Act 21 *Skills 20 and 21

## Section 7.1: Simplifying and Manipulating Trig Expressions

1. Let $x=7 \cot \theta$. Evaluate $\sin \theta$ in terms of $x$.
2. Use $x=4 \sin \theta$ to express $\frac{7 x^{2}}{\sqrt{16-x^{2}}}$ as a trigonometric function without radicals. Assume $\theta$ is an acute angle.
3. Factor and use identities to simplify completely: $\frac{\tan ^{4} x-\sec ^{4} x}{\sec ^{2} x+\tan ^{2} x}$
4. Multiply and use identities to reduce the number of terms in your final answer: $(1+\tan x)^{2}$

* 5. Let $x=6 \cot \theta$. Write $\sin \theta$ in terms of $x$.
* 6. Use $x=3 \tan \theta$. Write $\frac{6 x}{\sqrt{9+x^{2}}}$ as a trigonometric function without radicals. Assume $\theta$ is an acute angle.

7. Use identities to factor and simplify: $\frac{\csc ^{4} x-\cot ^{4} x}{\csc ^{2} x+\cot ^{2} x}$
8. Use an identity to rewrite $1+\cos x-2 \sin ^{2} x$ in terms of one trig function and then factor the expression.

## Section 7.1: Proving Trig Identities

Prove the following identities by working each side independently. Show every step!

1. $(\cos x+\sin x)(1-\sin x \cos x)=\cos ^{3} x+\sin ^{3} x$
2. $\cot x+\csc x=\frac{\sin x}{1-\cos x}$
3. $\frac{1+\sin t}{\cos t}+\frac{\cos t}{1+\sin t}=2 \sec t$
*4. $\frac{\sec t+1}{\tan t}=\frac{\tan t}{\sec t-1}$
4. $\frac{\cot A+\tan A}{\sec A+\csc A}=\frac{1}{\sin A+\cos A}$
5. $\sec ^{4} x-\tan ^{2} x=\tan ^{4} x+\sec ^{2} x$

## Section 7.2: Sum/Difference Identities

1. Let the coordinates on the unit circle of the standard angle $\alpha$ be $(3 / 5,4 / 5)$ and let $\cos \beta$ be $1 / 6$ with the terminal side of $\beta$ in quadrant IV. Evaluate $\cos (\alpha+\beta)$ exactly.
2. Suppose $\sin \alpha=-3 / 11$ with $\alpha$ a third quadrant angle, and $\cos \beta=1 / 5$, with $\beta$ a first quadrant angle. Evaluate $\cos (\alpha+\beta)$ exactly.
3. Let $\tan \gamma=2 / 5$ with $\gamma$ a first quadrant angle. Evaluate $\sin \left(\gamma-90^{\circ}\right)$ exactly. Use any method you wish.

* 4. Suppose $\sin \alpha=3 / 7$ with $\alpha$ a second quadrant angle, and $\cos \beta=1 / 8$, with $\beta$ a first quadrant angle. Evaluate $\sin (\alpha+\beta)$ exactly.

5. Evaluate $\cos \beta$ and $\sin \left(\frac{\pi}{2}-\beta\right)$ exactly if $\sin \beta>0$ and $\tan \beta=-15 / 4$.

* 6. $\sin \left(\tan ^{-1}(6 / 7)+\cos ^{-1}(-1 / 3)\right)$

7. Rewrite $\sin 23^{\circ} \cos 22^{\circ}+\cos 23^{\circ} \sin 22^{\circ}$ as a trig function of a single angle and then evaluate exactly.

## Section 7.3: Double Angle Identities

1. Let $\sin \delta=5 / 8$, where the terminal side of $\delta$ is in quadrant II. Evaluate $\sin 2 \delta$.
2. Evaluate $\sin (2 x)$ exactly, if $\cos x=8 / 9$ with $3 \pi / 2<x<2 \pi$
3. Prove: $\tan x+\cot x=2 \csc (2 x)$
4. Prove: $\frac{\cos ^{3} x+\sin ^{3} x}{\cos x+\sin x}=\frac{2-\sin 2 x}{2}$

## Half Angle Identities

1. Use a half angle formula to evaluate $\cos (11 \pi / 12)$ exactly.
2. Evaluate $\cos \left(15^{\circ}\right)$ exactly. Use any method you wish.
3. Evaluate $\sin \left(112.5^{\circ}\right)$ exactly.

* 4. $\cos \alpha=1 / 4$ and $\alpha \in\left(270^{\circ}, 360^{\circ}\right)$
a. Evaluate $\cos (\alpha / 2)$
b. Evaluate $\sin (\alpha / 2)$

More Problems
\#1 of this dept. handout http://mathdepartment.us/departmentHandouts/math122
/problems using trig_identities
Simplifying Trig Expressions http://mathdepartment.us/departmentHandouts/math122
/simplifying_trig_expressions

