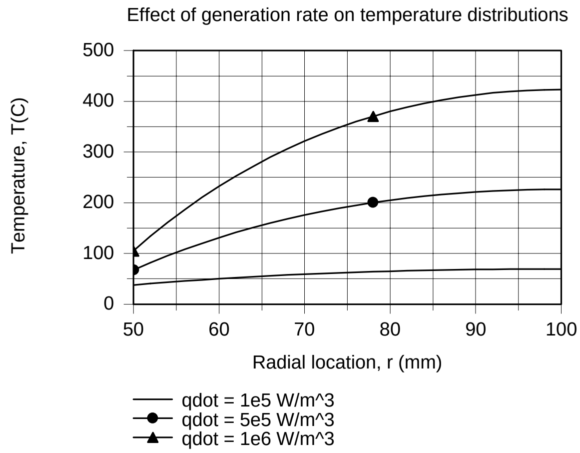
ANALYSIS: From Example 3.7, the temperature distribution in the tube is given by Eq. (7),

$$T(r) = T_{s,2} + \frac{\dot{q}}{4k}\left(r_2^2 - r^2\right) - \frac{\dot{q}}{2k} r_2^2 \ell n\left(\frac{r_2}{r}\right) \qquad r_1 \le r \le r_2 \qquad (1)$$

The temperature at the inner boundary, $T_{s,1}$, follows from the surface energy balance, Eq. (10),

$$\pi\dot{q}\left(r_2^2 - r_1^2\right) = h2\pi r_1\left(T_{s,1} - T_\infty\right) \qquad (2)$$

For the conditions prescribed in the schematic with $\dot{q} = 1\times10^5 W/m^3$, Eqs. (1) and (2), with r = r_1 and T(r) = $T_{s,1}$, are solved simultaneously to find $T_{s,2}$ = 69.3°C. Eq. (1), with $T_{s,2}$ now a known parameter, can be used to determine the temperature distribution, T(r). The results for different values of the generation rate are shown in the graph.



COMMENTS: (1) The temperature distributions are parabolic with a zero gradient at the insulated outer boundary, r = r_2. The effect of increasing $\dot{q}$ is to increase the maximum temperature in the tube, which always occurs at the outer boundary.

(2) The equations used to generate the graphical result in the *IHT Workspace* are shown below.

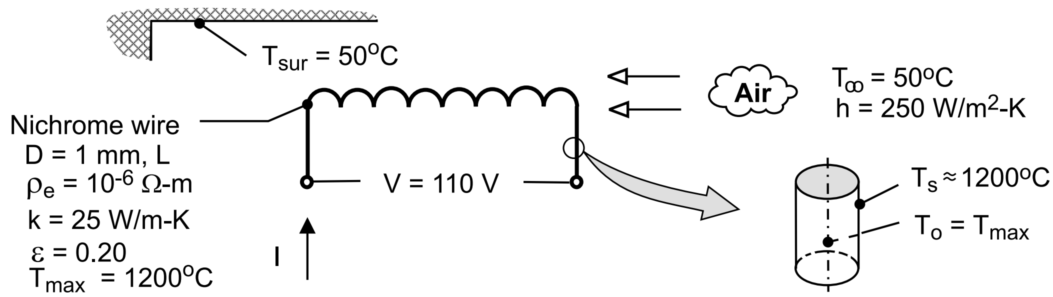
```
// The temperature distribution, from Eq. 7, Example 3.7
T_r = Ts2 + qdot/(4*k) * (r2^2 – r^2) – qgot / (2*k) * r2^2*ln (r2/r)
// The temperature at the inner surface, from Eq. 7
Ts1 = Ts2 + qdot / (4*k) * (r2^2 – r1^2) – qdot / (2*k) * r2^2 * ln (r2/r1)
// The energy balance on the surface, from Eq. 10
pi * qdot * (r2^2 – r1^2) = h * 2 * pi * r1 * (Ts1 – Tinf)
```

PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.55), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\,(1473 + 323)\,\text{K}\left(1473^2 + 323\right)^2 \text{K}^2 = 46.3\,\text{W/m}^2 \cdot \text{K}$$

$$h_t = (250 + 46.3)\,\text{W/m}^2 \cdot \text{K} = 296.3\,\text{W/m}^2 \cdot \text{K}$$

$$\dot{q}_{max} = \frac{2h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.3\,\text{W/m}^2 \cdot \text{K}\right)}{0.0005\text{m}}(1150°\text{C}) = 1.36 \times 10^9\,\text{W/m}^3$$

Hence, with $\qquad \dot{q} = \dfrac{I^2 R_e}{\forall} = \dfrac{I^2\left(\rho_e L/A_c\right)}{LA_c} = \dfrac{I^2 \rho_e}{A_c^2} = \dfrac{I^2 \rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2}\frac{\pi D^2}{4} = \left(\frac{1.36 \times 10^9\,\text{W/m}^3}{10^{-6}\,\Omega \cdot \text{m}}\right)^{1/2}\frac{\pi\,(0.001\text{m})^2}{4} = 29.0\,\text{A} \qquad <$$

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E \cdot A_c}{I_{max}\,\rho_e} = \frac{110\,\text{V}\left[\pi\,(0.001\text{m})^2/4\right]}{29.0\,\text{A}\left(10^{-6}\,\Omega \cdot \text{m}\right)} = 2.98\text{m} \qquad <$$

and the power rating is

$$P_{elec} = \Delta E \cdot I_{max} = 110\,\text{V}\,(29\,\text{A}) = 3190\,\text{W} = 3.19\,\text{kW} \qquad <$$

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.53) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \dfrac{\dot{q}\,r_o^2}{4k} + T_s = \dfrac{1.36 \times 10^9\,\text{W/m}^3\,(0.0005\text{m})^2}{4\,(25\,\text{W/m} \cdot \text{K})} + 1200°\text{C} = 1203°\text{C}$. With only a

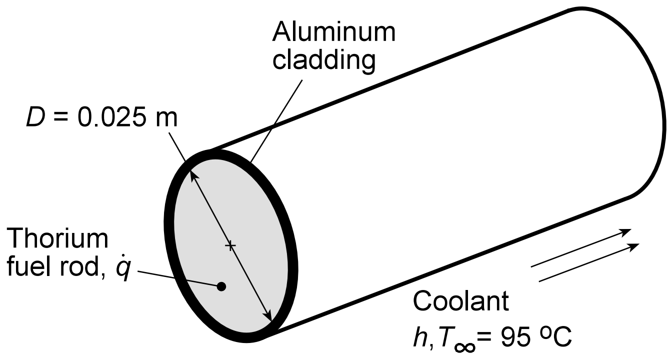
3°C temperature difference between the centerline and surface of the wire, the assumption is *excellent*.

PROBLEM 3.87

KNOWN: Energy generation in an aluminum-clad, thorium fuel rod under specified operating conditions.

FIND: (a) Whether prescribed operating conditions are acceptable, (b) Effect of $\dot{q}$ and h on acceptable operating conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r-direction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible temperature gradients in aluminum and contact resistance between aluminum and thorium.

PROPERTIES: *Table A-1*, Aluminum, pure: M.P. = 933 K; *Table A-1*, Thorium: M.P. = 2023 K, k ≈ 60 W/m·K.

ANALYSIS: (a) System failure would occur if the melting point of either the thorium or the aluminum were exceeded. From Eq. 3.53, the maximum thorium temperature, which exists at r = 0, is

$$T(0) = \frac{\dot{q} r_o^2}{4k} + T_s = T_{Th,max}$$

where, from the energy balance equation, Eq. 3.55, the surface temperature, which is also the aluminum temperature, is

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} = T_{Al}$$

Hence,

$$T_{Al} = T_s = 95^\circ C + \frac{7\times10^8\ W/m^3 \times 0.0125\ m}{14,000\ W/m^2\cdot K} = 720^\circ C = 993\ K$$

$$T_{Th,max} = \frac{7\times10^8\ W/m^3 (0.0125 m)^2}{4\times60\ W/m\cdot K} + 993\ K = 1449\ K \qquad \textless$$

Although $T_{Th,max} < M.P._{Th}$ and the thorium would not melt, $T_{al} > M.P._{Al}$ and the cladding would melt under the proposed operating conditions. The problem could be eliminated by *decreasing* $\dot{q}$, *increasing* h or using a cladding material with a higher melting point.

(b) Using the one-dimensional, steady-state conduction model (solid cylinder) of the IHT software, the following radial temperature distributions were obtained for parametric variations in $\dot{q}$ and h.

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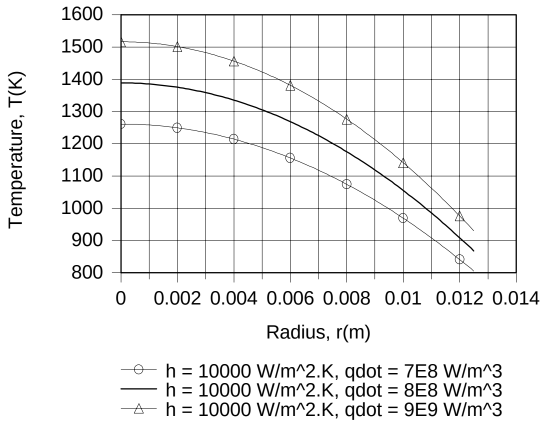
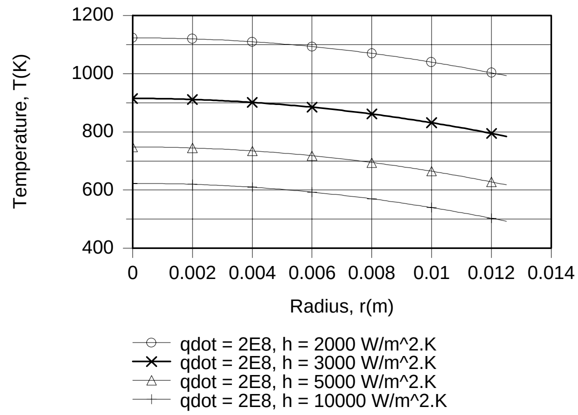
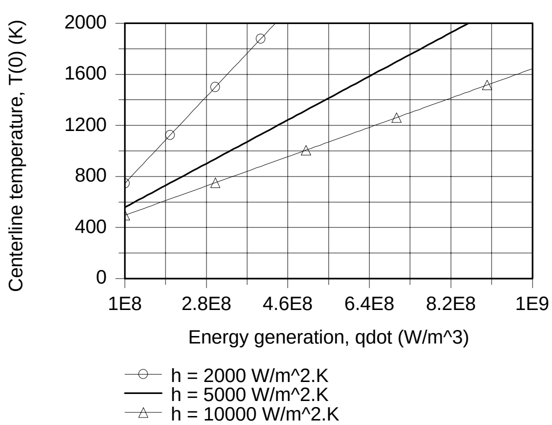
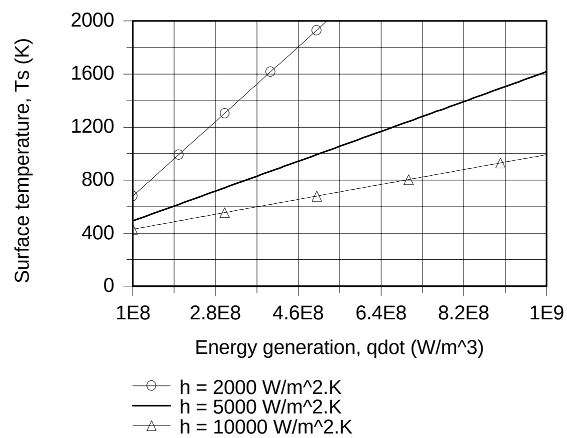




For h = 10,000 W/m^2·K, which represents a reasonable upper limit with water cooling, the temperature of the aluminum would be well below its melting point for $\dot{q} = 7 \times 10^8$ W/m^3, but would be close to the melting point for $\dot{q} = 8 \times 10^8$ W/m^3 and would exceed it for $\dot{q} = 9 \times 10^8$ W/m^3. Hence, under the best of conditions, $\dot{q} \approx 7 \times 10^8$ W/m^3 corresponds to the maximum allowable energy generation. However, if coolant flow conditions are constrained to provide values of h < 10,000 W/m^2·K, volumetric heating would have to be reduced. Even for $\dot{q}$ as low as 2×10^8 W/m^3, operation could not be sustained for h = 2000 W/m^2·K.

The effects of $\dot{q}$ and h on the centerline and surface temperatures are shown below.





For h = 2000 and 5000 W/m^2·K, the melting point of thorium would be approached for $\dot{q} \approx 4.4 \times 10^8$ and 8.5×10^8 W/m^3, respectively. For h = 2000, 5000 and 10,000 W/m^2·K, the melting point of aluminum would be approached for $\dot{q} \approx 1.6 \times 10^8$, 4.3×10^8 and 8.7×10^8 W/m^3. Hence, the envelope of acceptable operating conditions must call for a reduction in $\dot{q}$ with decreasing h, from a maximum of $\dot{q} \approx 7 \times 10^8$ W/m^3 for h = 10,000 W/m^2·K.

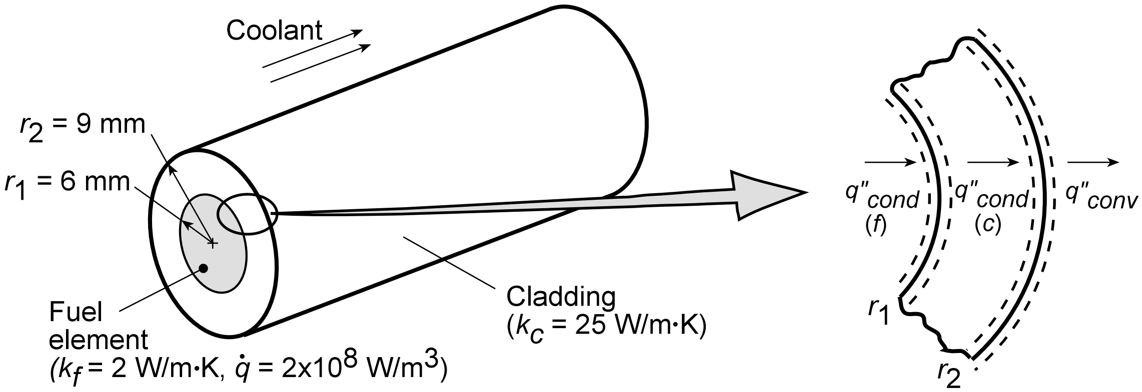
COMMENTS: Note the problem which would arise in the event of a *loss of coolant*, for which case h would *decrease* drastically.

PROBLEM 3.88

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.49 and 3.23, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(\frac{dT_f}{dr}\right) = -\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right) = 0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr} = -\frac{\dot{q}r}{2k_f} + \frac{C_1}{k_f r} \qquad\qquad T_f = -\frac{\dot{q}r^2}{4k_f} + \frac{C_1}{k_f}\ln r + C_2 \tag{1,2}$$

$$\frac{dT_c}{dr} = \frac{C_3}{k_c r} \qquad\qquad T_c = \frac{C_3}{k_c}\ln r + C_4 \tag{3,4}$$

The corresponding boundary conditions are:

$$dT_f/dr\big)_{r=0} = 0 \qquad\qquad T_f(r_1) = T_c(r_1) \tag{5,6}$$

$$-k_f \frac{dT_f}{dr}\bigg)_{r=r_1} = -k_c \frac{dT_c}{dr}\bigg)_{r=r_1} \qquad\qquad -k_c\frac{dT_c}{dr}\bigg)_{r=r_2} = h\left[T_c(r_2) - T_\infty\right] \tag{7,8}$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f = -\frac{\dot{q}r^2}{4k_f} + C_2 \tag{9}$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f} + C_2 = \frac{C_3 \ln r_1}{k_c} + C_4 \tag{10}$$

Continued...

Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \qquad (11)$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \qquad (12)$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h}T_\infty \qquad (13)$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty \qquad (14) \; <$$

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2h} + T_\infty\,. \qquad (15) \; <$$

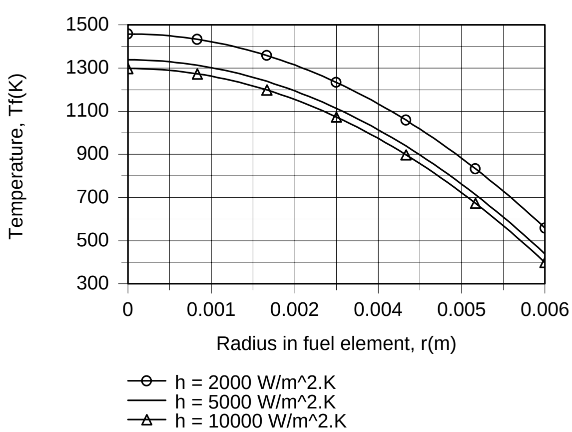
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 W/m^2·K is

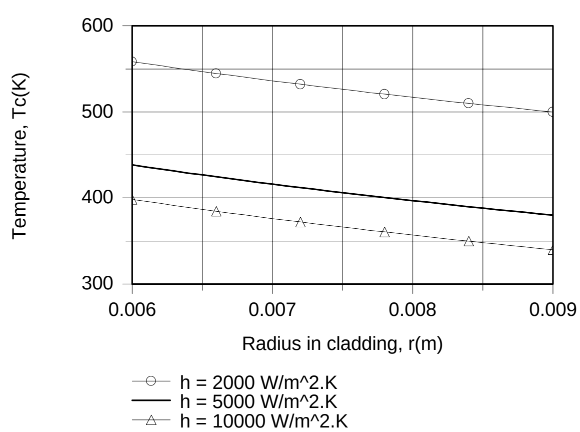
$$T_f(0) = \frac{2\times10^8\ \mathrm{W/m^3}\times(0.006\,\mathrm{m})^2}{4\times2\ \mathrm{W/m\cdot K}} + \frac{2\times10^8\ \mathrm{W/m^3}\times(0.006\,\mathrm{m})^2}{2\times25\ \mathrm{W/m\cdot K}}\ln\frac{0.009\,\mathrm{m}}{0.006\,\mathrm{m}}$$

$$+\frac{2\times10^8\ \mathrm{W/m^3}\,(0.006\,\mathrm{m})^2}{2\times(0.09\,\mathrm{m})\,2000\ \mathrm{W/m^2\cdot K}} + 300\,\mathrm{K}$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,\mathrm{K} = 1458\,\mathrm{K}\,. \qquad <$$

(c) Temperature distributions for the prescribed values of h are as follows:





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Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \to \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

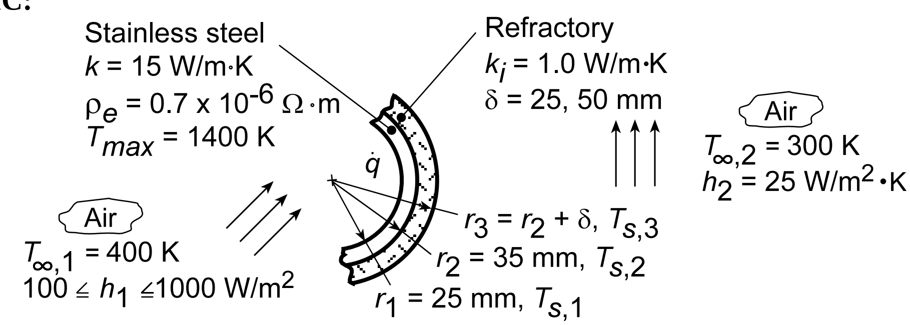
COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2 / 4k_f = (2\times10^8\ \text{W/m}^3)(0.006\ \text{m})^3/8\ \text{W/m}\cdot\text{K} = 900\ \text{K}$, in which case, with no cladding and $h \to \infty$, $T_f(0) = 1200$ K. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.

PROBLEM 3.89

KNOWN: Dimensions and properties of tubular heater and external insulation. Internal and external convection conditions. Maximum allowable tube temperature.

FIND: (a) Maximum allowable heater current for adiabatic outer surface, (3) Effect of internal convection coefficient on heater temperature distribution, (c) Extent of heat loss at outer surface.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform heat generation, (4) Negligible radiation at outer surface, (5) Negligible contact resistance.

ANALYSIS: (a) From Eqs. 7 and 10, respectively, of Example 3.7, we know that

$$T_{s,2} - T_{s,1} = \frac{\dot{q}}{2k} r_2^2 \ln\frac{r_2}{r_1} - \frac{\dot{q}}{4k}\left(r_2^2 - r_1^2\right) \qquad (1)$$

and

$$T_{s,1} = T_{\infty,1} + \frac{\dot{q}\left(r_2^2 - r_1^2\right)}{2h_1 r_1} \qquad (2)$$

Hence, eliminating $T_{s,1}$, we obtain

$$T_{s,2} - T_{\infty,1} = \frac{\dot{q}r_2^2}{2k}\left[\ln\frac{r_2}{r_1} - \frac{1}{2}\left(1 - r_1^2/r_2^2\right) + \frac{k}{h_1 r_1}\left(1 - r_1^2/r_2^2\right)\right]$$

Substituting the prescribed conditions ($h_1 = 100\ \text{W/m}^2\cdot\text{K}$),

$$T_{s,2} - T_{\infty,1} = 1.237\times10^{-4}\left(\text{m}^3\cdot\text{K/W}\right)\dot{q}\left(\text{W/m}^3\right)$$

Hence, with T_{max} corresponding to $T_{s,2}$, the maximum allowable value of $\dot{q}$ is

$$\dot{q}_{max} = \frac{1400 - 400}{1.237\times10^{-4}} = 8.084\times10^6\ \text{W/m}^3$$

with

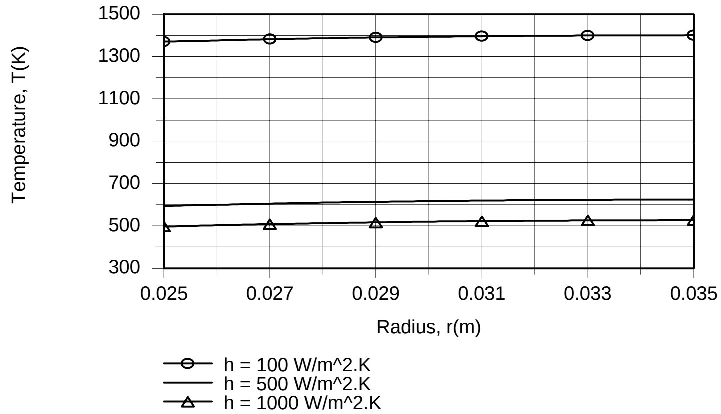
$$\dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2 \rho_e L/A_c}{LA_c} = \frac{\rho_e I^2}{\left[\pi\left(r_2^2 - r_1\right)\right]^2}$$

$$I_{max} = \pi\left(r_2^2 - r_1^2\right)\left(\frac{\dot{q}}{\rho_e}\right)^{1/2} = \pi\left(0.035^2 - 0.025^2\right)\text{m}^2\left(\frac{8.084\times10^6\ \text{W/m}^3}{0.7\times10^{-6}\,\Omega\cdot\text{m}}\right)^{1/2} = 6406\ \text{A} \quad <$$

Continued …..

PROBLEM 3.89 (Cont.)

(b) Using the one-dimensional, steady-state conduction model of IHT (hollow cylinder; convection at inner surface and adiabatic outer surface), the following temperature distributions were obtained.



The results are consistent with key implications of Eqs. (1) and (2), namely that the value of h_1 has no effect on the temperature drop across the tube ($T_{s,2}$ - $T_{s,1}$ = 30 K, irrespective of h_1), while $T_{s,1}$ decreases with increasing h_1. For h_1 = 100, 500 and 1000 $W/m^2 \cdot K$, respectively, the ratio of the temperature drop between the inner surface and the air to the temperature drop across the tube, $(T_{s,1} - T_{\infty,1})/(T_{s,2} - T_{s,1})$, decreases from 970/30 = 32.3 to 194/30 = 6.5 and 97/30 = 3.2. Because the outer surface is insulated, the heat rate to the airflow is fixed by the value of $\dot{q}$ and, irrespective of h_1,

$$q'(r_1) = \pi\left(r_2^2 - r_1^2\right)\dot{q} = -15,240\,\text{W}$$ <

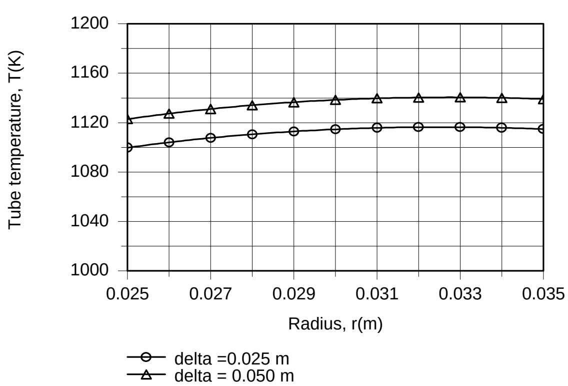
(c) Heat loss from the outer surface of the tube to the surroundings depends on the total thermal resistance

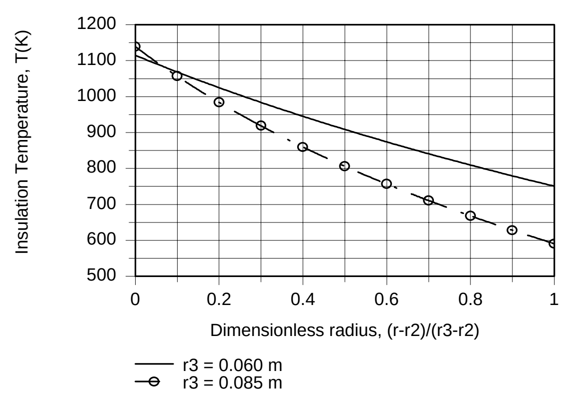
$$R_{tot} = \frac{\ln(r_3/r_2)}{2\pi L k_i} + \frac{1}{2\pi r_3 L h_2}$$

or, for a unit area on surface 2,

$$R''_{tot,2} = (2\pi r_2 L) R_{tot} = \frac{r_2 \ln(r_3/r_2)}{k_i} + \frac{r_2}{r_3 h_2}$$

Again using the capabilities of IHT (hollow cylinder; convection at inner surface and heat transfer from outer surface through $R''_{tot,2}$), the following temperature distributions were determined for the tube and insulation.





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PROBLEM 3.89 (Cont.)

Heat losses through the insulation, $q'(r_2)$, are 4250 and 3890 W/m for δ = 25 and 50 mm, respectively, with corresponding values of $q'(r_1)$ equal to -10,990 and -11,350 W/m. Comparing the tube temperature distributions with those predicted for an adiabatic outer surface, it is evident that the losses reduce tube wall temperatures predicted for the adiabatic surface and also shift the maximum temperature from r = 0.035 m to r ≈ 0.033 m. Although the tube outer and insulation inner surface temperatures, $T_{s,2} = T(r_2)$, increase with increasing insulation thickness, Fig. (c), the insulation outer surface temperature decreases.

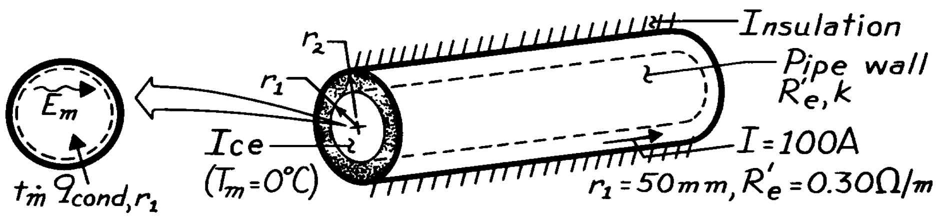
COMMENTS: If the intent is to maximize heat transfer to the airflow, heat losses to the ambient should be reduced by selecting an insulation material with a significantly smaller thermal conductivity.

PROBLEM 3.90

KNOWN: Electric current I is passed through a pipe of resistance R_e' to melt ice under steady-state conditions.

FIND: (a) Temperature distribution in the pipe wall, (b) Time to completely melt the ice.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation in the pipe wall, (5) Outer surface of the pipe is adiabatic, (6) Inner surface is at a constant temperature, T_m.

PROPERTIES: *Table A-3,* Ice (273K): $\rho = 920\ kg/m^3$; *Handbook Chem. & Physics,* Ice: Latent heat of fusion, $h_{sf} = 3.34\times10^5$ J/kg.

ANALYSIS: (a) The appropriate form of the heat equation is Eq. 3.49, and the general solution, Eq. 3.51 is

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

where

$$\dot{q} = \frac{I^2 R_e'}{\pi\left(r_2^2 - r_1^2\right)}.$$

Applying the boundary condition $(dT/dr)_{r_2} = 0,$ it follows that

$$0 = \frac{\dot{q}r_2}{2k} + \frac{C_1}{r_2}$$

Hence $$C_1 = \frac{\dot{q}r_2^2}{2k}$$

and $$T(r) = -\frac{\dot{q}}{4k}r^2 + \frac{\dot{q}r_2^2}{2k}\ln r + C_2.$$

Continued …..

Applying the second boundary condition, $T(r_1) = T_m$, it follows that

$$T_m = -\frac{\dot{q}}{4k}r_1^2 + \frac{\dot{q}r_2^2}{2k}\ln r_1 + C_2.$$

Solving for C_2 and substituting into the expression for T(r), find

$$T(r) = T_m + \frac{\dot{q}r_2^2}{2k}\ln\frac{r}{r_1} - \frac{\dot{q}}{4k}\left(r^2 - r_1^2\right). \qquad <$$

(b) Conservation of energy dictates that the energy required to completely melt the ice, E_m, must equal the energy which reaches the inner surface of the pipe by conduction through the wall during the melt period. Hence from Eq. 1.11b

$$\Delta E_{st} = E_{in} - E_{out} + E_{gen}$$

$$\Delta E_{st} = E_m = t_m \cdot q_{cond,r_1}$$

or, for a unit length of pipe,

$$\rho\left(\pi\, r_1^2\right)h_{sf} = t_m\left[-k\left(2\pi\, r_1\right)\left[\frac{dT}{dr}\right]_{r_1}\right]$$

$$\rho\left(\pi\, r_1^2\right)h_{sf} = -2\pi r_1 k t_m\left[\frac{\dot{q}r_2^2}{2kr_1} - \frac{\dot{q}r_1}{2k}\right]$$

$$\rho\left(\pi\, r_1^2\right)h_{sf} = -t_m\dot{q}\pi\left(r_2^2 - r_1^2\right).$$

Dropping the minus sign, which simply results from the fact that conduction is in the negative r direction, it follows that

$$t_m = \frac{\rho h_{sf} r_1^2}{\dot{q}\left(r_2^2 - r_1^2\right)} = \frac{\rho h_{sf}\pi\, r_1^2}{I^2 R_e'}.$$

With r_1 = 0.05m, I = 100 A and $R_e' = 0.30\ \Omega/\text{m}$, it follows that

$$t_m = \frac{920\text{kg/m}^3 \times 3.34\times10^5\text{J/kg}\times\pi\times(0.05\text{m})^2}{(100\text{A})^2\times0.30\Omega/\text{m}}$$

or $\quad t_m = 804\text{s}. \qquad <$

COMMENTS: The foregoing expression for t_m could also be obtained by recognizing that all of the energy which is generated by electrical heating in the pipe wall must be transferred to the ice. Hence,

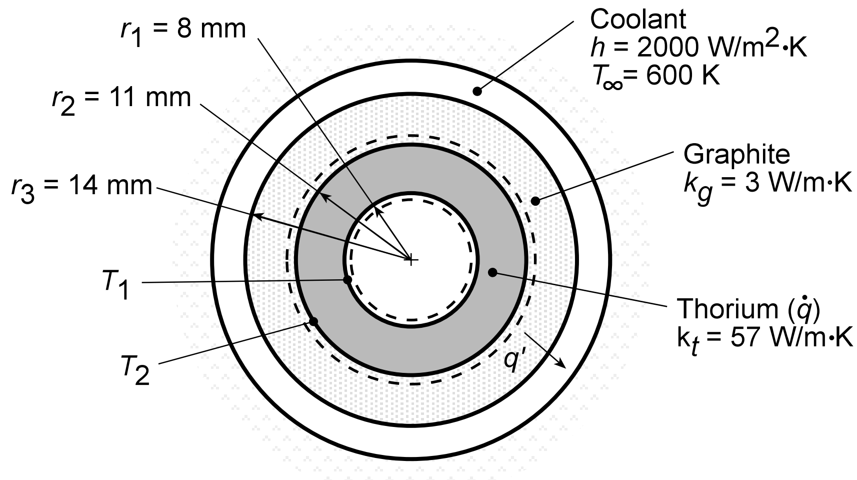
$$I^2 R_e' t_m = \rho h_{sf}\pi\, r_1^2.$$

PROBLEM 3.91

KNOWN: Materials, dimensions, properties and operating conditions of a gas-cooled nuclear reactor.

FIND: (a) Inner and outer surface temperatures of fuel element, (b) Temperature distributions for different heat generation rates and maximum allowable generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Thoriun: $T_{mp} \approx 2000$ K; *Table A.2*, Graphite: $T_{mp} \approx 2300$ K.

ANALYSIS: (a) The outer surface temperature of the fuel, T_2, may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{tot}}$$

where

$$R'_{tot} = \frac{\ln(r_3/r_2)}{2\pi\, k_g} + \frac{1}{2\pi r_3 h} = \frac{\ln(14/11)}{2\pi\,(3\,\mathrm{W/m\cdot K})} + \frac{1}{2\pi\,(0.014\,\mathrm{m})\left(2000\,\mathrm{W/m^2\cdot K}\right)} = 0.0185\,\mathrm{m\cdot K/W}$$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 10^8\,\mathrm{W/m^3} \times \pi\left(0.011^2 - 0.008^2\right)\mathrm{m^2} = 17{,}907\,\mathrm{W/m}$$

Hence,

$$T_2 = q' R'_{tot} + T_\infty = 17{,}907\,\mathrm{W/m}\,(0.0185\,\mathrm{m\cdot K/W}) + 600\,\mathrm{K} = 931\,\mathrm{K}$$ <

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

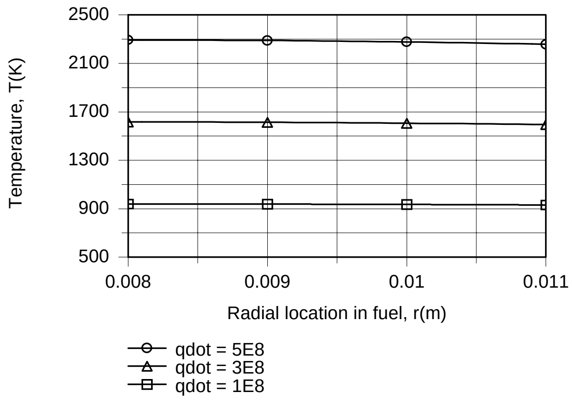
$$T_1 = T_2 + \frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q} r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right)$$

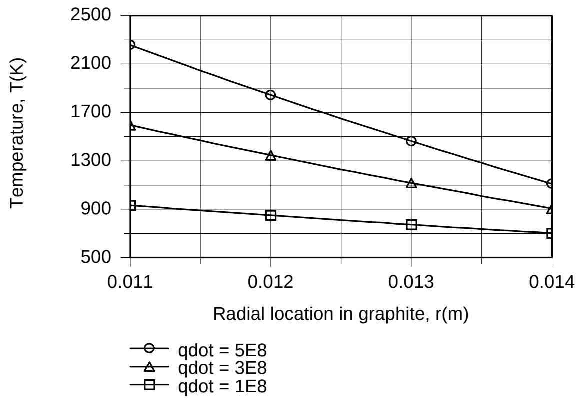
$$T_1 = 931\,\mathrm{K} + \frac{10^8\,\mathrm{W/m^3}\,(0.011\,\mathrm{m})^2}{4\times 57\,\mathrm{W/m\cdot K}}\left[1 - \left(\frac{0.008}{0.011}\right)^2\right] - \frac{10^8\,\mathrm{W/m^3}\,(0.008\,\mathrm{m})^2}{2\times 57\,\mathrm{W/m\cdot K}}\ln\left(\frac{0.011}{0.008}\right)$$

Continued...

$$T_1 = 931\,K + 25\,K - 18\,K = 938\,K \qquad <$$

(b) The temperature distributions may be obtained by using the IHT model for one-dimensional, steady-state conduction in a hollow tube. For the fuel element ($\dot{q} > 0$), an adiabatic surface condition is prescribed at r_1, while heat transfer from the outer surface at r_2 to the coolant is governed by the thermal resistance $R''_{tot,2} = 2\pi r_2 R'_{tot} = 2\pi(0.011\text{ m})0.0185\text{ m}\cdot\text{K/W} = 0.00128\text{ m}^2\cdot\text{K/W}$. For the graphite ($\dot{q}$ = 0), the value of T_2 obtained from the foregoing solution is prescribed as an inner boundary condition at r_2, while a convection condition is prescribed at the outer surface (r_3). For $1 \times 10^8 \le \dot{q} \le 5 \times 10^8\text{ W/m}^3$, the following distributions are obtained.





The comparatively large value of k_t yields small temperature variations across the fuel element, while the small value of k_g results in large temperature variations across the graphite. Operation at $\dot{q} = 5 \times 10^8\text{ W/m}^3$ is clearly unacceptable, since the melting points of thorium and graphite are exceeded and approached, respectively. To prevent softening of the materials, which would occur below their melting points, the reactor should not be operated much above $\dot{q} = 3 \times 10^8\text{ W/m}^3$.

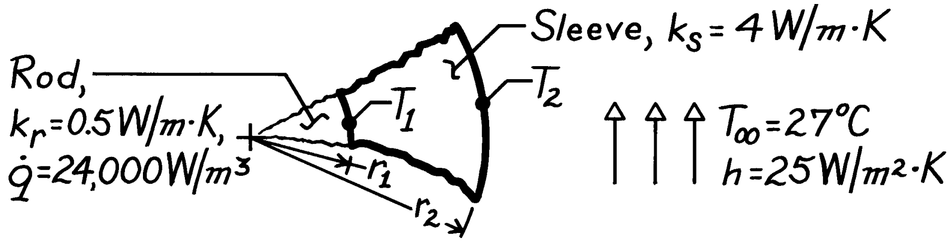
COMMENTS: A contact resistance at the thorium/graphite interface would increase temperatures in the fuel element, thereby reducing the maximum allowable value of $\dot{q}$.

PROBLEM 3.92

KNOWN: Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection.

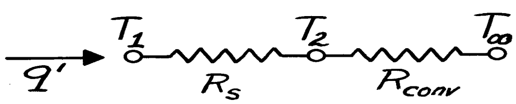
FIND: (a) Temperature at the interface between rod and sleeve and on the outer surface, (b) Temperature at center of rod.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in rod and sleeve, (2) Steady-state conditions, (3) Uniform volumetric generation in rod, (4) Negligible contact resistance between rod and sleeve.

ANALYSIS: (a) Construct a thermal circuit for the sleeve,



where

$$q' = \dot{E}'_{gen} = \dot{q}\pi\, D_1^2/4 = 24,000\ \text{W/m}^3 \times \pi \times (0.20\ \text{m})^2/4 = 754.0\ \text{W/m}$$

$$R'_s = \frac{\ln(r_2/r_1)}{2\pi\, k_s} = \frac{\ln(400/200)}{2\pi \times 4\ \text{W/m}\cdot\text{K}} = 2.758 \times 10^{-2}\,\text{m}\cdot\ \text{K/W}$$

$$R_{conv} = \frac{1}{h\pi\, D_2} = \frac{1}{25\ \text{W/m}^2\cdot\text{K} \times \pi \times 0.400\ \text{m}} = 3.183 \times 10^{-2}\,\text{m}\cdot\ \text{K/W}$$

The rate equation can be written as

$$q' = \frac{T_1 - T_\infty}{R'_s + R'_{conv}} = \frac{T_2 - T_\infty}{R'_{conv}}$$

$$T_1 = T_\infty + q'\left(R'_s + R'_{conv}\right) = 27^\circ\text{C} + 754\ \text{W/m}\left(2.758 \times 10^{-2} + 3.183 \times 10^{-2}\right)\text{K/W}\cdot\text{m} = 71.8^\circ\text{C} \quad <$$

$$T_2 = T_\infty + q' R'_{conv} = 27^\circ\text{C} + 754\ \text{W/m} \times 3.183 \times 10^{-2}\text{m}\cdot\ \text{K/W} = 51.0^\circ\text{C}. \quad <$$

(b) The temperature at the center of the rod is

$$T(0) = T_o = \frac{\dot{q} r_1^2}{4k_r} + T_1 = \frac{24,000\ \text{W/m}^3\,(0.100\ \text{m})^2}{4 \times 0.5\ \text{W/m}\cdot\text{K}} + 71.8^\circ\text{C} = 192^\circ\text{C}. \quad <$$

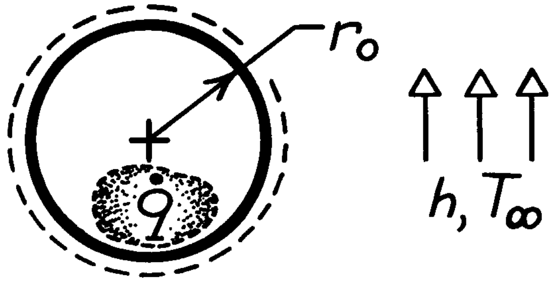
COMMENTS: The thermal resistances due to conduction in the sleeve and convection are comparable. Will increasing the sleeve outer diameter cause the surface temperature T_2 to increase or decrease?

PROBLEM 3.93

KNOWN: Radius, thermal conductivity, heat generation and convection conditions associated with a solid sphere.

FIND: Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation.

ANALYSIS: Integrating the appropriate form of the heat diffusion equation,

$$\frac{1}{r^2}\frac{d}{dr}\left[kr^2\frac{dT}{dr}\right]+\dot{q}=0 \qquad \text{or} \qquad \frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=-\frac{\dot{q}r^2}{k}$$

$$r^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3k}+C_1 \qquad \frac{dT}{dr}=-\frac{\dot{q}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2.$$

The boundary conditions are: $\left.\frac{dT}{dr}\right]_{r=0}=0$ hence $C_1=0$, and

$$-k\left.\frac{dT}{dr}\right]_{r_o}=h\left[T(r_o)-T_\infty\right].$$

Substituting into the second boundary condition ($r = r_o$), find

$$\frac{\dot{q}r_o}{3}=h\left[-\frac{\dot{q}r_o^2}{6k}+C_2-T_\infty\right] \qquad C_2=\frac{\dot{q}r_o}{3h}+\frac{\dot{q}r_o^2}{6k}+T_\infty.$$

The temperature distribution has the form

$$T(r)=\frac{\dot{q}}{6k}\left(r_o^2-r^2\right)+\frac{\dot{q}r_o}{3h}+T_\infty.$$ <

COMMENTS: To verify the above result, obtain $T(r_o) = T_s$,

$$T_s=\frac{\dot{q}r_o}{3h}+T_\infty$$

Applying energy balance to the control volume about the sphere,

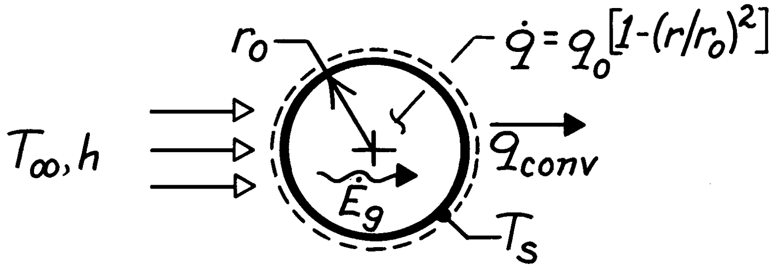
$$\dot{q}\left[\frac{4}{3}\pi\, r_o^3\right]=h4\pi\, r_o^2\left(T_s-T_\infty\right) \qquad \text{find} \qquad T_s=\frac{\dot{q}r_o}{3h}+T_\infty.$$

PROBLEM 3.94

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence
$$r^2\frac{dT}{dr} = -\frac{\dot{q}_o}{k}\left(\frac{r^3}{3}-\frac{r^5}{5r_o^2}\right)+C_1$$

$$T = -\frac{\dot{q}_o}{k}\left(\frac{r^2}{6}-\frac{r^4}{20r_o^2}\right)-\frac{C_1}{r}+C_2.$$

From the boundary conditions,

$$dT/dr|_{r=0} = 0 \quad \text{and} \quad -k\,dT/dr|_{r=r_o} = h\left[T(r_o)-T_\infty\right]$$

it follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3}-\frac{r_o}{5}\right) = h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6}-\frac{r_o^2}{20}\right)+C_2-T_\infty\right]$$

$$C_2 = \frac{2r_o\dot{q}_o}{15h}+\frac{7\dot{q}_o r_o^2}{60k}+T_\infty.$$

Hence
$$T(r) = T_\infty+\frac{2r_o\dot{q}_o}{15h}+\frac{\dot{q}r_o^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_o}\right)^2+\frac{1}{20}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty+\left(2r_o\dot{q}_o/15h\right).$$

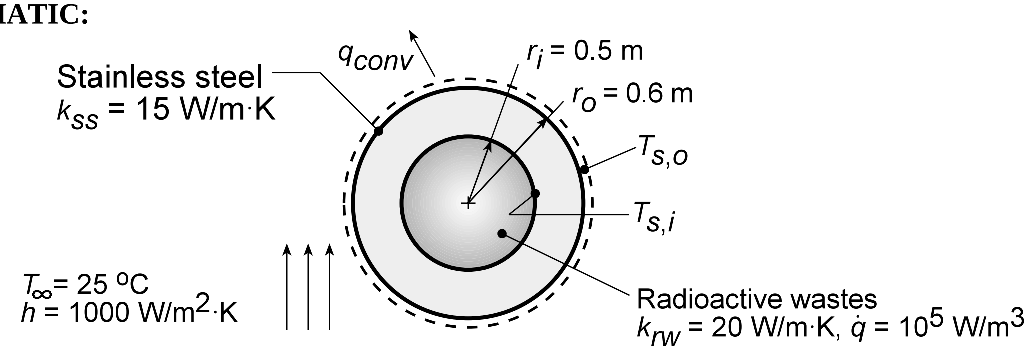
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.95

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m^3,

$$T_{s,o} = T_\infty + \frac{\dot{q}r_i^3}{3hr_o^2} = 25^\circ C + \frac{10^5\,W/m^2\,(0.5\,m)^3}{3000\,W/m^2\cdot K\,(0.6\,m)^2} = 36.6^\circ C. \qquad <$$

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$. Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right)r_o\left(T_{s,o} - T_\infty\right) = 36.6^\circ C + \frac{1000\,W/m^2\cdot K}{15\,W/m\cdot K}(0.2)0.6\,m\left(11.6^\circ C\right) = 129.4^\circ C. \qquad <$$

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right) \quad <$$

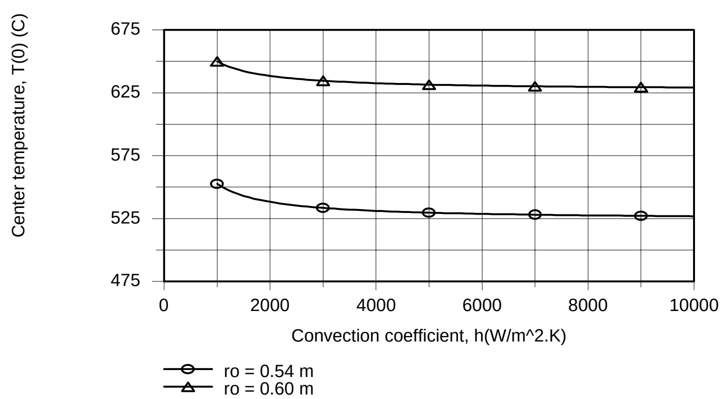
At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 129.4^\circ\text{C} + \frac{10^5\ \text{W/m}^3\,(0.5\,\text{m})^2}{6(20\,\text{W/m}\cdot\text{K})} = 337.7^\circ\text{C} \quad <$$

(d) The feasibility assessment may be performed by using the IHT model for one-dimensional, steady-state conduction in a solid sphere, with the surface boundary condition prescribed in terms of the total thermal resistance

$$R''_{tot,i} = \left(4\pi r_i^2\right)R_{tot} = R''_{cnd,i} + R''_{cnv,i} = \frac{r_i^2\left[(1/r_i) - (1/r_o)\right]}{k_{ss}} + \frac{1}{h}\left(\frac{r_i}{r_o}\right)^2$$

where, for r_o = 0.6 m and h = 1000 W/m^2·K, $R''_{cnd,i}$ = 5.56 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 6.94 × 10^{-4} m^2·K/W, and $R''_{tot,i}$ = 6.25 × 10^{-3} m^2·K/W. Results for the center temperature are shown below.



Clearly, even with r_o = 0.54 m = $r_{o,min}$ and h = 10,000 W/m^2·K (a practical upper limit), T(0) > 475°C and the desired condition can not be met. The corresponding resistances are $R''_{cnd,i}$ = 2.47 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 8.57 × 10^{-5} m^2·K/W, and $R''_{tot,i}$ = 2.56 × 10^{-3} m^2·K/W. The conduction resistance remains dominant, and the effect of reducing $R''_{cnv,i}$ by increasing h is small. *The proposed extension is not feasible.*

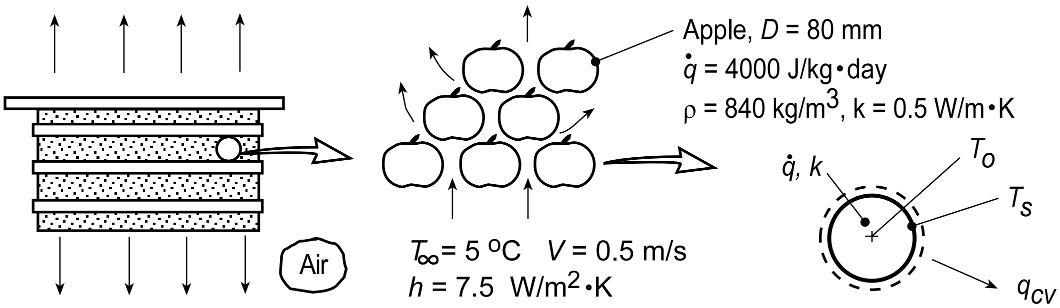
COMMENTS: A value of $\dot{q}$ = 1.79 × 10^5 W/m^3 would allow for operation at T(0) = 475°C with r_o = 0.54 m and h = 10,000 W/m^2·K.

PROBLEM 3.96

KNOWN: Carton of apples, modeled as 80-mm diameter spheres, ventilated with air at 5°C and experiencing internal volumetric heat generation at a rate of 4000 J/kg·day.

FIND: (a) The apple center and surface temperatures when the convection coefficient is 7.5 $W/m^2 \cdot K$, and (b) Compute and plot the apple temperatures as a function of air velocity, V, for the range $0.1 \le V \le 1$ m/s, when the convection coefficient has the form $h = C_1 V^{0.425}$, where $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$.

SCHEMATIC:



ASSUMPTIONS: (1) Apples can be modeled as spheres, (2) Each apple experiences flow of ventilation air at $T_\infty = 5°C$, (3) One-dimensional radial conduction, (4) Constant properties and (5) Uniform heat generation.

ANALYSIS: (a) From Eq. C.24, the temperature distribution in a solid sphere (apple) with uniform generation is

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (1)$$

To determine T_s, perform an energy balance on the apple as shown in the sketch above, with volume V = $4/3 \pi r_o^3$,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad -q_{cv} + \dot{q}\forall = 0$$

$$-h\left(4\pi r_o^2\right)\left(T_s - T_\infty\right) + \dot{q}\left(4/3\pi r_o^3\right) = 0 \qquad (2)$$

$$-7.5\,W/m^2 \cdot K\left(4\pi \times 0.040^2\,m^2\right)\left(T_s - 5^\circ C\right) + 38.9\,W/m^3\left(4/3\pi \times 0.040^3\,m^3\right) = 0$$

where the volumetric generation rate is

$$\dot{q} = 4000\,J/kg \cdot day$$

$$\dot{q} = 4000\,J/kg \cdot day \times 840\,kg/m^3 \times (1\,day/24\,hr) \times (1\,hr/3600\,s)$$

$$\dot{q} = 38.9\,W/m^3$$

and solving for T_s, find

$$T_s = 5.14^\circ C$$ <

From Eq. (1), at r = 0, with T_s, find

$$T(0) = \frac{38.9\,W/m^3 \times 0.040^2\,m^2}{6 \times 0.5\,W/m \cdot K} + 5.14^\circ C = 0.12^\circ C + 5.14^\circ C = 5.26^\circ C$$ <

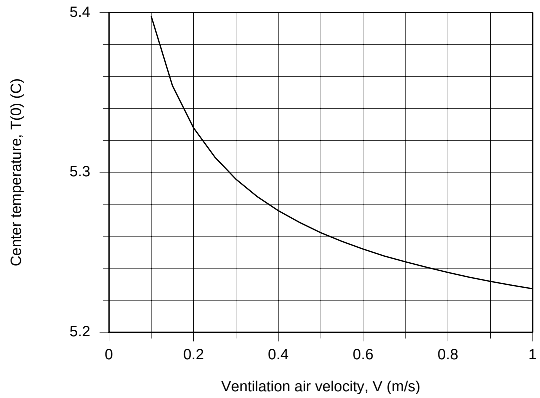
Continued...

PROBLEM 3.96 (Cont.)

(b) With the convection coefficient depending upon velocity,

$$h = C_1 V^{0.425}$$

with C_1 = 10.1 W/m^2·K·(m/s)$^{0.425}$, and using the energy balance of Eq. (2), calculate and plot T_s as a function of ventilation air velocity V. With very low velocities, the center temperature is nearly 0.5°C higher than the air. From our earlier calculation we know that T(0) - T_s = 0.12°C and is independent of V.



COMMENTS: (1) While the temperature within the apple is nearly isothermal, the center temperature will track the ventilation air temperature which will increase as it passes through stacks of cartons.

(2) The IHT Workspace used to determine T_s for the base condition and generate the above plot is shown below.

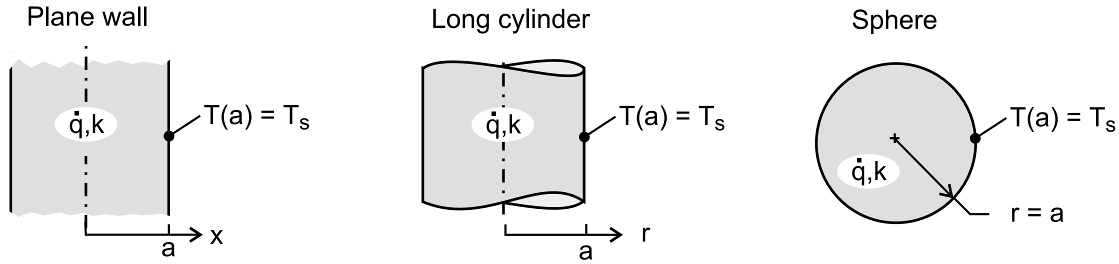
```
// The temperature distribution, Eq (1),
T_r = qdot * ro^2 / (4 * k) * ( 1- r^2/ro^2 )  +  Ts

// Energy balance on the apple, Eq (2)
- qcv + qdot * Vol = 0
Vol = 4 / 3 * pi *  ro ^3

// Convection rate equation:
qcv = h* As * ( Ts - Tinf )
As = 4 * pi * ro^2

// Generation rate:
qdot = qdotm * (1/24) * (1/3600) * rho          // Generation rate, W/m^3; Conversions: days/h and h/sec

// Assigned variables:
ro = 0.080                  // Radius of apple, m
k = 0.5                     // Thermal conductivity, W/m.K
qdotm = 4000                // Generation rate, J/kg.K
rho = 840                   // Specific heat, J/kg.K
r = 0                       // Center, m; location for T(0)
h =  7.5                    // Convection coefficient, W/m^2.K; base case, V = 0.5 m/s
//h = C1 * V^0.425          // Correlation
//C1 = 10.1
//V = 0.5                   // Air velocity, m/s; range 0.1 to 1 m/s
Tinf = 5                    // Air temperature, C
```

PROBLEM 3.97

KNOWN: Plane wall, long cylinder and sphere, each with characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.

FIND: (a) On the same graph, plot the dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[\dot{q}a^2/2k]$, *vs.* the dimensionless characteristic length, x/a or r/a, for each shape; (b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area; and (c) Which shape would be preferred for use as a nuclear fuel element? Explain why?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties and (4) Uniform volumetric generation.

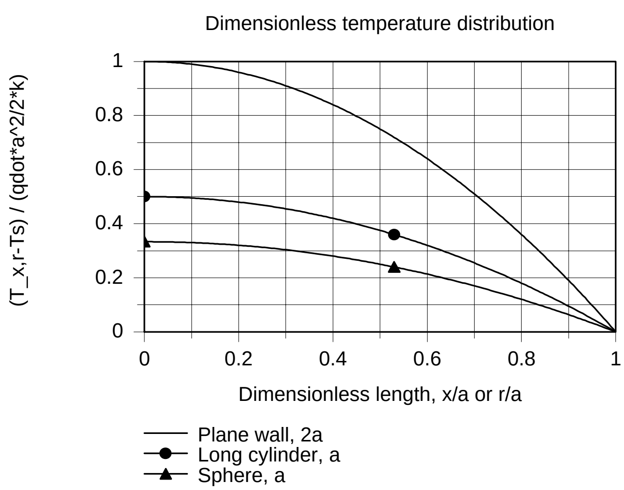
ANALYSIS: (a) For each of the shapes, with T(a) = T_s, the dimensionless temperature distributions can be written by inspection from results in Appendix C.3.

Plane wall, Eq. C.22
$$\frac{T(x)-T_s}{\dot{q}a^2/2k} = 1-\left(\frac{x}{a}\right)^2$$

Long cylinder, Eq. C.23
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1-\left(\frac{r}{a}\right)^2\right]$$

Sphere, Eq. C.24
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1-\left(\frac{r}{a}\right)^2\right]$$

The dimensionless temperature distributions using the foregoing expressions are shown in the graph below.



Continued …..

(b) The sphere shape has the smallest temperature difference between the center and surface, T(0) – T(a). The ratio of volume-to-surface-area, $\forall/A_s$, for each of the shapes is

Plane wall $$\frac{\forall}{A_s} = \frac{a(1\times1)}{(1\times1)} = a$$

Long cylinder $$\frac{\forall}{A_s} = \frac{\pi a^2 \times 1}{2\pi a \times 1} = \frac{a}{2}$$

Sphere $$\frac{\forall}{A_s} = \frac{4\pi a^3/3}{4\pi a^2} = \frac{a}{3}$$

The smaller the $\forall/A_s$ ratio, the smaller the temperature difference, T(0) – T(a).

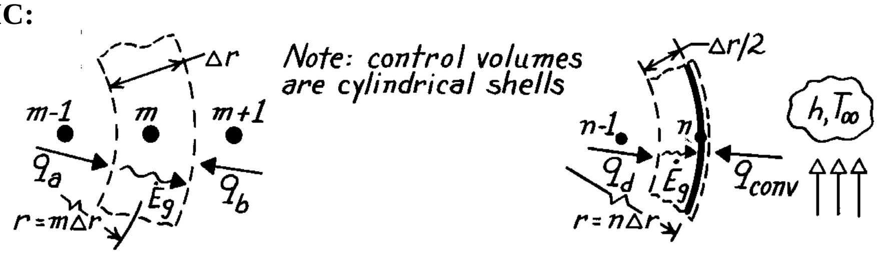
(c) The sphere would be the preferred element shape since, for a given $\forall/A_s$ ratio, which controls the generation and transfer rates, the sphere will operate at the lowest temperature.

PROBLEM 4.38

KNOWN: Conduction in a one-dimensional (radial) *cylindrical* coordinate system with volumetric generation.

FIND: Finite-difference equation for (a) Interior node, m, and (b) Surface node, n, with convection.

SCHEMATIC:



(a) Interior node, m (b) Surface node with convection, n

ASSUMPTIONS: (1) Steady-state, one-dimensional (radial) conduction in *cylindrical* coordinates, (2) Constant properties.

ANALYSIS: (a) The network has nodes spaced at equal Δr increments with m = 0 at the center; hence, r = mΔr (or nΔr). The control volume is $V = 2\pi\ r \cdot \Delta r \cdot \ell = 2\pi\ (m\Delta r)\,\Delta r \cdot \ell$. The energy balance is $\dot E_{in} + \dot E_g = q_a + q_b + \dot q V = 0$

$$k\left[2\pi\left[r-\frac{\Delta r}{2}\right]\ell\right]\frac{T_{m-1}-T_m}{\Delta r} + k\left[2\pi\left[r+\frac{\Delta r}{2}\right]\ell\right]\frac{T_{m+1}-T_m}{\Delta r} + \dot q\left[2\pi\,(m\Delta r)\,\Delta r\ell\right] = 0.$$

Recognizing that r = mΔr, canceling like terms, and regrouping find

$$\left[m-\frac{1}{2}\right]T_{m-1} + \left[m+\frac{1}{2}\right]T_{m+1} - 2mT_m + \frac{\dot q m\Delta r^2}{k} = 0.$$ <

(b) The control volume for the surface node is $V = 2\pi\ r\cdot(\Delta r/2)\cdot\ell$. The energy balance is $\dot E_{in} + \dot E_g = q_d + q_{conv} + \dot q V = 0$. Use Fourier's law to express q_d and Newton's law of cooling for q_{conv} to obtain

$$k\left[2\pi\left[r-\frac{\Delta r}{2}\right]\ell\right]\frac{T_{n-1}-T_n}{\Delta r} + h\left[2\pi\ r\ell\right](T_\infty - T_n) + \dot q\left[2\pi\,(n\Delta r)\frac{\Delta r}{2}\ell\right] = 0.$$

Let r = nΔr, cancel like terms and regroup to find

$$\left[n-\frac{1}{2}\right]T_{n-1} - \left[\left[n-\frac{1}{2}\right] + \frac{hn\Delta r}{k}\right]T_n + \frac{\dot q n\Delta r^2}{2k} + \frac{hn\Delta r}{k}T_\infty = 0.$$

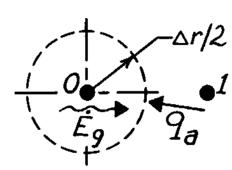
<

COMMENTS: (1) Note that when m or n becomes very large compared to ½, the finite-difference equation becomes independent of m or n. Then the cylindrical system approximates a rectangular one.

(2) The finite-difference equation for the center node (m = 0) needs to be treated as a special case. The control volume is

$V = \pi\,(\Delta r/2)^2\,\ell$ and the energy balance is

$$\dot E_{in} + \dot E_g = q_a + \dot q V = k\left[2\pi\left[\frac{\Delta r}{2}\right]\ell\right]\frac{T_1 - T_0}{\Delta r} + \dot q\left[\pi\left[\frac{\Delta r}{2}\right]^2\ell\right] = 0.$$



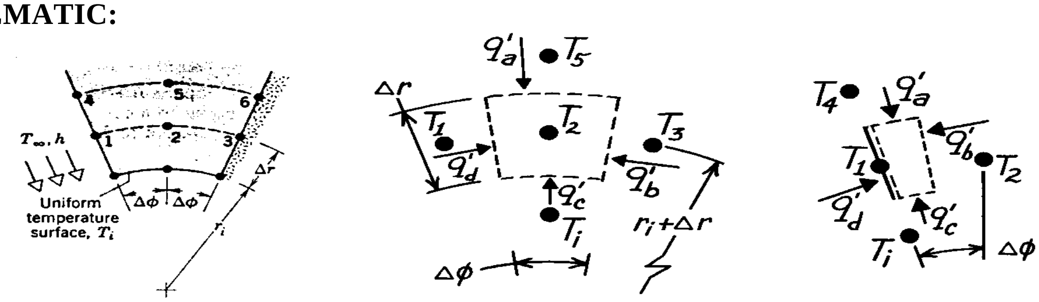
Regrouping, the finite-difference equation is $-T_o + T_1 + \frac{\dot q\Delta r^2}{4k} = 0.$

PROBLEM 4.39

KNOWN: Two-dimensional cylindrical configuration with prescribed radial (Δr) and angular ($\Delta\phi$) spacings of nodes.

FIND: Finite-difference equations for nodes 2, 3 and 1.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction in cylindrical coordinates (r,ϕ), (3) Constant properties.

ANALYSIS: The method of solution is to define the appropriate control volume for each node, to identify relevant processes and then to perform an energy balance.

(a) Node 2. This is an *interior* node with control volume as shown above. The energy balance is $\dot{E}_{in} = q'_a + q'_b + q'_c + q'_d = 0.$ Using Fourier's law for each process, find

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\Delta f\right]\frac{(T_5 - T_2)}{\Delta r} + k(\Delta r)\frac{(T_3 - T_2)}{(r_i + \Delta r)\Delta f} +$$

$$+ k\left[\left[r_i + \frac{1}{2}\Delta r\right]\Delta f\right]\frac{(T_i - T_2)}{\Delta r} + k(\Delta r)\frac{(T_1 - T_2)}{(r_i + \Delta r)\Delta f} = 0.$$

Canceling terms and regrouping yields,

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta f)^2}\frac{1}{(r_i + \Delta r)}\right]T_2 + \left[r_i + \frac{3}{2}\Delta r\right]T_5 + \frac{(\Delta r)^2}{(r_i + \Delta r)(\Delta f)^2}(T_3 + T_1) + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(b) Node 3. The adiabatic surface behaves as a symmetry surface. We can utilize the result of Part (a) to write the finite-difference equation by inspection as

$$-2\left[(r_i + \Delta r) + \frac{(\Delta r)^2}{(\Delta f)^2}\frac{1}{(r_i + \Delta r)}\right]T_3 + \left[r_i + \frac{3}{2}\Delta r\right]T_6 + \frac{2(\Delta r)^2}{(r_i + \Delta r)(\Delta f)^2}\cdot T_2 + \left[r_i + \frac{1}{2}\Delta r\right]T_i = 0.$$

(c) Node 1. The energy balance is $q'_a + q'_b + q'_c + q'_d = 0.$ Substituting,

$$k\left[\left[r_i + \frac{3}{2}\Delta r\right]\frac{\Delta f}{2}\right]\frac{(T_4 - T_1)}{\Delta r} + k(\Delta r)\frac{(T_2 - T_1)}{(r_i + \Delta r)\Delta f} +$$

$$+ k\left[\left[r_i + \frac{1}{2}\Delta r\right]\frac{\Delta f}{2}\right]\frac{(T_i - T_1)}{\Delta r} + h(\Delta r)(T_\infty - T_1) = 0$$ <

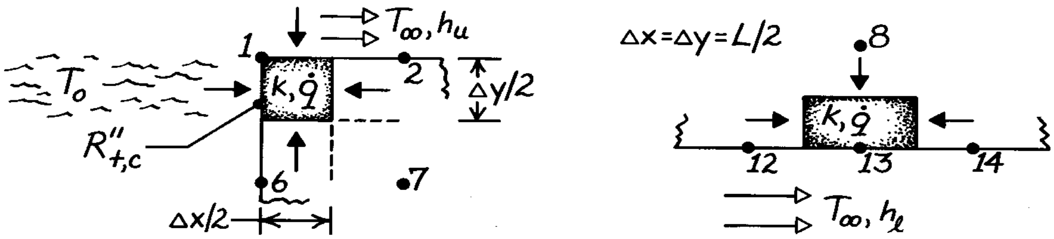
This expression could now be rearranged.

PROBLEM 4.40

KNOWN: Heat generation and thermal boundary conditions of bus bar. Finite-difference grid.

FIND: Finite-difference equations for selected nodes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) Performing an energy balance on the control volume, $(\Delta x/2)(\Delta y/2)\cdot 1$, find the FDE for node 1,

$$\frac{T_o - T_1}{R''_{t,c}/(\Delta y/2)1} + h_u\left(\frac{\Delta x}{2}\cdot 1\right)(T_\infty - T_1) + \frac{k(\Delta y/2\cdot 1)}{\Delta x}(T_2 - T_1)$$
$$+\frac{k(\Delta x/2\cdot 1)}{\Delta y}(T_6 - T_1) + \dot{q}\left[(\Delta x/2)(\Delta y/2)1\right] = 0$$

$$\left(\Delta x/kR''_{t,c}\right)T_o + \left(h_u \Delta x/k\right)T_\infty + T_2 + T_6$$
$$+\dot{q}(\Delta x)^2/2k - \left[\left(\Delta x/kR''_{t,c}\right) + \left(h_u \Delta x/k\right) + 2\right]T_1 = 0. \qquad <$$

(b) Performing an energy balance on the control volume, $(\Delta x)(\Delta y/2)\cdot 1$, find the FDE for node 13,

$$h_l(\Delta x\cdot 1)(T_\infty - T_{13}) + (k/\Delta x)(\Delta y/2\cdot 1)(T_{12} - T_{13})$$
$$+(k/\Delta y)(\Delta x\cdot 1)(T_8 - T_{13}) + (k/\Delta x)(\Delta y/2\cdot 1)(T_{14} - T_{13}) + \dot{q}(\Delta x\cdot \Delta y/2\cdot 1) = 0$$

$$(h_l \Delta x/k)T_\infty + 1/2(T_{12} + 2T_8 + T_{14}) + \dot{q}(\Delta x)^2/2k - (h_l \Delta x/k + 2)T_{13} = 0. \qquad <$$

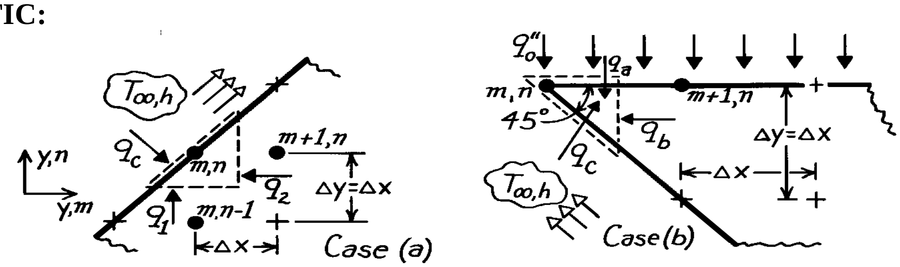
COMMENTS: For fixed T_o and T_∞, the relative amounts of heat transfer to the air and heat sink are determined by the values of h and $R''_{t,c}$.

PROBLEM 4.41

KNOWN: Nodal point configurations corresponding to a diagonal surface boundary subjected to a convection process and to the tip of a machine tool subjected to constant heat flux and convection cooling.

FIND: Finite-difference equations for the node m,n in the two situations shown.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The control volume about node m,n has triangular shape with sides Δx and Δy while the diagonal (surface) length is $\sqrt{2}\ \Delta x$. The heat rates associated with the control volume are due to conduction, q_1 and q_2, and to convection, q_c. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$k\left(\Delta x \cdot 1\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + k\left(\Delta y \cdot 1\right)\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h\left(\sqrt{2}\ \Delta x \cdot 1\right)\left(T_\infty - T_{m,n}\right) = 0.$$

Note that we have considered the tool to have unit depth normal to the page. Recognizing that Δx = Δy, dividing each term by k and regrouping, find

$$T_{m,n-1} + T_{m+1,n} + \sqrt{2}\ \cdot\frac{h\Delta x}{k}T_\infty - \left[2 + \sqrt{2}\ \cdot\frac{h\Delta x}{k}\right]T_{m,n} = 0. \qquad <$$

(b) The control volume about node m,n has triangular shape with sides Δx/2 and Δy/2 while the lower diagonal surface length is $\sqrt{2}\,(\Delta x/2)$. The heat rates associated with the control volume are due to the constant heat flux, q_a, to conduction, q_b, and to the convection process, q_c. Perform an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_a + q_b + q_c = 0$$

$$q_o'' \cdot\left[\frac{\Delta x}{2}\cdot 1\right] + k\cdot\left[\frac{\Delta y}{2}\cdot 1\right]\frac{T_{m+1,n} - T_{m,n}}{\Delta x} + h\cdot\left[\sqrt{2}\cdot\frac{\Delta x}{2}\right]\left(T_\infty - T_{m,n}\right) = 0.$$

Recognizing that Δx = Δy, dividing each term by k/2 and regrouping, find

$$T_{m+1,n} + \sqrt{2}\cdot\frac{h\Delta x}{k}\cdot T_\infty + q_o''\cdot\frac{\Delta x}{k} - \left(1 + \sqrt{2}\cdot\frac{h\Delta x}{k}\right)T_{m,n} = 0. \qquad <$$

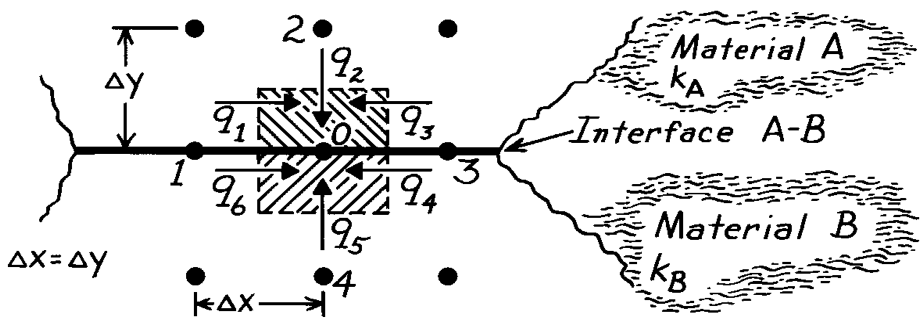
COMMENTS: Note the appearance of the term hΔx/k in both results, which is a dimensionless parameter (the *Biot number*) characterizing the relative effects of convection and conduction.

PROBLEM 4.42

KNOWN: Nodal point on boundary between two materials.

FIND: Finite-difference equation for steady-state conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional conduction, (3) Constant properties, (4) No internal heat generation, (5) Negligible thermal contact resistance at interface.

ANALYSIS: The control volume is defined about nodal point 0 as shown above. The conservation of energy requirement has the form

$$\sum_{i=1}^{6} q_i = q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 0$$

since all heat rates are shown as *into* the CV. Each heat rate can be written using Fourier's law,

$$k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} + k_A \cdot \Delta x \cdot \frac{T_2 - T_0}{\Delta y} + k_A \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x}$$
$$+ k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_3 - T_0}{\Delta x} + k_B \cdot \Delta x \cdot \frac{T_4 - T_0}{\Delta y} + k_B \cdot \frac{\Delta y}{2} \cdot \frac{T_1 - T_0}{\Delta x} = 0.$$

Recognizing that $\Delta x = \Delta y$ and regrouping gives the relation,

$$-T_0 + \frac{1}{4} T_1 + \frac{k_A}{2(k_A + k_B)} T_2 + \frac{1}{4} T_3 + \frac{k_B}{2(k_A + k_B)} T_4 = 0.$$ <

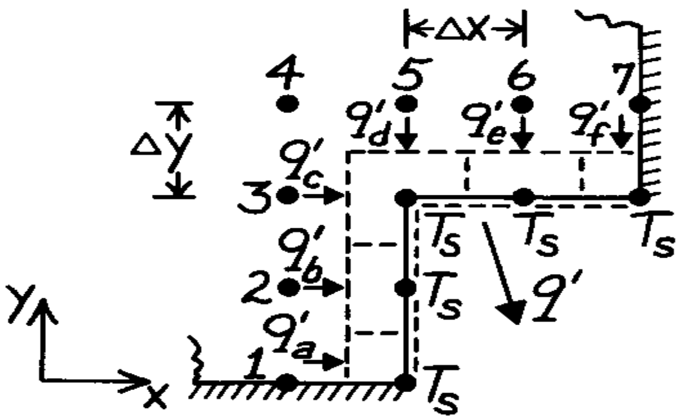
COMMENTS: Note that when $k_A = k_B$, the result agrees with Eq. 4.33 which is appropriate for an interior node in a medium of fixed thermal conductivity.

PROBLEM 4.43

KNOWN: Two-dimensional grid for a system with no internal volumetric generation.

FIND: Expression for heat rate per unit length normal to page crossing the isothermal boundary.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) Constant properties.

ANALYSIS: Identify the surface nodes (T_s) and draw control volumes about these nodes. Since there is no heat transfer in the direction parallel to the isothermal surfaces, the heat rate out of the constant temperature surface boundary is

$$q' = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$$

For each q'_i, use Fourier's law and pay particular attention to the manner in which the cross-sectional area and gradients are specified.

$$q' = k(\Delta y/2)\frac{T_1 - T_s}{\Delta x} + k(\Delta y)\frac{T_2 - T_s}{\Delta x} + k(\Delta y)\frac{T_3 - T_s}{\Delta x}$$
$$+ k(\Delta x)\frac{T_5 - T_s}{\Delta y} + k(\Delta x)\frac{T_6 - T_s}{\Delta y} + k(\Delta x/2)\frac{T_7 - T_s}{\Delta y}$$

Regrouping with $\Delta x = \Delta y$, find

$$q' = k\left[0.5T_1 + T_2 + T_3 + T_5 + T_6 + 0.5T_7 - 5T_s\right].$$ <

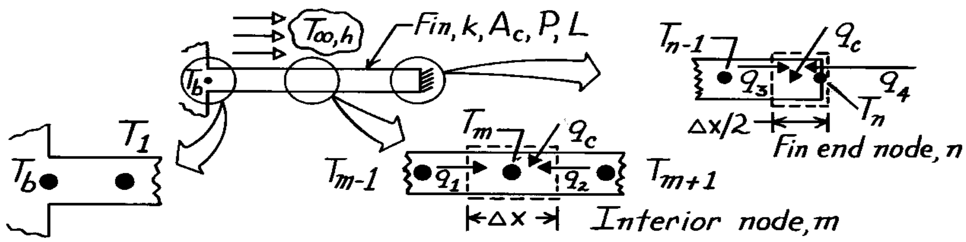
COMMENTS: Looking at the corner node, it is important to recognize the areas associated with q'_c and q'_d (Δy and Δx, respectively).

PROBLEM 4.44

KNOWN: One-dimensional fin of uniform cross section insulated at one end with prescribed base temperature, convection process on surface, and thermal conductivity.

FIND: Finite-difference equation for these nodes: (a) Interior node, m and (b) Node at end of fin, n, where x = L.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction.

ANALYSIS: (a) The control volume about node m is shown in the schematic; the node spacing and control volume length in the x direction are both Δx. The uniform cross-sectional area and fin perimeter are A_c and P, respectively. The heat transfer process on the control surfaces, q_1 and q_2, represent conduction while q_c is the convection heat transfer rate between the fin and ambient fluid. Performing an energy balance, find

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_1 + q_2 + q_c = 0$$

$$kA_c \frac{T_{m-1} - T_m}{\Delta x} + kA_c \frac{T_{m+1} - T_m}{\Delta x} + hP\Delta x\left(T_\infty - T_m\right) = 0.$$

Multiply the expression by $\Delta x/kA_c$ and regroup to obtain

$$T_{m-1} + T_{m+1} + \frac{hP}{kA_c} \cdot \Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c}\Delta x^2\right] T_m = 0 \qquad 1<m<n$$ <

Considering now the special node m = 1, then the m-1 node is T_b, the base temperature. The finite-difference equation would be

$$T_b + T_2 + \frac{hP}{kA_c}\Delta x^2 T_\infty - \left[2 + \frac{hP}{kA_c}\Delta x^2\right] T_1 = 0 \qquad m=1$$ <

(b) The control volume of length Δx/2 about node n is shown in the schematic. Performing an energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_3 + q_4 + q_c = 0$$

$$kA_c \frac{T_{n-1} - T_n}{\Delta x} + 0 + hP\frac{\Delta x}{2}\left(T_\infty - T_n\right) = 0.$$

Note that $q_4 = 0$ since the end (x = L) is insulated. Multiplying by $\Delta x/kA_c$ and regrouping,

$$T_{n-1} + \frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} T_\infty - \left[\frac{hP}{kA_c} \cdot \frac{\Delta x^2}{2} + 1\right] T_n = 0.$$ <

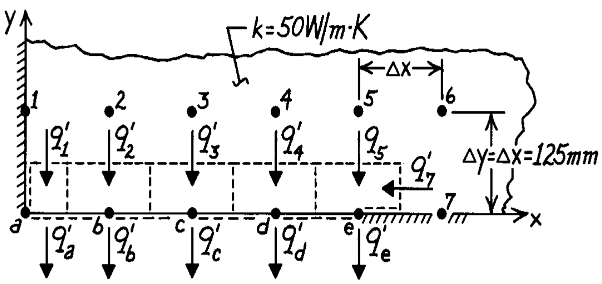
COMMENTS: The value of Δx will be determined by the selection of n; that is, Δx = L/n. Note that the grouping, hP/kA_c, appears in the finite-difference and differential forms of the energy balance.

PROBLEM 4.45

KNOWN: Two-dimensional network with prescribed nodal temperatures and thermal conductivity of the material.

FIND: Heat rate per unit length normal to page, q'.

SCHEMATIC:



Node	T_i(°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

ASSUMPTIONS: (1) Steady-state conditions, (2) Two-dimensional heat transfer, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: Construct control volumes around the nodes on the surface maintained at the uniform temperature T_s and indicate the heat rates. The heat rate per unit length is $q' = q'_a + q'_b + q'_c + q'_d + q'_e$ or in terms of conduction terms between nodes,

$$q' = q'_1 + q'_2 + q'_3 + q'_4 + q'_5 + q'_7.$$

Each of these rates can be written in terms of nodal temperatures and control volume dimensions using Fourier's law,

$$q' = k\cdot\frac{\Delta x}{2}\cdot\frac{T_1 - T_s}{\Delta y} + k\cdot\Delta x\cdot\frac{T_2 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_3 - T_s}{\Delta y} + k\cdot\Delta x\frac{T_4 - T_s}{\Delta y}$$
$$+ k\cdot\Delta x\frac{T_5 - T_s}{\Delta y} + k\cdot\frac{\Delta y}{2}\cdot\frac{T_7 - T_s}{\Delta x}.$$

and since $\Delta x = \Delta y$,

$$q' = k[(1/2)(T_1 - T_s) + (T_2 - T_s) + (T_3 - T_s)$$
$$+ (T_4 - T_s) + (T_5 - T_s) + (1/2)(T_7 - T_s)].$$

Substituting numerical values, find

$$q' = 50\ \text{W/m}\cdot\text{K}[(1/2)(120.55 - 100) + (120.64 - 100) + (121.29 - 100)$$
$$+ (123.89 - 100) + (134.57 - 100) + (1/2)(147.14 - 100)]$$

$$q' = 6711\ \text{W/m}.$$ <

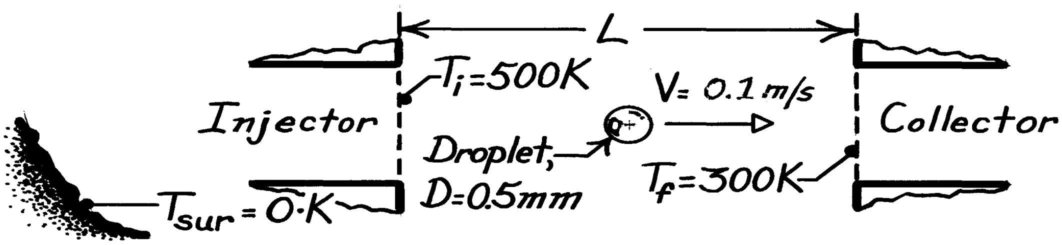
COMMENTS: For nodes a through d, there is no heat transfer into the control volumes in the x-direction. Look carefully at the energy balance for node e, $q'_e = q'_5 + q'_7$, and how q'_5 and q'_7 are evaluated.

PROBLEM 5.22

KNOWN: Droplet properties, diameter, velocity and initial and final temperatures.

FIND: Travel distance and rejected thermal energy.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible radiation from space.

PROPERTIES: Droplet (given): ρ = 885 kg/m^3, c = 1900 J/kg·K, k = 0.145 W/m·K, ε = 0.95.

ANALYSIS: To assess the suitability of applying the lumped capacitance method, use Equation 1.9 to obtain the maximum radiation coefficient, which corresponds to T = T_i.

$$h_r = \varepsilon\sigma T_i^3 = 0.95\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4 (500\text{ K})^3 = 6.73\text{ W/m}^2\cdot\text{K}.$$

Hence

$$Bi_r = \frac{h_r(r_o/3)}{k} = \frac{\left(6.73\text{ W/m}^2\cdot\text{K}\right)\left(0.25\times 10^{-3}\text{ m/3}\right)}{0.145\text{ W/m}\cdot\text{K}} = 0.0039$$

and the lumped capacitance method can be used. From Equation 5.19,

$$t = \frac{L}{V} = \frac{\rho c\left(\pi D^3/6\right)}{3\varepsilon\left(\pi D^2\right)\sigma}\left(\frac{1}{T_f^3} - \frac{1}{T_i^3}\right)$$

$$L = \frac{(0.1\text{ m/s})\,885\text{ kg/m}^3(1900\text{ J/kg}\cdot\text{K})\,0.5\times 10^{-3}\text{ m}}{18\times 0.95\times 5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4}\left(\frac{1}{300^3} - \frac{1}{500^3}\right)\frac{1}{\text{K}^3}$$

$$L = 2.52\text{ m.}$$ <

The amount of energy rejected by each droplet is equal to the change in its internal energy.

$$E_i - E_f = \rho Vc(T_i - T_f) = 885\text{ kg/m}^3\pi\frac{\left(5\times 10^{-4}\text{m}\right)^3}{6}1900\text{ J/kg}\cdot\text{K}(200\text{ K})$$

$$E_i - E_f = 0.022\text{ J.}$$ <

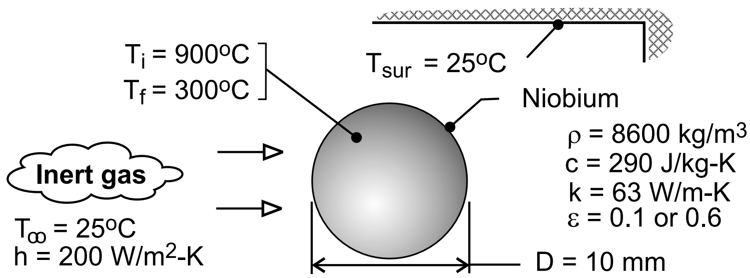
COMMENTS: Because some of the radiation emitted by a droplet will be intercepted by other droplets in the stream, the foregoing analysis overestimates the amount of heat dissipated by radiation to space.

PROBLEM 5.23

KNOWN: Initial and final temperatures of a niobium sphere. Diameter and properties of the sphere. Temperature of surroundings and/or gas flow, and convection coefficient associated with the flow.

FIND: (a) Time required to cool the sphere exclusively by radiation, (b) Time required to cool the sphere exclusively by convection, (c) Combined effects of radiation and convection.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform temperature at any time, (2) Negligible effect of holding mechanism on heat transfer, (3) Constant properties, (4) Radiation exchange is between a small surface and large surroundings.

ANALYSIS: (a) If cooling is exclusively by radiation, the required time is determined from Eq. (5.18). With $V = \pi D^3/6$, $A_{s,r} = \pi D^2$, and $\varepsilon = 0.1$,

$$t = \frac{8600\ kg/m^3\left(290\,J/kg\cdot K\right)0.01m}{24\left(0.1\right)5.67\times10^{-8}W/m^2\cdot K^4\left(298K\right)^3}\left\{\ln\left|\frac{298+573}{298-573}\right| - \ln\left|\frac{298+1173}{298-1173}\right|\right.$$

$$\left. +2\left[\tan^{-1}\left(\frac{573}{298}\right) - \tan^{-1}\left(\frac{1173}{298}\right)\right]\right\}$$

$$t = 6926s\left\{1.153 - 0.519 + 2\left(1.091 - 1.322\right)\right\} = 1190s \qquad \left(\varepsilon = 0.1\right)$$ <

If $\varepsilon = 0.6$, cooling is six times faster, in which case,

$$t = 199s \qquad \left(\varepsilon = 0.6\right)$$ <

(b) If cooling is exclusively by convection, Eq. (5.5) yields

$$t = \frac{\rho c D}{6h}\ln\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right) = \frac{8600\ kg/m^3\left(290\,J/kg\cdot K\right)0.010m}{1200\,W/m^2\cdot K}\ln\left(\frac{875}{275}\right)$$

$$t = 24.1s$$ <

(c) With both radiation and convection, the temperature history may be obtained from Eq. (5.15).

$$\rho\left(\pi D^3/6\right)c\frac{dT}{dt} = -\pi D^2\left[h\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Integrating numerically from $T_i = 1173$ K at t = 0 to T = 573K, we obtain

$$t = 21.0s$$ <

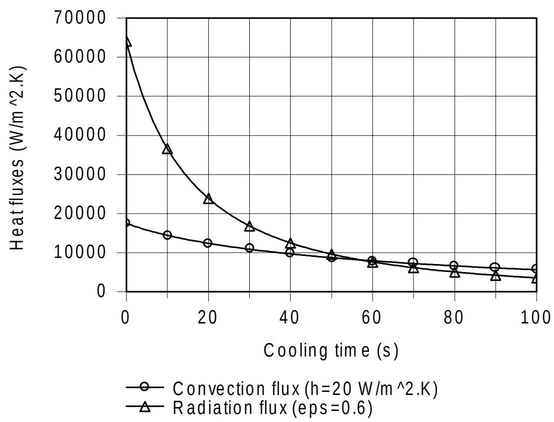
Continued …..

PROBLEM 5.23 (Cont.)

Cooling times corresponding to representative changes in ε and h are tabulated as follows

$h(W/m^2 \cdot K)$	200	200	20	500
ε	0.6	1.0	0.6	0.6
t(s)	21.0	19.4	102.8	9.1

For values of h representative of forced convection, the influence of radiation is secondary, even for a maximum possible emissivity of 1.0. Hence, to accelerate cooling, it is necessary to increase h. However, if cooling is by natural convection, radiation is significant. For a representative natural convection coefficient of $h = 20\ W/m^2 \cdot K$, the radiation flux exceeds the convection flux at the surface of the sphere during early to intermediate stages of the transient.



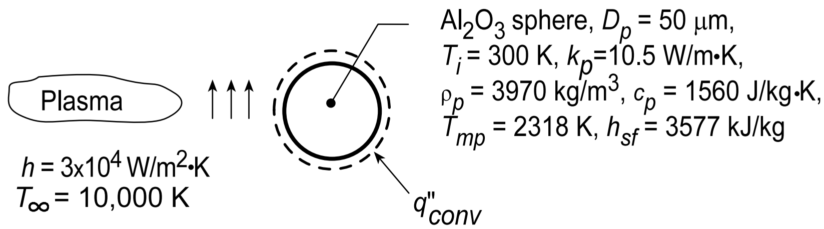
COMMENTS: (1) Even for h as large as $500\ W/m^2 \cdot K$, $Bi = h\,(D/6)/k = 500\ W/m^2 \cdot K\ (0.01m/6)/63\ W/m \cdot K = 0.013 < 0.1$ and the lumped capacitance model is appropriate. (2) The largest value of h_r corresponds to $T_i = 1173$ K, and for $\varepsilon = 0.6$ Eq. (1.9) yields $h_f = 0.6 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (1173 + 298)K\ (1173^2 + 298^2)K^2 = 73.3\ W/m^2 \cdot K$.

PROBLEM 5.24

KNOWN: Diameter and thermophysical properties of alumina particles. Convection conditions associated with a two-step heating process.

FIND: (a) Time-in-flight (t_{i-f}) required for complete melting, (b) Validity of assuming negligible radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Particle behaves as a lumped capacitance, (2) Negligible radiation, (3) Constant properties.

ANALYSIS: (a) The two-step process involves (i) the time t_1 to heat the particle to its melting point and (ii) the time t_2 required to achieve complete melting. Hence, $t_{i-f} = t_1 + t_2$, where from Eq. (5.5),

$$t_1 = \frac{\rho_p V c_p}{h A_s} \ln \frac{\theta_i}{\theta} = \frac{\rho_p D_p c_p}{6h} \ln \frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

$$t_1 = \frac{3970\,\text{kg/m}^3 \left(50 \times 10^{-6}\,\text{m}\right) 1560\,\text{J/kg}\cdot\text{K}}{6\left(30,000\,\text{W/m}^2\cdot\text{K}\right)} \ln \frac{(300 - 10,000)}{(2318 - 10,000)} = 4 \times 10^{-4}\,\text{s}$$

Performing an energy balance for the second step, we obtain

$$\int_{t_1}^{t_1+t_2} q_{conv}\, dt = \Delta E_{st}$$

where $q_{conv} = hA_s(T_\infty - T_{mp})$ and $\Delta E_{st} = \rho_p V h_{sf}$. Hence,

$$t_2 = \frac{\rho_p D_p}{6h} \frac{h_{sf}}{\left(T_\infty - T_{mp}\right)} = \frac{3970\,\text{kg/m}^3 \left(50 \times 10^{-6}\,\text{m}\right)}{6\left(30,000\,\text{W/m}^2\cdot\text{K}\right)} \times \frac{3.577 \times 10^6\,\text{J/kg}}{(10,000 - 2318)\,\text{K}} = 5 \times 10^{-4}\,\text{s}$$

Hence $t_{i-f} = 9 \times 10^{-4}\,\text{s} \approx 1\,\text{ms}$ <

(b) Contrasting the smallest value of the convection heat flux, $q''_{conv,min} = h\left(T_\infty - T_{mp}\right) = 2.3 \times 10^8\ \text{W/m}^2$ to the largest radiation flux, $q''_{rad,max} = \varepsilon\sigma\left(T_{mp}^4 - T_{sur}^4\right) = 6.5 \times 10^5\ \text{W/m}^2$, we conclude that radiation is, in fact, negligible.

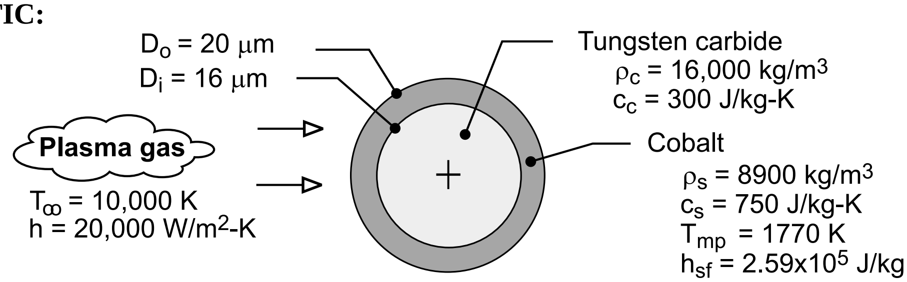
COMMENTS: (1) Since $Bi = (hr_p/3)/k \approx 0.05$, the lumped capacitance assumption is good. (2) In an actual application, the droplet should impact the substrate in a superheated condition ($T > T_{mp}$), which would require a slightly larger t_{i-f}.

PROBLEM 5.25

KNOWN: Diameters, initial temperature and thermophysical properties of WC and Co in composite particle. Convection coefficient and freestream temperature of plasma gas. Melting point and latent heat of fusion of Co.

FIND: Times required to reach melting and to achieve complete melting of Co.

SCHEMATIC:



ASSUMPTIONS: (1) Particle is isothermal at any instant, (2) Radiation exchange with surroundings is negligible, (3) Negligible contact resistance at interface between WC and Co, (4) Constant properties.

ANALYSIS: From Eq. (5.5), the time required to reach the melting point is

$$t_1 = \frac{(\rho Vc)_{tot}}{h\pi D_o^2}\ln\frac{T_i - T_\infty}{T_{mp} - T_\infty}$$

where the total heat capacity of the composite particle is

$$(\rho Vc)_{tot} = (\rho Vc)_c + (\rho Vc)_s = 16,000\,\text{kg/m}^3\left[\pi\left(1.6\times10^{-5}\,\text{m}\right)^3/6\right]300\,\text{J/kg}\cdot\text{K}$$

$$+8900\,\text{kg/m}^3\left\{\pi/6\left[\left(2.0\times10^{-5}\,\text{m}\right)^3 - \left(1.6\times10^{-5}\,\text{m}\right)^3\right]\right\}750\,\text{J/kg}\cdot\text{K}$$

$$= \left(1.03\times10^{-8} + 1.36\times10^{-8}\right)\text{J/K} = 2.39\times10^{-8}\,\text{J/K}$$

$$t_1 = \frac{2.39\times10^{-8}\,\text{J/K}}{\left(20,000\,\text{W/m}^2\cdot\text{K}\right)\pi\left(2.0\times10^{-5}\,\text{m}\right)^2}\ln\frac{(300-10,000)\,\text{K}}{(1770-10,000)\,\text{K}} = 1.56\times10^{-4}\,\text{s} \qquad <$$

The time required to melt the Co may be obtained by applying the first law, Eq. (1.11b) to a control surface about the particle. It follows that

$$E_{in} = h\pi D_o^2\left(T_\infty - T_{mp}\right)t_2 = \Delta E_{st} = \rho_s(\pi/6)\left(D_o^3 - D_i^3\right)h_{sf}$$

$$t_2 = \frac{8900\,\text{kg/m}^3(\pi/6)\left[\left(2\times10^{-5}\,\text{m}\right)^3 - \left(1.6\times10^{-5}\,\text{m}\right)^3\right]2.59\times10^5\,\text{J/kg}}{\left(20,000\,\text{W/m}^2\cdot\text{K}\right)\pi\left(2\times10^{-5}\,\text{m}\right)^2(10,000-1770)\,\text{K}} = 2.28\times10^{-5}\,\text{s} \qquad <$$

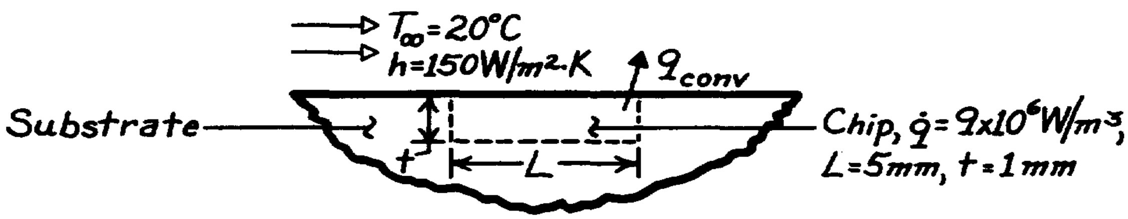
COMMENTS: (1) The largest value of the radiation coefficient corresponds to $h_r = \varepsilon\sigma\left(T_{mp} + T_{sur}\right)\left(T_{mp}^2 + T_{sur}^2\right)$. For the maximum possible value of $\varepsilon = 1$ and $T_{sur} = 300\text{K}$, $h_r = 378\ \text{W/m}^2\cdot\text{K} \ll h = 20,000\ \text{W/m}^2\cdot\text{K}$. Hence, the assumption of negligible radiation exchange is excellent. (2) Despite the large value of h, the small values of D_o and D_i and the large thermal conductivities (~ 40 W/m·K and 70 W/m·K for WC and Co, respectively) render the lumped capacitance approximation a good one. (3) A detailed treatment of plasma heating of a composite powder particle is provided by Demetriou, Lavine and Ghoniem (Proc. 5th ASME/JSME Joint Thermal Engineering Conf., March, 1999).

PROBLEM 5.26

KNOWN: Dimensions and operating conditions of an integrated circuit.

FIND: Steady-state temperature and time to come within 1°C of steady-state.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible heat transfer from chip to substrate.

PROPERTIES: Chip material (given): $\rho = 2000$ kg/m^3, c = 700 J/kg·K.

ANALYSIS: At steady-state, conservation of energy yields

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left(L^2\right)\left(T_f - T_\infty\right) + \dot{q}\left(L^2 \cdot t\right) = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{h}$$

$$T_f = 20°C + \frac{9\times10^6 \text{ W/m}^3 \times 0.001 \text{ m}}{150 \text{ W/m}^2 \cdot \text{K}} = 80°C. \qquad <$$

From the general lumped capacitance analysis, Equation 5.15 reduces to

$$\rho\left(L^2 \cdot t\right)c\frac{dT}{dt} = \dot{q}\left(L^2 \cdot t\right) - h\left(T - T_\infty\right)L^2.$$

With

$$a \equiv \frac{h}{\rho tc} = \frac{150 \text{ W/m}^2 \cdot \text{K}}{\left(2000 \text{ kg/m}^3\right)\left(0.001 \text{ m}\right)\left(700 \text{ J/kg} \cdot \text{K}\right)} = 0.107 \text{ s}^{-1}$$

$$b \equiv \frac{\dot{q}}{\rho c} = \frac{9\times10^6 \text{ W/m}^3}{\left(2000 \text{ kg/m}^3\right)\left(700 \text{ J/kg} \cdot \text{K}\right)} = 6.429 \text{ K/s}.$$

From Equation 5.24,

$$\exp\left(-at\right) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{\left(79 - 20 - 60\right)\text{K}}{\left(20 - 20 - 60\right)\text{K}} = 0.01667$$

$$t = -\frac{\ln\left(0.01667\right)}{0.107 \text{ s}^{-1}} = 38.3 \text{ s}. \qquad <$$

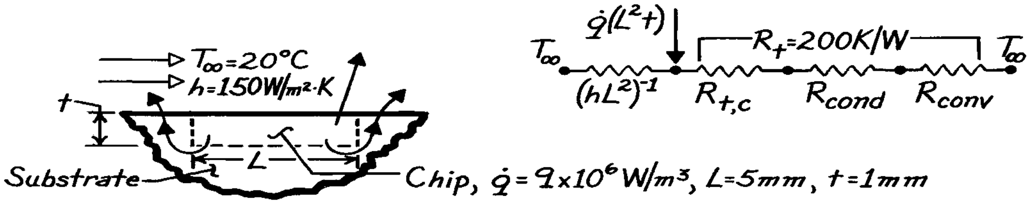
COMMENTS: Due to additional heat transfer from the chip to the substrate, the actual values of T_f and t are less than those which have been computed.

PROBLEM 5.27

KNOWN: Dimensions and operating conditions of an integrated circuit.

FIND: Steady-state temperature and time to come within 1°C of steady-state.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties.

PROPERTIES: Chip material (given): $\rho = 2000\ kg/m^3$, $c_p = 700\ J/kg\cdot K$.

ANALYSIS: The direct and indirect paths for heat transfer from the chip to the coolant are in parallel, and the equivalent resistance is

$$R_{equiv} = \left[hL^2 + R_t^{-1}\right]^{-1} = \left[\left(3.75\times10^{-3} + 5\times10^{-3}\right)W/K\right]^{-1} = 114.3\ K/W.$$

The corresponding overall heat transfer coefficient is

$$U = \frac{\left(R_{equiv}\right)^{-1}}{L^2} = \frac{0.00875\ W/K}{(0.005\ m)^2} = 350\ W/m^2\cdot K.$$

To obtain the steady-state temperature, apply conservation of energy to a control surface about the chip.

$$-\dot{E}_{out} + \dot{E}_g = 0 \qquad -UL^2\left(T_f - T_\infty\right) + \dot{q}\left(L^2\cdot t\right) = 0$$

$$T_f = T_\infty + \frac{\dot{q}t}{U} = 20^\circ C + \frac{9\times10^6\ W/m^3 \times 0.001\ m}{350\ W/m^2\cdot K} = 45.7^\circ C. \qquad <$$

From the general lumped capacitance analysis, Equation 5.15 yields

$$\rho\left(L^2 t\right)c\frac{dT}{dt} = \dot{q}\left(L^2 t\right) - U\left(T - T_\infty\right)L^2.$$

With

$$a \equiv \frac{U}{\rho\ tc} = \frac{350\ W/m^2\cdot K}{\left(2000\ kg/m^3\right)(0.001\ m)(700\ J/kg\cdot K)} = 0.250\ s^{-1}$$

$$b = \frac{\dot{q}}{\rho\ c} = \frac{9\times10^6\ W/m^3}{\left(2000\ kg/m^3\right)(700\ J/kg\cdot K)} = 6.429\ K/s$$

Equation 5.24 yields

$$\exp(-at) = \frac{T - T_\infty - b/a}{T_i - T_\infty - b/a} = \frac{(44.7 - 20 - 25.7)K}{(20 - 20 - 25.7)K} = 0.0389$$

$$t = -\ln(0.0389)/0.250\ s^{-1} = 13.0\ s. \qquad <$$

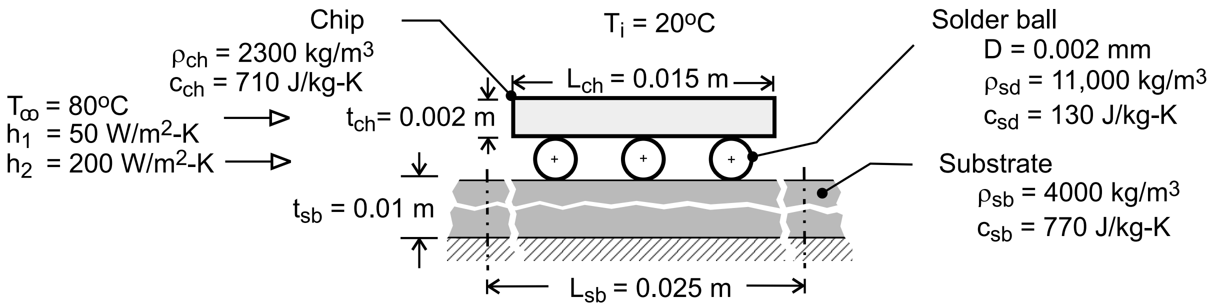
COMMENTS: Heat transfer through the substrate is comparable to that associated with direct convection to the coolant.

PROBLEM 5.28

KNOWN: Dimensions, initial temperature and thermophysical properties of chip, solder and substrate. Temperature and convection coefficient of heating agent.

FIND: (a) Time constants and temperature histories of chip, solder and substrate when heated by an air stream. Time corresponding to maximum stress on a solder ball. (b) Reduction in time associated with using a dielectric liquid to heat the components.

SCHEMATIC:



ASSUMPTIONS: (1) Lumped capacitance analysis is valid for each component, (2) Negligible heat transfer between components, (3) Negligible reduction in surface area due to contact between components, (4) Negligible radiation for heating by air stream, (5) Uniform convection coefficient among components, (6) Constant properties.

ANALYSIS: (a) From Eq. (5.7), $\tau_t = (\rho V c)/hA$

Chip: $V = \left(L_{ch}^2\right)t_{ch} = (0.015\text{m})^2(0.002\text{m}) = 4.50\times10^{-7}\text{m}^3, A_s = \left(2L_{ch}^2 + 4L_{ch}t_{ch}\right)$

$$= 2(0.015\text{m})^2 + 4(0.015\text{m})0.002\text{m} = 5.70\times10^{-4}\text{m}^2$$

$$\tau_t = \frac{2300\,\text{kg/m}^3\times4.50\times10^{-7}\text{m}^3\times710\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times5.70\times10^{-4}\text{m}^2} = 25.8\text{s} \qquad <$$

Solder: $V = \pi D^3/6 = \pi(0.002\text{m})^3/6 = 4.19\times10^{-9}\text{m}^3, A_s = \pi D^2 = \pi(0.002\text{m})^2 = 1.26\times10^{-5}\text{m}^2$

$$\tau_t = \frac{11{,}000\,\text{kg/m}^3\times4.19\times10^{-9}\text{m}^3\times130\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times1.26\times10^{-5}\text{m}^2} = 9.5\text{s} \qquad <$$

Substrate: $V = \left(L_{sb}^2\, t_{sb}\right) = (0.025\text{m})^2(0.01\text{m}) = 6.25\times10^{-6}\text{m}^3, A_s = L_{sb}^2 = (0.025\text{m})^2 = 6.25\times10^{-4}\text{m}^2$

$$\tau_t = \frac{4000\,\text{kg/m}^3\times6.25\times10^{-6}\text{m}^3\times770\,\text{J/kg}\cdot\text{K}}{50\,\text{W/m}^2\cdot\text{K}\times6.25\times10^{-4}\text{m}^2} = 616.0\text{s} \qquad <$$

Substituting Eq. (5.7) into (5.5) and recognizing that $(T - T_i)/(T_\infty - T_i) = 1 - (\theta/\theta_i)$, in which case $(T - T_i)/(T_\infty - T_i) = 0.99$ yields $\theta/\theta_i = 0.01$, it follows that the time required for a component to experience 99% of its maximum possible temperature rise is

$$t_{0.99} = \tau\ln(\theta_i/\theta) = \tau\ln(100) = 4.61\tau$$

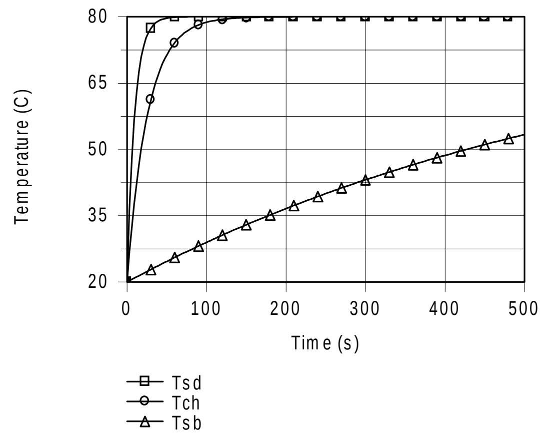
Hence,

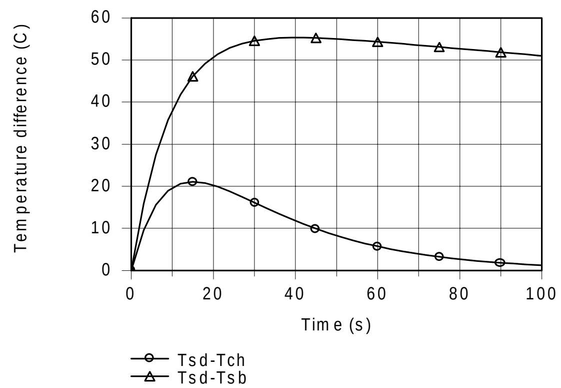
Chip: $t = 118.9\text{s}$, *Solder*: $t = 43.8\text{s}$, *Substrate*: $t = 2840$ $<$

Continued …..

PROBLEM 5.28 (Cont.)

Histories of the three components and temperature differences between a solder ball and its adjoining components are shown below.





Commensurate with their time constants, the fastest and slowest responses to heating are associated with the solder and substrate, respectively. Accordingly, the largest temperature difference is between these two components, and it achieves a maximum value of 55°C at

$$t(\text{maximum stress}) \approx 40\text{s} \qquad <$$

(b) With the 4-fold increase in h associated with use of a dielectric liquid to heat the components, the time constants are each reduced by a factor of 4, and the times required to achieve 99% of the maximum temperature rise are

Chip: $t = 29.5\text{s}$, *Solder*: $t = 11.0\text{s}$, *Substrate*: $t = 708\text{s}$ $<$

The time savings is approximately 75%.

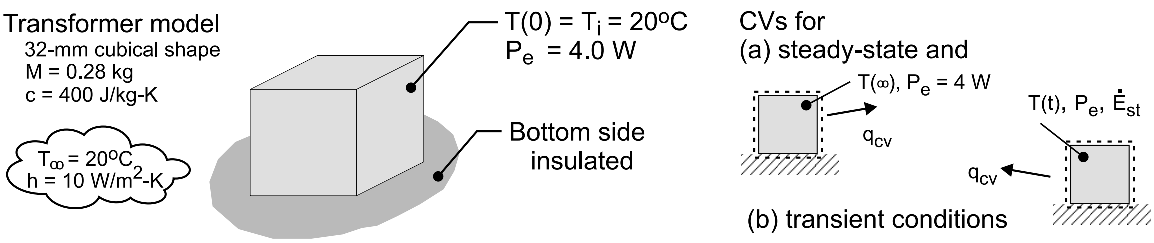
COMMENTS: The foregoing analysis provides only a first, albeit useful, approximation to the heating problem. Several of the assumptions are highly approximate, particularly that of a uniform convection coefficient. The coefficient will vary between components, as well as on the surfaces of the components. Also, because the solder balls are flattened, there will be a reduction in surface area exposed to the fluid for each component, as well as heat transfer between components, which reduces differences between time constants for the components.

PROBLEM 5.29

KNOWN: Electrical transformer of approximate cubical shape, 32 mm to a side, dissipates 4.0 W when operating in ambient air at 20°C with a convection coefficient of 10 $W/m^2 \cdot K$.

FIND: (a) Develop a model for estimating the steady-state temperature of the transformer, $T(\infty)$, and evaluate $T(\infty)$, for the operating conditions, and (b) Develop a model for estimating the temperature-time history of the transformer if initially the temperature is $T_i = T_\infty$ and suddenly power is applied. Determine the time required to reach within 5°C of its steady-state operating temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Transformer is spatially isothermal object, (2) Initially object is in equilibrium with its surroundings, (3) Bottom surface is adiabatic.

ANALYSIS: (a) Under steady-state conditions, for the control volume shown in the schematic above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0 \qquad 0 - q_{cv} + P_e = -h A_s \left[T(\infty) - T_\infty\right] + P_e = 0 \qquad (1)$$

where $A_s = 5 \times L^2 = 5 \times 0.032\text{m} \times 0.032\text{m} = 5.12 \times 10^{-3}\ \text{m}^2$, find

$$T(\infty) = T_\infty + P_e / h A_s = 20°C + 4\,W / \left(10\ W/m^2 \cdot K \times 5.12 \times 10^{-3} m^2\right) = 98.1°C \qquad <$$

(b) Under transient conditions, for the control volume shown above, the energy balance is

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st} \qquad 0 - q_{cv} + P_e = Mc\frac{dT}{dt} \qquad (2)$$

Substitute from Eq. (1) for P_e, separate variables, and define the limits of integration.

$$-h\left[T(t) - T_\infty\right] + h\left[T(\infty) - T_\infty\right] = Mc\frac{dT}{dt}$$

$$-h\left[T(t) - T(\infty)\right] = Mc\frac{d}{dt}\left(T - T(\infty)\right) \qquad \frac{h}{Mc}\int_0^{t_o} dt = -\int_{\theta_i}^{\theta_o}\frac{d\theta}{\theta}$$

where $\theta = T(t) - T(\infty)$; $\theta_i = T_i - T(\infty) = T_\infty - T(\infty)$; and $\theta_o = T(t_o) - T(\infty)$ with t_o as the time when θ_o = - 5°C. Integrating and rearranging find (see Eq. 5.5),

$$t_o = \frac{Mc}{h A_s}\ell n\frac{\theta_i}{\theta_o}$$

$$t_o = \frac{0.28\ kg \times 400\ J/kg \cdot K}{10\ W/m^2 \cdot K \times 5.12 \times 10^{-3} m^2}\ell n\frac{(20 - 98.1)°C}{-5°C} = 1.67\ \text{hour} \qquad <$$

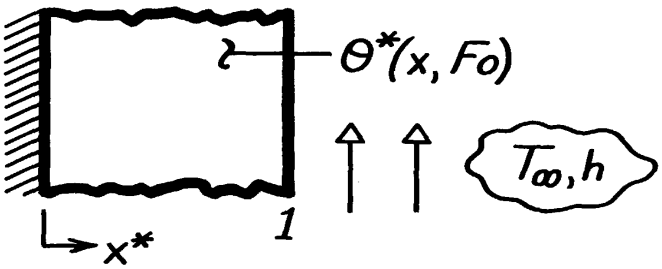
COMMENTS: The spacewise isothermal assumption may not be a gross over simplification since most of the material is copper and iron, and the external resistance by free convection is high. However, by ignoring internal resistance, our estimate for t_o is optimistic.

PROBLEM 5.30

KNOWN: Series solution, Eq. 5.39, for transient conduction in a plane wall with convection.

FIND: Midplane (x*=0) and surface (x*=1) temperatures θ* for Fo=0.1 and 1, using Bi=0.1, 1 and 10 with only the first four eigenvalues. Based upon these results, discuss the validity of the approximate solutions, Eqs. 5.40 and 5.41.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional transient conduction, (2) Constant properties.

ANALYSIS: The series solution, Eq. 5.39a, is of the form,

$$q^* = \sum_{n=1}^{\infty} C_n \exp\left(-z_n^2 Fo\right) \cos\left(z_n x^*\right)$$

where the eigenvalues, z_n, and the constants, C_n, are from Eqs. 5.39b and 5.39c.

$$z_n \tan z_n = Bi \qquad C_n = 4 \sin z_n / \left(2z_n + \sin\left(2z_n\right)\right).$$

The eigenvalues are tabulated in Appendix B.3; note, however, that z_1 and C_1 are available from Table 5.1. The values of z_n and C_n used to evaluate θ* are as follows:

Bi	z_1	C_1	z_2	C_2	z_3	C_3	z_4	C_4
0.1	0.3111	1.0160	3.1731	-0.0197	6.2991	0.0050	9.4354	-0.0022
1	0.8603	1.1191	3.4256	-0.1517	6.4373	0.0466	9.5293	-0.0217
10	1.4289	1.2620	4.3058	-0.3934	7.2281	0.2104	10.2003	-0.1309

Using z_n and C_n values, the terms of q^*, designated as q_1^*, q_2^*, q_3^* and q_4^*, are as follows:

Fo=0.1

	Bi=0.1		Bi=1.0		Bi=10	
x*	0	1	0	1	0	1
q_1^*	1.0062	0.9579	1.0393	0.6778	1.0289	0.1455
q_2^*	-0.0072	0.0072	-0.0469	0.0450	-0.0616	0.0244
q_3^*	0.0001	0.0001	0.0007	0.0007	0.0011	0.0006
q_4^*	-2.99×10^{-7}	3.00×10^{-7}	2.47×10^{-6}	2.46×10^{-7}	-3.96×10^{-6}	2.83×10^{-6}
q^*	0.9991	0.9652	0.9931	0.7235	0.9684	0.1705

Continued …..

PROBLEM 5.30(Cont.)

Fo=1

	Bi=0.1		Bi=1.0		Bi=10	
x*	0	1	0	1	0	1
q_1^*	0.9223	0.8780	0.5339	0.3482	0.1638	0.0232
q_2^*	8.35×10^{-7}	8.35×10^{-7}	-1.22×10^{-5}	1.17×10^{-6}	3.49×10^{-9}	1.38×10^{-9}
q_3^*	7.04×10^{-20}	-	4.70×10^{-20}	-	4.30×10^{-24}	-
q_4^*	4.77×10^{-42}	-	7.93×10^{-42}	-	8.52×10^{-47}	-
q^*	0.9223	0.8780	0.5339	0.3482	0.1638	0.0232

The tabulated results for $q^* = q^*\left(x^*, \text{Bi}, \text{Fo}\right)$ demonstrate that for Fo=1, the first eigenvalue is sufficient to accurately represent the series. However, for Fo=0.1, three eigenvalues are required for accurate representation.

A more detailed analysis would show that a practical criterion for representation of the series solution by one eigenvalue is Fo>0.2. For these situations the approximate solutions, Eqs. 5.40 and 5.41, are appropriate. For the midplane, $x^*=0$, the first two eigenvalues for Fo=0.2 are:

	Fo=0.2	x*=0	
Bi	0.1	1.0	10
q_1^*	0.9965	0.9651	0.8389
q_2^*	-0.00226	-0.0145	-0.0096
q^*	0.9939	0.9506	0.8293
Error,%	+0.26	+1.53	+1.16

The percentage error shown in the last row of the above table is due to the effect of the second term. For Bi=0.1, neglecting the second term provides an error of 0.26%. For Bi=1, the error is 1.53%.

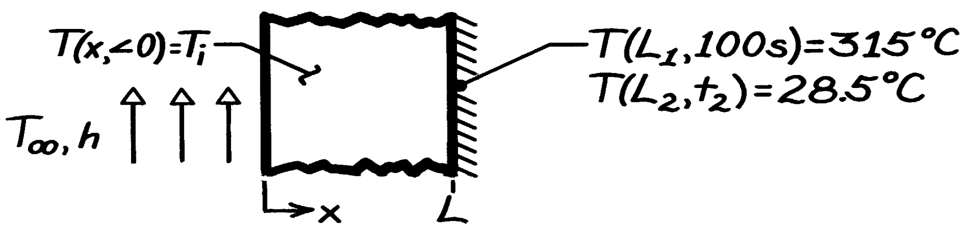
Hence we conclude that the approximate series solutions (with only one eigenvalue) provides systematically high results, but by less than 1.5%, for the Biot number range from 0.1 to 10.

PROBLEM 5.31

KNOWN: One-dimensional wall, initially at a uniform temperature, T_i, is suddenly exposed to a convection process (T_∞, h). For wall #1, the time (t_1 = 100s) required to reach a specified temperature at x = L is prescribed, $T(L_1, t_1)$ = 315°C.

FIND: For wall #2 of different thickness and thermal conditions, the time, t_2, required for $T(L_2, t_2)$ = 28°C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: The properties, thickness and thermal conditions for the two walls are:

Wall	L(m)	$\alpha(m^2/s)$	k(W/m·K)	T_i(°C)	T_∞(°C)	$h(W/m^2{\cdot}K)$
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

The dimensionless functional dependence for the one-dimensional, transient temperature distribution, Eq. 5.38, is

$$q^* = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = f\left(x^*, \text{Bi, Fo}\right)$$

where

$$x^* = x/L \qquad \text{Bi} = hL/k \qquad \text{Fo} = a\,t/L^2.$$

If the parameters x*, Bi, and Fo are the same for both walls, then $q_1^* = q_2^*$. Evaluate these parameters:

Wall	x*	Bi	Fo	θ*
1	1	0.40	0.150	0.85
2	1	0.40	$1.563\times10^{-4}\ t_2$	0.85

where

$$q_1^* = \frac{315-400}{300-400} = 0.85 \qquad q_2^* = \frac{28.5-20}{30-20} = 0.85.$$

It follows that

$$\text{Fo}_2 = \text{Fo}_1 \qquad 1.563\times10^{-4} t_2 = 0.150$$

$$t_2 = 960\text{s}.$$

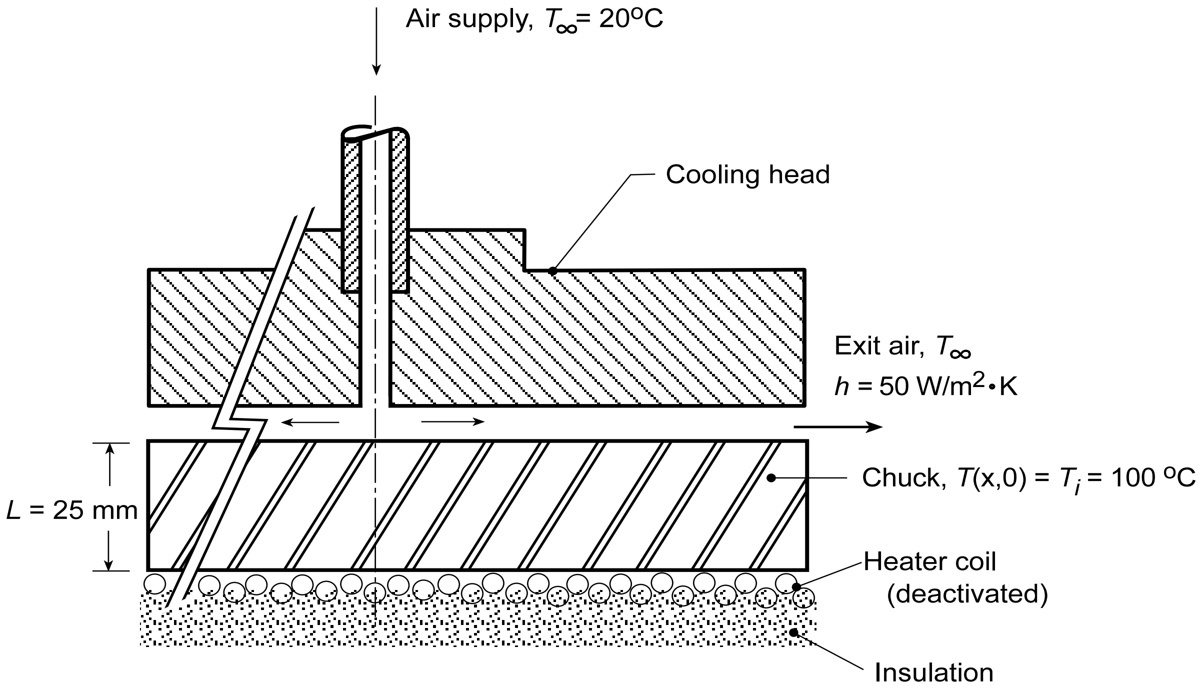
<

PROBLEM 5.32

KNOWN: The chuck of a semiconductor processing tool, initially at a uniform temperature of T_i = 100°C, is cooled on its top surface by supply air at 20°C with a convection coefficient of 50 W/m^2·K.

FIND: (a) Time required for the lower surface to reach 25°C, and (b) Compute and plot the time-to-cool as a function of the convection coefficient for the range $10 \leq h \leq 2000$ W/m^2·K; comment on the effectiveness of the head design as a method for cooling the chuck.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, transient conduction in the chuck, (2) Lower surface is perfectly insulated, (3) Uniform convection coefficient and air temperature over the upper surface of the chuck, and (4) Constant properties.

PROPERTIES: *Table A.1*, Aluminum alloy 2024 ((25 + 100)°C / 2 = 335 K): ρ = 2770 kg/m^3, c_p = 880 J/kg· K, k = 179 W/m·K.

ANALYSIS: (a) The Biot number for the chuck with h = 50 W/m^2·K is

$$\mathrm{Bi} = \frac{hL}{k} = \frac{50\,\mathrm{W/m^2 \cdot K} \times 0.025\,\mathrm{m}}{179\,\mathrm{W/m \cdot K}} = 0.007 \leq 0.1 \tag{1}$$

so that the lumped capacitance method is appropriate. Using Eq. 5.5, with $V/A_s = L$,

$$t = \frac{\rho V c}{h A_s} \ln \frac{\theta_i}{\theta} \qquad \theta = T - T_\infty \qquad \theta_i = T_i - T_\infty$$

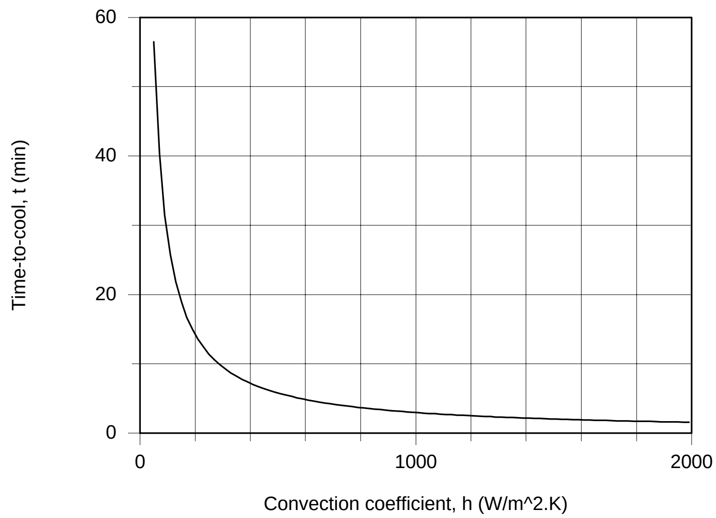
$$t = \left(2770\,\mathrm{kg/m^3} \times 0.025\,\mathrm{m} \times 880\,\mathrm{J/kg \cdot K} / 50\,\mathrm{W/m^2 \cdot K}\right) \ln \frac{(100-20)^\circ\,\mathrm{C}}{(25-20)^\circ\,\mathrm{C}}$$

$$t = 3379\,\mathrm{s} = 56.3\,\mathrm{min}$$ <

Continued...

PROBLEM 5.32 (Cont.)

(b) When h = 2000 W/m^2·K, using Eq. (1), find Bi = 0.28 > 0.1 so that the series solution, Section 5.51, for the plane wall with convection must be used. Using the *IHT Transient Conduction, Plane Wall Model*, the time-to-cool was calculated as a function of the convection coefficient. Free convection cooling conduction corresponds to h ≈ 10 W/m^2·K and the time-to-cool is 282 minutes. With the cooling head design, the time-to-cool can be substantially decreased if the convection coefficient can be increased as shown below.

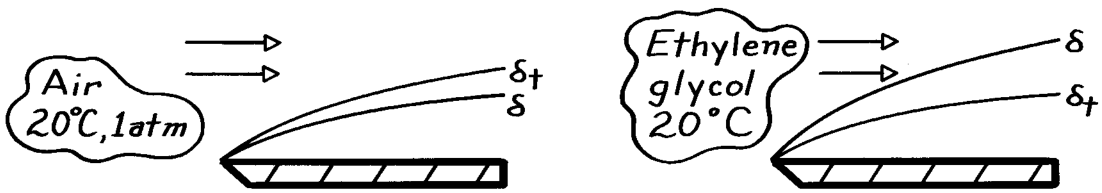


PROBLEM 6.33

KNOWN: Laminar boundary layer flow of air at 20°C and 1 atm having $\delta_t = 1.13\,\delta$.

FIND: Ratio δ / δ_t when fluid is ethylene glycol for same conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: *Table A-4,* Air (293K, 1 atm): Pr = 0.709; *Table A-5,* Ethylene glycol (293K): Pr = 211.

ANALYSIS: The Prandtl number strongly influences relative growth of the velocity, δ, and thermal, δ_t, boundary layers. For laminar flow, the approximate relationship is given by

$$\mathrm{Pr}^n \approx \frac{\delta}{\delta_t}$$

where n is a positive coefficient. Substituting the values for air

$$(0.709)^n = \frac{1}{1.13}$$

find that n = 0.355. Hence, for ethylene glycol it follows that

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{0.355} = 211^{0.355} = 6.69. \qquad <$$

COMMENTS: (1) For laminar flow, generally we find n = 0.33. In which case, $\delta / \delta_t = 5.85$.

(2) Recognize the physical importance of $\nu > \alpha$, which gives large values of the Prandtl number, and causes $\delta > \delta_t$.

PROBLEM 6.34

KNOWN: Air, water, engine oil or mercury at 300K in laminar, parallel flow over a flat plate.

FIND: Sketch of velocity and thermal boundary layer thickness.

ASSUMPTIONS: (1) Laminar flow.

PROPERTIES: For the fluids at 300K:

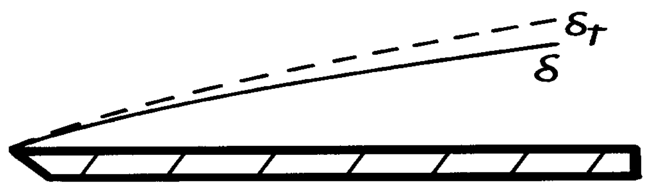
Fluid	Table	Pr
Air	A.4	0.71
Water	A.6	5.83
Engine Oil	A.5	6400
Mercury	A.5	0.025

ANALYSIS: For laminar, boundary layer flow over a flat plate.

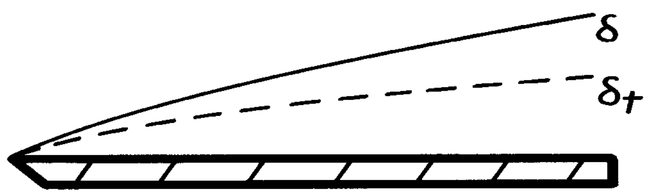
$$\frac{\delta}{\delta_t} \sim \mathrm{Pr}^n$$

where n > 0. Hence, the boundary layers appear as shown below.

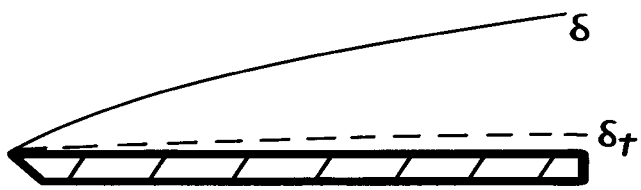
Air:



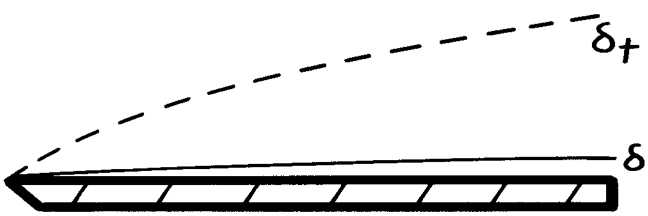
Water:



Engine Oil:



Mercury:



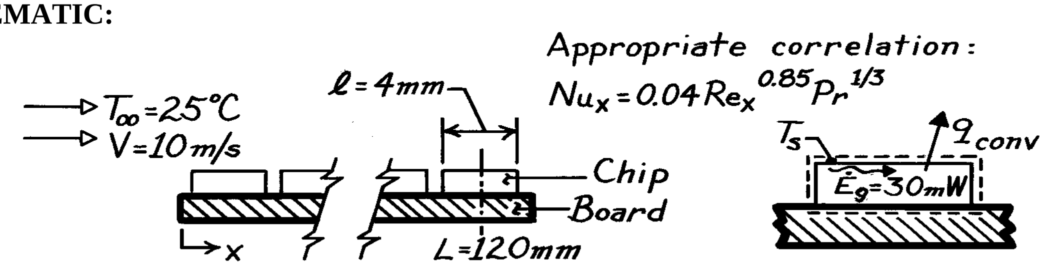
COMMENTS: Although Pr strongly influences relative boundary layer development in laminar flow, its influence is weak for turbulent flow.

PROBLEM 6.35

KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L.

PROPERTIES: *Table A-4*, Air (assume T_s = 45°C, T_f = (45 + 25)/2 = 35°C = 308K, 1atm): $\nu = 16.69 \times 10^{-6} m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip (see above),

$$q_{conv} = \dot{E}_g = 30W. \tag{1}$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \bar{h} A_{chip} \tag{2}$$

where $A_{chip} = \ell^2$. Assume that the *average* heat transfer coefficient $(\bar{h})$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L. That is, $\bar{h}_{chip} \approx h_x(L)$ where the local coefficient can be evaluated from the special correlation for this situation,

$$Nu_x = \frac{h_x x}{k} = 0.04 \left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

and substituting numerical values with x = L, find

$$h_x = 0.04 \frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_x = 0.04\left[\frac{0.0269\ W/m\cdot K}{0.120\ m}\right]\left[\frac{10\ m/s \times 0.120\ m}{16.69\times10^{-6}\ m^2/s}\right]^{0.85}(0.703)^{1/3} = 107\ W/m^2\cdot K.$$

The surface temperature of the chip is from Eq. (2),

$$T_s = 25°C + 30\times10^{-3}\ W/107\ W/m^2\cdot K\times(0.004m)^2 = 42.5°C. \quad <$$

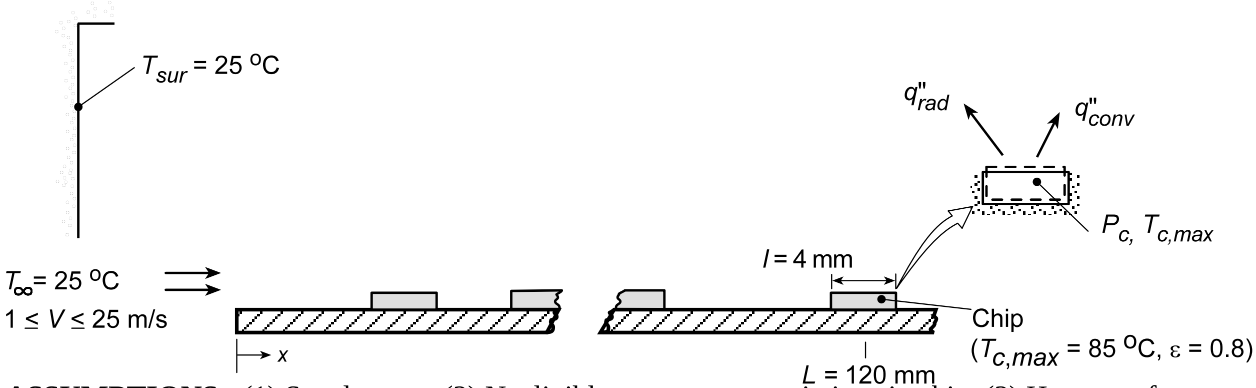
COMMENTS: (1) Note that the estimated value for T_f used to evaluate the air properties was reasonable. (2) Alternatively, we could have evaluated $\bar{h}_{chip}$ by performing the integration of the local value, h(x).

PROBLEM 6.36

KNOWN: Location and dimensions of computer chip on a circuit board. Form of the convection correlation. Maximum allowable chip temperature and surface emissivity. Temperature of cooling air and surroundings.

FIND: Effect of air velocity on maximum power dissipation, first without and then with consideration of radiation effects.

SCHEMATIC:



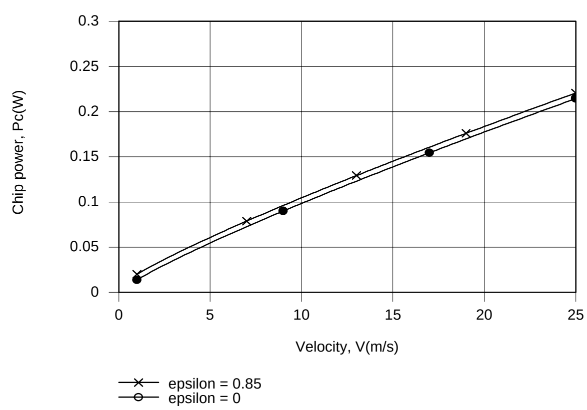
ASSUMPTIONS: (1) Steady-state, (2) Negligible temperature variations in chip, (3) Heat transfer exclusively from the top surface of the chip, (4) The local heat transfer coefficient at x = L provides a good approximation to the average heat transfer coefficient for the chip surface.

PROPERTIES: *Table A.4*, air ($\overline{T} = (T_\infty + T_c)/2$ = 328 K): $\nu = 18.71 \times 10^{-6}$ m^2/s, k = 0.0284 W/m·K, Pr = 0.703.

ANALYSIS: Performing an energy balance for a control surface about the chip, we obtain $P_c = q_{conv} + q_{rad}$, where $q_{conv} = \overline{h}A_s(T_c - T_\infty)$, $q_{rad} = h_r A_s (T_c - T_{sur})$, and $h_r = \varepsilon\sigma(T_c + T_{sur})(T_c^2 + T_{sur}^2)$. With $\overline{h} \approx h_L$, the convection coefficient may be determined from the correlation provided in Problem 6.35 ($Nu_L = 0.04\, Re_L^{0.85}\, Pr^{1/3}$). Hence,

$$P_c = \ell^2\left[0.04(k/L)Re_L^{0.85}\,Pr^{1/3}(T_c - T_\infty) + \varepsilon\sigma(T_c + T_{sur})(T_c^2 + T_{sur}^2)(T_c - T_{sur})\right]$$

where $Re_L = VL/\nu$. Computing the right side of this expression for ε = 0 and ε = 0.85, we obtain the following results.



Since h_L increases as $V^{0.85}$, the chip power must increase with V in the same manner. Radiation exchange increases P_c by a fixed, but small (6 mW) amount. While h_L varies from 14.5 to 223 W/m^2·K over the prescribed velocity range, h_r = 6.5 W/m^2·K is a constant, independent of V.

COMMENTS: Alternatively, $\overline{h}$ could have been evaluated by integrating h_x over the range 118 ≤ x ≤ 122 mm to obtain the appropriate average. However, the value would be extremely close to $h_{x=L}$.

PROBLEM 6.37

KNOWN: Form of Nusselt number for flow of air or a dielectric liquid over components of a circuit card.

FIND: Ratios of time constants associated with intermittent heating and cooling. Fluid that provides faster thermal response.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, $\nu = 2 \times 10^{-5}$ m^2/s, Pr = 0.71. Dielectric liquid: k = 0.064 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25.

ANALYSIS: From Eq. 5.7, the thermal time constant is

$$\tau_t = \frac{\rho \forall c}{\overline{h} A_s}$$

Since the only variable that changes with the fluid is the convection coefficient, where

$$\overline{h} = \frac{k}{L}\overline{Nu}_L = \frac{k}{L} C \, Re_L^m \, Pr^n = \frac{k}{L} C \left(\frac{VL}{\nu}\right)^m Pr^n$$

the desired ratio reduces to

$$\frac{\tau_{t,air(a)}}{\tau_{t,dielectric(d)}} = \frac{\overline{h}_d}{\overline{h}_a} = \frac{k_d}{k_a}\left(\frac{\nu_a}{\nu_d}\right)^m \left(\frac{Pr_d}{Pr_a}\right)^n$$

$$\frac{\tau_{t,a}}{\tau_{t,d}} = \frac{0.064}{0.026}\left(\frac{2 \times 10^{-5}}{10^{-6}}\right)^{0.8} \left(\frac{25}{0.71}\right)^{0.33} = 88.6$$

Since its time constant is nearly two orders of magnitude smaller than that of the air, the dielectric liquid is clearly the fluid of choice.

COMMENTS: The accelerated testing procedure suggested by this problem is commonly used to test the durability of electronic packages.

PROBLEM 6.38

KNOWN: Form of the Nusselt number correlation for forced convection and fluid properties.

FIND: Expression for figure of merit F_F and values for air, water and a dielectric liquid.

PROPERTIES: Prescribed. Air: k = 0.026 W/m·K, $\nu = 1.5 \times 10^{-5}$ m^2/s, Pr = 0.70. Water: k = 0.600 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 5.0. Dielectric liquid: k = 0.064 W/m·K, $\nu = 10^{-6}$ m^2/s, Pr = 25

ANALYSIS: With $Nu_L \sim Re_L^m Pr^n$, the convection coefficient may be expressed as

$$h \sim \frac{k}{L}\left(\frac{VL}{\nu}\right)^m Pr^n \sim \frac{V^m}{L^{1-m}}\left(\frac{kPr^n}{\nu^m}\right)$$

The figure of merit is therefore

$$F_F = \frac{kPr^n}{\nu^m}$$ <

and for the three fluids, with m = 0.80 and n = 0.33,

$F_F\left(W \cdot s^{0.8}/m^{2.6} \cdot K\right)$	Air	Water	Dielectric	
	167	64,400	11,700	<

Water is clearly the superior heat transfer fluid, while air is the least effective.

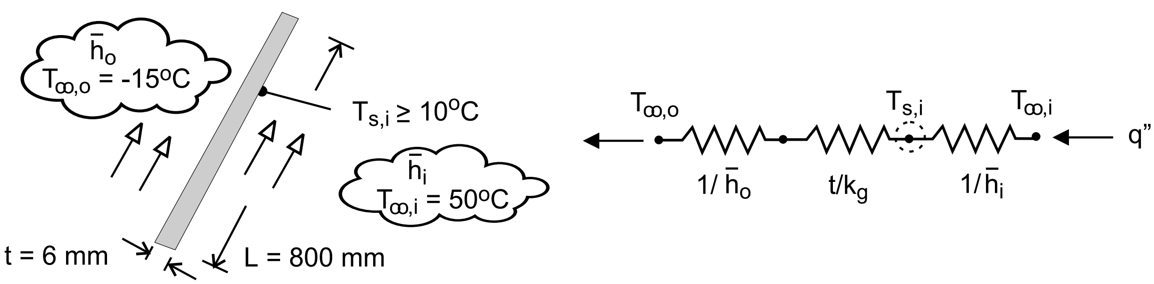
COMMENTS: The figure of merit indicates that heat transfer is enhanced by fluids of large k, large Pr and small ν.

PROBLEM 6.39

KNOWN: Ambient, interior and dewpoint temperatures. Vehicle speed and dimensions of windshield. Heat transfer correlation for external flow.

FIND: Minimum value of convection coefficient needed to prevent condensation on interior surface of windshield.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

PROPERTIES: *Table A-3*, glass: k_g = 1.4 W/m·K. Prescribed, air: k = 0.023 W/m·K, $\nu = 12.5 \times 10^{-6}$ m^2/s, Pr = 0.70.

ANALYSIS: From the prescribed thermal circuit, conservation of energy yields

$$\frac{T_{\infty,i} - T_{s,i}}{1/\bar{h}_i} = \frac{T_{s,i} - T_{\infty,o}}{t/k_g + 1/\bar{h}_o}$$

where $\bar{h}_o$ may be obtained from the correlation

$$\overline{Nu}_L = \frac{\bar{h}_o L}{k} = 0.030\, Re_L^{0.8}\, Pr^{1/3}$$

With V = (70 mph × 1585 m/mile)/3600 s/h = 30.8 m/s, Re_D = (30.8 m/s × 0.800 m)/12.5 × 10^{-6} m^2/s = 1.97 × 10^6 and

$$\bar{h}_o = \frac{0.023\,W/m\cdot K}{0.800\ m} 0.030\left(1.97\times10^6\right)^{0.8}(0.70)^{1/3} = 83.1\,W/m^2\cdot K$$

From the energy balance, with $T_{s,i} = T_{dp}$ = 10°C

$$\bar{h}_i = \frac{\left(T_{s,i} - T_{\infty,o}\right)}{\left(T_{\infty,i} - T_{s,i}\right)}\left(\frac{t}{k_g} + \frac{1}{\bar{h}_o}\right)^{-1}$$

$$\bar{h}_i = \frac{(10+15)°C}{(50-10)°C}\left(\frac{0.006\ m}{1.4\,W/m\cdot K} + \frac{1}{83.1\,W/m^2\cdot K}\right)^{-1}$$

$$\bar{h}_i = 38.3\,W/m^2\cdot K$$ <

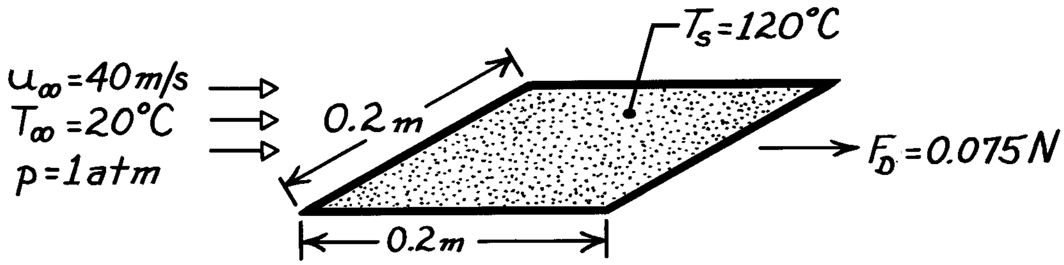
COMMENTS: The output of the fan in the automobile's heater/defroster system must maintain a velocity for flow over the inner surface that is large enough to provide the foregoing value of $\bar{h}_i$. In addition, the output of the heater must be sufficient to maintain the prescribed value of $T_{\infty,i}$ at this velocity.

PROBLEM 6.40

KNOWN: Drag force and air flow conditions associated with a flat plate.

FIND: Rate of heat transfer from the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Chilton-Colburn analogy is applicable.

PROPERTIES: *Table A-4*, Air (70°C,1 atm): ρ = 1.018 kg/m^3, c_p = 1009 J/kg·K, Pr = 0.70, $\nu = 20.22 \times 10^{-6}$ m^2/s.

ANALYSIS: The rate of heat transfer from the plate is

$$q = 2\bar{h}\left(L^2\right)\ \left(T_s - T_\infty\right)$$

where $\bar{h}$ may be obtained from the Chilton-Colburn analogy,

$$\bar{j}_H = \frac{\bar{C}_f}{2} = \overline{St}\,Pr^{2/3} = \frac{\bar{h}}{\rho\, u_\infty\, c_p} Pr^{2/3}$$

$$\frac{\bar{C}_f}{2} = \frac{1}{2}\frac{\bar{\tau}_s}{\rho\, u_\infty^2/2} = \frac{1}{2}\frac{(0.075\text{ N}/2)/(0.2\text{m})^2}{1.018\text{ kg/m}^3(40\text{ m/s})^2/2} = 5.76\times10^{-4}.$$

Hence,

$$\bar{h} = \frac{\bar{C}_f}{2}\rho\, u_\infty\, c_p\ Pr^{-2/3}$$

$$\bar{h} = 5.76\times10^{-4}\left(1.018\text{kg/m}^3\right)40\text{m/s}\ \left(1009\text{J/kg}\cdot\text{K}\right)\ (0.70)^{-2/3}$$

$$\bar{h} = 30\text{ W/m}^2\cdot\text{K}.$$

The heat rate is

$$q = 2\left(30\text{ W/m}^2\cdot\text{K}\right)\ (0.2\text{m})^2\ (120-20)^\circ\text{C}$$

$$q = 240\text{ W}.$$ <

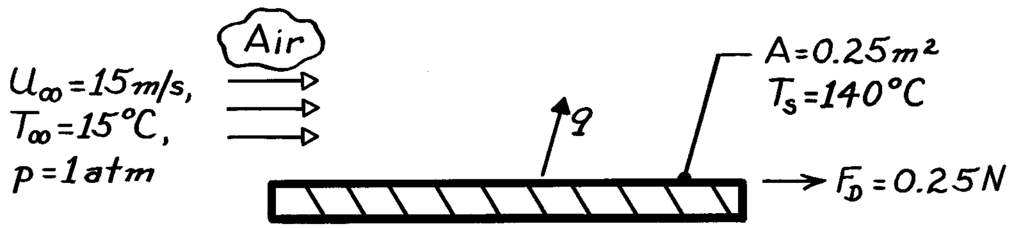
COMMENTS: Although the flow is laminar over the entire surface ($Re_L = u_\infty L/\nu$ = 40 m/s × 0.2m/20.22 × 10^{-6}m^2/s = 4.0 × 10^5), the pressure gradient is zero and the Chilton-Colburn analogy is applicable to *average*, as well as *local*, surface conditions. Note that the only contribution to the drag force is made by the surface shear stress.

PROBLEM 6.41

KNOWN: Air flow conditions and drag force associated with a heater of prescribed surface temperature and area.

FIND: Required heater power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Reynolds analogy is applicable, (3) Bottom surface is adiabatic.

PROPERTIES: *Table A-4*, Air (T_f = 350K, 1atm): ρ = 0.995 kg/m^3, c_p = 1009 J/kg·K, Pr = 0.700.

ANALYSIS: The average shear stress and friction coefficient are

$$\bar{\tau}_s = \frac{F_D}{A} = \frac{0.25\text{ N}}{0.25\text{ m}^2} = 1\text{ N/m}^2$$

$$\bar{C}_f = \frac{\bar{\tau}_s}{\rho u_\infty^2/2} = \frac{1\text{ N/m}^2}{0.995\text{ kg/m}^3(15\text{m/s})^2/2} = 8.93\times10^{-3}.$$

From the Reynolds analogy,

$$\overline{St} = \frac{\bar{h}}{\rho u_\infty c_p} = \frac{\bar{C}_f}{2}\text{Pr}^{-2/3}.$$

Solving for $\bar{h}$ and substituting numerical values, find

$$\bar{h} = 0.995\text{ kg/m}^3(15\text{m/s})\ 1009\text{ J/kg}\cdot\text{K}\left(8.93\times10^{-3}/2\right)\ (0.7)^{-2/3}$$

$$\bar{h} = 85\text{ W/m}^2\cdot\text{K}.$$

Hence, the heat rate is

$$q = \bar{h}A\left(T_s - T_\infty\right) = 85\text{W/m}^2\cdot\text{K}\left(0.25\text{m}^2\right)\ (140-15)^\circ\text{ C}$$

$$q = 2.66\text{ kW}.$$ <

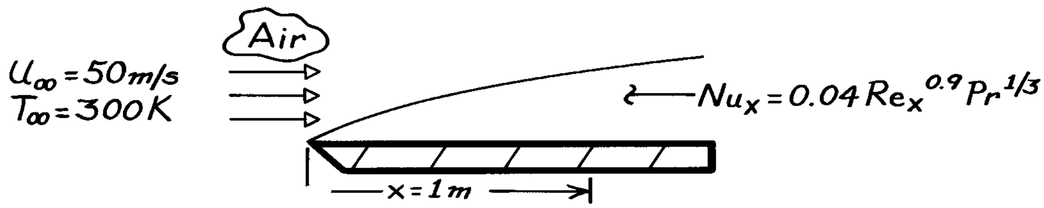
COMMENTS: Due to bottom heat losses, which have been assumed negligible, the actual power requirement would exceed 2.66 kW.

PROBLEM 6.42

KNOWN: Heat transfer correlation associated with parallel flow over a rough flat plate. Velocity and temperature of air flow over the plate.

FIND: Surface shear stress l m from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Modified Reynolds analogy is applicable, (2) Constant properties.

PROPERTIES: *Table A-4*, Air (300K, 1atm): $\nu = 15.89 \times 10^{-6} m^2/s$, Pr = 0.71, $\rho = 1.16$ kg/m^3.

ANALYSIS: Applying the Chilton-Colburn analogy

$$\frac{C_f}{2} = St_x\, Pr^{2/3} = \frac{Nu_x}{Re_x\, Pr} Pr^{2/3} = \frac{0.04\, Re_x^{0.9}\, Pr^{1/3}}{Re_x\, Pr} Pr^{2/3}$$

$$\frac{C_f}{2} = 0.04\, Re_x^{-0.1}$$

where

$$Re_x = \frac{u_\infty x}{\nu} = \frac{50\ m/s \times 1m}{15.89 \times 10^{-6} m^2/s} = 3.15 \times 10^6.$$

Hence, the friction coefficient is

$$C_f = 0.08\left(3.15 \times 10^6\right)^{-0.1} = 0.0179 = \tau_s / \left(\rho\, u_\infty^2 / 2\right)$$

and the surface shear stress is

$$\tau_s = C_f\left(\rho\, u_\infty^2 / 2\right) = 0.0179 \times 1.16 kg/m^3 (50\ m/s)^2 / 2$$

$$\tau_s = 25.96\ kg/m \cdot s^2 = 25.96\ N/m^2. \qquad <$$

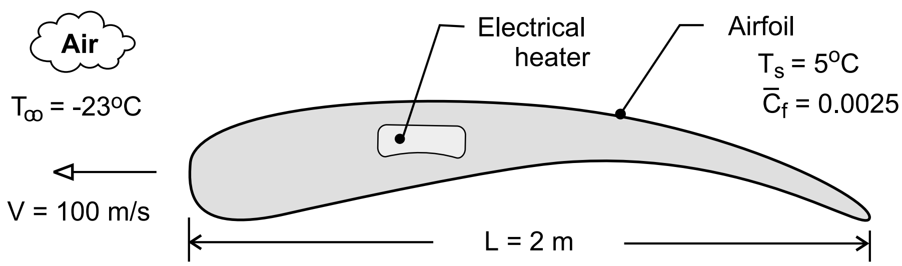
COMMENTS: Note that turbulent flow will exist at the designated location.

PROBLEM 6.43

KNOWN: Nominal operating conditions of aircraft and characteristic length and average friction coefficient of wing.

FIND: Average heat flux needed to maintain prescribed surface temperature of wing.

SCHEMATIC:



ASSUMPTIONS: (1) Applicability of modified Reynolds analogy, (2) Constant properties.

PROPERTIES: Prescribed, Air: $\nu = 16.3 \times 10^{-6}$ m^2/s, k = 0.022 W/m·K, Pr = 0.72.

ANALYSIS: The average heat flux that must be maintained over the surface of the air foil is $\bar{q}'' = \bar{h}\left(T_s - T_\infty\right)$, where the average convection coefficient may be obtained from the modified Reynolds analogy.

$$\frac{\bar{C}_f}{2} = \mathrm{St}\,\mathrm{Pr}^{2/3} = \frac{\overline{Nu}_L}{Re_L\,Pr}\,Pr^{2/3} = \frac{\overline{Nu}_L}{Re_L\,Pr^{1/3}}$$

Hence, with $Re_L = VL/\nu = 100\,\mathrm{m/s}\left(2\mathrm{m}\right)/16.3\times10^{-6}\mathrm{m}^2/\mathrm{s} = 1.23\times10^{7}$,

$$\overline{Nu}_L = \frac{0.0025}{2}\left(1.23\times10^{7}\right)\left(0.72\right)^{1/3} = 13{,}780$$

$$\bar{h} = \frac{k}{L}\overline{Nu}_L = \frac{0.022\,\mathrm{W/m\cdot K}}{2\mathrm{m}}\left(13{,}780\right) = 152\,\mathrm{W/m^2\cdot K}$$

$$\bar{q}'' = 152\,\mathrm{W/m^2\cdot K}\left[5-\left(-23\right)\right]^\circ\mathrm{C} = 4260\,\mathrm{W/m^2}$$ <

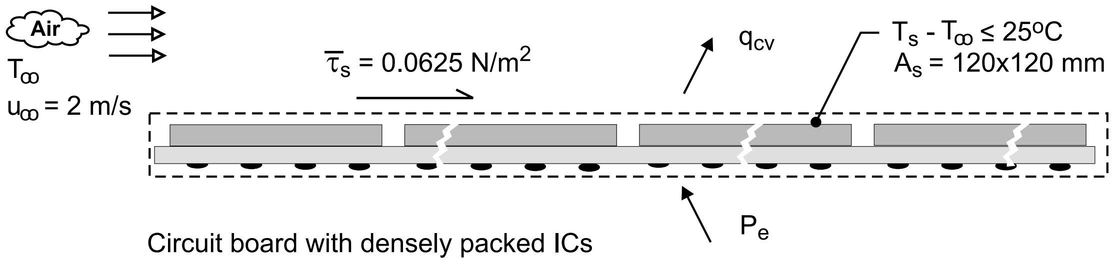
COMMENTS: If the flow is turbulent over the entire airfoil, the modified Reynolds analogy provides a good measure of the relationship between surface friction and heat transfer. The relation becomes more approximate with increasing laminar boundary layer development on the surface and increasing values of the magnitude of the pressure gradient.

PROBLEM 6.44

KNOWN: Average frictional shear stress of $\overline{\tau}_s = 0.0625\ N/m^2$ on upper surface of circuit board with densely packed integrated circuits (ICs)

FIND: Allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed a rise of 25°C above ambient air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) The modified Reynolds analogy is applicable, (3) Negligible heat transfer from bottom side of the circuit board, and (4) Thermophysical properties required for the analysis evaluated at 300 K,

PROPERTIES: *Table A-4*, Air (T_f = 300 K, 1 atm): $\rho = 1.161\ kg/m^3$, $c_p = 1007\ J/kg\cdot K$, Pr = 0.707.

ANALYSIS: The power dissipation from the circuit board can be calculated from the convection rate equation assuming an excess temperature $(T_s - T_\infty) = 25°C$.

$$q = \overline{h}\, A_s \left(T_s - T_\infty\right) \qquad (1)$$

The average convection coefficient can be estimated from the Reynolds analogy and the measured average frictional shear stress $\overline{\tau}_s$.

$$\frac{\overline{C}_f}{2} = \overline{St}\ Pr^{2/3} \qquad \overline{C}_f = \frac{\overline{\tau}_s}{\rho\ V^2/2} \qquad \overline{St} = \frac{\overline{h}}{\rho\ V\ c_p} \qquad (2,3,4)$$

With $V = u_\infty$ and substituting numerical values, find $\overline{h}$.

$$\frac{\overline{\tau}_s}{\rho\ V^2} = \frac{\overline{h}}{\rho\ V c_p} Pr^{2/3}$$

$$\overline{h} = \frac{\overline{\tau}_s\ c_p}{V} Pr^{-2/3}$$

$$\overline{h} = \frac{0.0625\ N/m^2 \times 1007\ J/kg\cdot K}{2\ m/s}\left(0.707\right)^{-2/3} = 39.7\ W/m^2\cdot K$$

Substituting this result into Eq. (1), the allowable power dissipation is

$$q = 39.7\ W/m^2\cdot K \times \left(0.120 \times 0.120\right) m^2 \times 25\ K = 14.3\ W$$ <

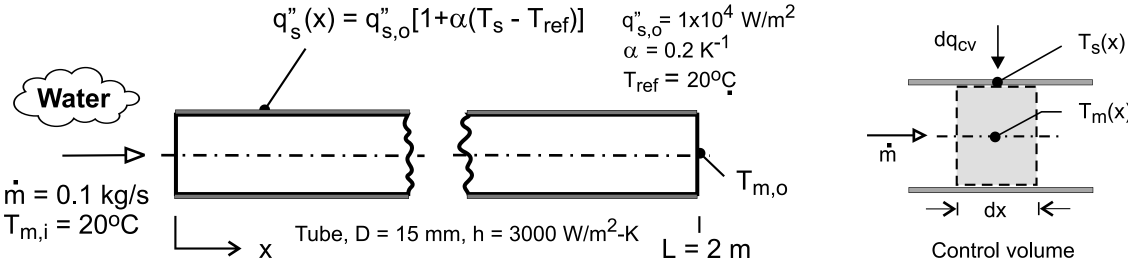
COMMENTS: For this analyses using the modified or Chilton-Colburn analogy, we found C_f = 0.0269 and St = 0.0170. Using the Reynolds analogy, the results are slightly different with $\overline{h} = 31.5\ W/m^2\cdot K$ and q = 11.3 W.

PROBLEM 8.21

KNOWN: Water is heated in a tube having a wall flux that is dependent upon the wall temperature.

FIND: (a) Beginning with a properly defined differential control volume in the tube, derive expressions that can be used to obtain the temperatures for the water and the wall surface as a function of distance from the inlet, $T_m(x)$ and $T_s(x)$, respectively; (b) Using a numerical integration scheme, calculate and plot the temperature distributions, $T_m(x)$ and $T_s(x)$, on the same graph. Identify and comment on the main features of the distributions; and (c) Calculate the total heat transfer rate to the water.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed flow and thermal conditions, (3) No losses to the outer surface of the tube, and (3) Constant properties.

PROPERTIES: *Table A-6*, Water $\left(\overline{T}_m = \left(T_{m,i} + T_{m,o}\right)/2 = 300\ \text{K}\right)$: $c_p = 4179$ J/kg·K

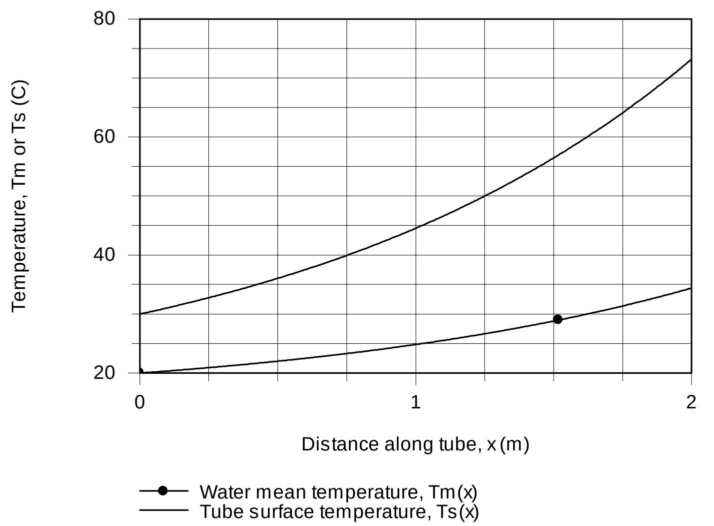
ANALYSIS: (a) The properly defined control volume of perimeter P = πD shown in the above schematic follows from Fig. 8.6. The energy balance on the CV includes advection, convection at the inner tube surface, and the heat flux dissipated in the tube wall. (See Eq. 8.38).

$$\dot{m}\, c_p \frac{dT_m}{dx} = q_s''(x) P = h P\left[T_s(x) - T_m(x)\right] \tag{1,2}$$

where $q_s''(x)$ is dependent upon $T_s(x)$ according to the relation

$$q_s''(x) = q_{s,o}''\left[1 + \alpha\left(T_s(x) - T_{ref}\right)\right] \tag{3}$$

(b) Eqs. (1 and 2) with Eq. (3) can be solved by numerical integration using the Der function in *IHT* as shown in Comment 1. The temperature distributions for the water and wall surface are plotted below.



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(c) The total heat transfer to the water can be evaluated from an overall energy balance on the water,

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \tag{4}$$

$$q = 0.1\ \text{kg/s} \times 4179\ \text{J/kg} \cdot \text{K} \left(34.4 - 20\right)\text{K} = 6018\ \text{W}$$ <

Alternatively, the heat rate can be evaluated by integration of the heat flux from the tube surface over the length of the tube,

$$q = \int_0^L q_s''(x) P dx \tag{5}$$

where $q_s''(x)$ is given by Eq. (3), and $T_s(x)$ and $T_m(x)$ are determined from the differential form of the energy equation, Eqs. (1) and (2). The result as shown in the *IHT* code below is 6005 W.

COMMENTS: (1) Note that $T_m(x)$ increases with distance greater than linearly, as expected since $q_s''(x)$ does. Also as expected, the difference, $T_s(x) - T_m(x)$, likewise increases with distance greater than linearly.

(2) In the foregoing analysis, c_p is evaluated at the mean fluid temperature $T_m = (T_{m,i} + T_{m,o})/2$.

(3) The *IHT* code representing the foregoing equations to calculate and plot the temperature distribution and to calculate the total heat rate to the water is shown below.

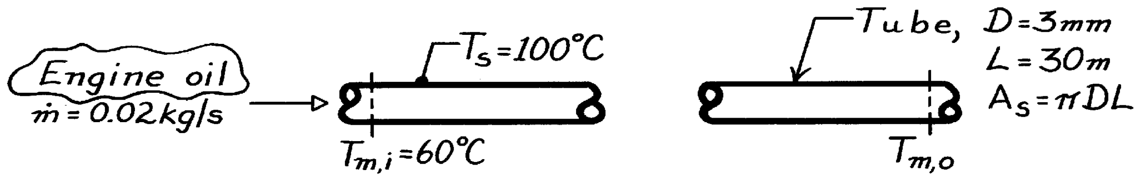
```
/* Results: integration for distributions; conditions at x = 2 m
F_xTs  Ts      q'      q''s_x    x      Tm
11.64  73.18   5483    1.164E5   2      34.39
3      30      1414    3E4       0      20   */

/* Results: heat rate by energy balances on fluid and tube surface
q_eb   q_hf
6018   6005    */

/* Results: for evaluating cp at Tm
Ts     cp      q''s_x    x      Tm
73.31  4179    1.166E5   2      34.44
30     4179    3E4       0      20      */

// Energy balances
mdot * cp * der(Tm,x) = q'                // Energy balance, Eq. 8.38
q' = q''s_x * P
q''s_x = q''o * F_xTs
q' = h * P * (Ts - Tm)                    // Convection rate equation
P = pi * D

// Surface heat flux specification
F_xTs = (1 + alpha * (Ts -Tref))
alpha = 0.2
Tref = 20

// Overall heat rate
// Energy balance on the fluid
q_eb = mdot * cp * (Tmo - Tmi)
Tmi = 20
Tmo = 34.4                 // From initial solve
// Integration of the surface heat flux
q_hf =  q''o * P * INTEGRAL(F_xTs, x)

// Input variables
mdot = 0.1
D = 0.015
h = 3000
q''o = 1.0e4
// L = 2                           // Limit of integration over x
// Tmi = 20                 // Initial condition for integration

// Water property functions :T dependence, From Table A.6
// Units: T(K), p(bars);
xx = 0              // Quality (0=sat liquid or 1=sat vapor)
cp = cp_Tx("Water",Tmm,xx)          // Specific heat, J/kg·K
Tmm = (20 + 34.4) / 2 + 273
```

PROBLEM 8.22

KNOWN: Flow rate of engine oil through a long tube.

FIND: (a) Heat transfer coefficient, $\bar{h}$, (b) Outlet temperature of oil, $T_{m,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Combined entry conditions exist.

PROPERTIES: *Table A-5*, Engine Oil (T_s = 100°C = 373K): $\mu_s = 1.73 \times 10^{-2}$ N·s/m^2; *Table A-5*, Engine Oil $\left(\bar{T}_m = 77^\circ C = 350K\right)$: c_p = 2118 J/kg·K, $\mu = 3.56 \times 10^{-2}$ N·s/m^2, k = 0.138 W/m·K, Pr = 546.

ANALYSIS: (a) The overall energy balance and rate equations have the form

$$q = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) \qquad q = \bar{h} A_s \Delta T_{1m} \tag{1,2}$$

Using Eq. 8.42b, with P = πD, and Eq. 8.6

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}\, c_p} \cdot \bar{h}\right). \tag{3}$$

$$Re_D = \frac{4\dot{m}}{p\, D m} = \frac{4 \times 0.02\ \text{kg/s}}{p \times 3 \times 10^{-3}\text{m} \times 3.56 \times 10^{-2}\text{N} \cdot \text{s/m}^2} = 238.$$

For laminar and combined entry conditions, use Eq. 8.57

$$\overline{Nu}_D = 1.86 \left(\frac{Re_D\, Pr}{L/D}\right)^{1/3} \left(\frac{m}{m_s}\right)^{0.14} = \left(\frac{238 \times 546}{30\text{m}/3 \times 10^{-3}\text{m}}\right)^{1/3} \left(\frac{3.56}{1.73}\right)^{0.14} = 4.83$$

$$\bar{h} = \overline{Nu}_D\, k/D = 4.83 \times 0.138\ \text{W/m} \cdot \text{K}/3 \times 10^{-3}\text{m} = 222\ \text{W/m}^2 \cdot \text{K}. \qquad <$$

(b) Using Eq. (3) with the foregoing value of $\bar{h}$,

$$\frac{\left(100 - T_{m,o}\right)^\circ C}{(100 - 60)^\circ C} = \exp\left(-\frac{p \times 3 \times 10^{-3}\text{m} \times 30\text{m}}{0.02\ \text{kg/s} \times 2118\ \text{J/kg} \cdot \text{K}} \times 222\text{W/m}^2 \cdot \text{K}\right) \quad T_{m,o} = 90.9^\circ C. \qquad <$$

COMMENTS: (1) Note that requirements for the correlation, Eq. 8.57, are satisfied.

(2) The assumption of $\bar{T}_m$ = 77°C for selecting property values was satisfactory.

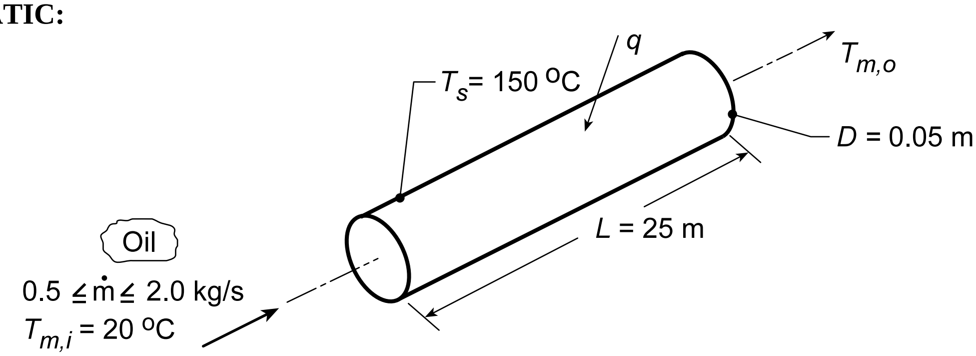
(3) For thermal entry effect only, Eq. 8.56, $\bar{h} = 201\ \text{W/m}^2 \cdot \text{K}$ and $T_{m,o}$ = 89.5°C.

PROBLEM 8.23

KNOWN: Inlet temperature and flowrate of oil flowing through a tube of prescribed surface temperature and geometry.

FIND: (a) Oil outlet temperature and total heat transfer rate, and (b) Effect of flowrate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature drop across tube wall, (2) Negligible kinetic energy, potential energy and flow work effects.

PROPERTIES: *Table A.5*, Engine oil (assume $T_{m,o}$ = 140°C, hence $\overline{T}_m$ = 80°C = 353 K): ρ = 852 kg/m^3, ν = 37.5 × 10^{-6} m^2/s, k = 138 × 10^{-3} W/m·K, Pr = 490, μ = ρ·ν = 0.032 kg/m·s, c_p = 2131 J/kg·K.

ANALYSIS: (a) For constant surface temperature the oil outlet temperature may be obtained from Eq. 8.42b. Hence

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\overline{h}\right)$$

To determine $\overline{h}$, first calculate Re_D from Eq. 8.6,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.5\,kg/s)}{\pi(0.05m)(0.032\,kg/m\cdot s)} = 398$$

Hence the flow is laminar. Moreover, from Eq. 8.23 the thermal entry length is

$$x_{fd,t} \approx 0.05D\,Re_D\,Pr = 0.05(0.05\,m)(398)(490) = 486\,m\,.$$

Since L = 25 m the flow is far from being thermally fully developed. However, from Eq. 8.3, $x_{fd,h} \approx 0.05DRe_D = 0.05(0.05\text{ m})(398) = 1\text{ m}$ and it is reasonable to assume fully developed hydrodynamic conditions throughout the tube. Hence $\overline{h}$ may be determined from Eq. 8.56

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D\,Pr}{1+0.04\left[(D/L)Re_D\,Pr\right]^{2/3}}.$$

With $(D/L)Re_DPr = (0.05/25)398 \times 490 = 390$, it follows that

$$\overline{Nu}_D = 3.66 + \frac{26}{1+2.14} = 11.95.$$

Hence, $\overline{h} = \overline{Nu}_D\dfrac{k}{D} = 11.95\dfrac{0.138\,W/m\cdot K}{0.05\,m} = 33\,W/m^2\cdot K$ and it follows that

Continued...

PROBLEM 8.23 (Cont.)

$$T_{m,o} = 150^\circ C - \left(150^\circ C - 20^\circ C\right)\exp\left[-\frac{\pi(0.05\,m)(25\,m)}{0.5\,kg/s \times 2131\,J/kg \cdot K} \times 33\,W/m^2 \cdot K\right]$$

$T_{m,o} = 35°C$. <

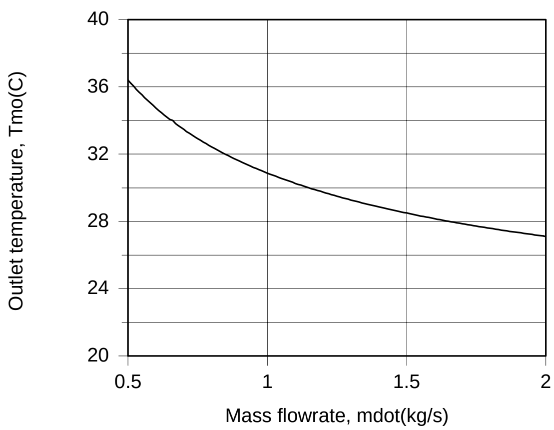
From the overall energy balance, Eq. 8.37, it follows that

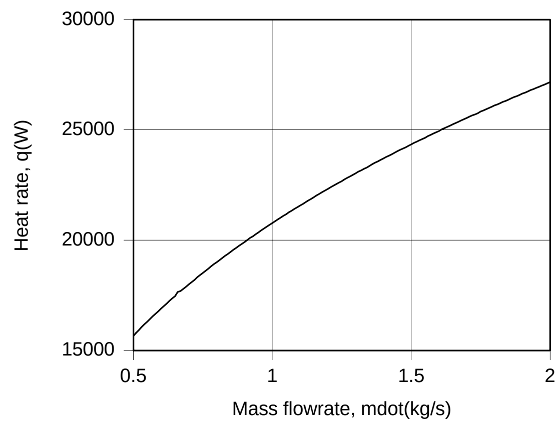
$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) = 0.5\,kg/s \times 2131\,J/kg \cdot K \times (35 - 20)^\circ C$$

$q = 15,980$ W. <

The value of $T_{m,o}$ has been grossly overestimated in evaluating the properties. The properties should be re-evaluated at $\overline{T} = (20 + 35)/2 = 27°C$ and the calculations repeated. Iteration should continue until satisfactory convergence is achieved between the calculated and assumed values of $T_{m,o}$. Following such a procedure, one would obtain $T_{m,o} = 36.4°C$, $Re_D = 27.8$, $\overline{h} = 32.8\ W/m^2 \cdot K$, and $q = 15,660$ W. The small effect of reevaluating the properties is attributed to the compensating effects on Re_D (a large decrease) and Pr (a large increase).

(b) The effect of flowrate on $T_{m,o}$ and q was determined by using the appropriate IHT *Correlations* and *Properties* Toolpads.





The heat rate increases with increasing $\dot{m}$ due to the corresponding increase in Re_D and hence $\overline{h}$. However, the increase is not proportional to $\dot{m}$, causing $\left(T_{m,o} - T_{m,i}\right) = q/\dot{m}c_p$, and hence $T_{m,o}$, to decrease with increasing $\dot{m}$. The maximum heat rate corresponds to the maximum flowrate ($\dot{m} = 0.20$ kg/s).

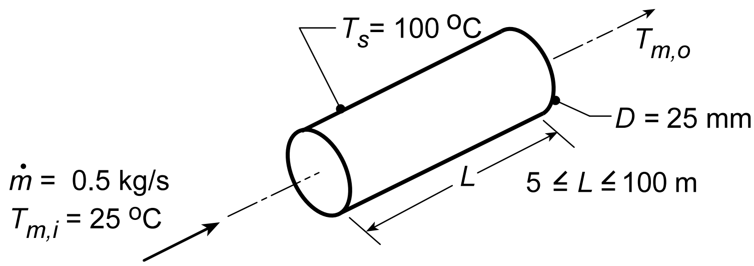
COMMENTS: Note that significant error would be introduced by assuming fully developed thermal conditions and $\overline{Nu}_D = 3.66$. The flow remains well within the laminar region over the entire range of $\dot{m}$.

PROBLEM 8.24

KNOWN: Inlet temperature and flowrate of oil moving through a tube of prescribed diameter and surface temperature.

FIND: (a) Oil outlet temperature $T_{m,o}$ for two tube lengths, 5 m and 100 m, and log mean and arithmetic mean temperature differences, (b) Effect of L on $T_{m,o}$ and $\overline{Nu}_D$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible kinetic energy, potential energy and flow work changes, (3) Constant properties.

PROPERTIES: *Table A.4*, Oil (330 K): c_p = 2035 J/kg·K, μ = 0.0836 N·s/m², k = 0.141 W/m·K, Pr = 1205.

ANALYSIS: (a) Using Eqs. 8.42b and 8.6

$$T_{m,o} = T_s - \left(T_s - T_{m,i}\right)\exp\left(-\frac{\pi DL}{\dot{m}c_p}\bar{h}\right)$$

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.5\,kg/s}{\pi\times 0.025\,m\times 0.0836\,N\cdot s/m^2} = 304.6$$

Since entry length effects will be significant, use Eq. 8.56

$$\bar{h} = \frac{k}{D}\left[3.66 + \frac{0.0688(D/L)Re_D\,Pr}{1+0.04\left[(D/L)Re_D\,Pr\right]^{2/3}}\right] = \frac{0.141\,W/m\cdot K}{0.025\,m}\left[3.66 + \frac{2.45\times 10^4\,D/L}{1+205(D/L)^{2/3}}\right]$$

For L = 5 m, $\bar{h} = 5.64(3.66+17.51) = 119\,W/m^2\cdot K$, hence

$$T_{m,o} = 100^\circ C - \left(75^\circ C\right)\exp\left(-\frac{\pi\times 0.025\,m\times 5\,m\times 119\,W/m^2\cdot K}{0.5\,kg/s\times 2035\,J/kg\cdot K}\right) = 28.4^\circ C \qquad <$$

For L = 100 m, $\bar{h} = 5.64(3.66+3.38) = 40\,W/m^2\cdot K$, $\quad T_{m,o}$ = 44.9°C. $\qquad <$

Also, *for* L = 5 m,

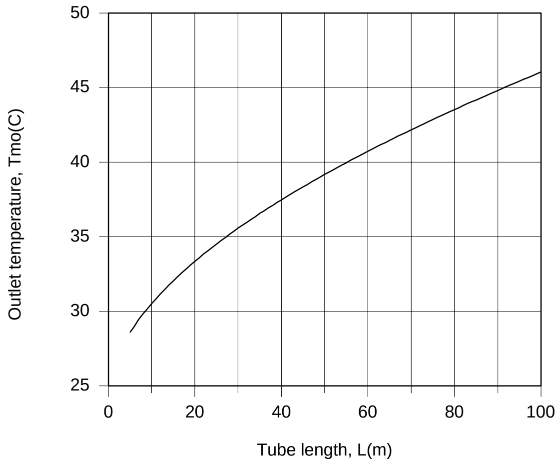
$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ell n\left(\Delta T_o/\Delta T_i\right)} = \frac{71.6-75}{\ell n\,(71.6/75)} = 73.3^\circ C \qquad \Delta T_{am} = \left(\Delta T_o + \Delta T_i\right)/2 = 73.3^\circ C \qquad <$$

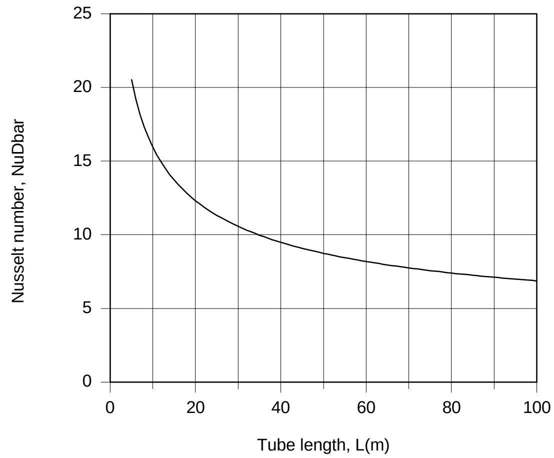
For L = 100 m, $\quad \Delta T_{\ell m} = 64.5^\circ C$, $\quad \Delta T_{am} = 65.1^\circ C \qquad <$

(b) The effect of tube length on the outlet temperature and Nusselt number was determined by using the *Correlations* and *Properties* Toolpads of IHT.

Continued...

PROBLEM 8.24 (Cont.)





The outlet temperature approaches the surface temperature with increasing L, but even for L = 100 m, $T_{m,o}$ is well below T_s. Although $\overline{Nu}_D$ decays with increasing L, it is still well above the fully developed value of $Nu_{D,fd} = 3.66$.

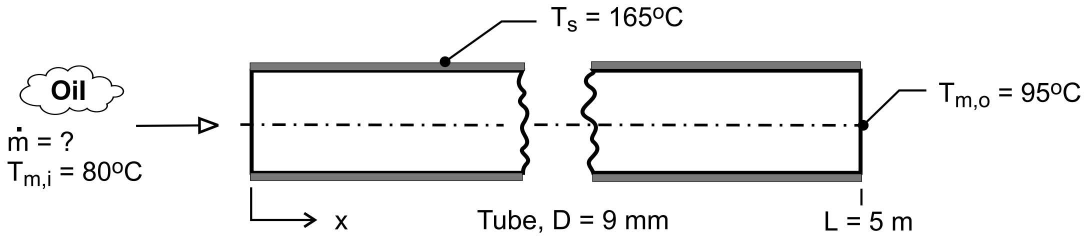
COMMENTS: (1) The average, mean temperature, $\overline{T}_m$ = 330 K, was significantly overestimated in part (a). The accuracy may be improved by evaluating the properties at a lower temperature. (2) Use of ΔT_{am} instead of $\Delta T_{\ell m}$ is reasonable for small to moderate values of $(T_{m,i} - T_{m,o})$. For large values of $(T_{m,i} - T_{m,o})$, $\Delta T_{\ell m}$ should be used.

PROBLEM 8.25

KNOWN: Oil at 80°C enters a single-tube preheater of 9-mm diameter and 5-m length; tube surface maintained at 165°C by swirling combustion gases.

FIND: Determine the flow rate and heat transfer rate when the outlet temperature is 95°C.

SCHEMATIC:



ASSUMPTIONS: (1) Combined entry length, laminar flow, (2) Tube wall is isothermal, (3) Negligible kinetic and potential energy, and flow work, (4) Constant properties.

PROPERTIES: *Table A-5*, Engine oil, new ($T_m = (T_{m,i} + T_{m,o})/2 = 361$ K): $\rho = 847.5$ kg/m^3, c_p = 2163 J/kg·K, $\nu = 2.931 \times 10^{-5}$ m^2/s, k = 0.1879 W/m·K, Pr = 3902; (T_s = 430 K): $\mu_s = 0.047$ N·s/m^2.

ANALYSIS: The overall energy balance, Eq. 8.37, and rate equation, Eq. 8.42b, are

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \tag{1}$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} c_p}\right) \tag{2}$$

Not knowing the flow rate $\dot{m}$, the Reynolds number cannot be calculated. Assume that the flow is laminar, and the combined entry length condition occurs. The average convection coefficient can be estimated using the Sieder-Tate correlation, Eq. 8.57,

$$\overline{Nu}_D = \frac{\bar{h} D}{k} = 1.86\left(\frac{Re_D\, Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \tag{3}$$

where all properties are evaluated at $T_m = (T_{m,i} + T_{m,o})/2$, except for μ_s at the wall temperature T_s. The Reynolds number follows from Eq. 8.6,

$$Re_D = 4\dot{m}/\pi D \mu \tag{4}$$

A tedious trial-and-error solution is avoided by using *IHT* to solve the system of equations with the following result:

Re_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	q(W)	$\dot{m}$ (kg/h)	
251	9.54	146	1432	159	<

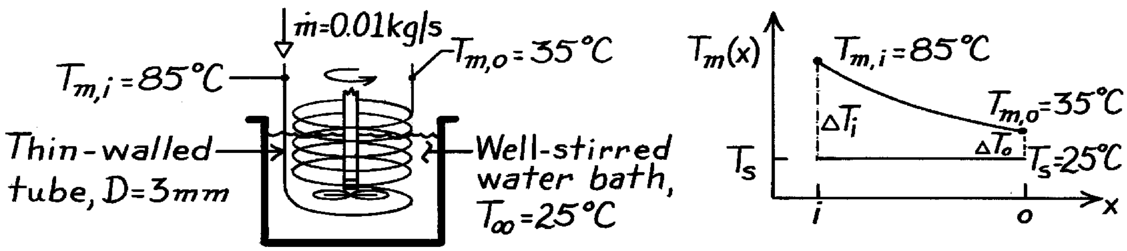
Note that the flow is laminar, and evaluating x_{fd} using Eq. 8.3, find $x_{fd,h}$ = 44 m so the combined entry length condition is appropriate.

PROBLEM 8.26

KNOWN: Ethylene glycol flowing through a coiled, thin walled tube submerged in a well-stirred water bath maintained at a constant temperature.

FIND: Heat rate and required tube length for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Tube wall thermal resistance negligible, (3) Convection coefficient on water side infinite; cooling process approximates constant wall surface temperature distribution, (4) KE, PE and flow work changes negligible, (5) Constant properties, (6) Negligible heat transfer enhancement associated with the coiling.

PROPERTIES: *Table A-5*, Ethylene glycol (T_m = (85 + 35)°C/2 = 60°C = 333 K): c_p = 2562 J/kg·K, $\mu = 0.522 \times 10^{-2}$ N·s/m^2, k = 0.260 W/m·K, Pr = 51.3.

ANALYSIS: From an overall energy balance on the tube,

$$q_{conv} = \dot{m}\, c_p \left(T_{m,o} - T_{m,i}\right) = 0.01\ \text{kg/s} \times 2562\ \text{J/kg}\left(35-85\right)^{\circ}\text{C} = -1281\ \text{W}. \qquad (1) <$$

For the constant surface temperature condition, from the rate equation,

$$A_s = q_{conv} / \bar{h}\Delta T_{\ell m} \qquad (2)$$

$$\Delta T_{\ell m} = \left(\Delta T_o - \Delta T_i\right)/\ell n \frac{\Delta T_o}{\Delta T_i} = \left[\left(35-25\right)^{\circ}\text{C} - \left(85-25\right)^{\circ}\text{C}\right] / \ell n \frac{35-25}{85-25} = 27.9^{\circ}\text{C}. \qquad (3)$$

Find the Reynolds number to determine flow conditions,

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.01\ \text{kg/s}}{\pi \times 0.003\ \text{m} \times 0.522 \times 10^{-2}\ \text{N}\cdot\text{s/m}^2} = 813. \qquad (4)$$

Hence, the flow is laminar and, assuming the flow is fully developed, the appropriate correlation is

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 3.66, \qquad \bar{h} = Nu\frac{k}{D} = 3.66 \times 0.260\frac{\text{W}}{\text{m}\cdot\text{K}}/0.003\text{m} = 317\ \text{W/m}^2\cdot\text{K}. \qquad (5)$$

From Eq. (2), the required area, A_s, and tube length, L, are

$$A_s = 1281\ \text{W}/317\ \text{W/m}^2\cdot\text{K} \times 27.9^{\circ}\text{C} = 0.1448\ \text{m}^2$$

$$L = A_s/\pi D = 0.1448\text{m}^2/\pi\left(0.003\text{m}\right) = 15.4\text{m}. \qquad <$$

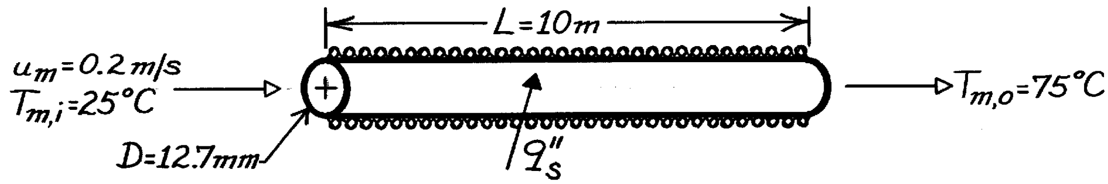
COMMENTS: Note that for fully developed laminar flow conditions, the requirement is satisfied: Gz^{-1} = (L/D) / Re_D Pr = (15.3/0.003) / (813 × 51.3) = 0.122 > 0.05. Note also the sign of the heat rate q_{conv} when using Eqs. (1) and (2).

PROBLEM 8.27

KNOWN: Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

FIND: Surface heat flux and temperatures at x = 0.5 and 10 m.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Negligible potential and kinetic energy changes and axial conduction.

PROPERTIES: Pharmaceutical (given): ρ = 1000 kg/m^3, c_p = 4000 J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.48 W/m·K, Pr = 10.

ANALYSIS: With

$$\dot{m} = r\,\mathrm{VA} = 1000\ \mathrm{kg/m^3}\,(0.2\ \mathrm{m/s})\,p\,(0.0127\ \mathrm{m})^2/4 = 0.0253\ \mathrm{kg/s}$$

Eq. 8.37 yields

$$q = \dot{m}\,c_p\left(T_{m,o} - T_{m,i}\right) = 0.0253\ \mathrm{kg/s}\,(4000\ \mathrm{J/kg\cdot K})\,50\ \mathrm{K} = 5060\ \mathrm{W}.$$

The required heat flux is then

$$q_s'' = q/A_s = 5060\ \mathrm{W}/p\,(0.0127\ \mathrm{m})\,10\ \mathrm{m} = 12{,}682\ \mathrm{W/m^2}.$$ <

With

$$\mathrm{Re_D} = r\,\mathrm{VD}/m = 1000\ \mathrm{kg/m^3}\,(0.2\ \mathrm{m/s})\,0.0127\ \mathrm{m}/2\times10^{-3}\ \mathrm{kg/s\cdot m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05\mathrm{Re_D}\,\mathrm{Pr}\,D = 0.05(1270)10(0.0127\ \mathrm{m}) = 8.06\ \mathrm{m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at x = 10 m, Eq. 8.53 yields

$$h\,(10\ \mathrm{m}) = \mathrm{Nu_{D,fd}}\,(k/D) = 4.36\,(0.48\ \mathrm{W/m\cdot K}/0.0127\ \mathrm{m}) = 164.8\ \mathrm{W/m^2\cdot K}.$$

Hence, from Newton's law of cooling,

$$T_{s,o} = T_{m,o} + \left(q_s''/h\right) = 75^\circ\mathrm{C} + \left(12{,}682\ \mathrm{W/m^2}/164.8\ \mathrm{W/m^2\cdot K}\right) = 152^\circ\mathrm{C}.$$ <

At x = 0.5 m, (x/D)/($\mathrm{Re_D}$Pr) = 0.0031 and Figure 8.9 yields $\mathrm{Nu_D} \approx 8$ for a thermal entry region with uniform surface heat flux. Hence, h(0.5 m) = 302.4 W/m^2·K and, since T_m increases linearly with x, T_m(x = 0.5 m) = $T_{m,i}$ + ($T_{m,o}$ - $T_{m,i}$) (x/L) = 27.5°C. It follows that

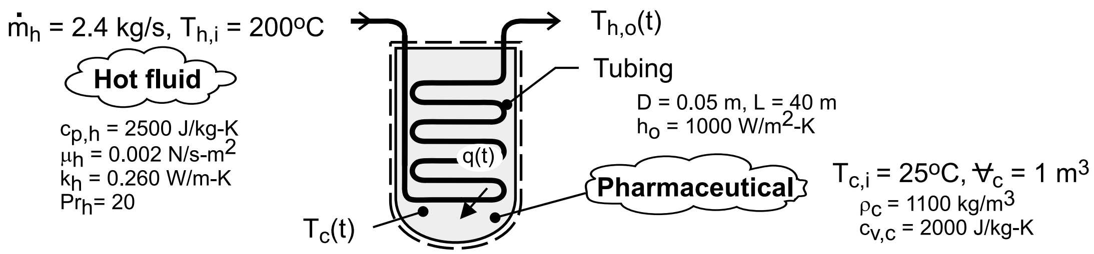
$$T_s\,(x = 0.5\ \mathrm{m}) \approx 27.5^\circ\mathrm{C} + \left(12{,}682\ \mathrm{W/m^2}/302.4\ \mathrm{W/m^2\cdot K}\right) = 69.4^\circ\mathrm{C}.$$ <

PROBLEM 8.28

KNOWN: Inlet temperature, flow rate and properties of hot fluid. Initial temperature, volume and properties of pharmaceutical. Heat transfer coefficient at outer surface and dimensions of coil.

FIND: (a) Expressions for $T_c(t)$ and $T_{h,o}(t)$, (b) Plots of $T_c(t)$ and $T_{h,o}(t)$ for prescribed conditions. Effect of flow rate on time for pharmaceutical to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible heat loss from vessel to surroundings, (3) Pharmaceutical is isothermal, (4) Negligible work due to stirring, (5) Negligible thermal energy generation (or absorption) due to chemical reactions associated with the batch process, (6) Negligible kinetic energy, potential energy and flow work changes for the hot fluid, (7) Negligible tube wall conduction resistance.

ANALYSIS: (a) Performing an energy balance for a control surface about the stirred liquid, it follows that

$$\frac{dU_c}{dt} = \frac{d}{dt}\left(\rho_c \forall_c c_{v,c} T_c\right) = \rho_c V_c c_{v,c}\frac{dT_c}{dt} = q(t) \qquad (1)$$

where, $$q(t) = \dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) \qquad (2)$$

or, $$q(t) = UA_s \Delta T_{\ell m} \qquad (3a)$$

where

$$\Delta T_{\ell m} = \frac{\left(T_{h,i} - T_c\right) - \left(T_{h,o} - T_c\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} = \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)} \qquad (3b)$$

Substituting (3b) into (3a) and equating to (2),

$$\dot{m}_h c_{p,h}\left(T_{h,i} - T_{h,o}\right) = UA_s \frac{\left(T_{h,i} - T_{h,o}\right)}{\ell n\left(\dfrac{T_{h,i} - T_c}{T_{h,o} - T_c}\right)}$$

Hence, $$\ell n\left(\frac{T_{h,i} - T_c}{T_{h,o} - T_c}\right) = \frac{UA_s}{\dot{m}_h c_{p,h}}$$

or, $$T_{h,o}(t) = T_c + \left(T_{h,i} - T_c\right)\exp\left(-UA_s / \dot{m}_h c_{p,h}\right) \qquad (4)$$ <

Substituting Eqs. (2) and (4) into Eq. (1),

Continued …..

$$\rho_c \forall_c c_{v,c} \frac{dT_c}{dt} = \dot{m}_h c_{p,h} \left[T_{h,i} - T_c - \left(T_{h,i} - T_c\right) \exp\left(-UA_s / \dot{m}_h c_{p,h}\right) \right]$$

$$\frac{dT_c}{dt} = \frac{\dot{m}_h c_{p,h} \left(T_{h,i} - T_c\right)}{\rho_c \forall_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right]$$

$$-\int_{T_{c,i}}^{T_c(t)} \frac{dT_c}{\left(T_c - T_{h,i}\right)} = \frac{\dot{m}_h c_{p,h}}{\rho_c \forall_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right] \int_o^t dt$$

$$-\ell n \left(\frac{T_c - T_{h,i}}{T_{c,i} - T_{h,i}} \right) = \frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}} \left[1 - \exp\left(-UA_s / \dot{m}_h c_{p,h}\right)\right] t$$

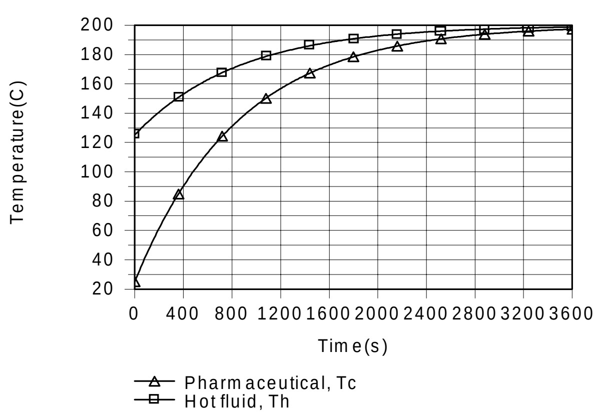
$$T_c(t) = T_{h,i} - \left(T_{h,i} - T_{c,i}\right) \exp\left\{ - \frac{\dot{m}_h c_{p,h} \left[1 - \exp\left(-UA / \dot{m}_h c_{p,h}\right)\right] t}{\rho_c \forall_c c_{v,c}} \right\} \qquad (5) \quad <$$

Eq. (5) may be used to determine $T_c(t)$ and the result used with (4) to determine $T_{h,o}(t)$.

(b) To evaluate the temperature histories, the overall heat transfer coefficient, $U = \left(h_o^{-1} + h_i^{-1}\right)^{-1}$, must first be determined. With $Re_D = 4\dot{m} / \pi D \mu = 4 \times 2.4\,kg/s / \pi\left(0.05m\right) 0.002\,N \cdot s/m^2 = 30,600$, the flow is turbulent and

$$h_i = \frac{k}{D} Nu_D = \frac{0.260\,W/m \cdot K}{0.05m} \left[0.023 (30,600)^{4/5} (20)^{0.3}\right] = 1140\,W/m^2 \cdot K$$

Hence, $U = \left[(1000)^{-1} + (1140)^{-1}\right]^{-1} W/m^2 \cdot K = 532\,W/m^2 \cdot K$. As shown below, the temperature of the pharmaceuticals increases with time due to heat transfer from the hot fluid, approaching the inlet temperature of the hot fluid (and its maximum possible temperature of 200°C) at t = 3600s.



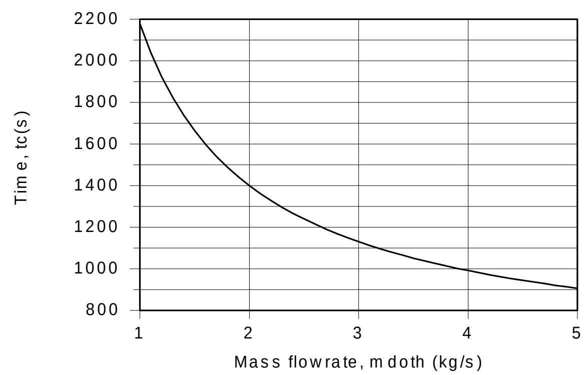
Continued …..

PROBLEM 8.28 (Cont.)

With increasing T_c, the rate of heat transfer from the hot fluid decreases (from 4.49×10^5 W at t = 0 to 6760 W at 3600s), in which case $T_{h,o}$ increases (from 125.2°C at t = 0 to 198.9°C at 3600s). The time required for the pharmaceuticals to reach a temperature of T_c = 160°C is

$$t_c = 1266\text{s}$$ <

With increasing $\dot{m}_h$, the overall heat transfer coefficient increases due to increasing h_i and the hot fluid maintains a higher temperature as it flows through the tube. Both effects enhance heat transfer to the pharmaceutical, thereby reducing the time to reach 160°C from 2178s for $\dot{m}_h = 1\,\text{kg/s}$ to 906s at 5 kg/s.



For $1 \le \dot{m}_h \le 5\,\text{kg/s}$, $12,700 \le Re_D \le 63,700$ and $565 \le h_i \le 2050\,\text{W/m}^2 \cdot \text{K}$.

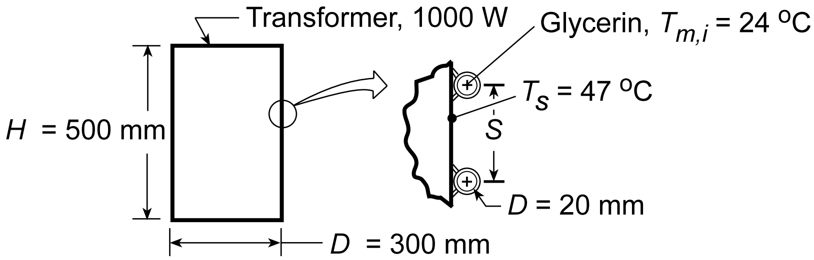
COMMENTS: Although design changes involving the length and diameter of the coil can be used to alter the heating rate, process control parameters are limited to $T_{h,i}$ and $\dot{m}_h$.

PROBLEM 8.29

KNOWN: Tubing with glycerin welded to transformer lateral surface to remove dissipated power. Maximum allowable temperature rise of coolant is 6°C.

FIND: (a) Required coolant rate $\dot{m}$, tube length L and lateral spacing S between turns, and (b) Effect of flowrate on outlet temperature and maximum power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) All heat dissipated by transformer transferred to glycerin, (3) Fully developed flow (part a), (4) Negligible kinetic and potential energy changes, (5) Negligible tube wall thermal resistance.

PROPERTIES: *Table A.5*, Glycerin ($\overline{T}_m \approx 300$ K): $\rho = 1259.9$ kg/m^3, $c_p = 2427$ J/kg·K, $\mu = 79.9 \times 10^{-2}$ N·s/m^2, $k = 286 \times 10^{-3}$ W/m·K, Pr = 6780.

ANALYSIS: (a) From an overall energy balance assuming the maximum temperature rise of the glycerin coolant is 6°C, find the flow rate as

$$q = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \qquad \dot{m} = q/c_p\left(T_{m,o} - T_{m,i}\right) = 1000\,\text{W}/2427\,\text{J/kg}\cdot\text{K}\left(6\,\text{K}\right) = 6.87\times10^{-2}\ \text{kg/s} \quad <$$

From Eq. 8.43, the length of tubing can be determined,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-PL\bar{h}/\dot{m}c_p\right)$$

where P = πD. For the tube flow, find

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times6.87\times10^{-2}\ \text{kg/s}}{\pi\times0.020\,\text{m}\times79.9\times10^{-2}\ \text{N}\cdot\text{s/m}^2} = 5.47$$

which implies laminar flow, and if fully developed,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 3.66 \qquad\qquad \bar{h} = \frac{3.66\times286\times10^{-3}\ \text{W/m}\cdot\text{K}}{0.020\,\text{m}} = 52.3\,\text{W/m}^2\cdot\text{K}$$

$$\frac{(47-30)^\circ\text{C}}{(47-24)^\circ\text{C}} = \exp\left[-\left(\pi\left(0.020\,\text{m}\right)\times52.3\,\text{W/m}^2\cdot\text{K}\times L\right)\Big/\left(6.87\times10^{-2}\ \text{kg/s}\times2427\,\text{J/kg}\cdot\text{K}\right)\right]$$

L = 15.3 m. <

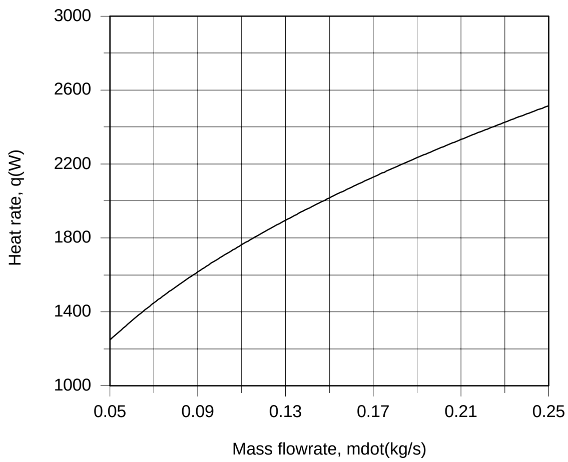
The number of turns of the tubing, N, is N = L/(πD) = (15.3 m)/π(0.3 m) = 16.2 and hence the spacing S will be

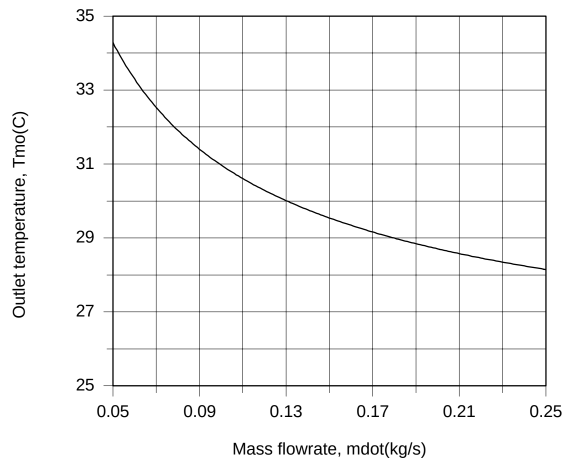
S = H/N = 500 mm/16.2 = 30.8 mm. <

Continued...

PROBLEM 8.29 (Cont.)

(b) Parametric calculations were performed using the IHT *Correlations* Toolpad based on Eq. 8.56 (a thermal entry length condition), and the following results were obtained.





With T_s maintained at 47°C, the maximum allowable transformer power (heat rate) and glycerin outlet temperature increase and decrease, respectively, with increasing $\dot{m}$. The increase in q is due to an increase in $\overline{Nu}_D$ (and hence $\bar{h}$) with increasing Re_D. The value of $\overline{Nu}_D$ increased from 5.3 to 9.4 with increasing $\dot{m}$ from 0.05 to 0.25 kg/s.

COMMENTS: Since $Gz_D^{-1} = (L/D)/Re_D\,Pr$ = (15.3 m/0.02 m)/(5.47 × 6780) = 0.0206 < 0.05, entrance length effects are significant, and Eq. 8.56 should be used to determine $\overline{Nu}_D$.