$$\Delta n = \nu_{q+1} - \nu_q = (q+1)c/2L - qc/2L = c/2L, \qquad (0\text{-}30)$$

so the separation between neighboring modes of a laser is constant and dependent only on the distance between the mirrors in the laser, as shown in **Fig. 0.33**. Since the amount of power obtained from small helium-neon lasers, such as those used for the projects described in this manual, is related to the **length** of the laser, the separation between mirrors is set by the laser manufacturers to produce the required power for the laser. But the band of wavelengths that can maintain stimulated emission is determined by the atomic physics of the lasing medium, in this case, neon. That band does not change radically for most helium-neon laser tubes. Therefore, the **number** of axial modes is mainly dependent on the distance between the mirrors, *L*. The farther apart the mirrors are, the closer are the axial mode frequencies. Thus, long high power helium-neon lasers have a large number of axial modes, whereas, the modest power lasers used in this **Projects in Optics** kit produce only a small number (usually three) of axial modes.

One of the other relations between neighboring laser modes, beside their separation, is that their polarization is orthogonal (crossed) to that of their neighbors (**Fig. 0.34**). Thus, if we examined a three-mode laser with the appropriate tools, we would expect to find that two of the modes would have one polarization and the other would have a perpendicular polarization. This means that, while axial modes are separated in frequency by $c/2L$, modes of the **same polarization** are separated by c/L.

Looking through a diffraction grating at the output of a three-mode laser, we see a single color. High resolution interferometers must be employed to display the axial modes of a laser. However, it is also possible to use a Michelson interferometer to investigate the modes without resorting to high resolution devices. This technique has special applications in the infrared region of the spectrum.

0.6.4 Coherence of a Laser

If we speak of something as being "coherent" in every-day life, we usually mean that it, a painting, a work of music, a plan of action, "makes sense." It "hangs together." There is in this concept the idea of consistency and predictability. The judgement of what is coherent, however, is one of individual taste. What one person may find coherent in heavy metal rock music, another person would hear in rhythm and blues ... or elevator music, perhaps. This concept of coherence as a predictable, consistent form of some idea or work of art has much the same meaning when applied to light

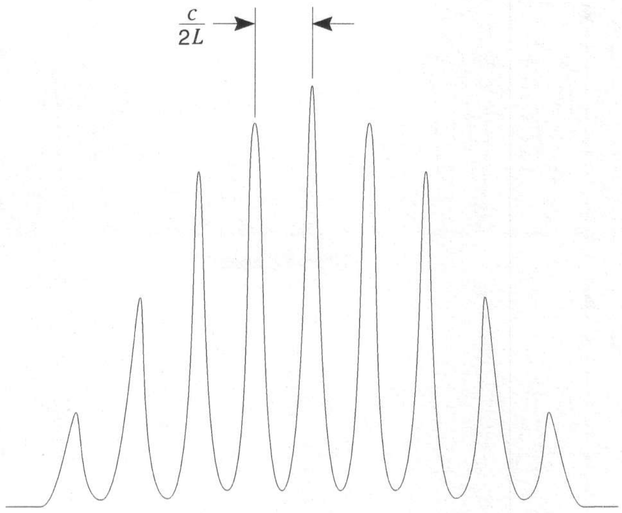

Figure 0.33. Laser mode distribution. Plot of power in laser output as a function of frequency.

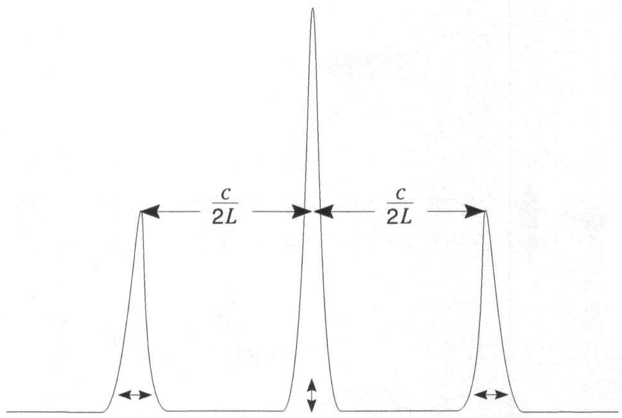

Figure 0.34. Output from a three mode laser. The relative polarization of each mode is indicated at its base.

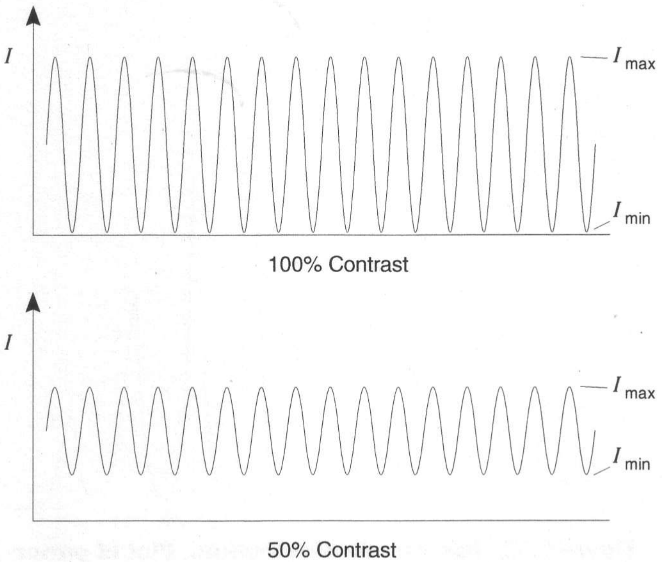

Figure 0.35. Contrast.

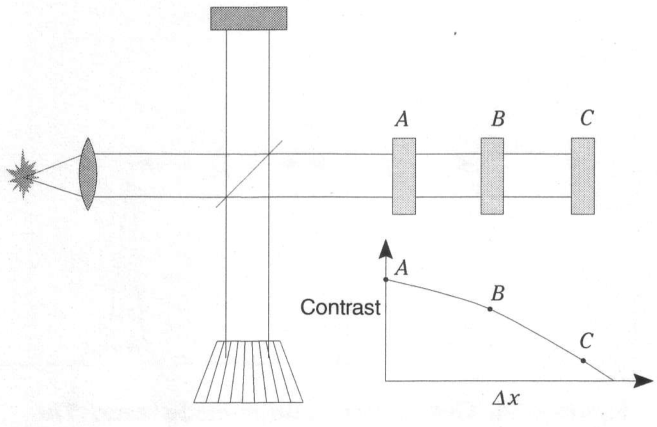

Figure 0.36. Visibility function.

sources. How consistent is a light field from one point to another? How do you make the comparison? The interference of the light beam with itself does the comparing. If there is a constant relation between one point on a laser beam and another point, then the interference of waves separated by that distance should produce a stable interference pattern. If, however, the amplitude or phase or wavelength changes between these two points, the interference, while it is still there at all times, will constantly vary with time. This unstable interference pattern may still exhibit fringes, but the fringes will be washed out. This loss of visibility of fringes as a function of the distance between the points of comparison is measure of the coherence of the light. This visibility can be measured by the **contrast** of the interference fringes. The contrast is defined by

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{0-31}$$

where $I_{\max}$ is the irradiance of the bright interference fringes and $I_{\min}$ is the irradiance of the dark interference fringes (**Fig. 0.35**). This contrast is determined by passing the light from the source through a Michelson interferometer with unequal arms. By changing the path length difference between the arm in the interferometer, the visibility of the fringes as a function of this difference can be recorded. From these observations, the measurement of the coherence of a source can be done using a Michelson interferometer.

If a source were absolutely monochromatic, there would be no frequency spread in its spectrum. That is, its frequency bandwidth would be zero. For this to be true, all parts of the wave exhibit the same sinusoidal dependence from one end of the wave to the other. Thus, a truly monochromatic wave would never show any lack of contrast in the fringes, no matter how large of a path length difference was made. But all sources, even laser sources contain a distribution of wavelengths. Therefore, as the path length difference is increased, the wavefront at one point on the beam gets out of phase with another point on the beam. A measure of the distance at which this occurs is the coherence length l_c of a laser. It is related to the frequency bandwidth of a laser by

$$\Delta\nu = c\,/l_c \tag{0-32}$$

Any measurement of the coherence length of a light source by observation of the visibility of fringes from a Michelson interferometer will yield information on the bandwidth of that source and, therefore, its coherence. For example, suppose the source is a laser with some broadening. As the length of the one of the arms in a

Michelson interferometer, as shown in **Fig. 0.36**, becomes unequal (mirror moved from A to B), the one part of a wave will interfere with another part that is delayed by a time equal to the difference in path length divided by the speed of light. Eventually the waves begin to get out of step and the fringe contrast begins to fall because the phase relations between the two waves is varying slightly due to the spread in frequencies in the light. The greater the broadening, the more rapidly the visibility of the fringes will go to zero.

One particularly interesting case consists of a source with only a few modes present as is the case for the three-mode helium-neon laser discussed above. Because only light of the same polarization can interfere, there will be two modes (λ_1, λ_3) in the laser that can interfere with each other. The third mode (λ_2) with orthogonal polarization is usually eliminated by passing the output of the laser through a polarizer. With the interferometer mirrors set at equal path length there are two sets of fringes, one from each mode. Since the path length difference is zero, these two sets of high contrast fringes overlap each other. But as the path length increases, the fringes begin to get out of step. Until, finally, the interference maximum of one set of fringes overlaps the interference minimum of the other set of fringes and the fringe contrast goes to zero. The calculation of this condition is fairly simple. The condition for an interference maximum is given by

$$L_1 - L_2 = m\lambda/2 \qquad m = \text{an integer} \tag{0-33}$$

and for an interference minimum by

$$L_1 - L_2 = m\lambda/4 \qquad m = \text{odd integers} \tag{0-34}$$

If we assume that the change in path length is from zero path length to the point where the visibility first goes to zero, then for one wavelength, λ_1,

$$L_1 - L_2 = m\lambda_1/2 \qquad m = \text{an integer} \qquad \text{(0-35)}$$

and for the other mode with the same polarization, there is a minimum.

$$L_1 - L_2 = m\lambda_3/2 + \lambda_3/4 \qquad \text{(0-35)}$$

Equating these two expressions and rearranging terms, gives

$$m\lambda_1/2 - m\lambda_3/2 = m(\lambda_1 - \lambda_3)/2 = \lambda_3/4. \qquad \text{(0-35)}$$

or

$$m\Delta\lambda = \lambda_3/2$$

Wavelength separation can be expressed as a frequency separation by Δv

$$\Delta\lambda = \lambda\Delta v/v \tag{0-36}$$

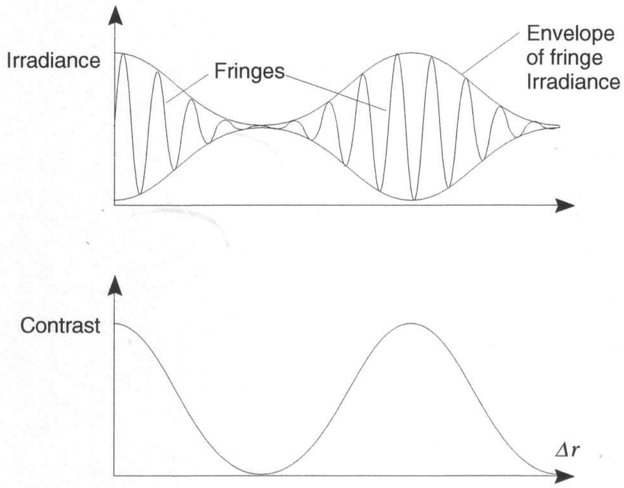

Figure 0.37. Visibility function for two mode system.

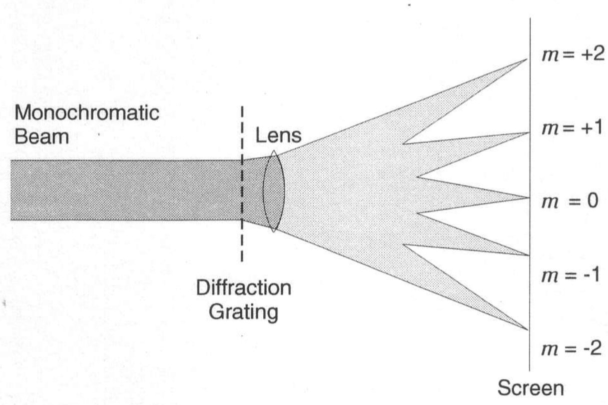

Figure 0.38. Diffraction orders.

where λ and v are the average values in the intervals $\Delta\lambda$ and Δv. Inserting this expression for $\Delta\lambda$, we obtain

$$\Delta v = v/2m\,. \tag{0-37}$$

The integer m is an extremely large number in most cases and is not easily determined, but it is related to the average wavelength of the source by $L_1 - L_2 = m\lambda/2$. If we set $\Delta L = L_1 - L_2$, solve for m and insert in the expression for Δv,

$$\Delta v = v/2m = \lambda v/4\Delta L = c/4\Delta L, \tag{0-38}$$

since $\lambda v = c$.

Thus the frequency separation between modes can be measured by determining the path length difference when the two interference fringe patterns are out of step with one another, causing the visibility to go to zero, as depicted in **Fig. 0.37**. It can also be demonstrated that there are additional minima in the visibility at $\Delta v = 3c/4\Delta L$, $5c/4\Delta L$, etc. Visibility maxima occur halfway between these minima as the two fringes patterns get back into step. In **Project#7**, this effect will enable you to determine the mode separations for the laser used in these projects. What has been derived here is a simple case of a much more involved application of this technique. It is possible to measure the fringe contrast as a function of mirror position (called an interferogram) and store it in the memory of a computer. It has been shown that a mathematical transformation (the same Fourier Transform that will be discussed in the next section) of the visibility function yields the frequency spectrum of the source.

While it might be considered difficult, the advent of powerful computers has reduced the cost and enhanced the utility of this technique, particularly in the far infrared part of the spectrum. These devices are known as Fourier transform spectrometers.

0.7 Abbe Theory of Imaging

The earlier discussion of imaging depended upon tracing a series of rays to determine the location and size of the image. It was shown that only a few rays were needed. This approach ignores the possibilities that the source could be monochromatic and sufficiently coherent that diffraction and interference effects could play a part in the formation of an image. What we will describe and then demonstrate in **Project #10**, is that after the light that will form an image has traversed the lens, we can intervene and change the image in very special ways. This approach to imaging has found use in a number of applications in modern optics. To begin to understand this concept, we need to review briefly the diffraction grating discussed in **Section 0.4.3**, since the grating is one of the simplest illustrations of this new way of thinking about imaging. Consider a diffraction grating consisting of a series of equally spaced, narrow absorbing and transmitting (black and white) bands. It is possible to determine mathematically not only the directions of the diffracted orders

$$\sin\theta_m = m\,\lambda/d \qquad m = \text{an integer}, \tag{0-39}$$

but also the relative irradiances of the diffracted spots to one another. If we insert a lens after the diffraction grating, we can relocate the orders of the diffraction grating from infinity to the back focal plane of the lens **(Fig. 0.38)**. We will see how this can be used to understand imaging.

0.7.1 Spatial Frequencies

We are used to the idea of repetitions in time. Electrical and audio sources of signals with single frequencies, particularly as they relate to sound are used to test equipment for their response. A good high fidelity system will reproduce a wide range of frequencies ranging from the deep bass around 20 Hz (cycles per second), that is as much felt as it is heard, to the nearly impossible to hear 15,000 Hz, depending on how well you have treated your ears during life. As noted earlier, the frequency of the electromagnetic field determines whether the radiation is visible to the eye. Again, this periodic variation in the electric field takes place in time. Just as it is possible to speak about variation of electrical waves and sound with time, in optics, variations in space can be expressed as **spatial frequencies**. These are usually expressed in cycles/mm (or mm^{-1}). They indicate the rapidity with which an object or image varies in space instead of time. An example that shows a number of spatial frequencies is given in **Fig. 0.39.**

As in the case of many sounds and electrical signals, most spatially repetitive patterns do not consist of a single frequency, but as a musical chord, are made up of some fundamental frequency plus its overtones, or higher harmonics. The discussion of spatial frequencies in optics is based on some interesting, but relatively complicated mathematics. You may want to read this section once to get the general ideas, then come back later after you have done **Project #10**. Certainly, here is a case where hands-on work will improve your understanding of the discussion of the subject.

An example of an object with a few spatial frequencies is the diffraction grating. If the grating just discussed consisted of a sinusoidal variation, as shown in **Fig. 0.40(a)**, there would only be a zero order and the first

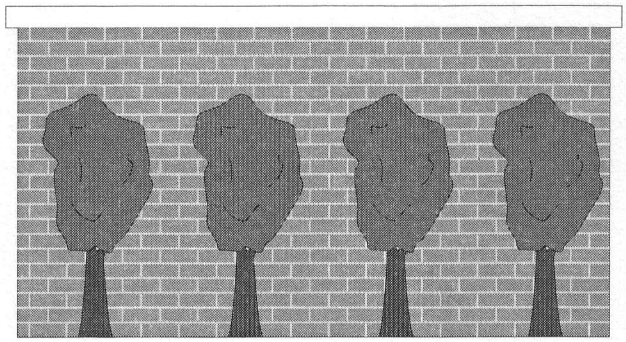

Figure 0.39. Spatial frequencies in an object.

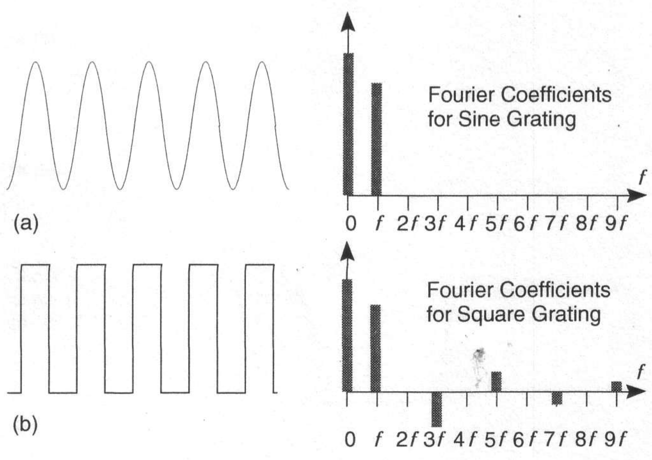

Figure 0.40. Sinusoidal grating versus black and white grating (Fourier analysis).

orders ($m = \pm 1$). As repetitive patterns depart from sinusoidal, additional diffraction orders appear and in the case of the black and white grating, a whole series of diffraction orders are present **(Fig. 0.40(b))**.

All of this can be expressed mathematically in terms of **Fourier (Four-ee-ay) Theory**. We will not go into the mathematical expression of the theory, but only graphically express the result as simply as possible.

Any periodic (repeating) function can be expressed as a series of sine and cosine functions consisting of the fundamental periodic frequency (f) and its higher harmonics (those frequencies that are multiples of the fundamental frequency, f ($2f$, $3f$, $4f$, ...) The amount that each frequency contributes to the original function can be calculated using some standard integral calculus expressions. The decomposition of the periodic pattern into its harmonics is referred to as **Fourier Analysis**. This analysis determines the amplitude of each harmonic contribution to the original function and its phase relative to the fundamental (in phase or 180° out of phase).

The procedure can be, in a sense, reversed. If a pattern at the fundamental frequency is combined with the appropriate amounts of the higher harmonics, it is possible to approximate any function with a repetition frequency of the fundamental. This is referred to as **Fourier Synthesis**. To completely synthesize a function such as our example of an alternating black and white grating, an infinite number of harmonics would be needed. If only frequencies up to some specific value are used, the synthesized function will resemble the function, but it will have edges that are not as sharp as the original. A simple example **(Fig. 0.41)** using only a fundamental and two harmonics shows the beginning of the synthesis of a square wave function, similar to our black and white grating. What you will be investigating in **Project #10** are optical techniques that use Fourier analysis and synthesis in creating images.

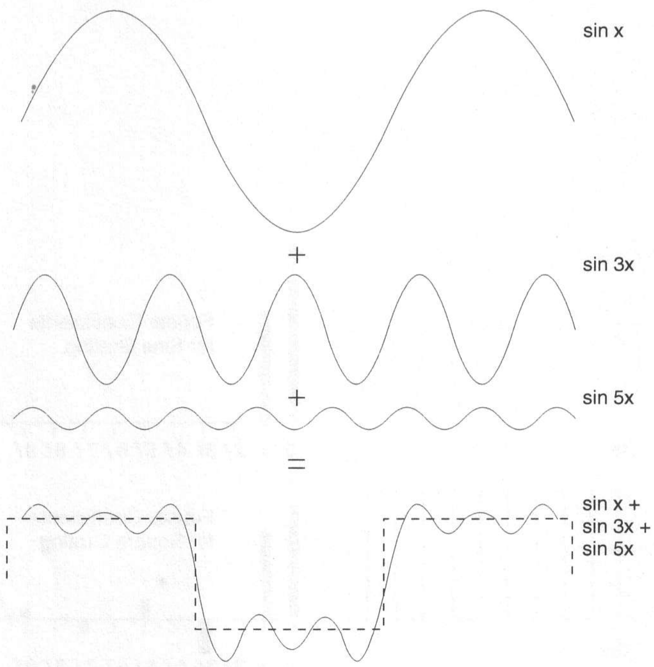

Figure 0.41. Fourier synthesis.

0.7.2 Image Formation

If the black and white grating is illuminated with plane waves of monochromatic light, a number of diffraction orders will be generated by the grating. These plane wave beams diffracted at different angles given by Eq. 0-39, can be focused with a lens located behind the diffraction grating, as shown in **Fig. 0.42**. The focused spots have intensities that are proportional to the square of the amplitudes that we could calculate for this diffraction grating. In effect, the laser plus lens combination serves as an optical Fourier analyzer for a diffractive object.