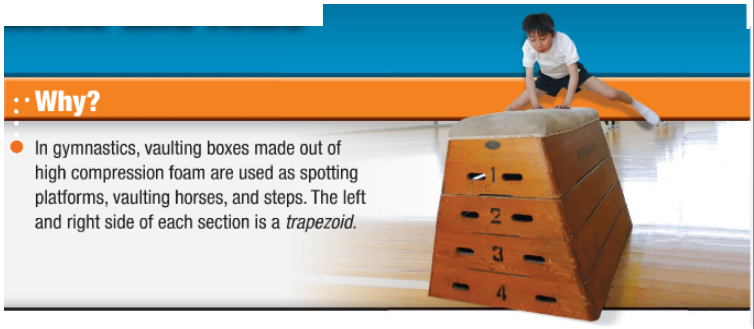
6-6 Trapezoids and Kites

Learning Targets:
I can apply properties of trapezoids
I can apply properties of kites



What characteristic differentiates a trapezoid from a parallelogram?

Why would a vaulting horse shaped like a trapezoid be more stable than one that is shaped like a rectangle?

Look at the illustration of the four staked vaulting boxes. What conjectures can you make about the angles of the trapezoids that form the end of the box?

Jan 23-7:07 AM

① $(x^2y - 3x^2) + (-2y + 6)$
$x^2(y-3) - 2(y-3)$
$(y-3)(x^2-2)$

② $g^3 - 25g$
$g(g^2-25)$
$g(g-5)(g+5)$

③ $12n^3 - 8n$
$4n(3n^2-2)$

④ $x^2 - 4x + 3$
$(x-3)(x-1)$
1, 3

⑤ $7x^2 - 30x + 8$
$x^2 - 30x + 56$
1, 56
2, 28
$(x-2)(x-28)$
$(7x-2)(x-4)$

Feb 5-11:37 AM

(9) $3x-1 = x+9$

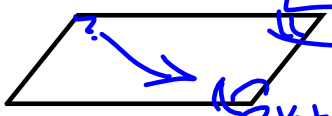
$2x = 10$

$x = 5$

$3(5)-1 = 14$

28

(11)



$2x-7+2x+3=180$

$4x-4=180$

$4x=184$

$x=46$

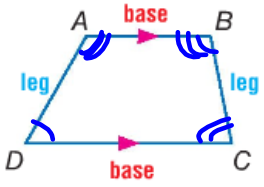
$2x+3$

$92+3=95$

Feb 5-11:47 AM

Trapezoid: a quadrilateral with exactly one pair of parallel sides.

Bases: the parallel sides
Legs: the nonparallel sides
Base angles: formed by the base and the one of the legs



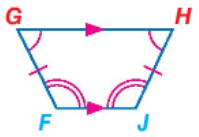
Isosceles trapezoid: a trapezoid that has congruent legs

Jan 23-7:17 AM

Theorems Isosceles Trapezoids

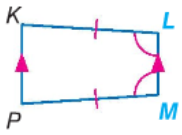
6.21 If a trapezoid is isosceles, then each pair of base angles is congruent.

Example If trapezoid *FGHJ* is isosceles, then $\angle G \cong \angle H$ and $\angle F \cong \angle J$.



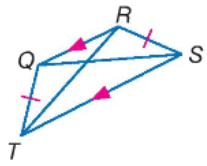
6.22 If a trapezoid has one pair of congruent base angles, then it is an isosceles trapezoid.

Example If $\angle L \cong \angle M$, then trapezoid *KLMP* is isosceles.



6.23 A trapezoid is isosceles if and only if its diagonals are congruent.

Example If trapezoid *QRST* is isosceles, then $\overline{QS} \cong \overline{RT}$. Likewise, if $\overline{QS} \cong \overline{RT}$, then trapezoid *QRST* is isosceles.

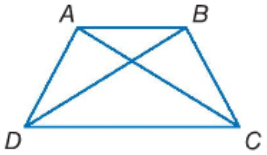


Jan 23-7:27 AM

Proof Part of Theorem 6.23

Given: *ABCD* is an isosceles trapezoid.

Prove: $\overline{AC} \cong \overline{BD}$



$ABCD$ is an isosceles trapezoid. — Given

- $\overline{DC} \cong \overline{CD}$ — Reflexive Property
- $\overline{AD} \cong \overline{BC}$ — Def. Isos. Trapezoid
- $\angle ADC \cong \angle BCD$ — Base $\measuredangle$s of trapezoid are $\cong$.

$\triangle ADC \cong \triangle BCD$ — SAS

$\overline{AC} \cong \overline{BD}$ — CPCTC

Jan 23-7:30 AM

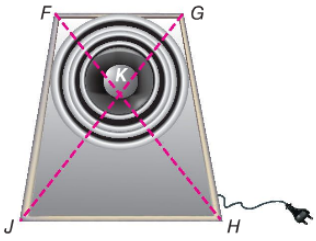
Real-WorldLink

Speakers are amplifiers that intensify sound waves so that they are audible to the unaided ear. Amplifiers exist in devices such as televisions, stereos, and computers.

Source: How Stuff Works

Real-World Example 1 Use Properties of Isosceles Trapezoids

MUSIC The speaker shown is an isosceles trapezoid. If $m\angle FJH = 85$, $FK = 8$ inches, and $JG = 19$ inches, find each measure.



a. $m\angle FGH$

Since $FGHJ$ is an isosceles trapezoid, $\angle FJH$ and $\angle GHJ$ are congruent base angles. So, $m\angle GHJ = m\angle FJH = 85$.

Since $FGHJ$ is a trapezoid, $\overline{FG} \parallel \overline{JH}$.

$m\angle FGH + m\angle GHJ = 180$	Consecutive Interior Angles Theorem
$m\angle FGH + 85 = 180$	Substitution
$m\angle FGH = 95$	Subtract 85 from each side.

b. KH

Since $FGHJ$ is an isosceles trapezoid, diagonals $\overline{FH}$ and $\overline{JG}$ are congruent.

$FH = JG$	Definition of congruent
$FK + KH = JG$	Segment Addition
$8 + KH = 19$	Substitution
$KH = 11$ cm	Subtract 8 from each side.

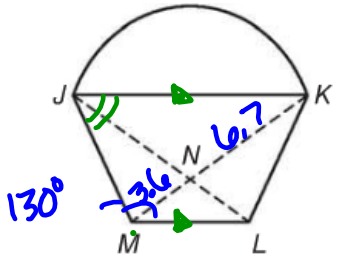
Jan 23-7:31 AM

Example 1:

Each side of the basket shown is an isosceles trapezoid. If m<*JML* = 130, *KN* = 6.7 feet, and *MN* = 3.6 feet, find each measure.

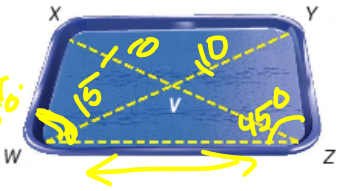
1. m<*MJK* $130 + x = 180$, $-130 \quad -130$, $x = 50°$

2. *JL* $= 3.6 + 6.7 = 10.3$



Jan 23-7:35 AM

To save space at a square table, cafeteria trays often incorporate trapezoids into their design. If *WXYZ* is an isosceles trapezoid and m<*YZW* = 45, *WV* = 15 centimeters, and *VY* = 10 centimeters, find each measure.



3. m< *XWZ* = 45

4. m<*WXY* 180 − 45 = 135° consecutive interiors

5. *XZ* = 15 + 10 = 25

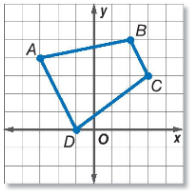
6. *XV* = 10

Jan 23-7:39 AM

Example 2 Isosceles Trapezoids and Coordinate Geometry

COORDINATE GEOMETRY **Quadrilateral *ABCD* has vertices *A*(−3, 4), *B*(2, 5), *C*(3, 3), and *D*(−1, 0). Show that *ABCD* is a trapezoid and determine whether it is an isosceles trapezoid.**

Graph and connect the vertices of *ABCD*.

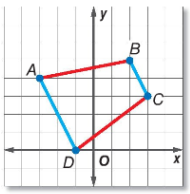


Step 1 Use the Slope Formula to compare the slopes of opposite sides $\overline{BC}$ and $\overline{AD}$ and of opposite sides $\overline{AB}$ and $\overline{DC}$. A quadrilateral is a trapezoid if exactly one pair of opposite sides are parallel.

Opposite sides $\overline{BC}$ and $\overline{AD}$:

slope of $\overline{BC} = 3 - \frac{5}{3} - 2 = -\frac{2}{1}$ or -2

slope of $\overline{AD} = \frac{0-4}{-1-(-3)} = \frac{-4}{2}$ or -2



Since the slopes of $\overline{BC}$ and $\overline{AD}$ are equal, $\overline{BC} \parallel \overline{AD}$.

Opposite sides $\overline{AB}$ and $\overline{DC}$:

slope of $\overline{AB} = \frac{5-4}{2-(-3)} = \frac{1}{5}$ slope of $\overline{DC} = \frac{0-3}{-1-3} = \frac{-3}{-4}$ or $\frac{3}{4}$

Since the slopes of $\overline{AB}$ and $\overline{DC}$ are *not* equal, $\overline{BC} \nparallel \overline{AD}$. Since quadrilateral *ABCD* has only one pair of opposite sides that are parallel, quadrilateral *ABCD* is a trapezoid.

Step 2 Use the Distance Formula to compare the lengths of legs $\overline{AB}$ and $\overline{DC}$. A trapezoid is isosceles if its legs are congruent.

$AB = \sqrt{(-3-2)^2 + (4-5)^2}$ or $\sqrt{26}$

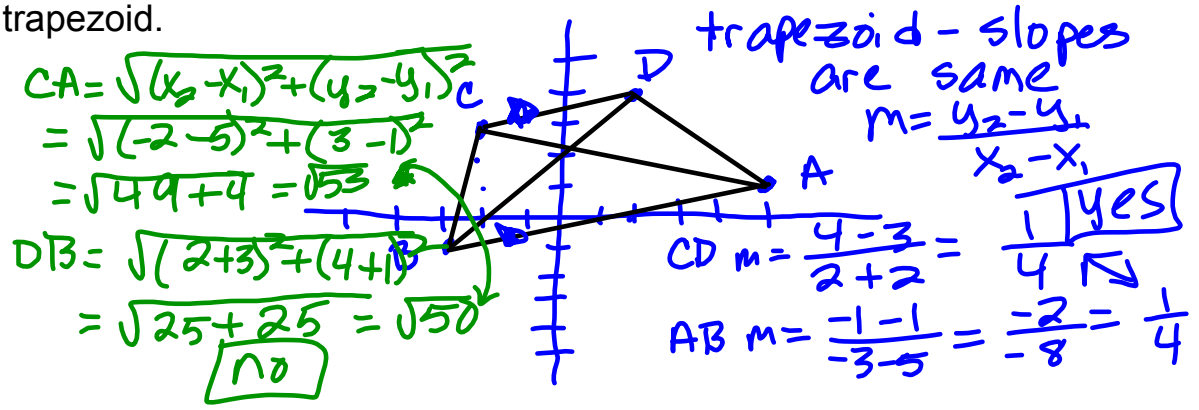
$DC = \sqrt{(-1-3)^2 + (0-3)^2} = \sqrt{25}$ or 5

Since $AB \neq DC$, legs $\overline{AB}$ abd $\overline{DC}$ are *not* congruent. Therefore, trapezoid *ABCD* is not isosceles.

Jan 23-7:47 AM

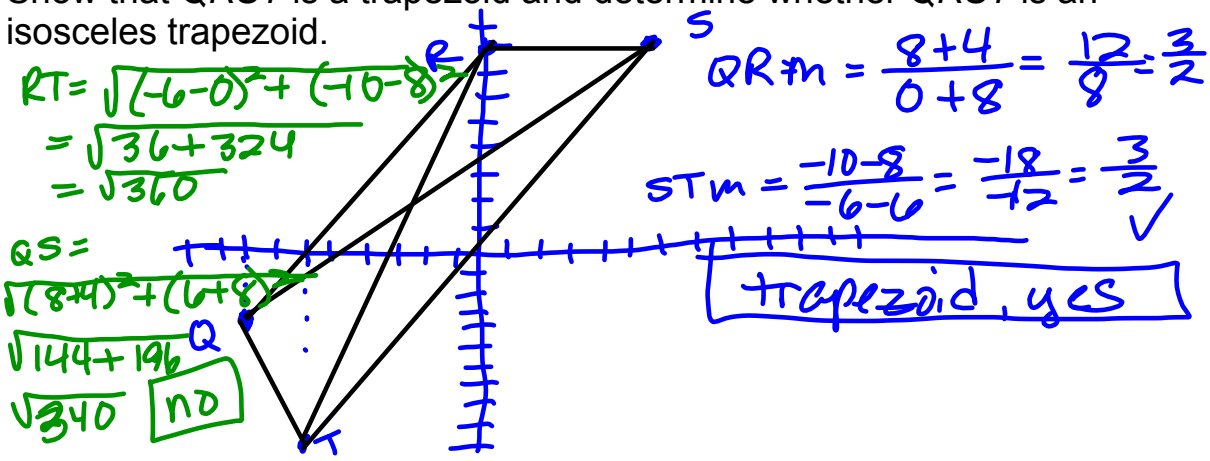
Example 2:

7. Quadrilateral *ABCD* has vertices *A*(5, 1), *B*(-3, -1), *C*(-2, 3), *D*(2, 4). Show that *ABCD* is a trapezoid and determine whether it is an isosceles trapezoid.



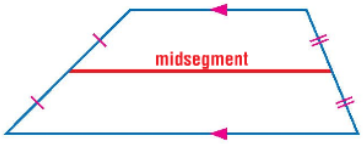
Jan 23-7:50 AM

8. Quadrilateral *QRST* has vertices *Q*(-8, -4), *R*(0, 8), *S*(6, 8), *T*(-6, -10). Show that *QRST* is a trapezoid and determine whether *QRST* is an isosceles trapezoid.



Jan 23-7:55 AM

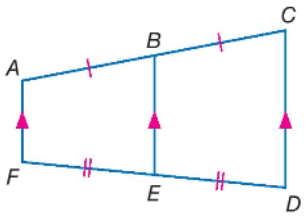
Midsegment of a trapezoid: the segment that connects the midpoints of the legs of the trapezoid.



ReadingMath
Midsegment A midsegment of a trapezoid can also be called a *median*.

Theorem 6.24 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base and its measure is one half the sum of the lengths of the bases.

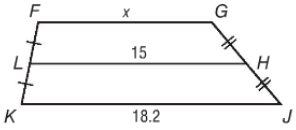


Example If $\overline{BE}$ is the midsegment of trapezoid $ACDF$, then $\overline{AF} \parallel \overline{BE}$, $\overline{CD} \parallel \overline{BE}$, and $BE = \frac{1}{2}(AF + CD)$.

Jan 23-8:04 AM

Standardized Test Example 3 Midsegment of a Trapezoid

GRIDDED RESPONSE In the figure, $\overline{LH}$ is the midsegment of trapezoid $FGJK$. What is the value of x?



Note: The figure is not drawn to scale.

Read the Test Item

You are given the measure of the midsegment of a trapezoid and the measure of one of its bases. You are asked to find the measure of the other base.

Solve the Test Item

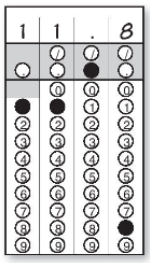
$LH = \frac{1}{2}(FG + KJ)$ Trapezoid Midsegment Theorem

$5 = \frac{1}{2}(x + 18.2)$ Substitution

$30 = x + 18.2$ Multiply each side by 2.

$11.8 = x$ Subtract 18.2 from each side.

Grid In Your Answer



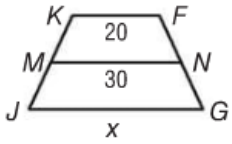
- You can align the numerical answer by placing the first digit in the left answer box or by putting the last digit in the right answer box.
- Do not leave blank boxes in the middle of an answer.
- Fill in **one** bubble for each filled answer box. Do not fill more than one bubble for an answer box. Do not fill in a bubble for blank answer boxes.

Test-TakingTip
Gridded Responses
Rational answers can often be gridded in more than one way. An answer such as $\frac{8}{5}$ could be gridded as 8/5 or 1.6, but not as 1 3/5.

Jan 23-8:08 AM

Example 3:

9. In the figure, $\overline{MN}$ is the midsegment of trapezoid *FGJK*. What is the value of x?



$\frac{2}{1} \cdot \frac{1}{2}(20 + x) = (30)2$

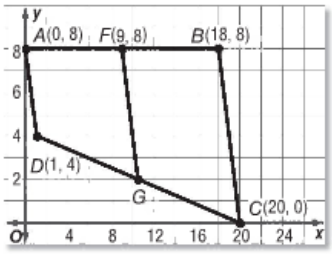
$20 + x = 60$

-20

$x = 40$

Jan 23-8:14 AM

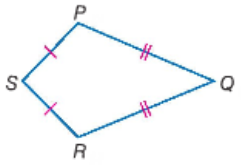
10. Trapezoid *ABCD* is shown. If $\overline{FG}$ is parallel to $\overline{AD}$, what is the x-coordinate of point *G*?



Jan 23-8:19 AM

Kite: a quadrilateral with exactly two pairs of consecutive congruent sides.

Opposite sides of a kite are not congruent or parallel.



Jan 23-8:24 AM

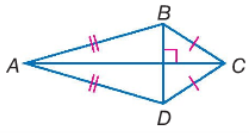
StudyTip

Kites The congruent angles of a kite are included by the non-congruent adjacent sides.

Theorems Kites

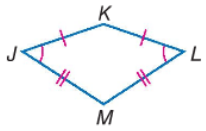
6.25 If a quadrilateral is a kite, then its diagonals are perpendicular.

Example If quadrilateral *ABCD* is a kite, then $\overline{AC} \perp \overline{BD}$.

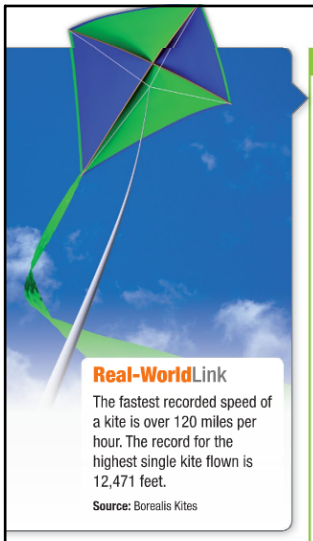


6.26 If a quadrilateral is a kite, then exactly one pair of opposite angles is congruent.

Example If quadrilateral *JKLM* is a kite, $\overline{JK} \cong \overline{KL}$, and $\overline{JM} \cong \overline{LM}$, then $\angle J \cong \angle L$ and $\angle K \ncong \angle M$.



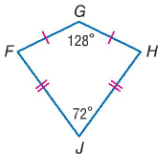
Jan 23-8:26 AM



Real-WorldLink
The fastest recorded speed of a kite is over 120 miles per hour. The record for the highest single kite flown is 12,471 feet.
Source: Borealis Kites

Example 4 Use Properties of Kites

a. If *FGHJ* is a kite, find $m\angle GFJ$.



Since a kite can only have one pair of opposite congruent angles and $\angle G \ncong \angle J$, then $\angle F \cong \angle H$. So, $m\angle F = m\angle H$. Write and solve an equation to find $m\angle F$.

$m\angle F + m\angle G + m\angle H + m\angle J = 360$ Polygon Interior Angles Sum Theorem

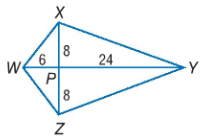
$m\angle F + 128 + m\angle F + 72 = 360$ Substitution

$2m\angle F + 200 = 360$ Simplify.

$2m\angle F = 160$ Subtract 200 from each side.

$m\angle F = 80$ Divide each side by 2.

b. If *WXYZ* is a kite, find *ZY*.



Since the diagonals of a kite are perpendicular, they divide *WXYZ* into four right triangles. Use the Pythagorean Theorem to find *ZY*, the length of the hypotenuse of right $\triangle YPZ$.

$PZ^2 + PY^2 = ZY^2$ Pythagorean Theorem

$8^2 + 24^2 = ZY^2$ Substitution

$640 = ZY^2$ Simplify.

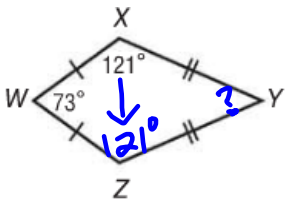
$\sqrt{640} = ZY$ Take the square root of each side.

$8\sqrt{10} = ZY$ Simplify.

Jan 23-8:27 AM

Example 4:

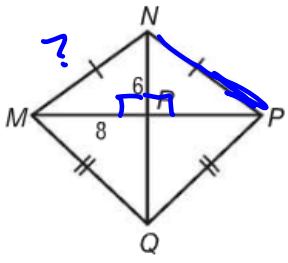
11. If *WXYZ* is a kite, find m<*XYZ*.



$121 + 73 + 121 + X = 360$

315 $X = 45^\circ$

12. If *MNPQ* is a kite, find *NP*. 10



$c^2 = a^2 + b^2$

$c^2 = 6^2 + 8^2$

$c^2 = 36 + 64$

$\sqrt{c^2} = \sqrt{100}$ $c = 10$

Jan 23-8:31 AM

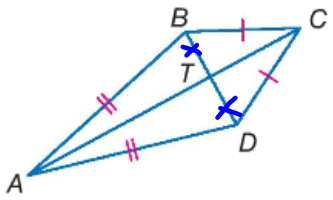
13. If m<*BAD* = 38 and m<*BCD* = 50, find m<*ADC*.

$x + x + 38 + 50 = 360$

$2x + 88 = 360$ -88

$2x = 272$

$x = 136 \div 2 = \boxed{68^\circ}$



14. If *BT* = 5 and *TC* = 8, find *CD*.

$c^2 = a^2 + b^2$

$c^2 = 5^2 + 8^2$

$c^2 = 25 + 64$ $\boxed{c = \sqrt{89}}$

$\sqrt{c^2} = \sqrt{89}$

Jan 23-8:36 AM