A Dynamical Systems Approach to Cryptocurrency Stability

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Recently, the notion of cryptocurrencies has come to the fore of public interest. These assets that exist only in electronic form, with no underlying value, offer the owners some protection from tracking or seizure by government or creditors. We model these assets from the perspective of asset flow equations developed by Caginalp and Balenovich, and investigate their stability under various parameters, as classical finance methodology is inapplicable. By utilizing the concept of liquidity price and analyzing stability of the resulting system of ordinary differential equations, we obtain conditions under which the system is linearly stable. We find that trend-based motivations and additional liquidity arising from an uptrend are destabilizing forces, while anchoring through value assumed to be fairly recent price history tends to be stabilizing.

Introduction

Blockchain technology enables large numbers of participants to make electronic transactions directly without intermediaries, and has led, in recent years to a new form of payment, and essentially to a new set of currencies called cryptocurrencies. During 2017 the spectacular nine-fold rise in the price of Bitcoin focused the spotlight of public attention on cryptocurrencies that evolved into a new asset class. Following the pattern of other nascent assets, speculators dominated trading and pushed prices toward a bubble.

As with some other asset bubbles of the past, notably the dot-com frenzy of the late 1990s, the emergence of a new technology clouded judgements about the basic value of the asset.

In almost all cases, unlike traditional equities, cryptocurrencies have no tangible value, and even their creation is often shrouded in mystery (3). For Bitcoin, the creation of additional units is relegated to a process termed mining, named after the old technology of mining gold. In this electronic version, computing power is used to solve complex mathematical problems, and once solutions are found, the miner is rewarded with some units of the cryptocurrency. It is peculiar that this brand new technology is coupled with the organizational hierarchy of centuries ago that featured the hegemony of gold miners and traders who held power over the economic lives of people without any accountability. Thus, the advances in technology are coupled with a regressive organizational structure. Major currencies such as the US dollar are controlled by officials appointed by elected representatives; assets such as common stocks are governed by a board of directors that are elected by the shareholders whose rights are assured by law, albeit through a circuitous process. Changes made by cryptocurrencies are often made at an ad hoc meeting of the developers or miners, reminiscent of tribal chiefs meetings of more primitive eras.

In certain types of securities, i.e. exchange-traded funds (ETFs), a certain group of traders known as authorized participants have the capacity to demand the underlying shares, preventing the price of the ETF from straying too far from its fundamental value. Securities held in a brokerage account are insured up to a value of several million dollars by the federal government against circumstances such as hacking or company insolvency.

Conversely, for cryptocurrencies, many of these rules have been lacking and a few have been recently instituted. First, these cryptocurrencies have no underlying assets. Further, there exists no mechanism by which one can redeem any value from a bank or other institution. Second, even the country of residence (let alone a business address) of the creators is merely speculative, making it unclear to which court one could possibly appeal in the case of a grievance. The original "developer" for Bitcoin, for example, is known only by a pseudonym. Third, the individual investor has no influence over something as simple as the number of Bitcoins in existence. Instead, these decisions are generally relegated to the heads of electronic Bitcoin mining operations, whose interests may be disjoint from investors'. Bitcoin is currently set to be capped at 21 million units, but there is no legal obstacle to prevent an increase at the whim of the miners. Fourth, many individual investors and some academicians tacitly assume that the laws and protections afforded by the purchase of securities through stock exchanges such as NYSE must also apply to cryptocurrencies. Recently, the Securities and Exchange Commission (SEC) has shuttered some initial coin offerings (ICOs) (4, 34) and announced future regulations to attempt to inject more clarity into the marketplace. Finally, Bitcoin is vulnerable to hacking or simply forgetfulness. Many holders of Bitcoin have their information stored in exchanges, i.e. platforms that handle transaction and storage of the cryptocurrency. These are hijacked with a disturbing regularity, and litigation can be pending years later, as in the infamous Mt. Gox hacking of 2014 (27).

Cryptocurrencies offer both opportunities and risks to society. On the one hand, cryptocur-

rencies and technology underpinning them – if designed appropriately – could be used to make transactions faster, safer and cheaper, alongside other societal benefits (15, 22). A less apparent feature is that they can make it more difficult (though not theoretically impossible; see (5, 18)) for totalitarian governments to expropriate savings, either directly or indirectly through currency inflation, thereby depriving savers of a large fraction of their assets. In this way, a proper cryptocurrency could lead to greater economic freedom, and render more difficult the financing of a dictatorship. Indeed, this can be modelled by a choice of two alternatives: either their home currency or cryptocurrency that cannot easily be seized (20, 39).

The risks presented by existing cryptocurrencies are multi-faceted. The difficulty in tracing transactions facilitate illicit activity and its financing. A less obvious – and possibly the most significant – risk arises from the instability of prices of major cryptocurrencies. As the market capitalization (number of units times the price of each unit) of the cryptocurrencies rises, there is growing risk that a sharp drop in the price of a cryptocurrency could have a cascading effect on other sectors of world economy, particularly if borrowing is involved. During the period October 2017 to April 2018, the price of a Bitcoin unit rose from 6,000 to 220,000 and back to 6,000. The market capitalization of all cryptocurrencies during that time period increased from 170 billion to 330 billion, peaking together with Bitcoin in December 2017. While attention is often focused on the rise and fall of the trading prices of these assets, the magnitude of the problem of stability has increased significantly during this six month period. As people become more accustomed to using these instruments, the market capitalization may increase to several trillion – i.e., a few percent of the 75 trillion Gross World Product – and many of the challenges will be critical.

Generally, the features of a financial instrument that might make it attractive to speculators are undesirable to those who seek to use it as a currency in daily transactions. Speculators see a greater opportunity in a volatile market, as they can use technical analysis and expertise to profit at the expense of the layperson. Conversely, large fluctuations on a day-to-day basis create obstacles for common purchases or the pricing of service contracts (*38*). Without stability in the marketplace, the cryptocurrencies may simply become "a mechanism for a transfer of wealth from the late-comers to the early entrants and nimble traders" (7). Thus, a set of questions of critical importance deals with the potential stability (or lack thereof) of Bitcoin or other cryptocurrencies, which is the main topic of our paper.

The turbulence arising from the collapse of the housing bubble was a major challenge for markets, but from a scientific perspective, it could be addressed largely with classical methods (19, 32, 35). However, classical methods are not readily adaptable to studying cryptocurrencies, as discussed below. We use a modern approach whereby an equilibrium price can be determined and the stability properties established within a dynamical system setting (6, 9, 10, 17, 23, 24, 28, 30, 35-37, 42).

1 Modelling prices and stability

Most of classical finance such as the Black-Scholes option pricing model has its origin in the basic equation

$$\frac{1}{P}dP = \mu dt + \sigma dW \tag{1}$$

for the change in the relative price $P^{-1}dP$ in terms of the expected return, μ , the standard deviation of the return, σ , and independent increments of Brownian motion, dW. It is widely acknowledged that this equation does not arise from compelling microeconomic considerations, nor empirical data. But rather, it is mathematically convenient and elegant for expressing and proving theorems (see (11) for discussion). Much of risk assessment is based upon this model with an increasing array of adjustments.

The limitations of this basic model are apparent, for example, if one examines the standard deviation of daily relative changes in the S&P 500 index, which is typically around 0.75%. This leads to the conclusion that a 4.5% drop is a sixth standard deviation event, i.e., it occurs once every billion trading days, while empirical data shows it is on the order of a few times per thousand (2).

Thus identifying risk on a large time scale based on the variance of a small time scale can vastly underestimates risk.

Furthermore, the modeling of asset prices is generally based on the underlying assumption of infinite arbitrage. While there may be some investors who are prone to cognitive errors or bias in assessing value, the impact of their trades will be marginalized by more savvy investors who manage a large pool of money. Of course the inherent assumption is that there is some value to an asset, based for example on the projection of the dividend stream, replacement value, etc., and that the shareholder has a vote that allows him ultimately to extract this value. For assets such as US Treasury bills, the model works quite well, as the owner is assured of receiving a particular dollar amount from the US at a specified time.

Herein lies the central problem for the application of classical theory to cryptocurrencies: there is no underlying asset value, as noted above. Cryptocurrencies constitute the opposite end of the market spectrum to US Treasury Bills, in which an arbitrageur can confidently buy or sell short based on a clear contract that will deliver a fixed amount of cash at a predetermined time.

If fact, classical game theory would conclude that since everyone knows the structure of the cryptocurrency, and understands that everyone else is also aware, then the price should never deviate much from zero. Furthermore, classical finance expressed through (1) would suggest that there is some measure of risk based on the historical average of σ , which will be less helpful that it is for stock indexes as discussed above.

Our analysis begins with the fact that despite the absence of underlying assets or backing, various groups have incentive to use it over traditional currencies. In particular there are large groups who need to make transactions outside of the usual banking system. Among these are (i) people with poor credit who cannot obtain a credit or even a debit card, (ii) citizens of totalitarian countries who fear expropriation of their savings, (iii) citizens of countries with high

inflation and a much lower interest rate, (iv) people engaged in illicit activity, (v) people who espouse utilizing a new idea or technology.

Collectively, these groups constitute a core ownership of cryptocurrencies, investing a sum that gradually grows with familiarity (14, 16, 25). Meanwhile, the rising prices catch the attention of speculators who provide additional cash into the system, but also bring motivations inherent in speculation, namely momentum trading, or the tendency to buy as prices rise, and analogously sell as prices fall (41).

We assume a single cryptocurrency and that the price is established by supply and demand without infinite arbitrage, and apply a modern theory of asset flow (10). This alternative approach relies on the notion of liquidity price. The experimental asset markets presented a puzzle to the economics community by demonstrating the endogenous price bubbles in which prices soared well above any possible expectation of outcome (28). Caginalp and Balenovich (10) observed that in addition to the trading price and fundamental value (defined clearly by the experimental setup), there was an additional important quantity with the same units: the total cash in the system divided by the number of shares. Denoting this by liquidity value or price, L, they adapted earlier versions (8) of the asset flow model.

This approach leads to a system of ordinary differential equations, as summarized below, whereupon equilibrium points can be evaluated and their stability established as a function of the basic parameters.

2 Modeling Cryptocurrency with Asset Flow Equations

For brevity, we first present the full model which will be a nonlinear evolutionary system that is based on (10) but with some key differences for cryptocurrencies. We can then consider simpler models in which some features are marginalized by setting parameters to zero and obtaining 2×2 or 3×3 systems, enabling us to understand the key factors in stability.

We denote the trading price by P(t), the number of units by N(t), the amount of cash available by M(t), and the liquidity price by L(t) = M(t) / N(t). With B as the fraction of wealth in the cryptocurrency, i.e., B = NP / (NP + M), the supply and demand are given by S = (1 - k) B, D = k (1 - B) respectively, where k is the transition rate from cash to the asset. Using a standard price equation (42) we write

$$\tau_0 \frac{1}{P} \frac{dP}{dt} = \frac{D}{S} - 1. \tag{2}$$

It follows that $B(1-B)^{-1} = NP/M = P/L$, so that the price equation is

$$\tau_0 \frac{1}{P} \frac{dP}{dt} = \frac{k}{1-k} \frac{L}{P} - 1.$$
(3)

The variable k is assumed to be a linearization of a tanh type function and involves the motivations of the traders which are expressed through sentiment, $\zeta = \zeta_1 + \zeta_2$ where ζ_1 is the trend component and ζ_2 is the value component. This construct has been studied, for example, in closed-end funds (1, 12, 13, 26) which frequently trade either at a discount or premium to their net asset value. Writing the term k/(1-k) in terms of the ζ_1 and ζ_2 and linearizing we have then

$$\frac{k}{1-k} = 1 + 2\zeta_1 + 2\zeta_2 \tag{4}$$

and the price equation is then

$$\tau_0 \frac{1}{P} \frac{dP}{dt} = (1 + 2\zeta_1 + 2\zeta_2) \frac{L}{P} - 1.$$
(5)

One defines ζ_1 through two parameters, c_1 , that expressed the time scale of the trend following and q_1 the amplitude of this factor, as

$$\zeta_{1}(t) = \frac{q_{1}}{c_{1}} \int_{-\infty}^{t} e^{-(t-\tau)/c_{1}} \frac{1}{P(\tau)} \tau_{0} \frac{dP(\tau)}{d\tau} d\tau$$
(6)

Note that L and ζ_1 are linear functions of one another, but we retain L as a variable so the system is more easily generalized to incorporate a time-dependent L_0 . The valuation is more subtle for a cryptocurrency. The only concept of value relates to fairly recent trading prices. The first purchase with Bitcoin was for two slices of pizza for 10,000 Bitcoins (29). The sense of value at that time was probably much less than 2018 when people became accustomed to prices in the thousands of dollars. We thus stipulate the definitions

$$P_{a}(t) = \frac{1}{c_{3}} \int_{-\infty}^{t} e^{-(t-\tau)/c_{3}} P(\tau) d\tau,$$
(7)

$$\zeta_{2}(t) = \frac{q_{2}}{c_{2}} \int_{-\infty}^{t} e^{-(t-\tau)/c_{2}} \frac{P_{a}(\tau) - P(\tau)}{P(\tau)} d\tau,$$
(8)

i.e., ζ_2 represents the motivation to buy based on the discount from the perceived value of the cryptocurrency. Finally, the liquidity will not be constant but will be the sum of the core group's capital L_0 plus the additional amounts arriving from speculators that is correlated with the recent trend:

$$L(t) = L_0 + \frac{L_0}{c} q \int_{-\infty}^t e^{-(t-\tau)/c} \frac{\tau_0}{P(\tau)} \frac{dP(\tau)}{d\tau} d\tau$$
(9)

We assume that L_0 is constant, but one can easily adapt the model to include temporal changes in L_0 due to, for example, greater public acceptance of cryptocurrencies. By differentiating (6-9) and combining the resulting equations with (5) we obtain the 5x5 system of ordinary differential equations:

$$c_{3}P'_{a} = P - P_{a},$$

$$c_{2}\zeta'_{2} = q_{2}\frac{P_{a}(t) - P(t)}{P_{a}(t)} - \zeta_{2},$$

$$\tau_{0}P' = (1 + 2\zeta_{1} + 2\zeta_{2})L - P,$$

$$cL' = 1 - L + q\left\{(1 + 2\zeta_{1} + 2\zeta_{2})L - P\right\},$$

$$c_{1}\zeta'_{1} = q_{1}\left((1 + 2\zeta_{1} + 2\zeta_{2})\frac{L}{P} - 1\right) - \zeta_{1}.$$
(10)

We find a unique equilibrium at $(P, P_a, L, \zeta_1, \zeta_2) = (1, 1, 1, 0, 0)$. In other words, the only steady-state of the system occurs when the price, the anchoring notion of fundamental value, and liquidity price all coincide with the base liquidity value L_0 (33). The time scale for price adjustment will be short as markets adjust rapidly to supply/demand changes. Much longer will be the time scale for observing the trend and reacting to under or over-valuation, and the assessment of valuation anchored through weighted price averages. Moreover, one might expect that the valuation is on an even longer time scale. Thus one expects three time scales such that $\tau_0 \ll c, c_1, c_2 \ll c_3$, which we can scale as $c = c_1 = c_2 = 1$, and we allow arbitrary τ_0, c_3 in the analysis.

$$\begin{pmatrix} \tau_0 P \\ c_3 P_a \\ L \\ \zeta_1 \\ \zeta_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 & 2 & 2 \\ 1 & -1 & 0 & 0 & 0 \\ -q & 0 & q - 1 & 2q & 2q \\ -q_1 & 0 & q_1 & 2q_1 - 1 & 2q_1 \\ -q_2 & q_2 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} P \\ P_a \\ L \\ \zeta_1 \\ \zeta_2 \end{pmatrix}.$$
(11)

Thus, the system is determined entirely by three parameters: q, the attention to trend; q_1 , a measure of the influence of delay times; and q_2 , the influence of fundamental value, along with the time parameters τ_0 and c_3 . The question of stability can be investigated by calculating the eigenvalues in the relevant parameter space, i.e. $(q, q_1, q_2) \in \mathbb{R}^3_+$ (the first octant), along with τ_0 and c_3 . In particular, the main question is whether the maximal real part of the eigenvalues is positive, leading to instability, or if they are all negative, yielding stability. One sees that there is a double eigenvalue at $\lambda = -1$, and the other three eigenvalues remain negative if the Routh-Hurwitz conditions (40) below are satisfied

$$\frac{1}{\tau_0} + \frac{1}{c_3} + Q > 0,$$

$$\left(\frac{Q}{c_3} + \frac{1}{\tau_0} + 2\frac{q_2}{\tau_0} + \frac{1}{\tau_0 c_3}\right) \left(\frac{1}{\tau_0} + \frac{1}{c_3} + Q\right) > \frac{1}{c_3 \tau_0}.$$
(12)

where we have set $Q = 1 - q - 2q_1$. A sufficient set of conditions for (12) to hold is the

following:

$$\frac{1}{c_3} + \frac{1}{\tau_0} > q + 2q_1 =: K,$$

$$\frac{1}{c_3} + \frac{1}{\tau_0} > \frac{K}{c_3} - 2\frac{q_2}{\tau_0}.$$
 (13)

However, one can observe numerically that (13) are not necessary conditions to satisfy (12). Also, if we set $q_2 = 0$, we obtain the simpler condition

$$\frac{1}{c_3} + \frac{1}{\tau_0} > K \tag{14}$$

for stability, which we will see describes a simpler model that excludes valuation and the component of investor sentiment associated with it. We sketch various cross-sections holding one of these parameters constant and numerically computing eigenvalues across values of the other two. Note in Figure 1 below that increasing q_2 induces a stabilizing effect, while large K serves to make the system less stable. We choose various values of τ_0 and c_3 in Figure 2.

This yields a number of results. First, as market participants focus greater attention to the deviation of the asset from the acquired fundamental value driven from the liquidity price, there is less room for prices to stray from equilibrium. In addition, for a fixed q_2 , the asset would experience stability given that K is large enough. Finally, for K large enough, one sees that we have instability for a large range of q, i.e., if investors place too much emphasis on the relative trend, the asset price becomes unstable. The shaded regions indicate the range of parameters for which the system (11) is stable.

When we set $q_2 = 0$, the model simplifies somewhat, leaving a linear interface between the regions of stability and instability. We then have the following theorem. We define $Q := 1 - q - 2q_1$.

Theorem 1 Consider the system (11). With $q_2 = 0$, one has stability of the system (11) if and only if

$$Q + \frac{1}{\tau_0} > 0 \tag{15}$$

Proof. Setting $q_2 = 0$, the necessary conditions become

$$\left(Q + \frac{1}{\tau_0}\right)\left(\frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2}\right) > 0 \text{ and } Q + \frac{1}{\tau_0} + \frac{1}{c_3} > 0$$
(16)

We prove this is equivalent to $Q + \frac{1}{\tau_0} > 0$.

(i) Assume $Q + \frac{1}{\tau_0} > 0$. Then clearly the second inequality in (16) is satisfied. Also, one has

$$\frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2} = \left(Q + \frac{1}{\tau_0}\right)\left(\frac{1}{c_3}\right) + \frac{1}{\tau_0} + \frac{1}{c_3^2} > 0, \tag{17}$$

satisfying the first inequality.

(ii) Suppose (16) holds. Then clearly

$$0 < \left(Q + \frac{1}{\tau_0} + \frac{1}{c_3}\right)\frac{1}{c_3} + \frac{1}{\tau_0} = \frac{1}{\tau_0} + \frac{Q}{c_3} + \frac{1}{c_3\tau_0} + \frac{1}{c_3^2},\tag{18}$$

implying (15). ■

3 The effect of liquidity with or without sentiment

In order to isolate the effect of liquidity, we eliminate the role of investor sentiment and value by setting the associated parameters to zero. To this end, we are left with the system

$$\tau_0 P' = L - P,$$

$$cL' = 1 + (q - 1) L - qP.$$
(19)

One readily calculates that there will be positive eigenvalues of the linearized system if and only if $q > 1 + \frac{c}{\tau_0}$ In other words, in a system where only price and liquidity are relevant, a large amplitude q of liquidity is destabilizing while a large time scale for the liquidity is stabilizing. The stability is illustrated in the Figure 2.

Another nontrivial subcase is obtained from examining the full model (10) in the case where we set the value component of the sentiment, ζ_2 , and the fundamental value equal to zero. We then have the system of equations

$$\tau_{0} \frac{dP}{dt} = (1 + 2\zeta_{1}) L - P$$

$$c \frac{dL}{dt} = 1 - L + q (1 + 2\zeta_{1}) L - qP$$

$$c_{1} \frac{d\zeta_{1}}{dt} = q_{1} (1 + 2\zeta) \frac{L}{P} - q_{1} - \zeta_{1}$$
(20)

One then observes that the only equilibrium point is $L = P = L_0$ and $\zeta = 0$. Recalling that $Q := 1 - q - 2q_1$, one has the following.

Theorem 2 The system (20) incorporating valuation and sentiment (with $c := c_1$) is stable if and only if

$$Q + \frac{c}{\tau_0} > 0, \tag{21}$$

i.e. if the perturbations from trend and valuation sentiment are sufficiently small as a relative comparison to the timescale of reaction with respect to price.

Proof. By scaling, assume without loss of generality that $c_1 = c = 1$; then we can linearize the system as follows:

$$\begin{pmatrix} P\\L\\\zeta \end{pmatrix}' = \begin{pmatrix} -1/\tau_0 & 1/\tau_0 & 2/\tau_0\\ -q & q-1 & 2q\\ -q_1 & q_1 & 2q_1-1 \end{pmatrix} \begin{pmatrix} P\\L\\\zeta \end{pmatrix} =: A \begin{pmatrix} P\\L\\\zeta \end{pmatrix}.$$
 (22)

Leaving aside the eigenvalue of -1 that is present for all values of the parameters, the matrix A has eigenvalues with positive real part if and only if

$$q + 2q_1 > 1 + \frac{1}{\tau_0}.$$
(23)

After rescaling, this is the statement of the theorem. \blacksquare

Furthermore, we have either zero or two roots with positive real parts, so that we will have a stable spiral for $Q + \frac{c}{\tau_0} > 0$ and an unstable spiral for $Q - \frac{c}{\tau_0} < 0$ for the equilibrium point at (1, 1, 0). This matches our intuition from an economics perspective since one has instability when $q + 2q_1 > 1 + \frac{c}{\tau_0}$, i.e., there will be stability if $q + 2q_1 < 1$ regardless of c and τ_0 . For $q + 2q_1 > 1$, one sees that instability arises when $\frac{c}{\tau_0}$ is sufficiently small, i.e. traders are focused on short term trends.

The analysis above clearly shows that the potential stability of a crypto-asset may be contingent on several parameters that one may be able to influence. With this information, further research may be useful to examine the correlations and fit of these parameters with the effects of news and government policy. A problem of future interest would be whether, and if so how, governmental policy might be developed to diminish the volatility in cryptocurrencies. Another alternative would be a decentralized cryptocurrency with a concrete value. A good index to base this on would be either current or future gross world product (which could be estimated via futures markets). For a nominal fee, holders of this currency would be able to demand a basket of underlying currencies (such as dollar, euro, yen, etc.), which would keep the value of such a currency relatively close to its true fundamental value.

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Acknowledgments

N/A

Supplementary materials

N/A

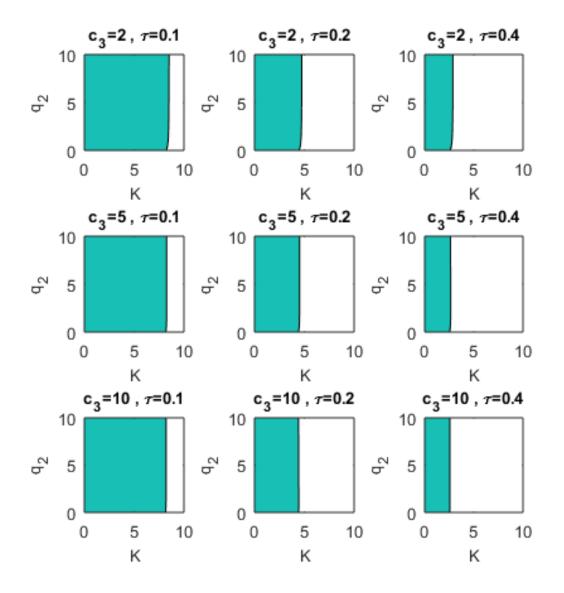


Figure 1: Stability of the 5×5 system in the $K - q_2$ plane for different values of the time scales c_3 and τ . Increasing c_3 and decreasing τ_0 increases the region of linear stability for the equations.

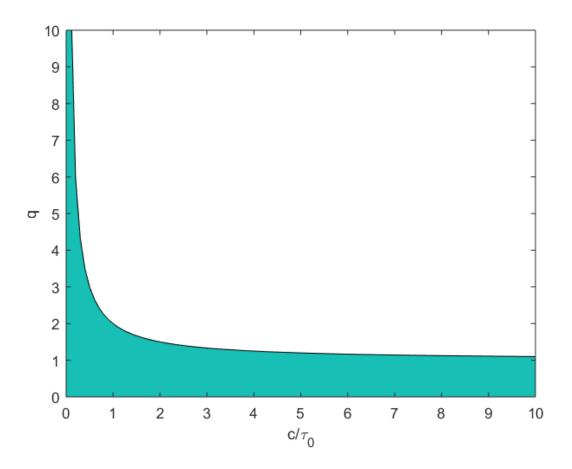


Figure 2: Stability for our simplified model without the presence of fundamental value or sentiment. The system is stable in the shaded region for the parameters q and $\frac{c}{\tau_0}$.