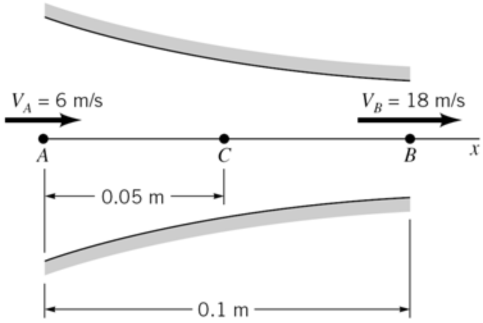
1. The fluid velocity along the x axis shown in figure changes from 6 m/s at point A to 18 m/s at point B. It is also known that the velocity is a linear function of distance along the streamline. Determine the acceleration at points A, B, and C. Assume steady flow.



Sol) From the definition of acceleration $\vec{a}$,

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} \qquad \text{with } u = u(x),\ v = 0, \text{ and } w = 0 \ \text{(flow along the } x \text{ axis)}$$

☞ $$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{i} = u\frac{\partial u}{\partial x}\hat{i} \qquad \text{(Steady flow)}$$

Since u is a linear function of x, $u = ax + b$ where a and b are constant

At point A, $u_A = a(0) + b = b = 6$ (m/s)

At point B, $u_B = a(0.1) + b = 0.1a + 6 = 18$ (m/s) ☞ $a = 120$

Thus, $u = 120x + 6$ (m/s)

From the equation of acceleration,

$$\vec{a} = u\frac{\partial u}{\partial x}\hat{i} = (120x + 6)\frac{\partial}{\partial x}(120x + 6)\hat{i} = 120(120x + 6)\hat{i} \ \ (\text{m/s}^2)$$

Finally,

for $x_A = 0$, $\vec{a}_A = 120(120 \cdot (0) + 6)\hat{i} = 720\hat{i}$ (m/s^2)

for $x_C = 0.05$m, $\vec{a}_C = 120(120 \cdot (0.05) + 6)\hat{i} = 1440\hat{i}$ (m/s^2)

for $x_B = 0.1$m, $\vec{a}_B = 120(120 \cdot (0.1) + 6)\hat{i} = 2160\hat{i}$ (m/s^2) **(ANSWER)**

2. A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. The flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$.
   (a) Determine the local acceleration, the convective acceleration, and the acceleration at points (1) and (2).
   (b) Repeat Prob. (a) with the assumption that the flow is not steady, but at the time when $V_1 = 10$ m/s and $V_2 = 25$ m/s, it is known that $\partial V_1 / \partial t = 20$ m/s^2 and $\partial V_2 / \partial t = 60$ m/s^2.

Sol)

a) With $u = ax + b$, $v = 0$, and $w = 0$, ($\vec{V}$: Linear velocity along x-axis)

the acceleration can be written as

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = u\frac{\partial u}{\partial x}\hat{i} = a_x\hat{i}$$

because $\vec{V}$: Independent to time and $\frac{\partial \vec{V}}{\partial y} = \frac{\partial \vec{V}}{\partial z} = 0$

Since $V_1 = u_1 = 10$ m/s at $x = 0$ ☞ $10 = a(0) + b$ ☞ $b = 10$

and $V_2 = u_2 = 25$ m/s at $x = 1$ ☞ $25 = a(1) + 10$ ☞ $a = 15$

That is, $u = 15x + 10$ m/s, so that the acceleration becomes

$$\vec{a} = (15x + 10)(15)\hat{i}$$

- Local acceleration: $\frac{\partial \vec{V}}{\partial t} = 0$
- Convective acceleration: $\vec{V} \cdot \nabla \vec{V} = u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} = (225x + 150)\hat{i}$ m/s^2
- Acceleration at (1) & (2): $\vec{a} = (150)\hat{i}$ m/s^2 at $x = 0$
  $\vec{a} = (225 + 150)\hat{i} = 375\hat{i}$ m/s^2 at $x = 1$ **(ANSWER)**

b) The acceleration

$$\vec{a} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{i} = a_x\hat{i}$$

This means, $\dfrac{\partial u}{\partial t} = \dfrac{\partial}{\partial t}(ax+b) \neq 0$ because of unsteadiness ☞ $a = a(t)$ and $b = b(t)$

At the time ($t = t_0$) when $V_1 = 10$ m/s (at $x = 0$) and $V_2 = 25$ m/s (at $x = 1$),

i.e. $V_1 = u_1 = a(t_0)(0) + b(t_0) = 10$ ☞ $b(t_0) = 10$

and $V_2 = u_2 = a(t_0)(1) + 10 = 25$ ☞ $a(t_0) = 15$

Also $\dfrac{\partial V_1}{\partial t} = 20$ m/s^2 at $x = 0$ and $t = t_0$ (Local acceleration, **ANSWER**)

$\dfrac{\partial V_2}{\partial t} = 60$ m/s^2 at $x = 1$ and $t = t_0$ (Local acceleration, **ANSWER**)

In addition,

- Convective acceleration:

At $x = 0$ and $t = t_0$ $\quad \vec{V}\cdot\nabla\vec{V} = u_1\dfrac{\partial u_1}{\partial x} = 15[15(0)+10]\hat{i} = 150\hat{i}$ m/s^2 (**ANSWER**)

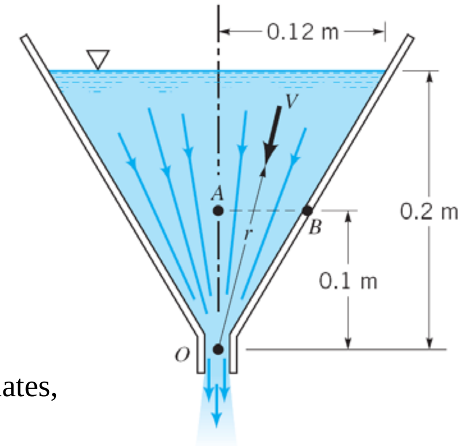
At $x = 1$ and $t = t_0$ $\quad \vec{V}\cdot\nabla\vec{V} = u_2\dfrac{\partial u_2}{\partial x} = 15[15(1)+10]\hat{i} = 375\hat{i}$ m/s^2 (**ANSWER**)

- Fluid acceleration:

At $x = 0$ and $t = t_0$ $\quad a = \left(\dfrac{\partial u_1}{\partial t} + u_1\dfrac{\partial u_1}{\partial x}\right)\hat{i} = (20+150)\hat{i} = 170\hat{i}$ m/s^2 (**ANSWER**)

At $x = 1$ and $t = t_0$ $\quad \vec{a} = \left(\dfrac{\partial u_2}{\partial t} + u_2\dfrac{\partial u_2}{\partial x}\right)\hat{i} = (60+375)\hat{i} = 435\hat{i}$ m/s^2 (**ANSWER**)

3. Water flows steadily through the funnel shown in figure. Throughout most of the funnel the flow is approximately radial (along rays from O) with a velocity of $V = c/r^2$, where r is the radial coordinate and c is a constant. If the velocity is 0.4 m/s when $r = 0.1$ m, determine the acceleration at points A and B.



Sol) From the equation of acceleration in the streamline coordinates,

$$\vec{a} = a_n \hat{n} + a_s \hat{s}$$

$$\text{where} \quad a_n = \frac{V^2}{R} = 0 \quad \text{since } R = \infty \quad \text{(For the straight streamline)}$$

$$a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r} \qquad \text{where } V = \frac{c}{r^2} \text{ and } \hat{s} = -\hat{r}$$

Since $V = 0.4$ m/s when $r = 0.1$ m,

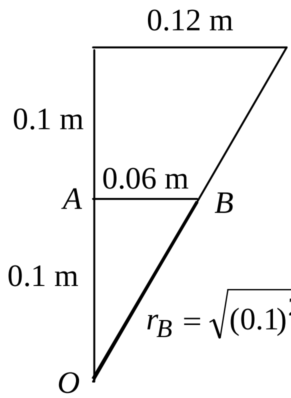
$$c = Vr^2 = (0.4)(0.1)^2 = 4\times10^{-3} \ \text{m}^3/\text{s} \qquad ☞ \qquad V = \frac{4\times10^{-3}}{r^2} \ \text{m/s}$$

Thus,

$$a_s = -\left(\frac{c}{r^2}\right)\frac{\partial}{\partial r}\left(\frac{c}{r^2}\right) = -\left(\frac{c}{r^2}\right)\left(-\frac{2c}{r^3}\right) = \frac{2c^2}{r^5} \ \text{m/s}^2$$

At point A, $$a_s = \frac{2(4\times10^{-3})^2}{(0.1)^5} = 3.20 \ \text{m/s}^2$$

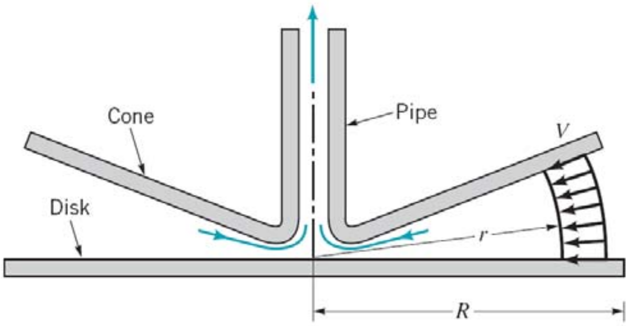
At point B, $$a_s = \frac{2(4\times10^{-3})^2}{(0.1167)^5} = 1.48 \ \text{m/s}^2$$



(ANSWER)

$$r_B = \sqrt{(0.1)^2 + (0.06)^2} = 0.1167 \ \text{m}$$

4. Air flows from a pipe into the region between a circular disk and a cone as shown in figure. The fluid velocity in the gap between the disk and the cone is closely approximated by $V = V_0 R^2 / r^2$, where R is the radius of the disk, r is the radial coordinate, and V_0 is the fluid velocity at the edge of the disk. Determine the acceleration for $r = 0.5$ and 2 ft if $V_0 = 5$ ft/s and $R = 2$ ft.



Sol) From the equation of acceleration in the streamline coordinates,

$$\vec{a} = a_n \hat{n} + a_s \hat{s}$$

where $a_n = \dfrac{V^2}{R} = 0$ since $R = \infty$ (For the straight streamline)

$$a_s = V\frac{\partial V}{\partial s} = -V\frac{\partial V}{\partial r} \quad \text{where } V = \frac{V_0 R^2}{r^2} \text{ and } \hat{s} = -\hat{r}$$

Thus,

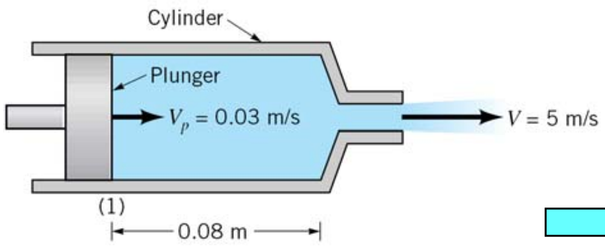
$$a_s = -\left(\frac{V_0 R^2}{r^2}\right)\frac{\partial}{\partial r}\left(\frac{V_0 R^2}{r^2}\right) = -\left(\frac{V_0 R^2}{r^2}\right)\left(-\frac{2V_0 R^2}{r^3}\right) = \frac{2{V_0}^2 R^4}{r^5} \text{ ft/s}^2$$

$$= \frac{2(5)^2(2)^4}{r^5} = \frac{800}{r^5} \text{ ft/s}^2$$

At point $r = 0.5$ ft, $\quad a_s = \dfrac{800}{(0.5)^5} = 25{,}600$ ft/s^2

At point $r = 2$ ft, $\quad a_s = \dfrac{800}{(2)^5} = 25$ ft/s^2 **(ANSWER)**

5. Water is squirted from a syringe with a speed of $V = 5\,\text{m/s}$ by pushing in the plunger with a speed of $V_p = 0.03\,\text{m/s}$ as shown in figure. The surface of the deforming control volume consists of the sides and end of the cylinder and the end of the plunger. The system consists of the water in the syringe at $t = 0$ when the plunger is at section (1) as shown. Make a sketch to indicate the control surface and the system when $t = 0.5$.



Sol) During the $t = 0.5\,\text{s}$ time interval the plunger moves $l_p = V_p \delta t = (0.03)(0.5) = 0.015\,\text{m}$ and the water initially at the exit moves $l_w = V\delta t = (5)(0.5) = 2.5\,\text{m}$. The corresponding control surfaces and the systems at $t = 0$ and $t = 0.5\,\text{s}$ shown in the figure below.