## Lesson 18

Objective: Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.

## Suggested Lesson Structure

| $\square$ | Fluency Practice |
| :--- | :--- |
| (12 minutes) |  |
| Application Problem | (6 minutes) |
| Concept Development | $(32$ minutes) |
| Student Debrief | $(10$ minutes) |
| Total Time | $(60$ minutes) |



## Fluency Practice (12 minutes)

- Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers 5.NBT. 7 (9 minutes)
- Multiply by Multiples of 10 and 100 5.NBT. 2


## Sprint: Divide Whole Numbers by Fractions and Fractions by Whole Numbers (9 minutes)

Materials: (S) Divide Whole Numbers by Fractions and Fractions by Whole Numbers Sprint
Note: This fluency activity reviews Module 4.

## Multiply by Multiples of 10 and 100 ( 3 minutes)

Note: This fluency activity reviews Modules 1-2.
T: $\quad($ Write $42 \times 10=$ $\qquad$ .) Say the multiplication sentence.
S: $42 \times 10=420$.
T: $\quad($ Write $42 \times 10=420$. Below it, write $420 \times 2=$ $\qquad$ .) Say the multiplication sentence.
S: $\quad 420 \times 2=840$.
T: (Write $420 \times 2=840$. Below it, write $42 \times 20=42 \times$ $\qquad$ $\times$ $\qquad$ $=$ $\qquad$ .) Say $42 \times 20$ as a threefactor multiplication sentence with 10 as one of the factors.
S: $42 \times 10 \times 2=840$.
Follow the same process for $23 \times 30$.
T: (Write $213 \times 30=213 \times$ $\qquad$ $\times$ $\qquad$ $=$ $\qquad$ .) Write $213 \times 30$ as a three-factor multiplication sentence with 10 as one of the factors, and solve.
S: $\quad$ (Write $213 \times 30=213 \times 10 \times 3=6,390$.)

Repeat the process for 4,213 $\times 20$.
T: $\quad($ Write $31 \times 100=$ $\qquad$ .) Say the multiplication sentence.
S: $\quad 31 \times 100=3,100$.
T: $\quad$ Write $31 \times 100=3,100$. Below it, write $3,100 \times 3=$ $\qquad$ .) Say the multiplication sentence.
S: $\quad 3,100 \times 3=9,300$.
T: $\quad$ Write $3,100 \times 3=9,300$. Below it, write $31 \times 300=$ $\qquad$ .) Say $31 \times 300$ as a three-factor multiplication sentence with 100 as one of the factors.
S: $\quad 31 \times 100 \times 3=9,300$.
T: $\quad$ (Write $31 \times 300=9,300$.
Direct students to solve $43 \times 300$ using the same method.

## Application Problem (6 minutes)

How many 2-inch cubes are needed to build a rectangular prism that measures 10 inches by 14 inches by 6 inches?



Note: Today's Application Problem requires that students reason about volume concepts from earlier in this module.

## Concept Development (32 minutes)

Materials: (T) Quadrilateral hierarchy with square: color (Template 2) (S) Ruler, set square or square template, protractor, scissors, quadrilateral hierarchy with square (Template 1)

## Problem 1

a. Draw a rhombus, and articulate the definition.
b. Measure and label its angles to explore their relationships.
c. Measure to explore diagonals of rhombuses.

T: Give the least specific name for all the shapes we have drawn so far.
S: Quadrilaterals.

T: Tell your partner a more specific name for a shape we have drawn, and explain what property it has that gives it that name.
S: Trapezoids because we have drawn shapes with at least one pair of parallel sides. $\rightarrow$ Some of the quadrilaterals could be called trapezoids and parallelograms. Parallelograms have two pairs of parallel sides.
T: How did we start drawing the trapezoids and parallelograms?
S: By drawing a pair of parallel sides.
T: If we wanted to draw a parallelogram that is also a rhombus, what would we need to think about?
S: It would need to have four sides the same length. $\rightarrow$ It would need another pair of parallel sides, but we would need to measure to be sure we drew all the sides the same length.
T: Draw an angle with sides that are equal length. Then, label the vertex as $B$ and the endpoints of the sides as $A$ and $C$.

S: (Draw an angle.)
T: Draw a line parallel to one of the sides through the endpoint of the other side.
S: (Draw a parallel line.)
T: Now, do the same for the second side.
S: (Draw a second parallel line.)
T: Label the last angle as $D$.
S: (Label the angle.)
T: Measure the sides, and compare your figure with your partner's. What is the most specific name for this shape? How do you know?
S: My sides were two inches long. My partner's were three inches long, but they both have two sets of parallel sides, and
 the sides are all the same length. So, we both drew a rhombus. $\rightarrow$ It's a parallelogram with four equal sides. $\rightarrow$ Mine is a parallelogram with equal sides, but my partner's is a square. We both drew a rhombus with four equal sides, even though I started with an acute angle and he started with a right angle.

T: Measure the angles, and mark them inside the rhombus.
S: (Measure and mark the angles.)
T: What do you notice? Turn and talk.


S: The angles that are beside each other add up to a straight angle. $\rightarrow$ There are two pairs of angles. Each pair adds up to $180^{\circ}$. $\rightarrow$ Angles between parallel lines equal $180^{\circ} . \rightarrow$ The opposite angles are the same size.
T: Use your ruler to draw the diagonals of your rhombus. Then, measure them and the distance from each corner to the point where they intersect. Tell your partner what you notice.
S: These diagonals are equal. $\rightarrow$ The diagonals bisect each other. $\rightarrow$ The point where they cross is the midpoint of both diagonals.


T: Now, measure the angles formed by the diagonals. What is the measure?
S: They are right angles. $\rightarrow$ The angles are all $90^{\circ}$.
T: What is the name for lines that intersect at a right angle?
S: Perpendicular lines.
T : Because they bisect each other at a $90^{\circ}$ angle, we call these diagonals perpendicular bisectors.
T: From our drawing, what attribute needs to be present to call this parallelogram a rhombus?
S : All four sides must be equal.


T: What else did we discover about the diagonals of a rhombus?
S: The diagonals are perpendicular bisectors.

## Problem 2

a. Draw a rectangle according to the definition of a rectangle.
b. Measure and label its angles to explore their relationships.
c. Measure to explore the diagonals of rectangles.

T : If I want to draw a parallelogram that is also a rectangle, what must I include in my drawing?
S: Rectangles are parallelograms, so they need two sets of parallel sides. $\rightarrow$ Rectangles have right angles and opposite sides that are parallel and equal.

T: Use your ruler and set square to draw a rectangle.
S: (Draw a rectangle.)
T: Cut out your rectangle, and by folding, confirm that the angles are all $90^{\circ}$ and the opposite sides are the same lengths.
S : (Cut and fold the rectangle.)
T: Now, measure the diagonals, the segments of the diagonals, and the angles around the intersection point. Record your measurements on the figure.
S : (Measure and record on the figure.)
T: What do you notice? Turn and talk.
S: The diagonals are equal lengths. $\rightarrow$ The segments of the diagonals are equal. $\rightarrow$ The angles between the parallel lines equal $180^{\circ} . \rightarrow$ The diagonals are equal and bisect each other.
T : Are the diagonals perpendicular bisectors? How do you know?
S: They are not perpendicular bisectors because they don't form right angles.
T : What properties must be present for a parallelogram to also be a rectangle?
$\mathrm{S}: \quad$ The sides across from each other have to be the same length. $\rightarrow$ All angles are $90^{\circ} . \rightarrow$ Diagonals bisect each other.

T: (Distribute the Problem Set to students.) Let's practice drawing more rhombuses and rectangles and think about their attributes by completing the Problem Set.
S: (Complete the Problem Set.)

## Problem Set

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Note: Along with today's Problem Set, the Problem Sets from Lessons 16 and 17 should be used in today's Student Debrief.

## Student Debrief ( 10 minutes)

Lesson Objective: Draw rectangles and rhombuses to clarify their attributes, and define rectangles and rhombuses based on those attributes.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- (Allow students to share all the different rhombuses and rectangles that are produced in Problem 1 of the Problem Set.) What attributes do all rhombuses share? What attributes appear on the rhombus list that were not on the list for parallelograms? What attributes do all rectangles share? Is this true for all quadrilaterals? Rhombuses? Rectangles? (Use the rhombuses and rectangles produced in Problem 1 to articulate the formal definitions. Continue posting definitions for comparisons.)
- When can a quadrilateral also be called a rhombus? When can a quadrilateral also be called a rectangle?
- Respond to the following statements with true or false. Explain your reasoning.
- All parallelograms are rhombuses. (False. Parallelograms have pairs of opposite sides that are the same length, but both pairs do not necessarily have the same length.)
- All rhombuses are parallelograms. (True. Rhombuses have all sides that are equal in length, and the opposite sides are also parallel.)
- All parallelograms are rectangles. (False. It is possible to have a quadrilateral with two pairs of parallel sides that do not have four right angles.)
- All rectangles are parallelograms. (True. If a quadrilateral has four right angles, it must have two pairs of parallel sides.)
- All trapezoids are rhombuses. (False. Trapezoids have at least one pair of parallel sides, but the lengths of the sides do not have to be equal.)
- All rhombuses are trapezoids. (True. If a quadrilateral has four equal sides, then the opposite sides will be parallel.)
- All trapezoids are rectangles. (False. All rectangles must have four right angles. The defining attribute of a trapezoid is that it must have at least one pair of parallel sides.)
- All rectangles are trapezoids. (True. In order to be a trapezoid, a quadrilateral needs to have at least one pair of parallel sides, and all rectangles have two pairs of parallel sides.)
- Continue the construction of the hierarchy diagram from Lessons 16 and 17. Students might draw or glue examples of rhombuses and rectangles and list attributes within the diagram. Encourage them to explain their placement of the figures in the hierarchy.
- Continue exploring the formal definition of a quadrilateral through the examination of counterexamples. (See the text box with the definition.)


## A quadrilateral:

- Consists of four different points, $A, B, C, D$, in the plane and four segments, $\overline{A B}, \overline{B C}, \overline{C D}, \overline{D A}$,
- Is arranged so that the segments intersect only at their endpoints, and
- Has no two adjacent segments that are collinear.

Step 1: Begin by asking students to tell what they know about a quadrilateral. Today's response should be a polygon with four straight sides that lie in the same plane and segments that only intersect at their endpoints.
Step 2: Follow the first and second bullets in the definition verbatim to draw four straight segments in the same plane that only intersect at their endpoints but have collinear endpoints as shown to the right.
Ask: "Is this figure also a quadrilateral? What must we
 add to our definition to eliminate the possibility of this figure?"
Step 3: Lead students to see that a four-sided figure is only a quadrilateral if all four segments lie in the same plane, the segments intersect only at their endpoints, and no two segments are collinear. Then, add the third bullet of the definition to that written in Lesson 17.

## Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Number Correct: $\qquad$

Divide Whole Numbers by Fractions and Fractions by Whole Numbers

| 1. | $1 / 2 \div 2=$ |  |
| :---: | :---: | :---: |
| 2. | $1 / 2 \div 3=$ |  |
| 3. | $1 / 2 \div 4=$ |  |
| 4. | $1 / 2 \div 7=$ |  |
| 5. | $7 \div 1 / 2=$ |  |
| 6. | $6 \div 1 / 2=$ |  |
| 7. | $5 \div 1 / 2=$ |  |
| 8. | $3 \div 1 / 2=$ |  |
| 9. | $2 \div 1 / 5=$ |  |
| 10. | $3 \div 1 / 5=$ |  |
| 11. | $4 \div 1 / 5=$ |  |
| 12. | $7 \div 1 / 5=$ |  |
| 13. | $1 / 5 \div 7=$ |  |
| 14. | $1 / 3 \div 2=$ |  |
| 15. | $2 \div 1 / 3=$ |  |
| 16. | $1 / 4 \div 2=$ |  |
| 17. | $2 \div 1 / 4=$ |  |
| 18. | $1 / 5 \div 2=$ |  |
| 19. | $2 \div 1 / 5=$ |  |
| 20. | $3 \div 1 / 4=$ |  |
| 21. | $1 / 4 \div 3=$ |  |
| 22. | $1 / 4 \div 4=$ |  |


| 23. | $4 \div 1 / 4=$ |  |
| :---: | :---: | :---: |
| 24. | $1 / 3 \div 3=$ |  |
| 25. | $2 / 3 \div 3=$ |  |
| 26. | $1 / 4 \div 2=$ |  |
| 27. | $3 / 4 \div 2=$ |  |
| 28. | $1 / 5 \div 2=$ |  |
| 29. | $3 / 5 \div 2=$ |  |
| 30. | $1 / 6 \div 2=$ |  |
| 31. | $5 / 6 \div 2=$ |  |
| 32. | $5 / 6 \div 3=$ |  |
| 33. | $1 / 6 \div 3=$ |  |
| 34. | $3 \div 1 / 6=$ |  |
| 35. | $6 \div 1 / 6=$ |  |
| 36. | $7 \div 1 / 7=$ |  |
| 37. | $8 \div 1 / 8=$ |  |
| 38. | $9 \div 1 / 9=$ |  |
| 39. | $1 / 8 \div 7=$ |  |
| 40. | $9 \div 1 / 8=$ |  |
| 41. | $1 / 8 \div 7=$ |  |
| 42. | $7 \div 1 / 6=$ |  |
| 43. | $9 \div 1 / 7=$ |  |
| 44. | $1 / 8 \div 9=$ |  |

Number Correct: $\qquad$
Improvement: $\qquad$
Divide Whole Numbers by Fractions and Fractions by Whole Numbers

| 1. | $1 / 2 \div 2=$ | 23. | $3 \div 1 / 3=$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 2. | $1 / 5 \div 3=$ | 24. | $1 / 4 \div 4=$ |  |
| 3. | $1 / 5 \div 4=$ | 25. | $3 / 4 \div 4=$ |  |
| 4. | $1 / 5 \div 7=$ | 26. | $1 / 3 \div 3=$ |  |
| 5. | $7 \div 1 / 5=$ | 27. | $2 / 3 \div 3=$ |  |
| 6. | $6 \div 1 / 5=$ | 28. | $1 / 6 \div 2=$ |  |
| 7. | $5 \div 1 / 5=$ | 29. | $5 / 6 \div 2=$ |  |
| 8. | $3 \div 1 / 5=$ | 30. | $1 / 5 \div 5=$ |  |
| 9. | $2 \div 1 / 2=$ | 31. | $3 / 5 \div 5=$ |  |
| 10. | $3 \div 1 / 2=$ | 32. | $3 / 5 \div 4=$ |  |
| 11. | $4 \div 1 / 2=$ | 33. | $1 / 5 \div 6=$ |  |
| 12. | $7 \div 1 / 2=$ | 34. | $6 \div 1 / 5=$ |  |
| 13. | $1 / 2 \div 7=$ | 35. | $6 \div 1 / 4=$ |  |
| 14. | $1 / 4 \div 2=$ | 36. | $7 \div 1 / 6=$ |  |
| 15. | $2 \div 1 / 4=$ | 37. | $8 \div 1 / 7=$ |  |
| 16. | $1 / 3 \div 2=$ | 38. | $9 \div 1 / 8=$ |  |
| 17. | $2 \div 1 / 3=$ | 39. | $1 / 8 \div 8=$ |  |
| 18. | $1 / 2 \div 2=$ | 40. | $9 \div 1 / 9=$ |  |
| 19. | $2 \div 1 / 2=$ | 41. | $1 / 9 \div 8=$ |  |
| 20. | $4 \div 1 / 3=$ | 42. | $7 \div 1 / 7=$ |  |
| 21. | $1 / 3 \div 4=$ | 43. | $9 \div 1 / 6=$ |  |
| 22. | $1 / 3 \div 3=$ | 44. | $1 / 8 \div 6=$ |  |

Name $\qquad$ Date $\qquad$

1. Draw the figures in each box with the attributes listed.

| a. Rhombus with no right angles | b. Rectangle with not all sides equal |
| :--- | :--- |
|  |  |
| c. Rhombus with 1 right angle |  |

2. Use the figures you drew to complete the tasks below.
a. Measure the angles of the figures with your protractor, and record the measurements on the figures.
b. Use a marker or crayon to circle pairs of angles inside each figure with a sum equal to $180^{\circ}$. Use a different color for each pair.
3. Draw a rhombus and a rectangle below.
a. Draw the diagonals, and measure their lengths. Record the measurements on the figure.
b. Measure the length of each segment of the diagonals from the vertex to the intersection point of the diagonals. Using a marker or crayon, color segments that have the same length. Use a different color for each different length.
4. a. List the properties that are shared by all of the rhombuses that you worked with today.
b. List the properties that are shared by all of the rectangles that you worked with today.
c. When can a trapezoid also be called a rhombus?
d. When can a parallelogram also be called a rectangle?
e. When can a quadrilateral also be called a rhombus?

Name $\qquad$ Date $\qquad$

1. Draw a rhombus.
2. Draw a rectangle.

Name
Date $\qquad$

1. Use the grid paper to draw.
a. A rhombus with no right angles

b. A rhombus with 4 right angles

c. A rectangle with not all sides equal

d. A rectangle with all sides equal
 rectangles and rhombuses based on those attributes.
2. A rhombus has a perimeter of 217 cm . What is the length of each side of the rhombus?
3. List the properties that all rhombuses share.
4. List the properties that all rectangles share.

quadrilateral hierarchy with square

