## The Unit Circle



## TRIGONOMETRIC IDENTITIES

## RECIPROCAL IDENTITIES

$$
\begin{aligned}
& \cot x=\frac{1}{\tan x}=\frac{\cos x}{\sin x} \\
& \csc x=\frac{1}{\sin x} \\
& \sec x=\frac{1}{\cos x}
\end{aligned}
$$

PYTHAGOREAN IDENTITIES

$$
\begin{aligned}
& \begin{aligned}
& \sin ^{2} x+\cos ^{2} x=1 \\
& \sin ^{2} x=1-\cos ^{2} x \\
& \cos ^{2} x=1-\sin ^{2} x \\
& 1+ \tan ^{2} x
\end{aligned}=\sec ^{2} x \\
& 1+\cot ^{2} x=\csc ^{2} x
\end{aligned}
$$

SUM AND DIFFERENCE IDENTITIES
$\sin (x \pm y)=\sin x \cos y \pm \cos x \sin y$ $\cos (x \pm y)=\cos x \cos y \mp \sin x \sin y$ $\tan (x \pm y)=\frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

DOUBLE-ANGLE IDENTITIES

$$
\begin{aligned}
& \sin 2 x=2 \sin x \cos x=\frac{2 \tan x}{1+\tan ^{2} x} \\
& \begin{aligned}
\cos 2 x & =\cos ^{2} x-\sin ^{2} x \\
& =2 \cos ^{2} x-1 \\
& =1-2 \sin ^{2} x=\frac{1-\tan ^{2} x}{1+\tan ^{2} x} \\
\tan 2 x & =\frac{2 \tan x}{1-\tan ^{2} x} \\
\cot 2 x & =\frac{\cot ^{2} x-1}{2 \cot x}
\end{aligned} .
\end{aligned}
$$

HALF-ANGLE IDENTITIES

$$
\begin{aligned}
& \sin \frac{x}{2}= \pm \sqrt{\frac{1-\cos x}{2}} \\
& \cos \frac{x}{2}= \pm \sqrt{\frac{1+\cos x}{2}} \\
& \tan \frac{x}{2}= \pm \sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}
\end{aligned}
$$

