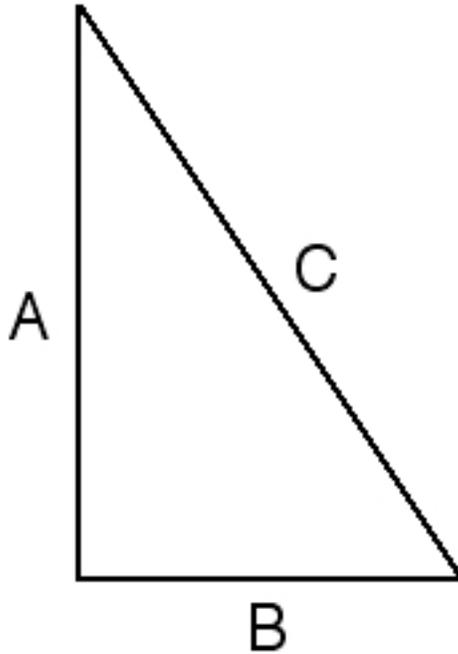


PART 3 MODULE 7

PROBLEMS INVOLVING DISTANCE AND THE PYTHAGOREAN THEOREM

THE PYTHAGOREAN THEOREM

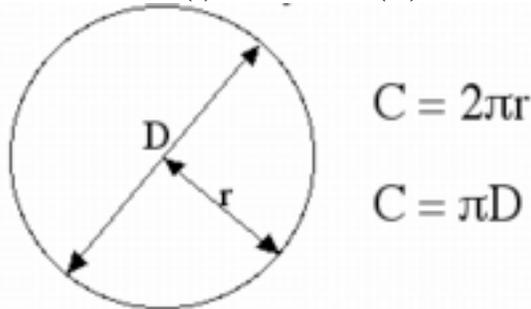
The Pythagorean Theorem states the relationship between the lengths of the three sides of a right triangle:



$C^2 = A^2 + B^2$, where A and B are the lengths of the two shorter sides (the legs) and C is the length of the longer side (the hypotenuse).

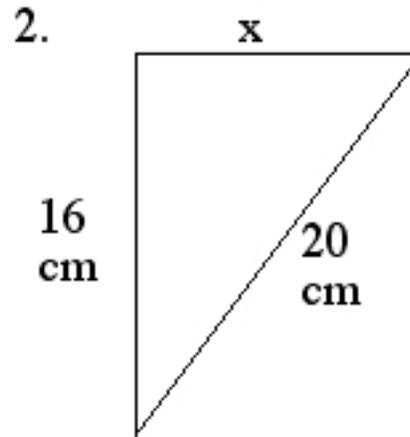
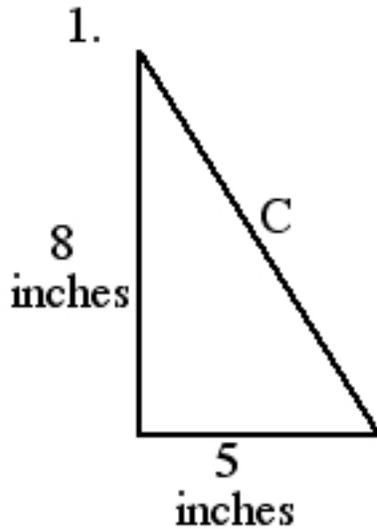
The CIRCUMFERENCE of a circle

The distance around a circle is called its *circumference* C, and is determined by the circle's radius (r) or diameter (D):



EXAMPLE 3.7.1

Find the missing side length for each triangle shown below.

**EXAMPLE 3.7.1 solutions**

1. We need to find the length of the hypotenuse of a right triangle where one leg measures 8 inches and the other leg measures 5 inches. According to the Pythagorean Theorem,

$$8^2 + 5^2 = C^2$$

$$64 + 25 = C^2$$

$$89 = C^2$$

$$\sqrt{89} = C$$

$$C \approx 9.43398 \text{ inches}$$

2. We need to find the length of one leg of a right triangle where the other leg measures 16 cm and the hypotenuse measures 20 cm. According to the Pythagorean Theorem,

$$20^2 = x^2 + 16^2$$

$$400 = x^2 + 256 \quad (\text{We want to isolate } x \text{ on one side of the equals sign.})$$

$$400 - 256 = x^2$$

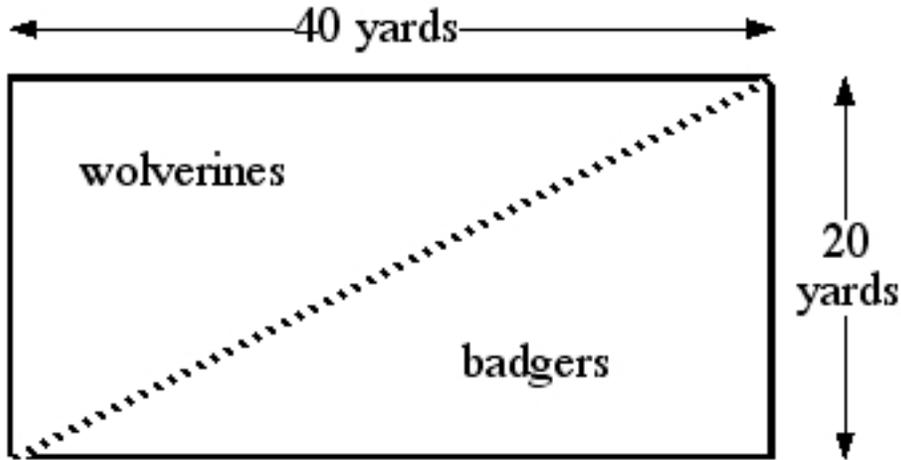
$$144 = x^2$$

$$\sqrt{144} = x$$

$$x = 12 \text{ inches}$$

EXAMPLE 3.7.2

The diagram below shows the rectangular pen in which Gomer confines his wolverines and badgers. In order to prevent the wolverines from dating the badgers, Gomer is going to build a fence running from one corner of the pen to the opposite corner, thus dividing the pen into two smaller pens. Assuming that construction of such a fence will cost \$1.25 per foot, find the total cost of this fence.



- A. \$4500 B. \$500 C. \$168 D. \$56

EXAMPLE 3.7.2 solution

We need to find the length of the fence (in feet) and multiply the length by the cost factor of \$1.25 per foot.

We can use the Pythagorean Theorem to find the length of the fence, since the fence is the hypotenuse of a right triangle whose legs measure 40 yards and 20 yards respectively. Since we want the length in feet, rather than yards, we will convert those measurements to feet before using the Pythagorean Theorem.

$$(40 \text{ yards})(3 \text{ feet per yard}) = 120 \text{ feet.}$$

$$(20 \text{ yards})(3 \text{ feet per yard}) = 60 \text{ feet.}$$

Now let L be the length of the fence. According to the Pythagorean Theorem:

$$L^2 = 120^2 + 60^2$$

$$L^2 = 14,400 + 3,600$$

$$L^2 = 18,000$$

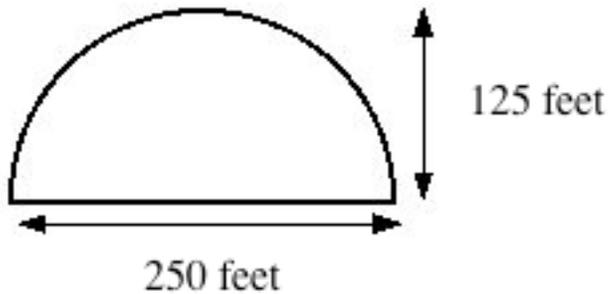
$$L = \sqrt{18,000}$$

$$L \approx 134 \text{ feet}$$

The length of the fence is roughly 134 feet, and the cost is \$1.25 per foot, so the total cost is $(\$134)(\$1.25) = \$167.50$

EXAMPLE 3.7.3

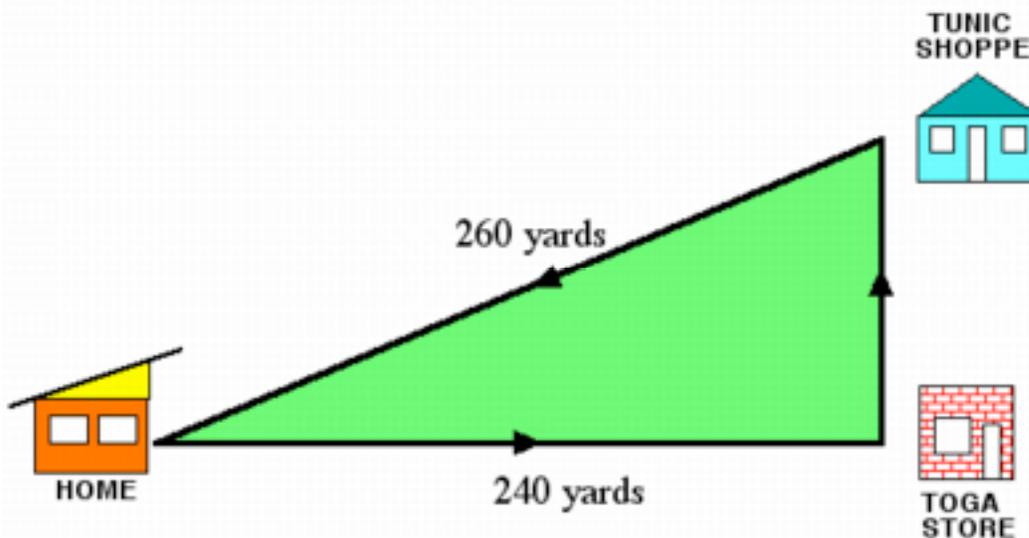
Study the race course shown below. If Gomer runs 62 laps around this course, how many miles will he have run?



- A. 4.6 miles B. 9.2 miles C. 7.5 miles D. 4.4 miles

EXAMPLE 3.7.4

The diagram below shows the path the Plato takes when he goes for a philosophical stroll. Plato starts at home, proceeds to the toga shop, then heads north to the tunic store, then returns home. On average, Plato thinks one profound thought for every 10 yards that he walks. Find the total number of profound thoughts that he will think during this walk.

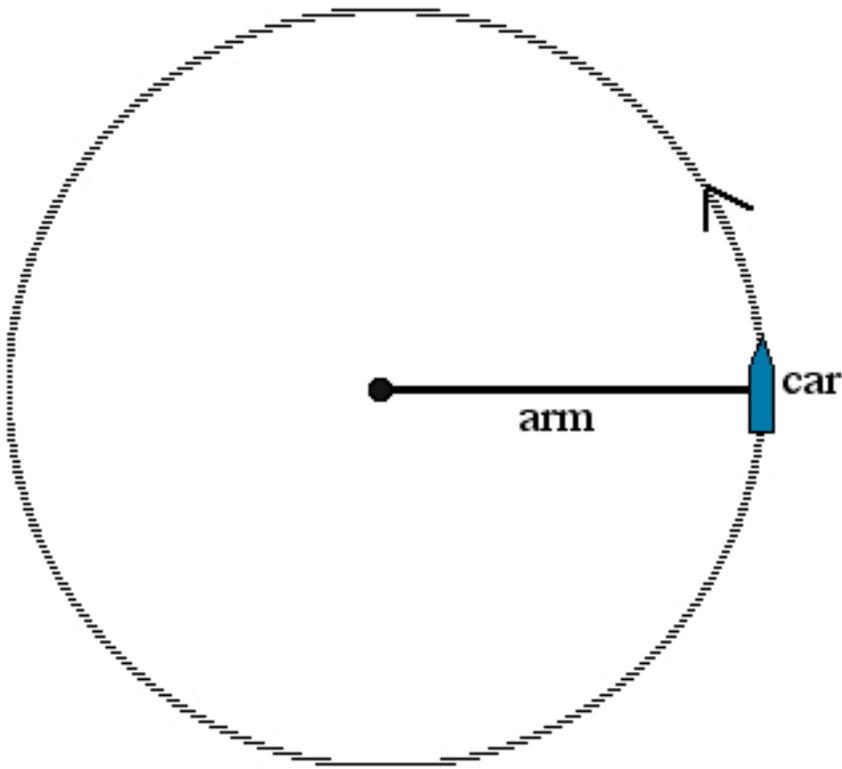


- A. 60 B. 10 C. 100 D. 258 E. None of these

EXAMPLE 3.7.5

The figure below shows an aerial view of The Hurl-O-Matic, a carnival ride in which the passengers are seated in a car, attached to the end of an arm which rotates rapidly around a central hub. Suppose that the length of the arm is 64 feet, and that, at full speed, it takes 10 seconds for the car to complete one revolution. Find the speed of the car.

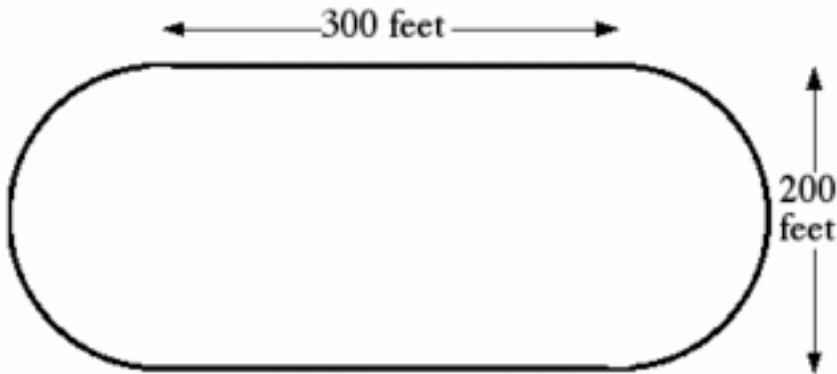
Hurl-O-Matic ride



- A. 40 miles per hour
- B. 10 miles per hour
- C. 27 miles per hour
- D. 21 miles per hour
- E. 37 miles per hour

EXAMPLE 3.7.6

Find the distance around the racetrack shown below.



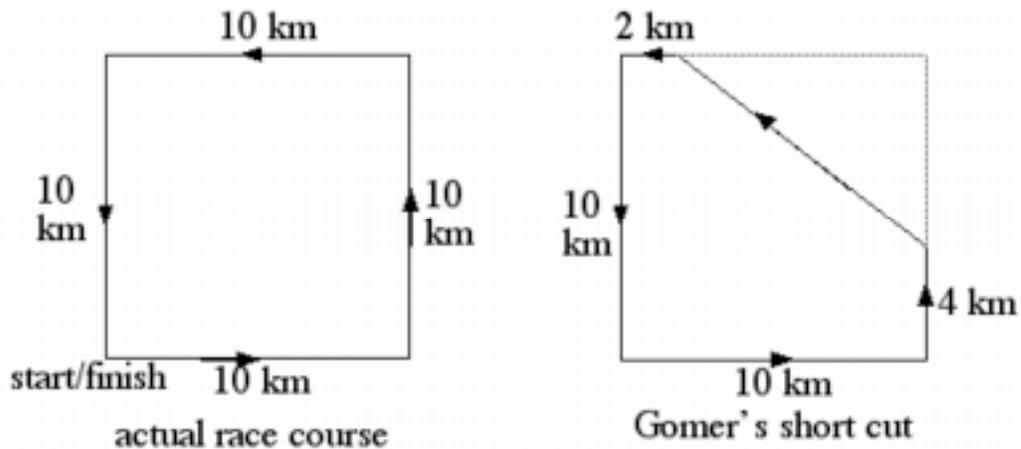
- A. 0.23 miles B. 0.48 miles C. 0.0067 miles D. 0.35 miles

EXAMPLE 3.7.7

How fast (in miles per hour) does the Earth travel as it orbits the sun? (Note: the Earth's orbit is approximately circular, with a radius of 93,000,000 miles.)

EXAMPLE 3.7.8

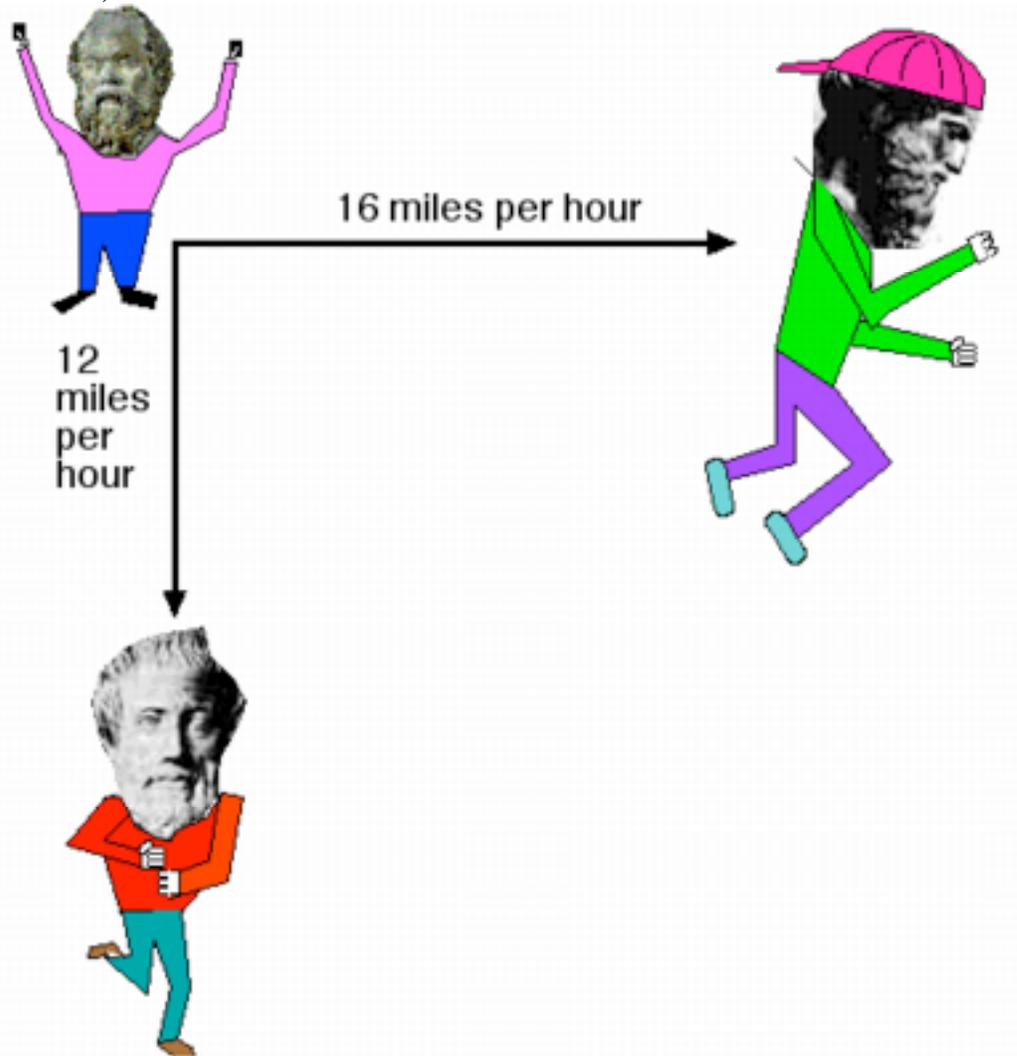
The diagram on the left below shows the race course for the 40-K Wolverine Day Fun Run. The diagram on the right shows the course modified by the short-cut that Gomer uses. What distance does Gomer cover if he runs the race using his short-cut?



- A. 36 km B. 26 km C. 10 km D. 16 km

EXAMPLE 3.7.9

Plato and Aristotle are loitering on the street corner, when suddenly Socrates (to whom they owe money) shows up. Plato takes off skating eastward at a rate of 16 miles per hour, and Aristotle runs southward at a rate of 12 miles per hour. How far apart (direct distance) are Plato and Aristotle after 15 minutes?



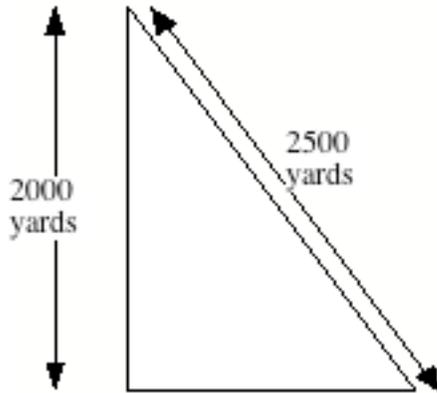
WORLD WIDE WEB NOTE

For practice on problems involving distance and the Pythagorean Theorem, visit the companion website and try THE GEOMETRIZER.

PRACTICE EXERCISES

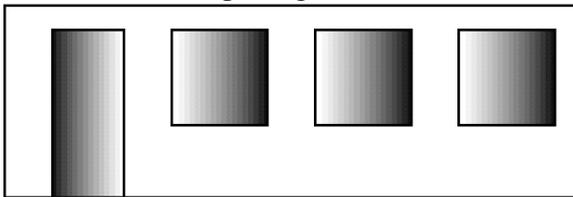
1. Plato exercises by walking laps around a circular track that is 200 feet in diameter. If he walks 20 laps, approximately how far will he have walked
- A. 2.4 miles B. 119 miles C. 5.9 miles D. 11.8 miles

2.



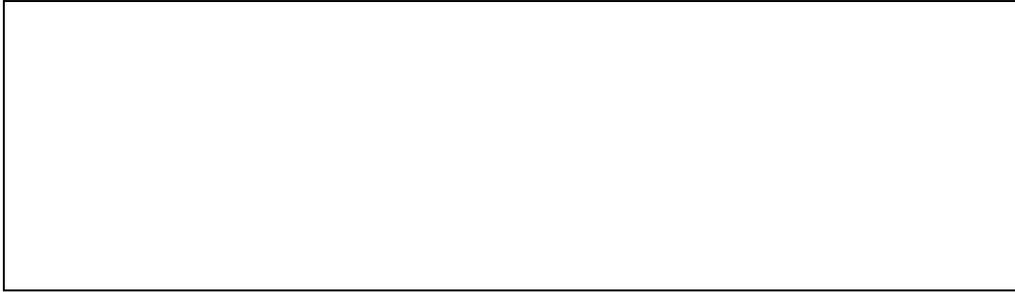
The diagram at left shows a triangular parcel of property that will be enclosed by a fence. If the fence material costs \$10 per foot, how much will this cost?

- A. \$60,000
B. \$45,000
C. \$15,000
D. \$180,000
3. Two boats leave the dock at 12:00 noon, one of them moving northward at 6 miles per hour, and the other moving westward at 8 mph. How far apart are the boats after 2 hours?
- A. 20 miles B. 28 miles C. 10 miles D. 14 miles
4. The diagram below shows one exterior wall of a house. The wall has a door that measures 3 ft. by 7 feet, and three windows which each measure 4 ft. by 4 ft. What is the perimeter of the door opening?

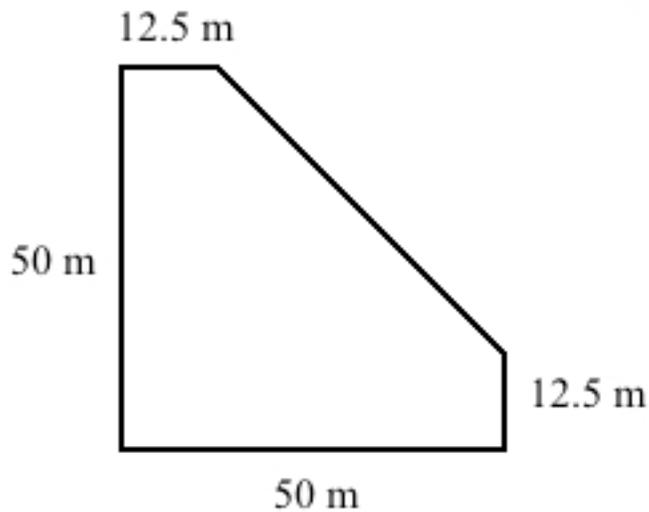


- A. 53 inches B. 65 inches C. 240 inches D. 4.42 inches

5. The diagram below shows Aristotle's stroll. He starts at his home, proceeds to the cheese shop, then to the toga store, and then returns home. What is the total distance (in feet) of his journey?



6. The figure below shows the parcel of land on which Homer the rancher confines his hippos. The parcel will be enclosed by a fence, at a cost of \$1.5 per meter. Find the total cost.



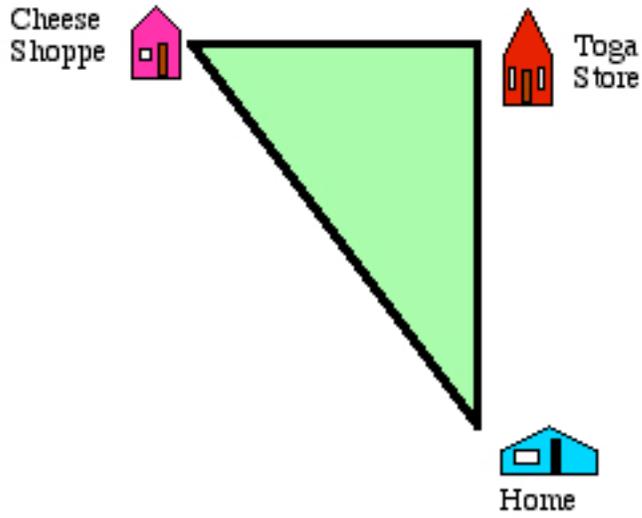
A. \$67.60

B. \$214.02

C. \$267.05

D. \$187.50

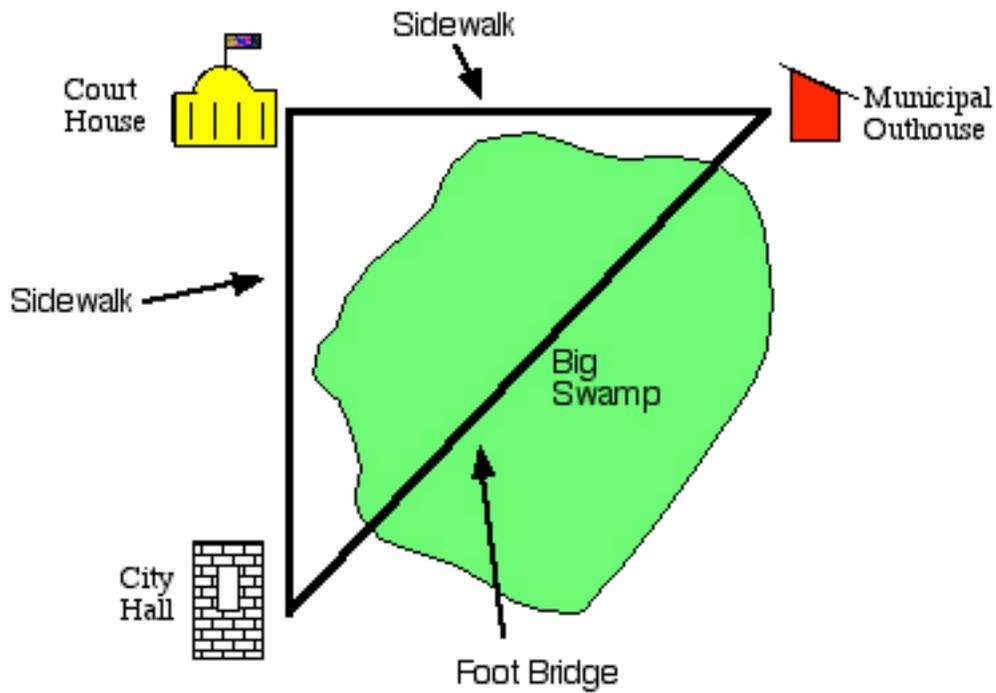
7. Study the figure below. The distance from the Cheese Shoppe to Diogenes's home is 1575 feet, and the distance from the Toga Store to the Cheese Shoppe is 945 feet.



Find the distance from the Toga Store to Diogenes's home.

- A. 1837 feet B. 3780 feet C. 1260 feet D. 630 feet

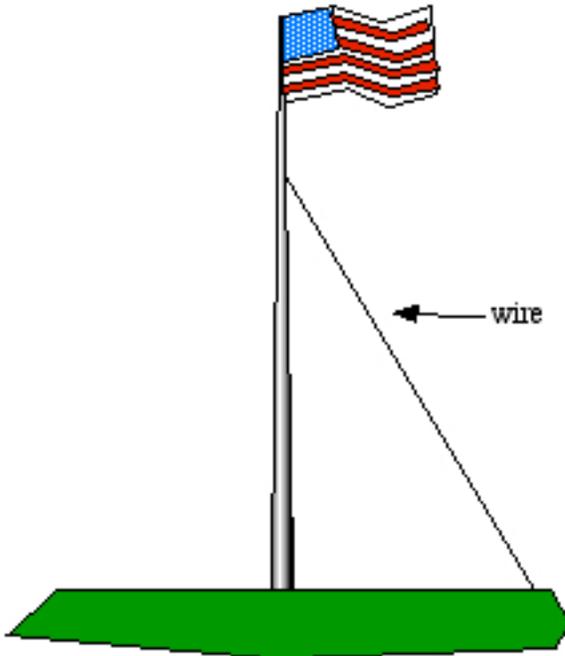
8. Study the figure below, which illustrates a dilemma facing the Gainesville City Council. They are going to build a footbridge connecting City Hall to the Municipal Outhouse, because a number of citizens have perished while crossing Big Swamp. The distance from City Hall to the Court House is 1440 feet, and the distance from the Court House to the Municipal Outhouse is 1512 feet.



Find the cost of the footbridge, assuming that such a structure costs \$21 per foot.

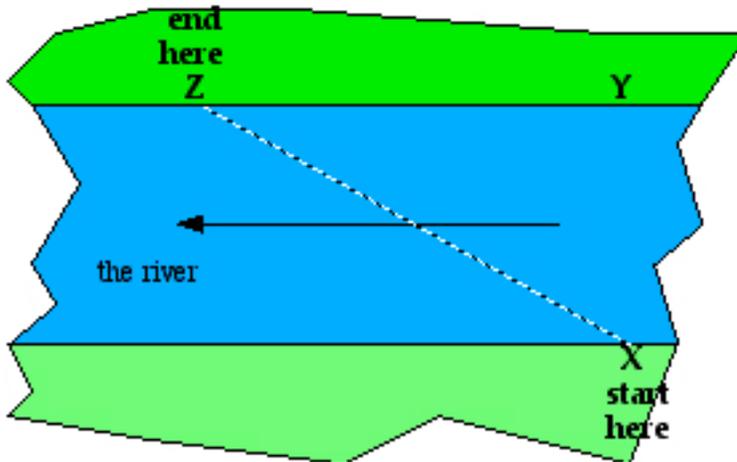
- A. \$43848 B. \$54810 C. \$61992 D. \$65772

9. Plato has raised a 91-foot-high flag pole. The flag pole is supported by 5 wires, each of which is attached to the flag pole at a place that is 19 feet from the top of the pole and attached to the ground at a place that is 54 feet from the base of the pole. Find the total length of all 5 wires.



- A. 450 feet B. 225 feet C. 338 feet D. 90 feet

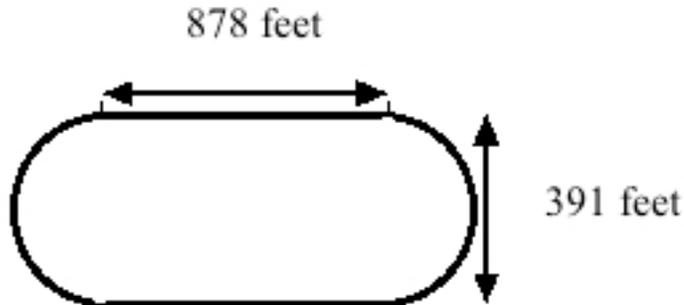
10. Study the figure below (which is not drawn to scale). Euclid has spent the afternoon sunbathing at point X on the south bank of the river. However, directly across the river at point Y he sees his buddies drinking beer. He decides to swim across to where they are, but the swift current carries him downstream so that he arrives at point Z instead.



Assuming that the distance from X to Y is 402 feet and the distance from X to Z is 670 feet, how far from his intended destination did Euclid end up?

- A. 536 feet B. 781 feet C. 668 feet D. 268 feet

11. Find the distance around the racetrack shown below.



- A. 2238.3 feet B. 105423616.9 feet
 C. 1492.2 feet D. 2984.4 feet

ANSWERS TO LINKED EXAMPLES

- EXAMPLE 3.7.3** C
EXAMPLE 3.7.4 A
EXAMPLE 3.7.5 C
EXAMPLE 3.7.6 A
EXAMPLE 3.7.7 About 66,700 miles per hour
EXAMPLE 3.7.9 5 miles

ANSWERS TO PRACTICE EXERCISES

- 1. A** **2. D** **3. A** **4. C** **5. B** **6. C**
7. C **8. A** **9. A** **10. A** **11. D**